Observable Matter Flows In Szekeres Spacetimes

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The Project

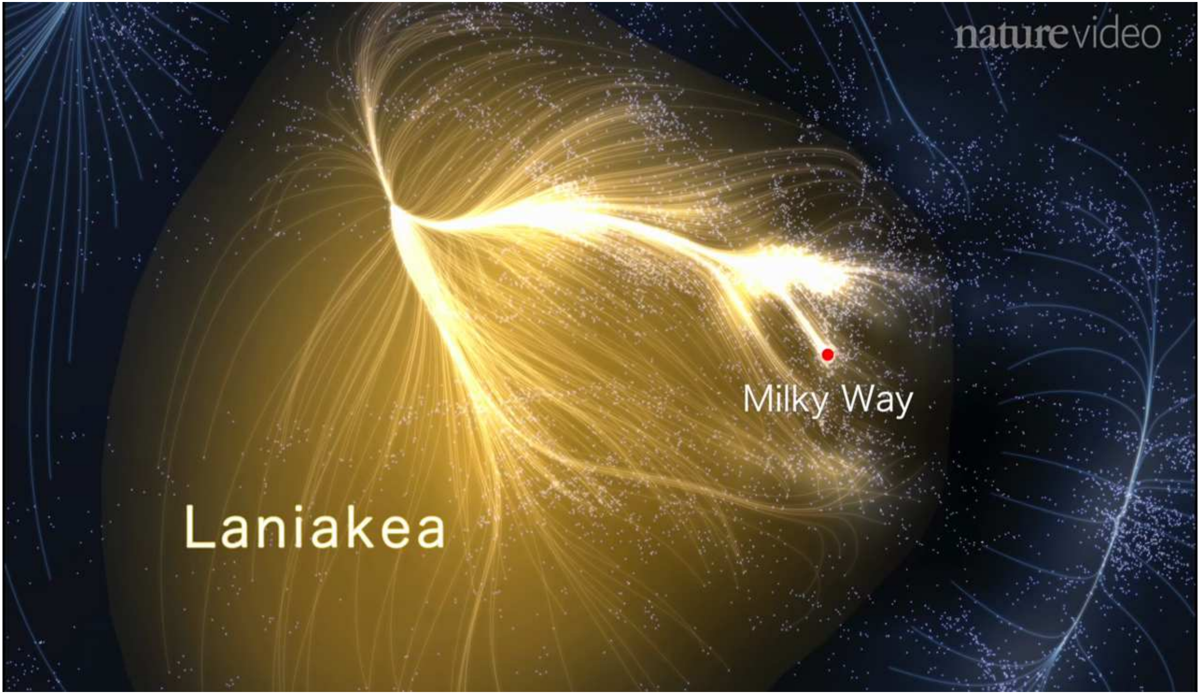
- Lay out a general framework for calculating cosmological observables in a given inhomogeneous model

- Integrate down PNC of arbitrarily placed observer in given spacetime
- Propagate observer's coordinates by Lie dragging
- Convert null geodesic eq & geodesic deviation eq to numerical form
- Calculate redshift, proper motions, diameter distance, (image distortion, etc)

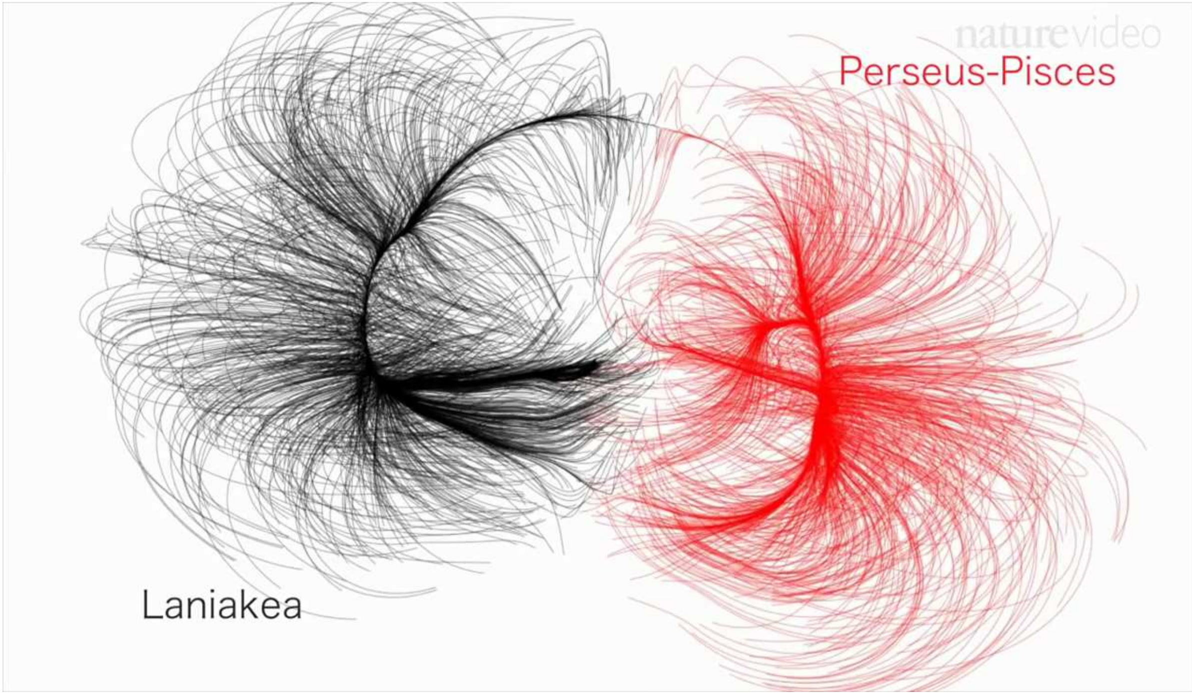
- Apply to Szekeres Metric and test — Tony
- Explore various Szekeres models and their observational patterns — in process.

