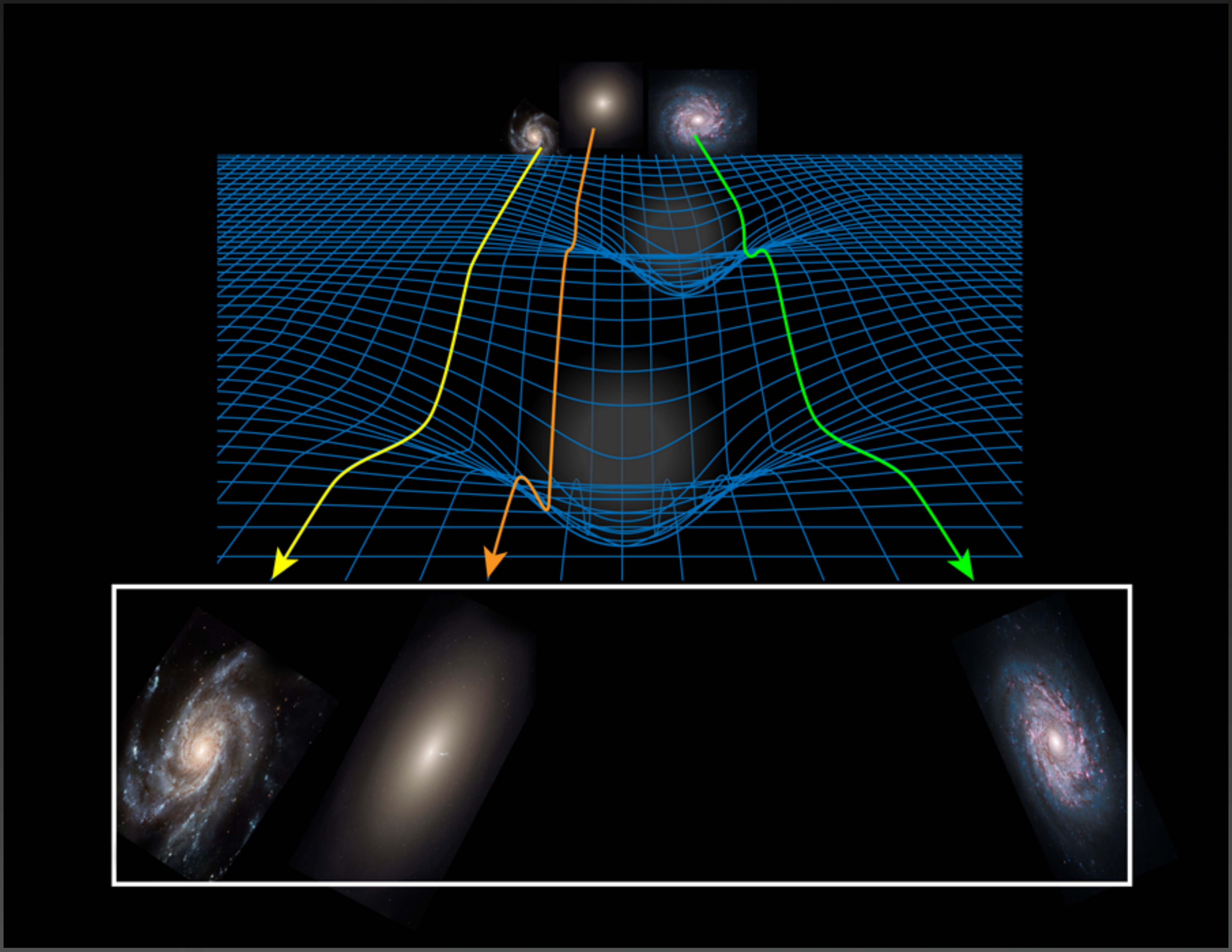
Michele Grasso
CFT Pan, Warsaw

Ray tracing, Parallax, Position and Redshift drift in Numerical Relativity

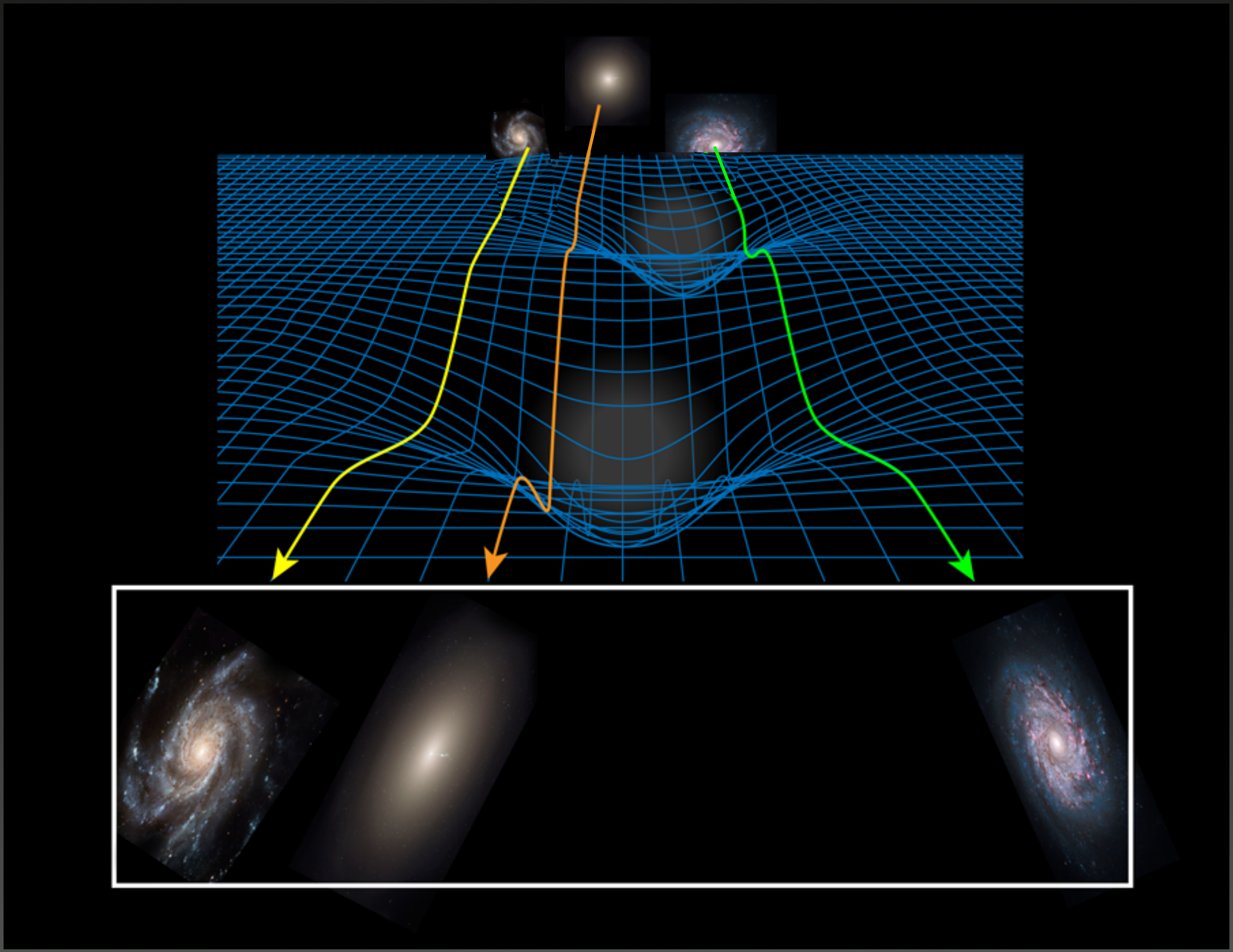
Inhomogeneous Cosmologies IV

July 18th 2019, Torun

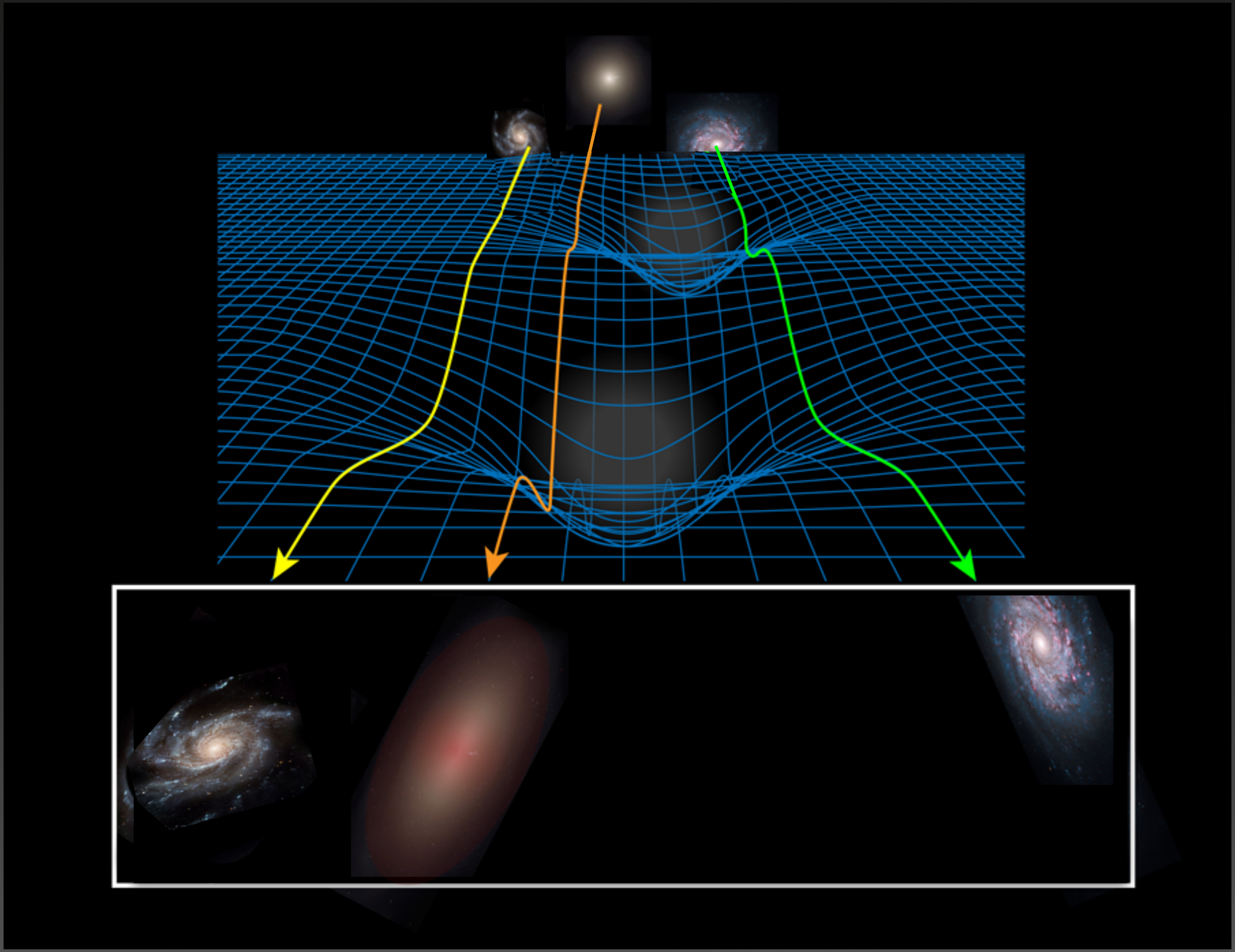
A long time ago in a galaxy far far away…



A long time ago in a galaxy far far away...

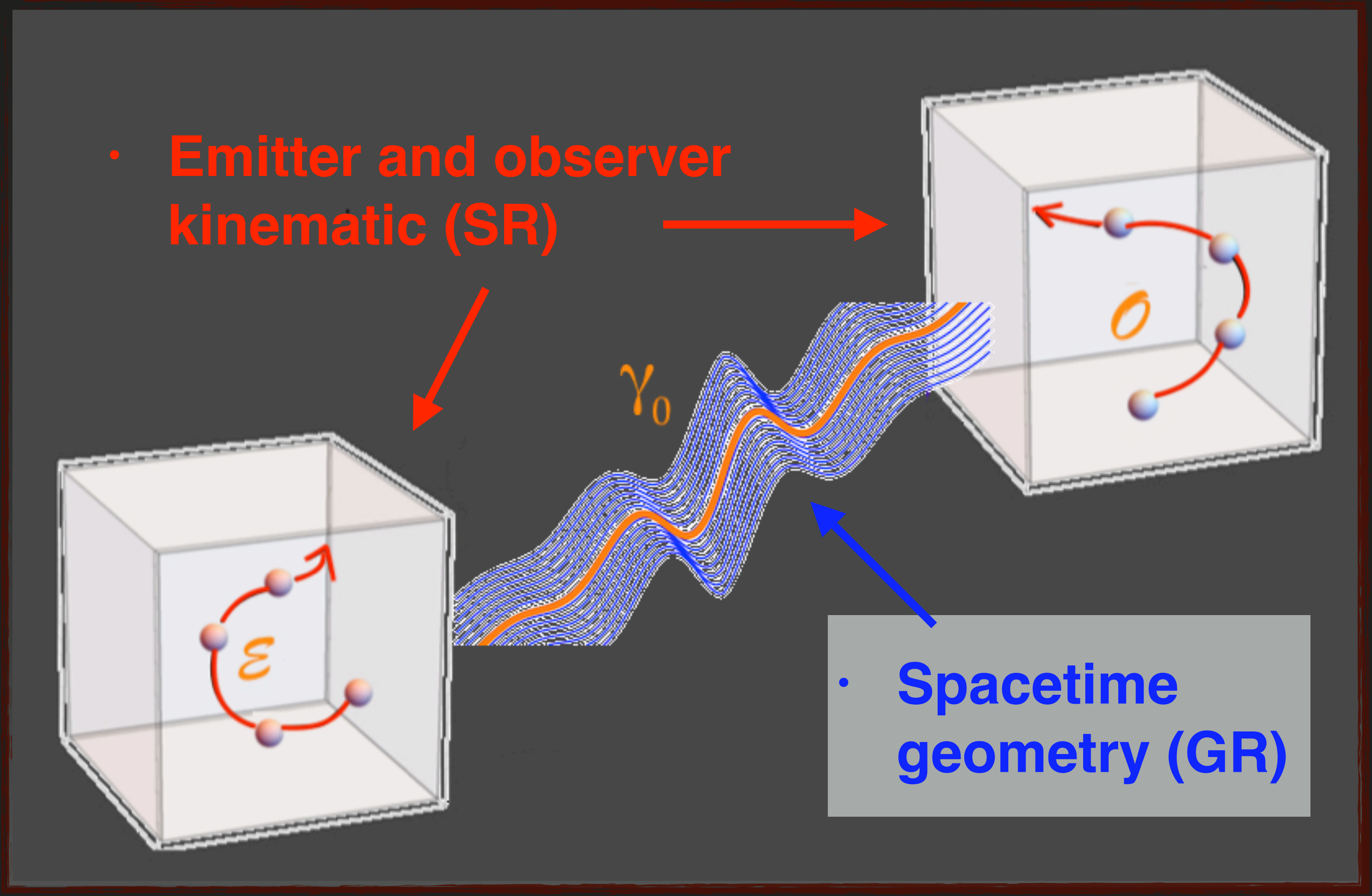


A long time ago in a galaxy far far away…



Advantage of our approach

Observables depending on:



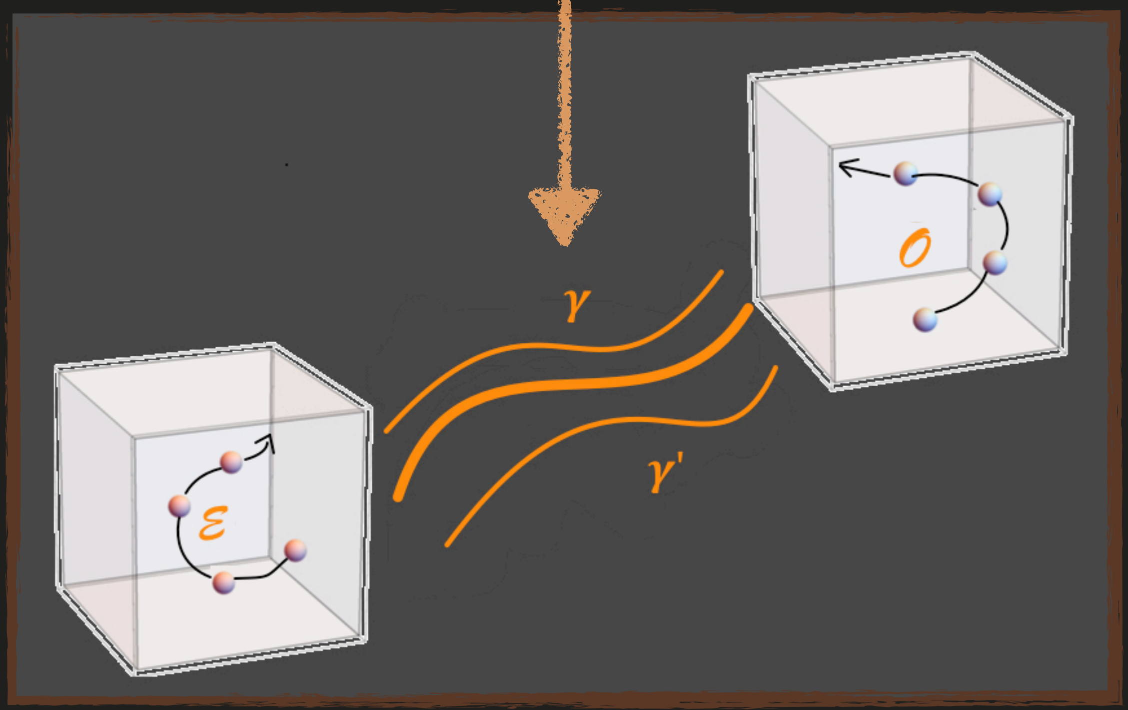
We separate the **effects of motion** from the **effects of geometry**