Motivation

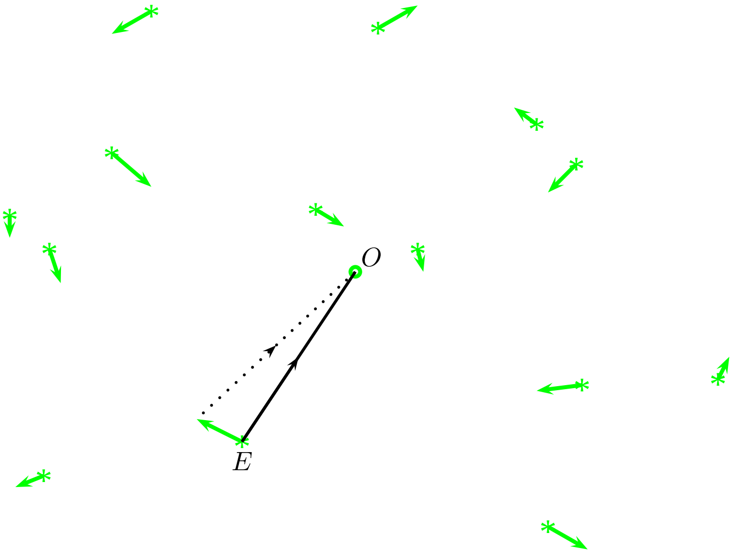
- Generate expected observations for a proposed inhomogneity & compare.
- Explore observational effects of other geometries & topologies.
- Exact solution methods valid where other methods aren't
  [strong variation, finite lightspeed, etc]
- Complement & check other methods.
- Future: Do inverse: Calculate metric from obs for more general models than LT
  'Metric of the Cosmos'
- Generate test data for a Metric of the Cosmos scheme.



Tully et al, Nature, 513, n 7516, p 71, 2014


naturevideo
Perseus-Pisces
Laniakea

Light Paths Don't Repeat

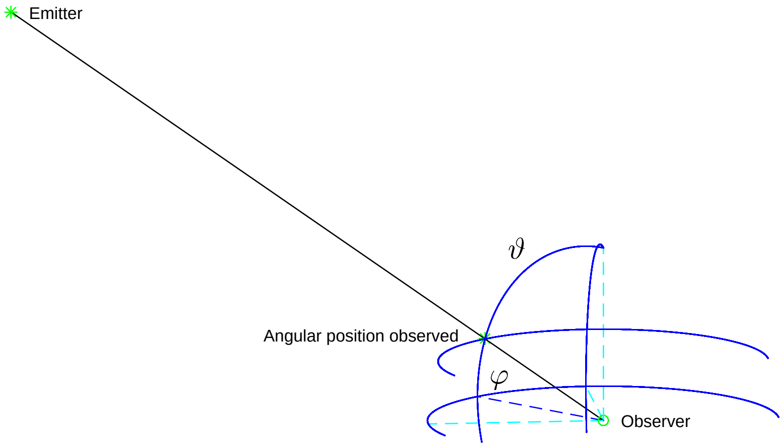


– For flow, need change in apparent position.
– Intervening spacetime has changed.
– How do you find the light ray from the **same** emitter at a **later** time?
– Numerical trial & error? Burdensome. **Try to avoid this.**

- **Solution** Use geodesic deviation eq to get instantaneous rates of change at initial time; repeat for each successive time.

Observer's Past Null Basis — Setup

- Observer uses angle on sky + time of observation
- Set up observer's coordinates in general inhomogeneous model



Observer's Past Null Basis — Define

- Metric & coords (general): g_{ab} , x^c ,
- Observer position (arbitrary): $x^c\big|_o$,
- Orthonormal basis at obs: $\overline{\mathbf{e}}_i\Big|_o = \Big[\overline{e}_i{}^a \,\boldsymbol{\partial}_a\Big]_o$, *(mark with overbar)*
- Spherical basis at obs: *(mark with tilde)*

$$\tilde{\mathbf{e}}_{\tilde{\tau}} = \overline{\mathbf{e}}_0$$
$$\tilde{\mathbf{e}}_{\tilde{r}} = \sin\tilde{\vartheta}\cos\tilde{\varphi}\,\overline{\mathbf{e}}_1 + \sin\tilde{\vartheta}\sin\tilde{\varphi}\,\overline{\mathbf{e}}_2 + \cos\tilde{\vartheta}\,\overline{\mathbf{e}}_3$$
$$\tilde{\mathbf{e}}_{\tilde{\vartheta}} = \tilde{r}\cos\tilde{\vartheta}\cos\tilde{\varphi}\,\overline{\mathbf{e}}_1 + \tilde{r}\cos\tilde{\vartheta}\sin\tilde{\varphi}\,\overline{\mathbf{e}}_2 - \tilde{r}\sin\tilde{\vartheta}\,\overline{\mathbf{e}}_3$$
$$\tilde{\mathbf{e}}_{\tilde{\varphi}} = -\tilde{r}\sin\tilde{\vartheta}\sin\tilde{\varphi}\,\overline{\mathbf{e}}_1 + \tilde{r}\sin\tilde{\vartheta}\cos\tilde{\varphi}\,\overline{\mathbf{e}}_2 \ ,$$

- Convert to past-null spherical basis at obs: *(mark with hat)*

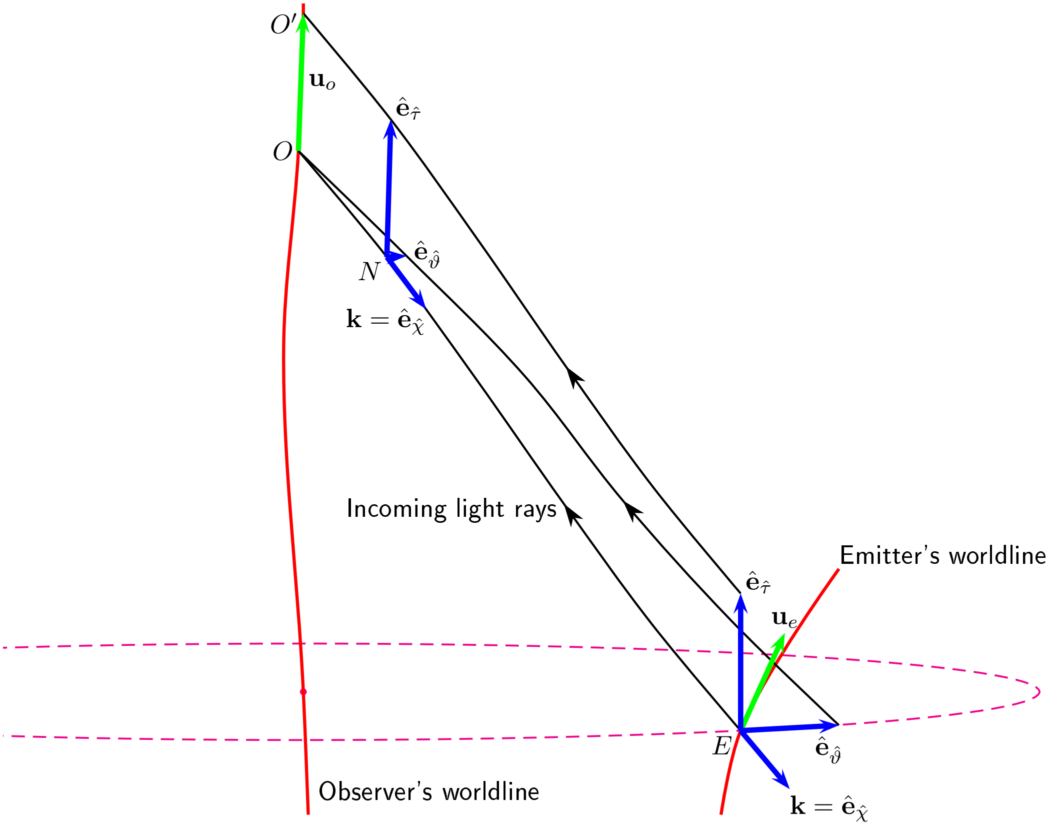
$$\hat{\tau} = \tilde{r} + \tilde{\tau} \ , \quad \hat{\chi} = \tilde{r} \qquad \leftrightarrow \qquad \tilde{\tau} = \hat{\tau} - \hat{\chi} \ , \quad \tilde{r} = \hat{\chi} \ ,$$