MG,
Korzyński, M.
Serbenta, J.
Phys.Rev. D, 99(6), 064038

The BGO's formalism

$$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\alpha\beta\nu} l^\alpha l^\beta \xi^\nu = 0 \quad , \quad \left\{ \begin{array}{c} \xi^\nu(\lambda_0) = \delta x^\nu_{\mathcal{O}} \\ \nabla_l \xi^\nu(\lambda_0) = \Delta l^\nu_{\mathcal{O}} \end{array} \right.$$

$$\begin{pmatrix} \delta x_{\mathcal{E}} \\ \Delta l_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x_{\mathcal{O}} \\ \Delta l_{\mathcal{O}} \end{pmatrix}$$

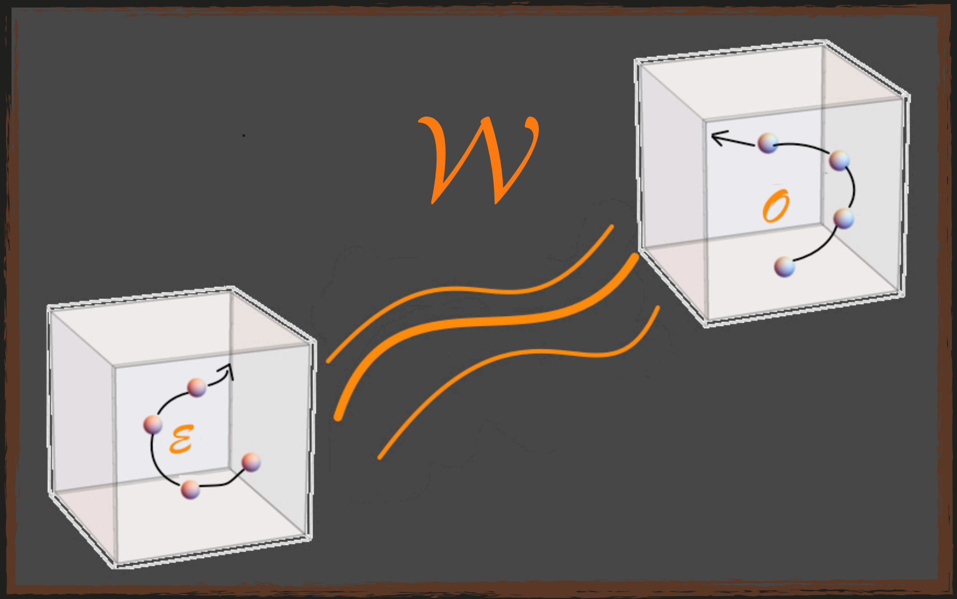


MG, Korzyński, M. Serbenta, J. Phys.Rev. D, 99(6), 064038

Clear separation between curvature and E-O motion contributions in the observable expressions.

The BGO's formalism

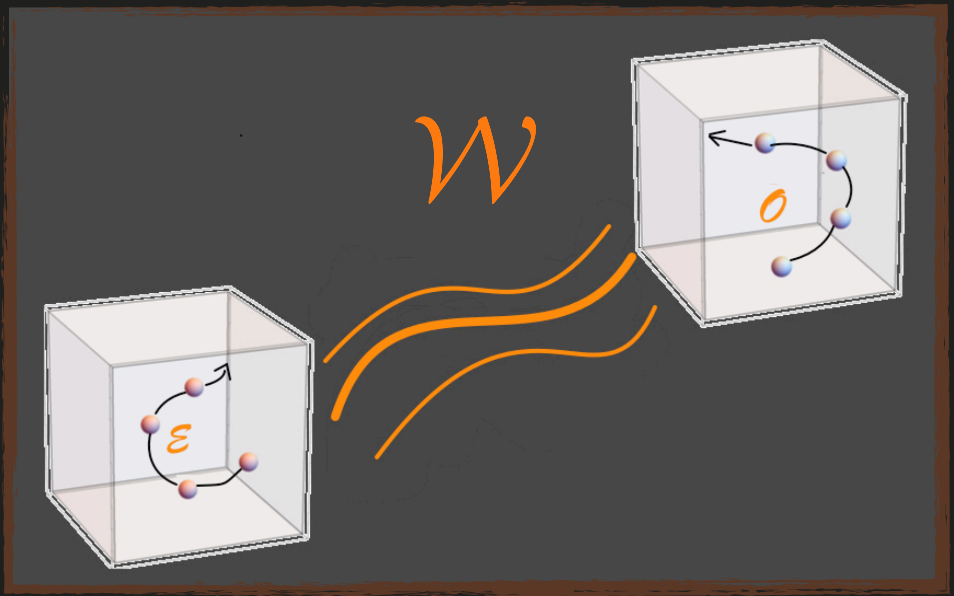
$$\begin{pmatrix} \delta x_{\mathcal{E}} \\ \Delta l_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x_{\mathcal{O}} \\ \Delta l_{\mathcal{O}} \end{pmatrix}$$



$$\mathcal{W} = \begin{pmatrix} W_{XX} & W_{XL} \\ W_{LX} & W_{LL} \end{pmatrix}$$

The BGO's formalism

$$\begin{pmatrix} \delta x_{\mathcal{E}} \\ \Delta l_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x_{\mathcal{O}} \\ \Delta l_{\mathcal{O}} \end{pmatrix}$$

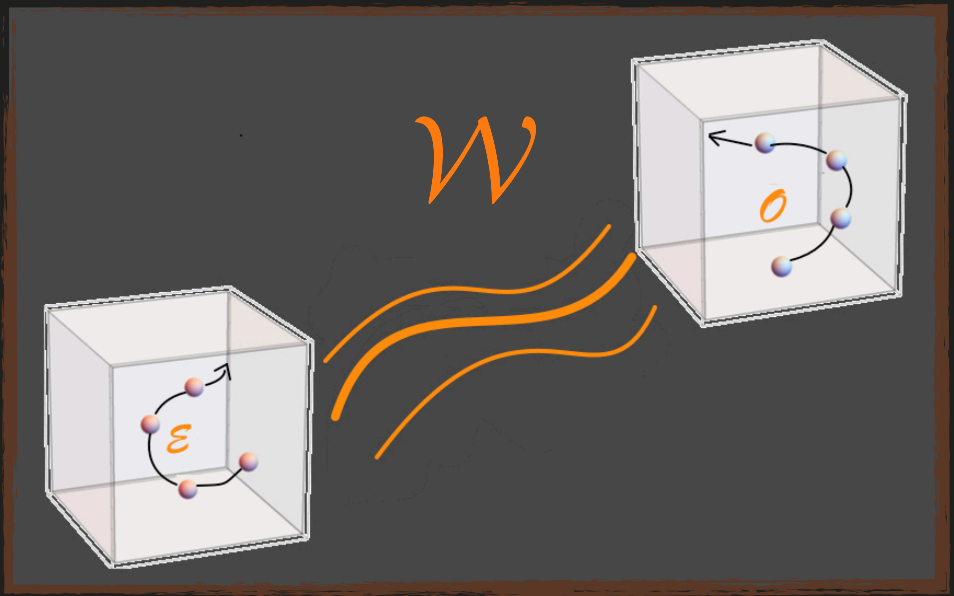


- Distorsion / Magnification
- Angular Distance

$$\mathcal{W} = \begin{pmatrix} W_{XX} & \underline{W_{XL}} \\ W_{LX} & W_{LL} \end{pmatrix}$$