$$\to \qquad \begin{aligned} \hat{\mathbf{e}}_{\hat{\tau}} &= \tilde{\mathbf{e}}_{\tilde{\tau}} = \overline{\mathbf{e}}_0 \\ \hat{\mathbf{e}}_{\hat{\chi}} &= -\tilde{\mathbf{e}}_{\tilde{\tau}} + \tilde{\mathbf{e}}_{\tilde{r}} = -\overline{\mathbf{e}}_0 + \sin\hat{\vartheta}\cos\hat{\varphi}\,\overline{\mathbf{e}}_1 + \sin\hat{\vartheta}\sin\hat{\varphi}\,\overline{\mathbf{e}}_2 + \cos\hat{\vartheta}\,\overline{\mathbf{e}}_3 \\ \hat{\mathbf{e}}_{\hat{\vartheta}} &= \tilde{\mathbf{e}}_{\tilde{\vartheta}} = \hat{\chi}\cos\hat{\vartheta}\cos\hat{\varphi}\,\overline{\mathbf{e}}_1 + \hat{\chi}\cos\hat{\vartheta}\sin\hat{\varphi}\,\overline{\mathbf{e}}_2 - \hat{\chi}\sin\hat{\vartheta}\,\overline{\mathbf{e}}_3 \\ \hat{\mathbf{e}}_{\hat{\varphi}} &= \tilde{\mathbf{e}}_{\tilde{\varphi}} = -\hat{\chi}\sin\hat{\vartheta}\sin\hat{\varphi}\,\overline{\mathbf{e}}_1 + \hat{\chi}\sin\hat{\vartheta}\cos\hat{\varphi}\,\overline{\mathbf{e}}_2 \ , \end{aligned}$$

- Propagate these down the observer's PNC.

Propagation Scheme

- Affine distance χ down each incoming light ray
- Keep $\hat{\vartheta}$, $\hat{\varphi}$ const along each light ray
  i.e. Lie drag coords & basis down incoming null geodesics.
- Exactly the set-up for geodesic deviation eq to hold.


O'
$\mathbf{u}_o$
$\hat{\mathbf{e}}_{\hat{\tau}}$
O
$\hat{\mathbf{e}}_{\hat{\vartheta}}$
N
$\mathbf{k} = \hat{\mathbf{e}}_{\hat{\chi}}$
Incoming light rays
Emitter's worldline
$\hat{\mathbf{e}}_{\hat{\tau}}$
$\mathbf{u}_e$
E
$\hat{\mathbf{e}}_{\hat{\vartheta}}$
Observer's worldline
$\mathbf{k} = \hat{\mathbf{e}}_{\hat{\chi}}$

Propagation Equations

- Geodesic Eq — light ray paths

$$\frac{\delta k^a}{\delta \hat{\chi}} = 0 \qquad \text{(good for tensor calcs)}$$

$$\frac{\mathrm{d}k^a}{\mathrm{d}\hat{\chi}} = -k^b \Gamma^a{}_{bc} k^c \ , \quad k^a k_a = 0 \ , \quad \frac{\mathrm{d}x^a}{\mathrm{d}\chi} = k^a \qquad \text{(good for numerics)}$$

- Geodesic Deviation Eq — past-null-obs basis propagation

$$\frac{\delta^2 \hat{e}_\alpha{}^a}{\delta \hat{\chi}^2} = -R^a{}_{bcd} \, k^b \, \hat{e}_\alpha{}^c \, k^d \qquad \text{(good for tensor calcs)}$$

$$\frac{\mathrm{d}^2 \hat{e}_\alpha{}^a}{\mathrm{d}\hat{\chi}^2} = -k^b \left(2\Gamma^a{}_{bc} \frac{\mathrm{d}\hat{e}_\alpha{}^c}{\mathrm{d}\hat{\chi}} + \hat{e}_\alpha{}^c k^d \Gamma^a{}_{db,c} \right) \qquad \text{(good for numerics)}$$

$$\hat{\mathbf{e}}_\alpha \ \equiv \ \{ \, \hat{\mathbf{e}}_\tau \ , \ \ \hat{\mathbf{e}}_{\hat{\chi}} = \mathbf{k} \ , \ \ \hat{\mathbf{e}}_{\hat{\vartheta}} \ , \ \ \hat{\mathbf{e}}_{\hat{\varphi}} \ \}$$

- Propagated $(\hat{\tau}, \hat{\chi}, \hat{\vartheta}, \hat{\varphi})$ is a coord system
- Propagated $\hat{\mathrm{e}}_\alpha$ is coord basis — provide transformation between metric and observer's coordinates,

$$\hat{e}^{\alpha}{}_{c} = e_{c}{}^{\alpha} = \frac{\partial \hat{x}^{\alpha}}{\partial x^{c}} \ , \qquad e^{c}{}_{\alpha} = \hat{e}_{\alpha}{}^{c} = \frac{\partial x^{c}}{\partial \hat{x}^{\alpha}} \ .$$

- What we actually need (later) is not $\hat{e}_\alpha{}^a$ but its inverse $\hat{e}^\alpha{}_a$.

Propagation — Initial Conditions

Geodesic Eq

- In orthonormal frame, initial $k^\alpha = (-1, 1, 0, 0)$, i.e.

$$|k^b u_b u^a|_o = 1 = |k^a(\delta^c_a + u^c u_a)|_o$$

Geodesic Deviation Eq

- $\hat{\mathbf{e}}_{\hat{\tau}}|_o = \mathbf{u}_o$
- Take $\hat{\chi} \to 0$ limits of $\hat{\mathbf{e}}_\alpha$ near-observer expressions
- Fermi-propagate $\mathbf{k}$ along $\mathbf{u}|_o$

$$\left.\frac{\delta k^a}{\delta \tau}\right|_{\hat{\chi}=0} = \left[u^b_o \nabla_b k^a - k_b a^b_o u^a_o + k_b u^b_o a^a_o\right]_{\hat{\chi}=0} = 0$$

and use $$\left.\frac{\delta k^a}{\delta \hat{\tau}}\right|_{\hat{\chi}=0} = \left.\frac{\delta \hat{e}_{\hat{\tau}}{}^a}{\delta \hat{\chi}}\right|_{\hat{\chi}=0} \quad \to \quad \left.\frac{\mathrm{d}\hat{e}_{\hat{\tau}}{}^a}{\mathrm{d}\hat{\chi}}\right|_o$$

- Take $\hat{\vartheta}$ & $\hat{\varphi}$ derivatives of near-observer $\hat{\mathbf{e}}_{\hat{\chi}} = \mathbf{k}$ and use e.g.

$$\left.\frac{\delta k^a}{\delta \hat{\vartheta}}\right|_{\hat{\chi}=0} = \left.\frac{\delta \hat{e}_{\hat{\vartheta}}{}^a}{\delta \hat{\chi}}\right|_{\hat{\chi}=0} \quad \to \quad \left.\frac{\mathrm{d}\hat{e}_{\hat{\vartheta}}{}^a}{\mathrm{d}\hat{\chi}}\right|_o$$

Observables - Redshift & Proper Motion

- Rate of change of observed angle with respect to observer time

$$\left.\frac{\mathrm{d}\tilde{x}^m}{\mathrm{d}\tilde{\tau}}\right|_o = \left[\frac{\partial \tilde{x}^m}{\partial \hat{x}^\beta}\frac{\mathrm{d}\hat{x}^\beta}{\mathrm{d}\hat{\tau}}\frac{\mathrm{d}\hat{\tau}}{\mathrm{d}\tilde{\tau}}\right]_o = \left[\hat{e}_\beta{}^m \frac{\mathrm{d}\hat{x}^\beta}{\mathrm{d}\hat{\tau}}\right]_o$$

where $$\left[\frac{\mathrm{d}\hat{x}^\beta}{\mathrm{d}\hat{\tau}}\right]_o = \left[\frac{\mathrm{d}\hat{x}^\beta}{\mathrm{d}\hat{\tau}}\right]_e = \left[\frac{\partial \hat{x}^\beta}{\partial x^a}\frac{\mathrm{d}x^a}{\mathrm{d}\tau_e}\frac{\mathrm{d}\tau_e}{\mathrm{d}\hat{\tau}}\right]_e = \frac{\left[\hat{e}^\beta{}_a u^a\right]_e}{(1+z)}$$

τ_e = the source proper time,
$\tilde{\tau}_o$ = observer's proper time,
$\hat{\tau}$ = its extension down the PNC.

- Hence we get redshift

$$1 = \left.\frac{\mathrm{d}\hat{\tau}}{\mathrm{d}\hat{\tau}}\right|_o = \frac{\left[\hat{e}^{\hat{\tau}}{}_a u^a\right]_e}{(1+z)} \qquad \rightarrow \qquad 1+z = \left[\hat{e}^{\hat{\tau}}{}_a u^a\right]_e$$

and observed proper motions

$$\left.\frac{\mathrm{d}\tilde{\vartheta}}{\mathrm{d}\tilde{\tau}}\right|_o = \frac{\left[\hat{e}^{\hat{\vartheta}}{}_a u^a\right]_e}{(1+z)}$$

$$\left.\frac{\mathrm{d}\tilde{\varphi}}{\mathrm{d}\tilde{\tau}}\right|_o = \frac{\left[\hat{e}^{\hat{\varphi}}{}_a u^a\right]_e}{(1+z)} .$$

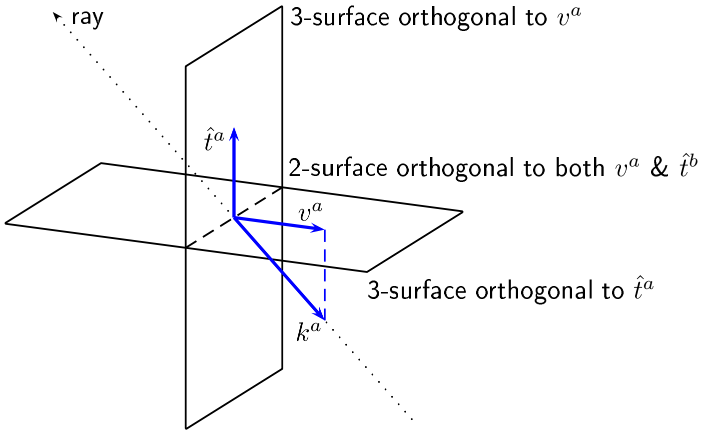
- Note dual basis vectors $\hat{\mathrm{e}}^\alpha$ actually what's needed.

Tracking sources

- Once we have $\left.\frac{\mathrm{d}\tilde{\vartheta}}{\mathrm{d}\tilde{\tau}}\right|_o$ & $\left.\frac{\mathrm{d}\tilde{\varphi}}{\mathrm{d}\tilde{\tau}}\right|_o$, we can estimate $\hat{\vartheta}$ & $\hat{\varphi}$ to use for next τ value.