The BGO's formalism

$$\begin{pmatrix} \delta x_{\mathcal{E}} \\ \Delta l_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x_{\mathcal{O}} \\ \Delta l_{\mathcal{O}} \end{pmatrix}$$

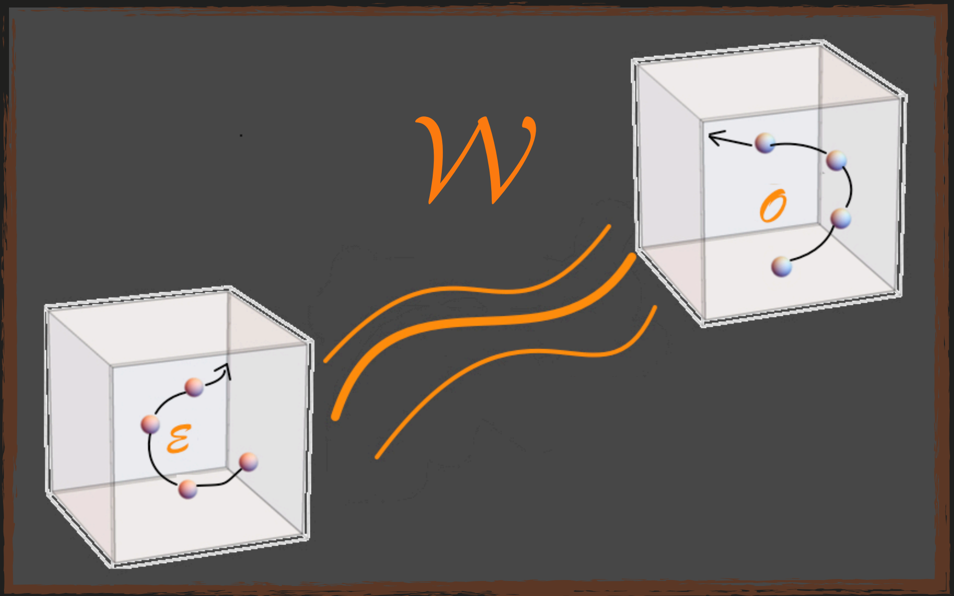


$$\mathcal{W} = \begin{pmatrix} \underline{W_{XX}} & \underline{W_{XL}} \\ W_{LX} & W_{LL} \end{pmatrix}$$

- Parallax
- Position Drift

The BGO's formalism

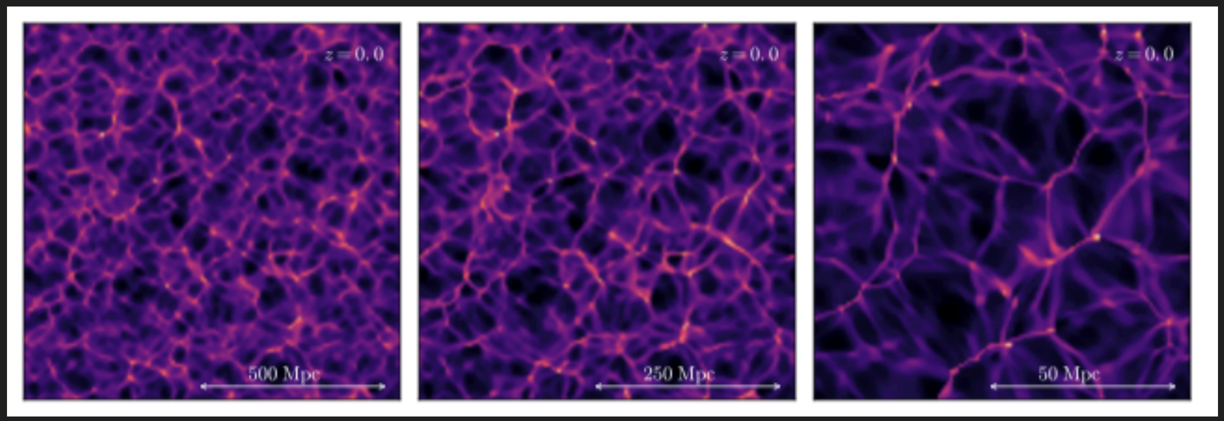
$$\begin{pmatrix} \delta x_{\mathcal{E}} \\ \Delta l_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x_{\mathcal{O}} \\ \Delta l_{\mathcal{O}} \end{pmatrix}$$



$$\mathcal{W} = \begin{pmatrix} W_{XX} & W_{XL} \\ \underline{W_{LX}} & \underline{W_{LL}} \end{pmatrix}$$

- Redshift Drifts
- Visual Prospective
- ...

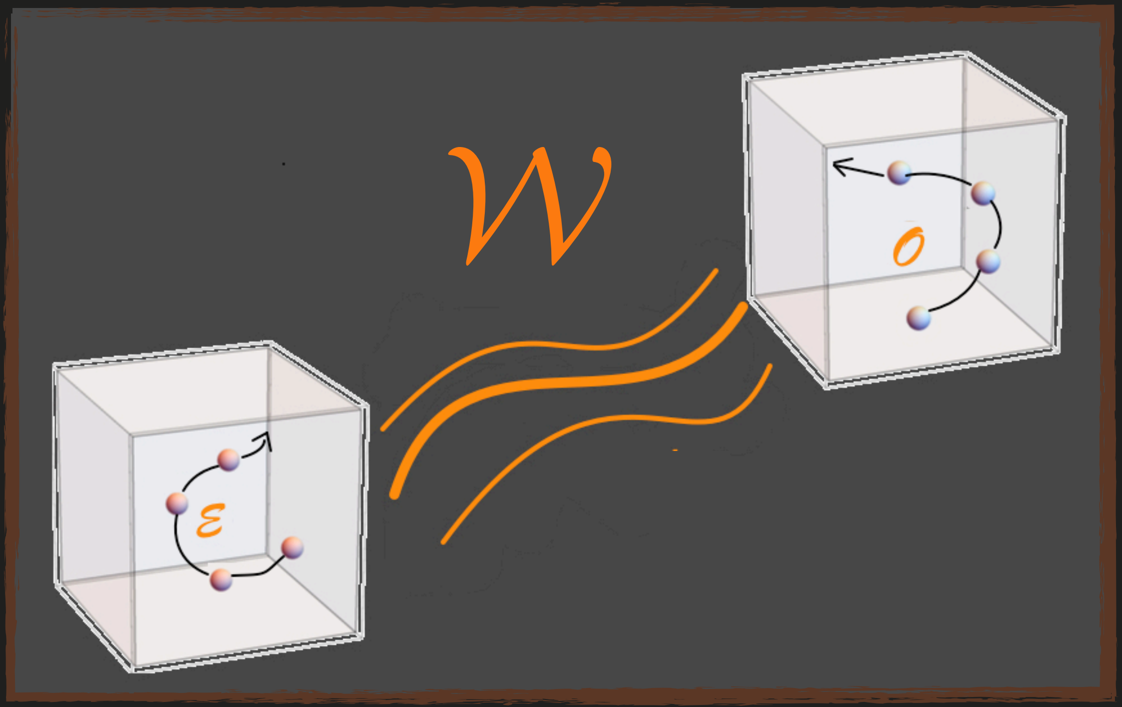
Numerical Relativity and Light propagation