Viewing Plane

- Split ray direction into t^a along local matter flow & $v^a \perp$ to it



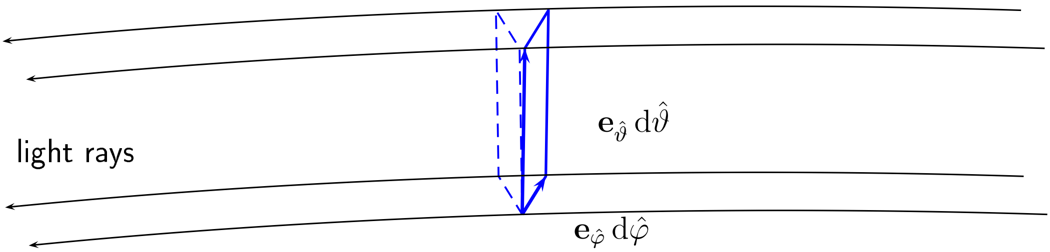
$$v^a = \frac{k^a}{A} + \hat{t}^a \,, \qquad \text{where} \;\; A = k_c \hat{t}^c \,, \qquad A < 0$$

proj $\perp \hat{t}^a$: $\quad h^a_b = \delta^a_b + \hat{t}^a \hat{t}_b$

proj $\perp v^a$: $\quad j^b_c = \delta^b_c - v^b v_c$

proj $\perp$ both: $\quad h^a_b j^b_c = \delta^a_c - \frac{k^a k_c}{A^2} - \frac{(\hat{t}^a k_c + k^a \hat{t}_c)}{A}$

Observables - Area Distance



physical area: $\mathrm{d}A_{\hat{\vartheta}\hat{\varphi}} = \left|(\hat{\mathbf{e}}_{\hat{\vartheta}}\,\mathrm{d}\hat{\vartheta}) \wedge (\hat{\mathbf{e}}_{\hat{\varphi}}\,\mathrm{d}\hat{\vartheta})\right|$

projected area: $\mathrm{d}A_{\perp} = \left|\eta_{abcd}\,t^a\,v^b\,(\hat{e}_{\hat{\vartheta}}{}^c\,\mathrm{d}\hat{\vartheta})\,(\hat{e}_{\hat{\varphi}}{}^d\,\mathrm{d}\hat{\vartheta})\right|$

solid angle subtended: $\mathrm{d}\Omega = \sin\hat{\vartheta}\,\mathrm{d}\hat{\vartheta}\,\mathrm{d}\hat{\varphi}$

area distance: $d_A = \dfrac{\mathrm{d}A_{\perp}}{\mathrm{d}\Omega}$

- NOTE: If basis vectors $\hat{\mathbf{e}}_{\hat{\vartheta}}$, $\hat{\mathbf{e}}_{\hat{\varphi}}$ not $\perp$ rays $\mathbf{k}$, then
  projected area $\mathrm{d}A_{\perp} \neq$ area spanned by basis vectors $\mathrm{d}A_{\hat{\vartheta}\hat{\varphi}}$
  [Tony currently checking this]

Flow Magnitude

$$\text{proper motion} \approx \frac{\text{physical transverse velocity}}{\text{area distance}}$$

$$\begin{aligned} \text{Area distance has a maximum} \quad &\rightarrow \quad \text{angluar size has a minimum} \\ &\rightarrow \quad \text{proper motion has a minimum} \end{aligned}$$

Apply to Szekeres Metric

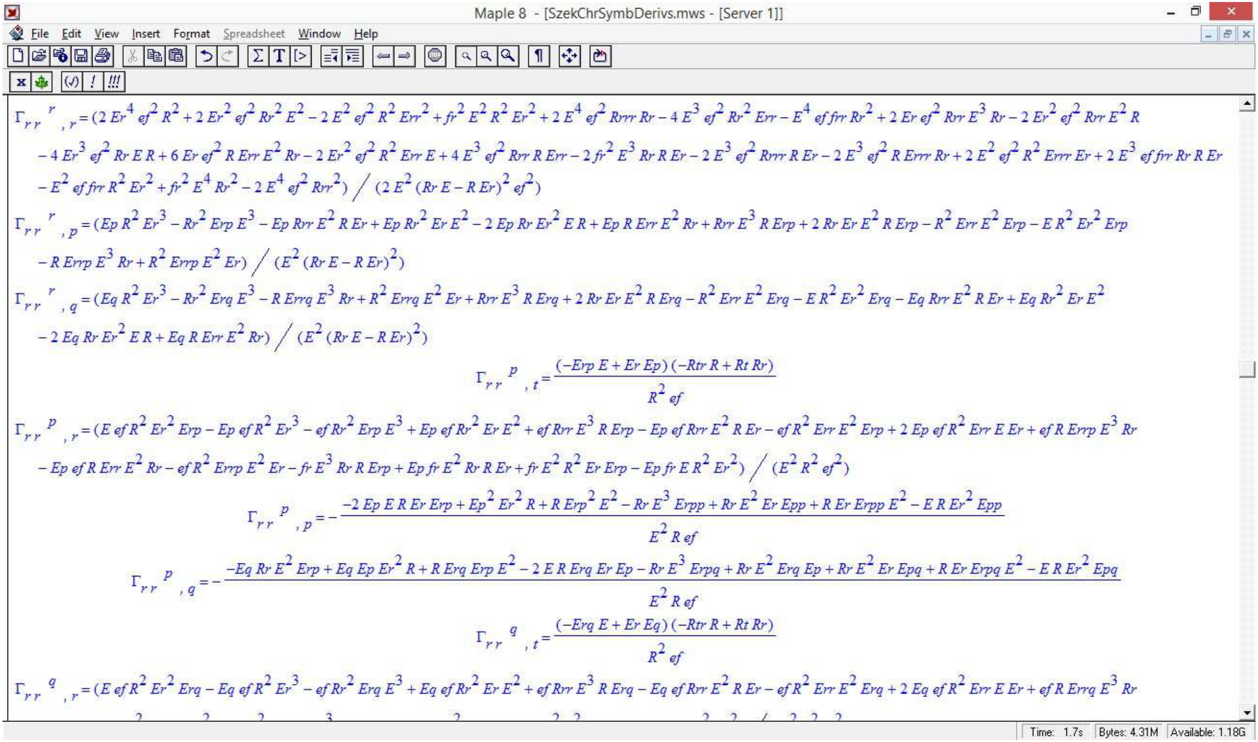
$$ds^2 = -\mathrm{d}t^2 + \frac{\left(R' - \frac{RE'}{E}\right)^2 \, \mathrm{d}r^2}{\epsilon + f} + R^2 \frac{(dp^2 + dq^2)}{E^2}$$

- $E = E(r, p, q)$ — dipole function
- $\epsilon = +1, 0, -1$ — foliation type (spheres, planes, hyperboloids)
- Evolution function $R(t, r)$ same as for LT.
- 6 arbitrary functions of r (f, M, a, S, P, Q)
  - f = local geometry/energy function
  - M = gravitational mass within comoving sphere at r
  - a = local time of bang
  - (S, P, Q) control non-symmetry - strength & orientation of the "dipole"
- No Killing vectors
- Very interesting metric, not well explored

Numerical Implementation with Szekeres (Tony Walters)

- Specify Observer position (t, r, p, q)
- Specify Arbitrary Functions:
  - "LT functions": f, M, a
  - "Szekeres functions": S, P, Q which characterise deviation from G_3 symmetry
  - 1st & 2nd derivatives
- Calculate $R(t, r)$ & $E(r, p, q)$ up to 3rd partial derivatives
  in terms of arbitrary functions f, M, a, S, P, Q
- Analytically calculate and simplify
  Christoffel Symbols (30 non-zero)
  & partial derivatives (104)
  Express in terms of R & f & E & derivatives
- Analytic simplification greatly reduces numerical cancellation error

10 Pages of Maple/GRTensor output:



$$\Gamma_{rr}{}^{r}{}_{,r} = (2\,Er^4\,ef^2\,R^2 + 2\,Er^2\,ef^2\,Rr^2\,E^2 - 2\,E^2\,ef^2\,R^2\,Err^2 + fr^2\,E^2\,R^2\,Er^2 + 2\,E^4\,ef^2\,Rrrr\,Rr - 4\,E^3\,ef^2\,Rr^2\,Err - E^4\,ef\,frr\,Rr^2 + 2\,Er\,ef^2\,Rrr\,E^3\,Rr - 2\,Er^2\,ef^2\,Rrr\,E^2\,R - 4\,Er^3\,ef^2\,Rr\,E\,R + 6\,Er\,ef^2\,R\,Err\,E^2\,Rr - 2\,Er^2\,ef^2\,R^2\,Err\,E + 4\,E^3\,ef^2\,Rrr\,R\,Err - 2\,fr^2\,E^3\,Rr\,R\,Er - 2\,E^3\,ef^2\,Rrrr\,R\,Er - 2\,E^3\,ef^2\,R\,Errr\,Rr + 2\,E^2\,ef^2\,R^2\,Errr\,Er + 2\,E^3\,ef\,frr\,Rr\,R\,Er - E^2\,ef\,frr\,R^2\,Er^2 + fr^2\,E^4\,Rr^2 - 2\,E^4\,ef^2\,Rrr^2) \Big/ (2\,E^2\,(Rr\,E - R\,Er)^2\,ef^2)$$

$$\Gamma_{rr}{}^{r}{}_{,p} = (Ep\,R^2\,Er^3 - Rr^2\,Erp\,E^3 - Ep\,Rrr\,E^2\,R\,Er + Ep\,Rr^2\,Er\,E^2 - 2\,Ep\,Rr\,Er^2\,E\,R + Ep\,R\,Err\,E^2\,Rr + Rrr\,E^3\,R\,Erp + 2\,Rr\,Er\,E^2\,R\,Erp - R^2\,Err\,E^2\,Erp - E\,R^2\,Er^2\,Erp - R\,Errp\,E^3\,Rr + R^2\,Errp\,E^2\,Er) \Big/ (E^2\,(Rr\,E - R\,Er)^2)$$

$$\Gamma_{rr}{}^{r}{}_{,q} = (Eq\,R^2\,Er^3 - Rr^2\,Erq\,E^3 - R\,Errq\,E^3\,Rr + R^2\,Errq\,E^2\,Er + Rrr\,E^3\,R\,Erq + 2\,Rr\,Er\,E^2\,R\,Erq - R^2\,Err\,E^2\,Erq - E\,R^2\,Er^2\,Erq - Eq\,Rrr\,E^2\,R\,Er + Eq\,Rr^2\,Er\,E^2 - 2\,Eq\,Rr\,Er^2\,E\,R + Eq\,R\,Err\,E^2\,Rr) \Big/ (E^2\,(Rr\,E - R\,Er)^2)$$

$$\Gamma_{rr}{}^{p}{}_{,t} = \frac{(-Erp\,E + Er\,Ep)\,(-Rtr\,R + Rt\,Rr)}{R^2\,ef}$$

$$\Gamma_{rr}{}^{p}{}_{,r} = (E\,ef\,R^2\,Er^2\,Erp - Ep\,ef\,R^2\,Er^3 - ef\,Rr^2\,Erp\,E^3 + Ep\,ef\,Rr^2\,Er\,E^2 + ef\,Rrr\,E^3\,R\,Erp - Ep\,ef\,Rrr\,E^2\,R\,Er - ef\,R^2\,Err\,E^2\,Erp + 2\,Ep\,ef\,R^2\,Err\,E\,Er + ef\,R\,Errp\,E^3\,Rr - Ep\,ef\,R\,Err\,E^2\,Rr - ef\,R^2\,Errp\,E^2\,Er - fr\,E^3\,Rr\,R\,Erp + Ep\,fr\,E^2\,Rr\,R\,Er + fr\,E^2\,R^2\,Er\,Erp - Ep\,fr\,E\,R^2\,Er^2) \Big/ (E^2\,R^2\,ef^2)$$

$$\Gamma_{rr}{}^{p}{}_{,p} = -\frac{-2\,Ep\,E\,R\,Er\,Erp + Ep^2\,Er^2\,R + R\,Erp^2\,E^2 - Rr\,E^3\,Erpp + Rr\,E^2\,Er\,Epp + R\,Er\,Erpp\,E^2 - E\,R\,Er^2\,Epp}{E^2\,R\,ef}$$

$$\Gamma_{rr}{}^{p}{}_{,q} = -\frac{-Eq\,Rr\,E^2\,Erp + Eq\,Ep\,Er^2\,R + R\,Erq\,Erp\,E^2 - 2\,E\,R\,Erq\,Er\,Ep - Rr\,E^3\,Erpq + Rr\,E^2\,Erq\,Ep + Rr\,E^2\,Er\,Epq + R\,Er\,Erpq\,E^2 - E\,R\,Er^2\,Epq}{E^2\,R\,ef}$$

$$\Gamma_{rr}{}^{q}{}_{,t} = \frac{(-Erq\,E + Er\,Eq)\,(-Rtr\,R + Rt\,Rr)}{R^2\,ef}$$

$$\Gamma_{rr}{}^{q}{}_{,r} = (E\,ef\,R^2\,Er^2\,Erq - Eq\,ef\,R^2\,Er^3 - ef\,Rr^2\,Erq\,E^3 + Eq\,ef\,Rr^2\,Er\,E^2 + ef\,Rrr\,E^3\,R\,Erq - Eq\,ef\,Rrr\,E^2\,R\,Er - ef\,R^2\,Err\,E^2\,Erq + 2\,Eq\,ef\,R^2\,Err\,E\,Er + ef\,R\,Errq\,E^3\,Rr$$

- For each view direction, numerically integrate down an incoming light ray:
  - null geodesic equation for k^a
  - geodesic deviation equations for the $\hat{e}_\beta{}^b$
  - $\hat{\mathbf{e}}_{\hat{\varphi}}$ is divided by $\sin\hat{\vartheta}$ to remove problems at the poles
  - 40 initial conditions
  - integration is relative to affine parameter χ
- Choose a set of z values for mapping the data
- To get data for the same redshift z on all rays, interpolate on each ray
- At each z, invert the basis, and calculate the observables
- Produce sky maps for each z

Testing

- k^a should stay null
- k^a & $\hat{e}_{\hat{\chi}}{}^a$ should coincide
- FLRW special case of Szekeres — should match direct calculations from FLRW formulas
- Szekeres-coords FLRW case — should still match FLRW results
- LT case — should be axially symmetric about direction from observer to LT centre
- All tests passed within set tolerances.