H. Macpherson et. al., arXiv:1807.01711

Cosmic web

from Numerical Relativity



To Do

einstein toolkit

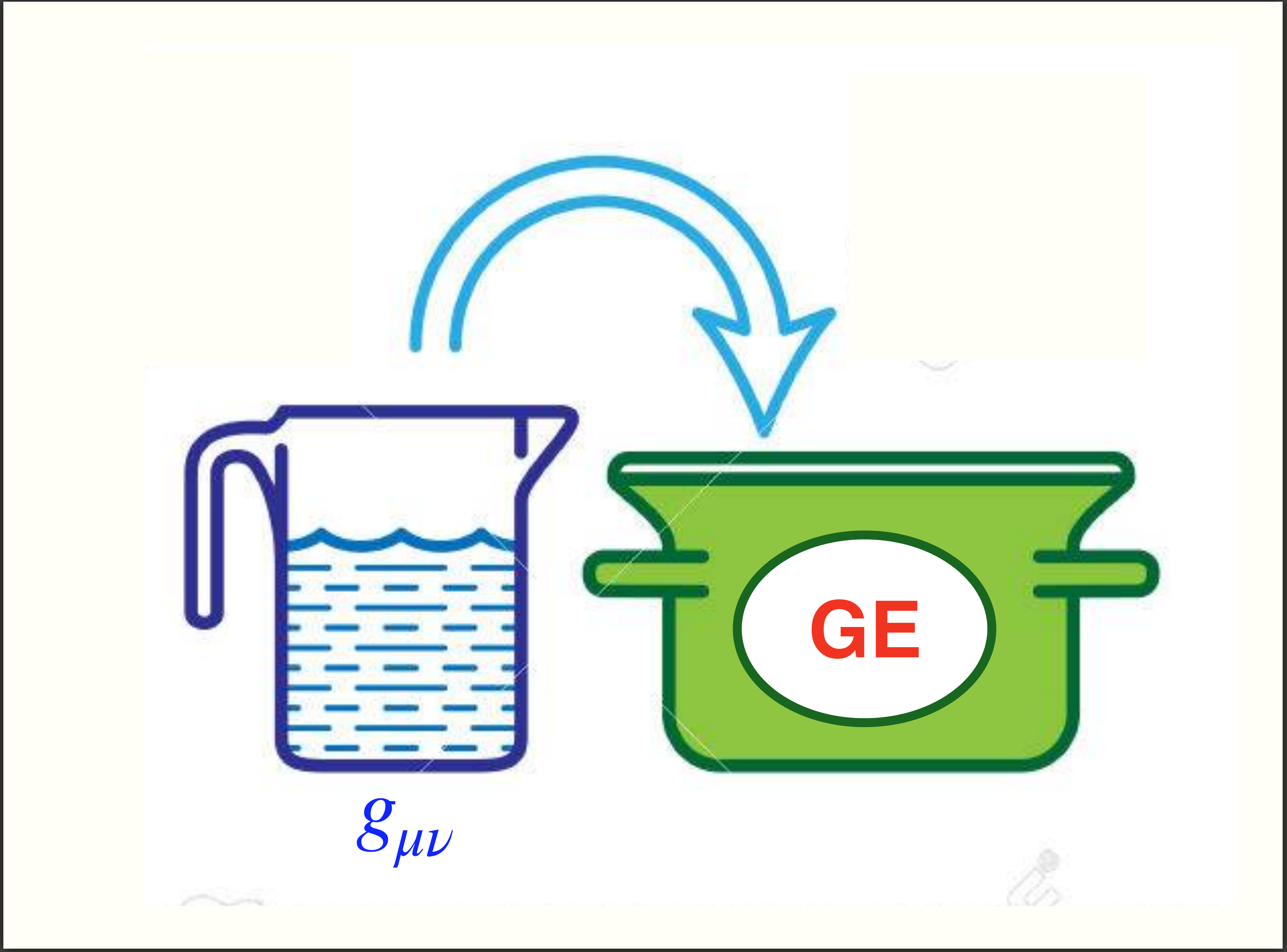
Light Propagation
in Numerical Relativity

Recipe for cooking observables using BGO's

Recipe for cooking observables using BGO's

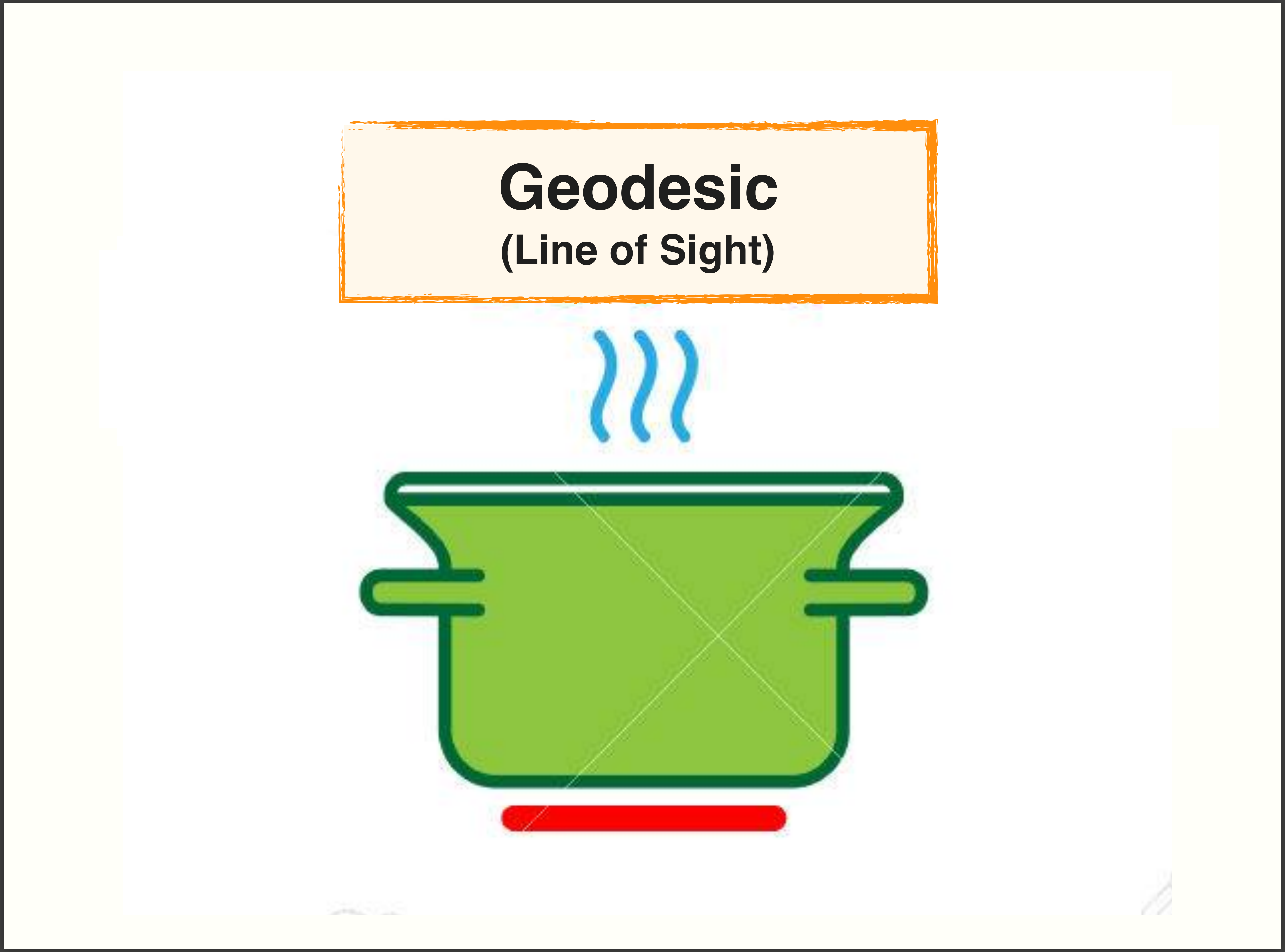
Necessary ingredient:

- **Spacetime geometry**



Recipe for cooking observables using BGO’s

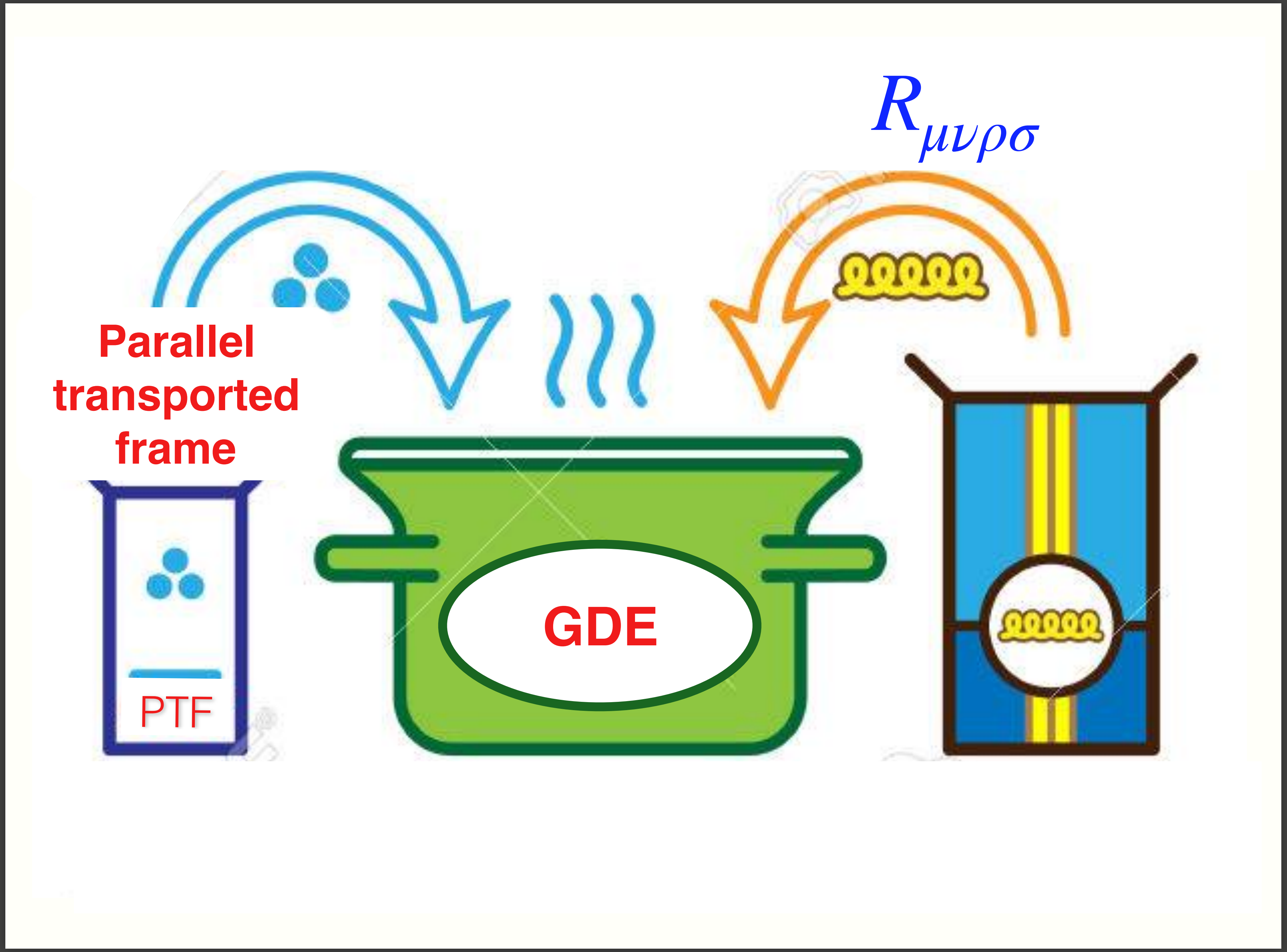
after some time…



Recipe for cooking observables using BGO’s

Next ingredients:

- **Curvature**
- **a way to connect $\mathcal{O}$ and $\mathcal{E}$**



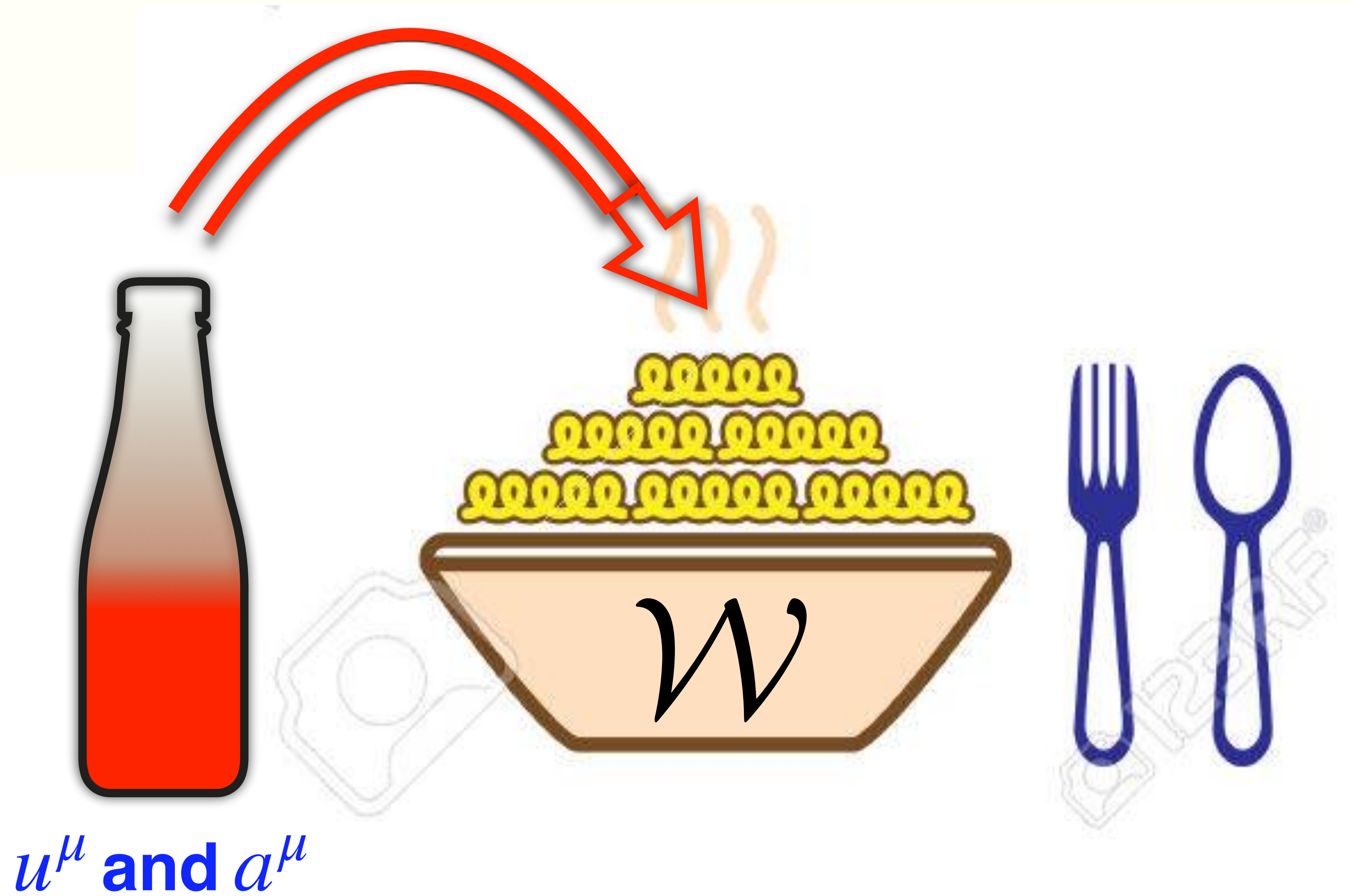
Recipe for cooking observables using BGO's

after some time…

Recipe for cooking observables using BGO's

Last ingredient:

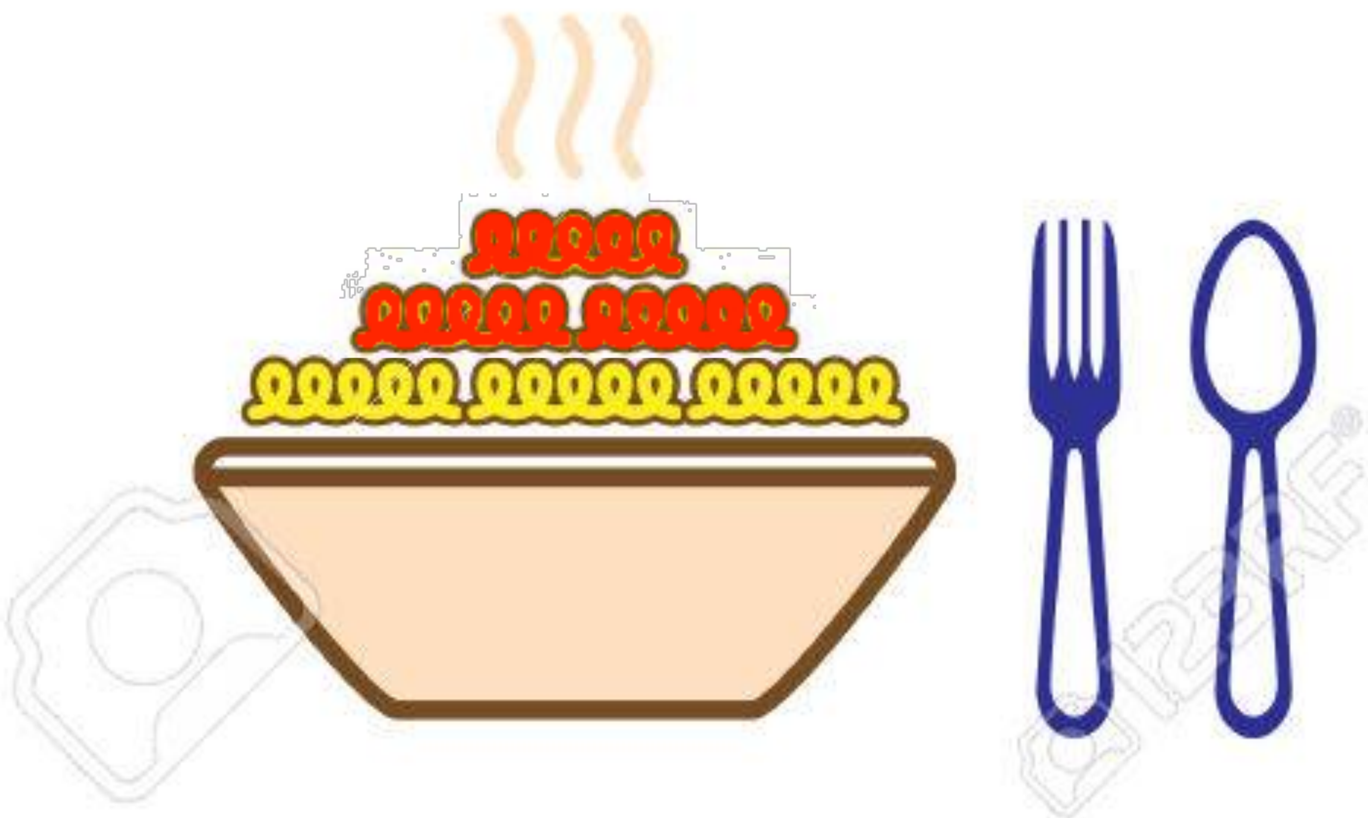
- **Observer and source motion**



Recipe for cooking observables using BGO’s

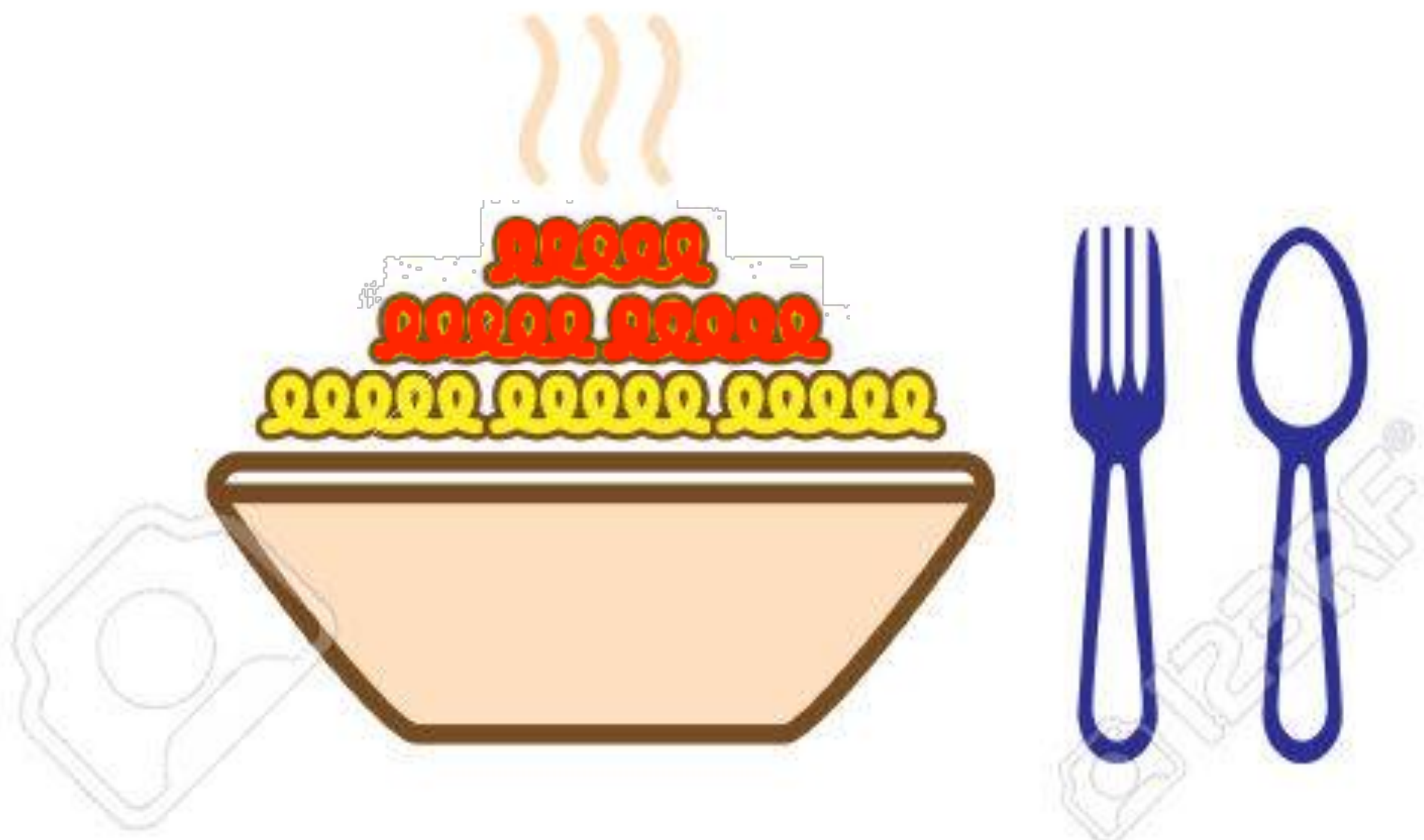
Distances
Parallax
Position drift
Redshift drift
…