A Sample Model

$$M = \frac{r^3(M_0 + M_\infty C_5 r)}{1 + C_5 r}\,, \qquad M_0 = 10, M_\infty = 20, C_5 = 2$$

$$f = \frac{r^2(f_0 + f_\infty C_6 r)}{1 + C_6 r}\,, \qquad f_0 = 1, f_\infty = 3, C_6 = 1.6$$

$$a = \frac{a_0 + a_\infty C_7 r^2}{1 + C_7 r^2}\,, \qquad a_0 = -1, a_\infty = -2, C_7 = 1.8$$

$$\epsilon = +1$$

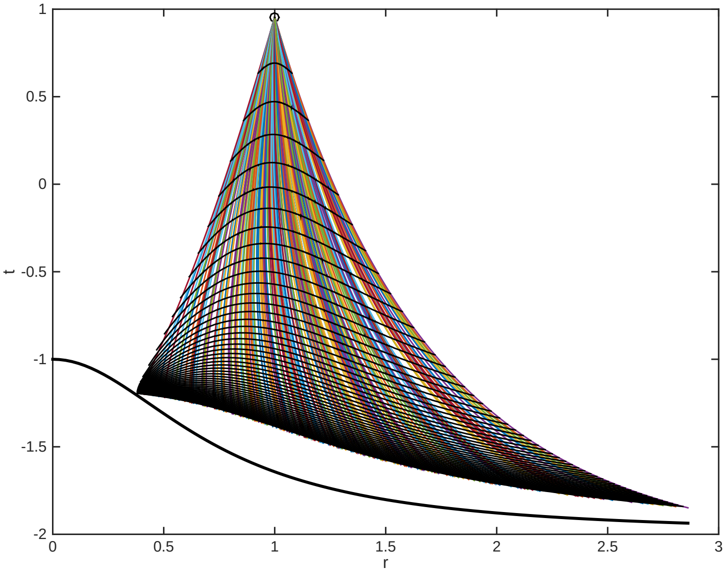
$$S = \frac{S_0 + S_\infty C_1 r}{1 + C_1 r}\,, \qquad S_0 = 1, S_\infty = 9, C_1 = 2.1$$

$$P = \frac{P_0 + P_\infty C_1 r}{1 + C_1 r}\,, \qquad P_0 = 0, P_\infty = 4$$

$$Q = \frac{Q_0 + Q_\infty C_1 r}{1 + C_1 r}\,, \qquad Q_0 = 0, Q_\infty = -2$$

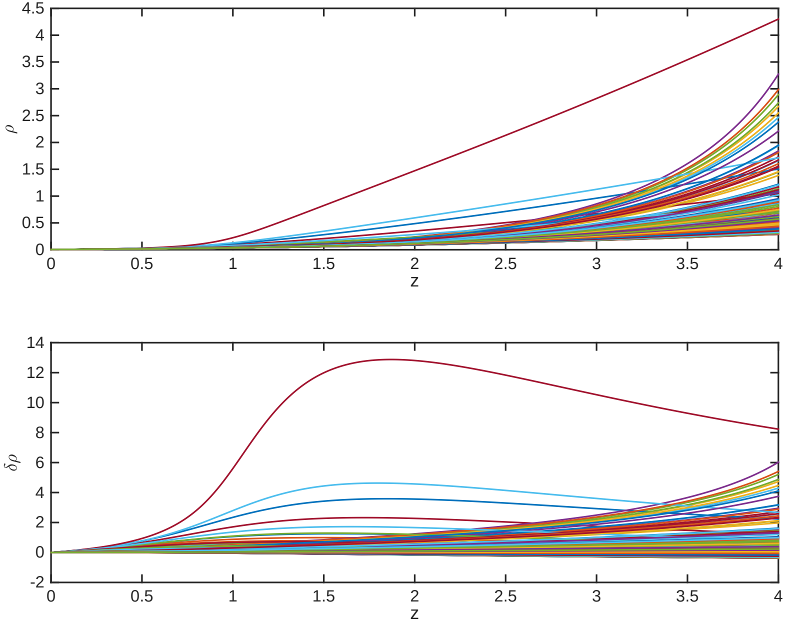
No attempt here to model a realistic structure

Sample Run #3a — Past Null Cone Ray Loci in (t, r)



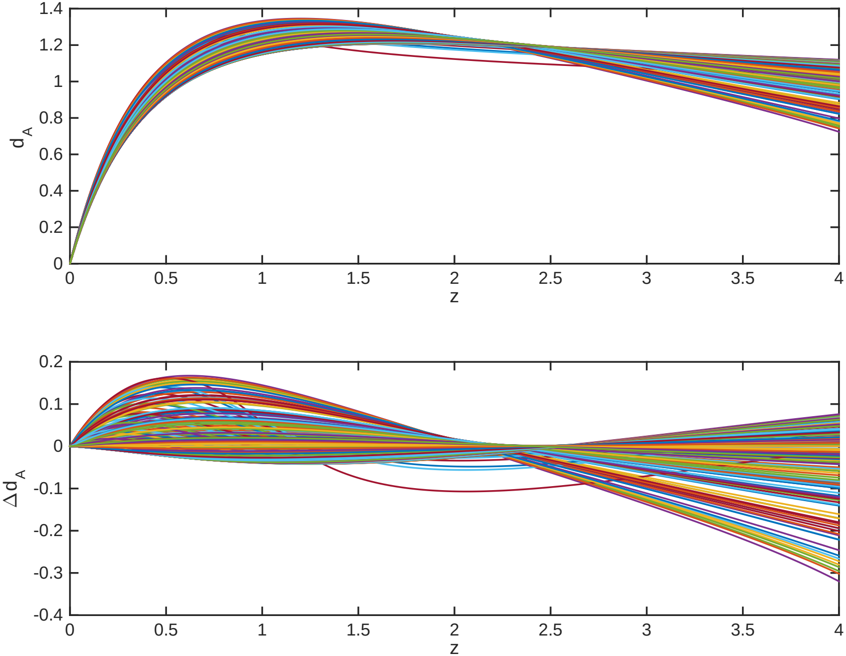
Observer: circle at $(t, r) \approx (0.86, 1)$
Coloured lines: light rays arriving from different directions
Black horizontal lines: surfaces of constant redshift
Thick black line: bang at $t = 0 = t_b$

Sample Run #3a — Line of Sight Density



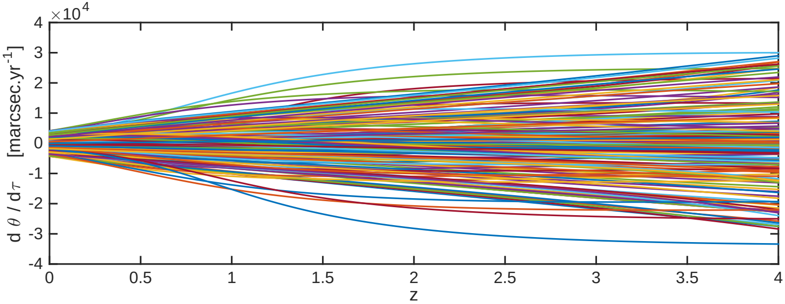
Top: density along each line of sight
Bottom: Ray density relative to chosen reference ray
Ref ray is directed away from inhomogeneity features & is roughly background

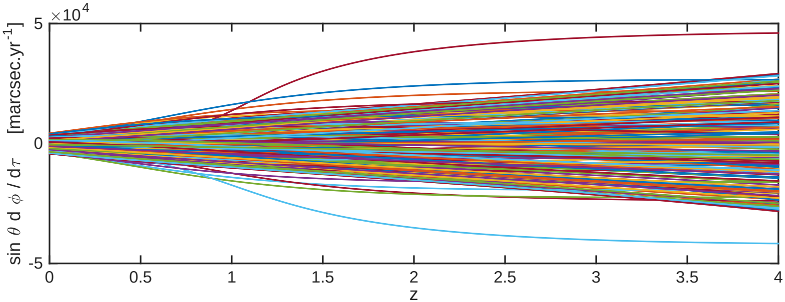
Sample Run #3a — Area Distance vs Redshift



Top: area distance vs. redshift along various incoming rays
Bottom: difference between each ray and the reference ray

Sample Run #3a — Proper Motion vs Redshift



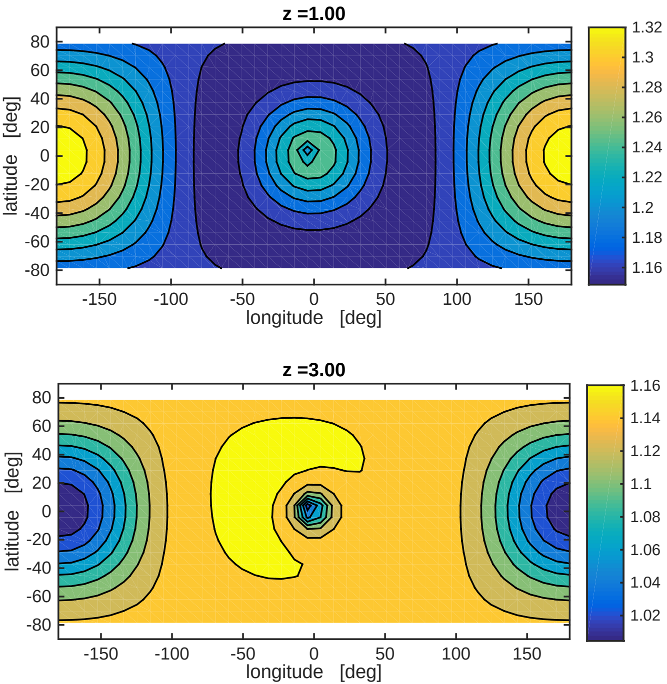


Top: proper motion — $\hat{\vartheta}$ component

Bottom: Proper motion — $\hat{\varphi}$ component

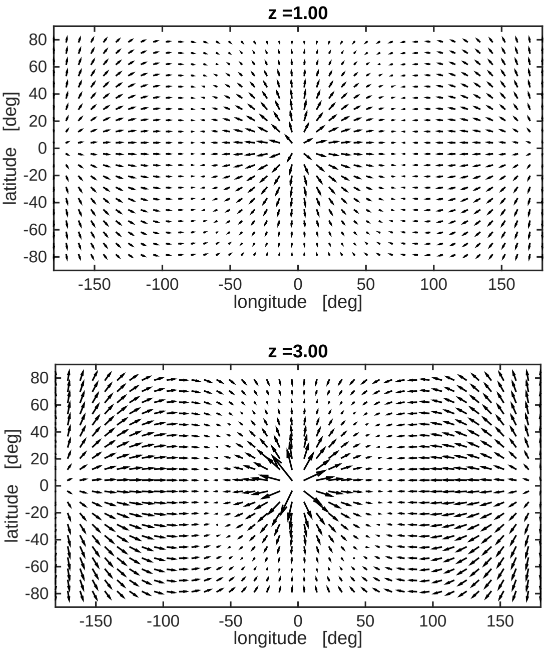
Flow rates ~ 1.5 arcseconds/yr, up to 10s of arcsecs/yr

Sample Run #3a — Area Distance Map



Top: area distance over the observer's sky at $z = 0.3$
Bottom: same at $z = 0.7$.

Sample Run #3a — Flow Map



Top: proper motion over the observer's sky at $z = 0.3$

Bottom: same at $z = 0.7$

Discussion

- Algorithm for calculating observational features for a given observer in a given model
- Facilitates exploring models and what they'd look like
- Try out interesting geometries
- Instantaneous angular flow rates greatly assist in locating ray directions to same sources at later times
- Important complement to the Metric of the Cosmos Project
  — enables generation of very realistic fake data for testing
- Applied to Szekeres model; Matlab code developed & thoroughly tested
- Works for quasi-hyperboloidal & quasi-planar as well as quasi-spherical Szekeres models (all ϵ values)
- In process of creating a variety of Szekeres models

Dziękuję