Cosmological Observables

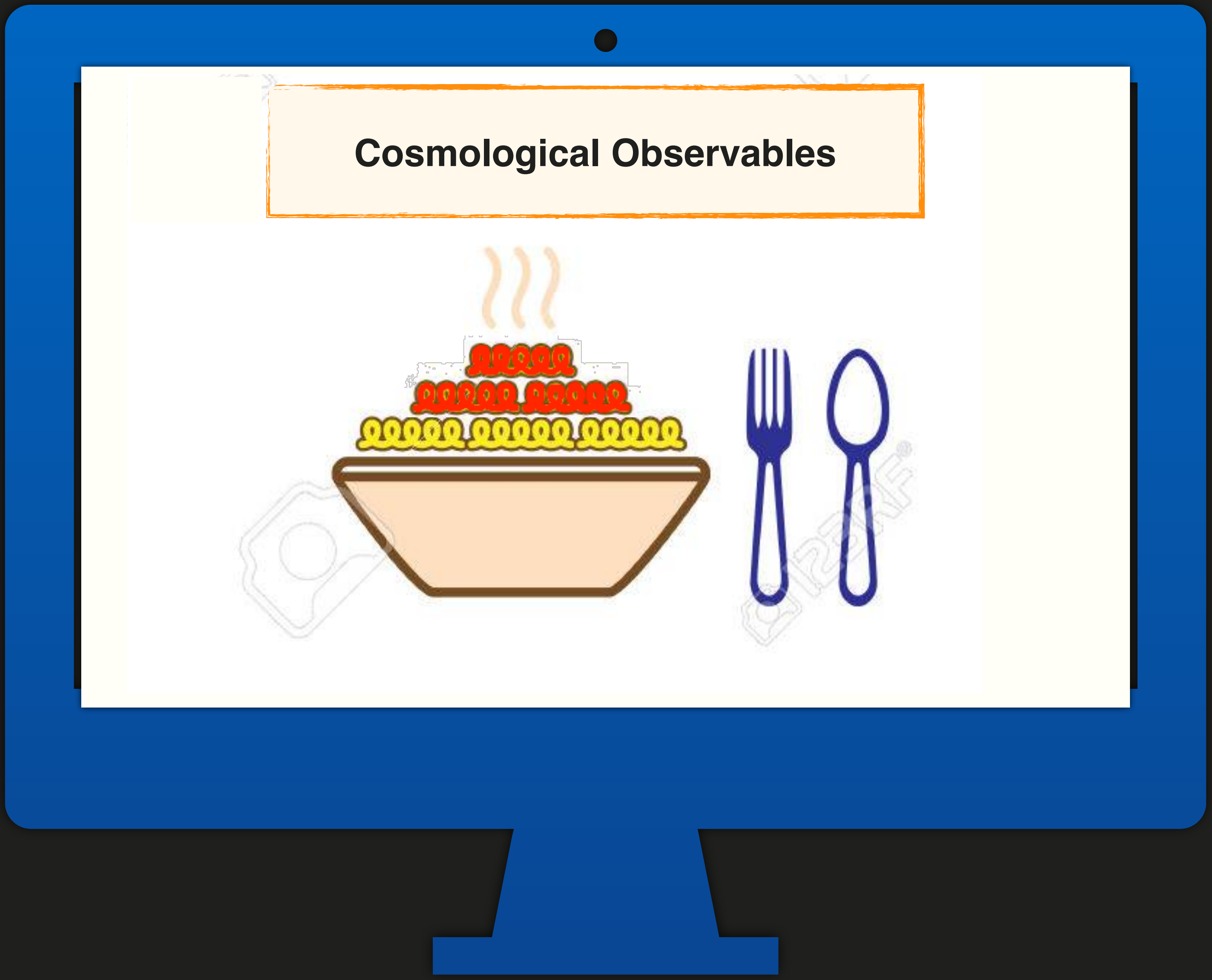


The actual computation

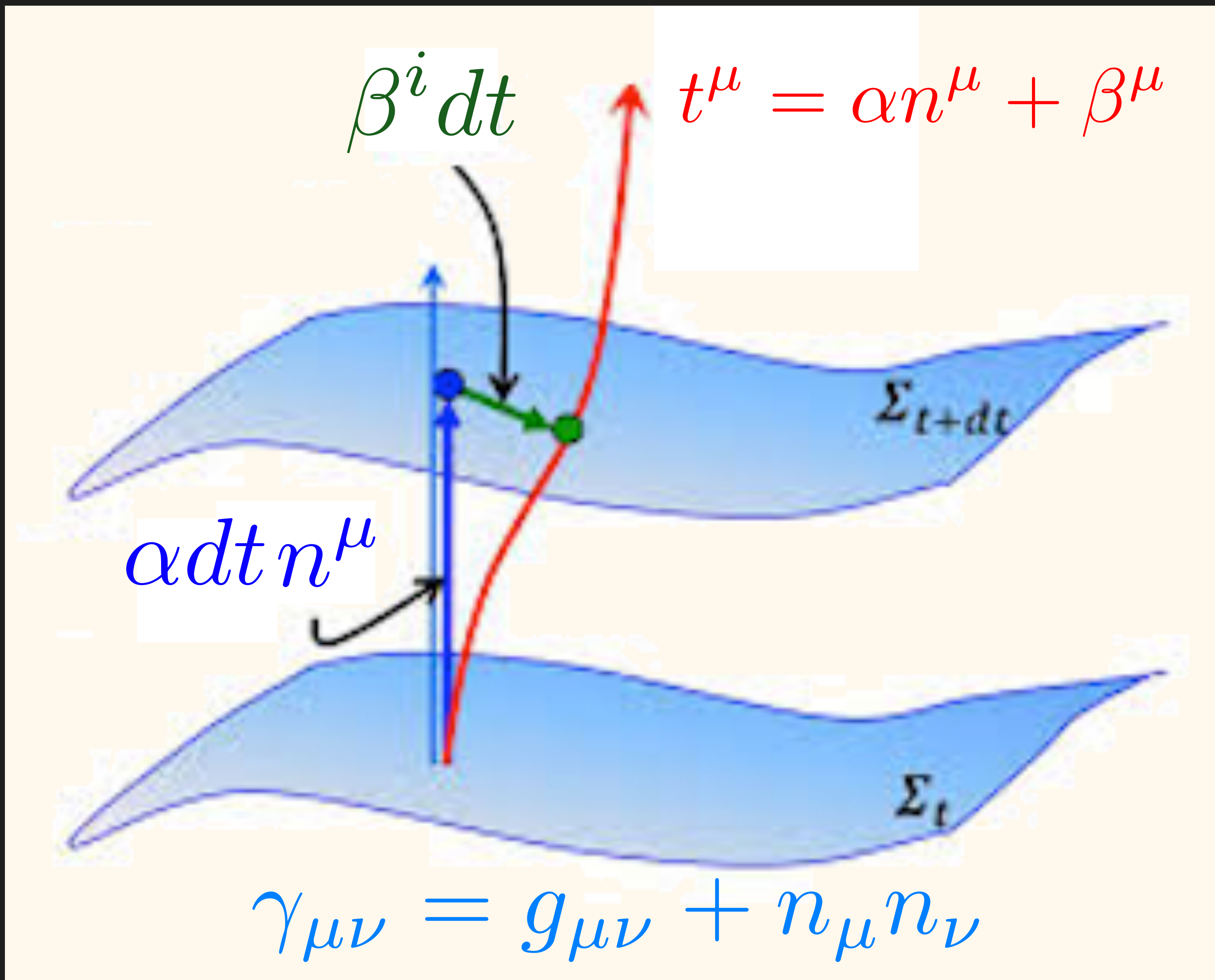
Cosmological Observables



The actual computation



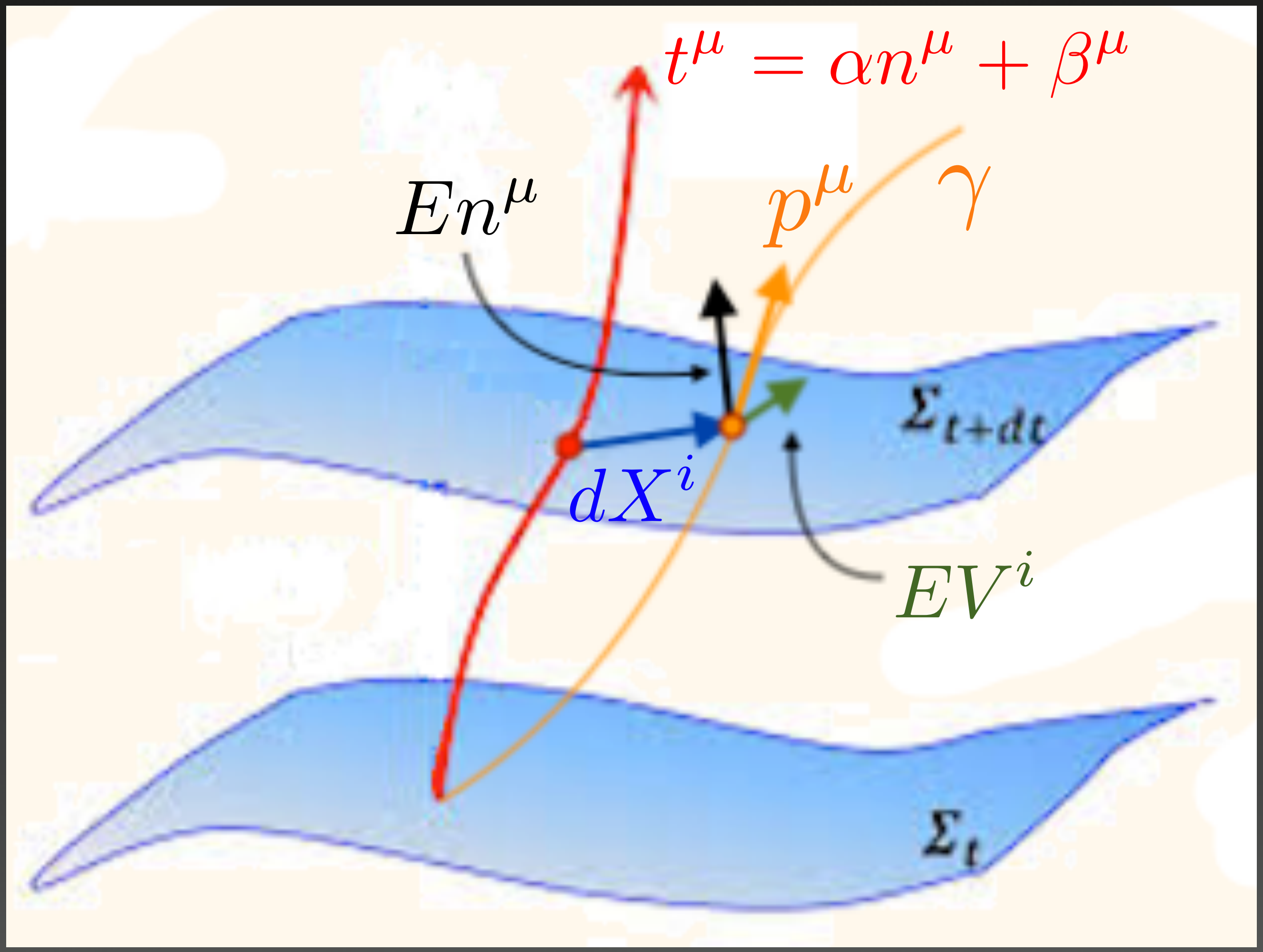
3+1 Foliation of Spacetime



$$(\mathcal{M}, g_{\mu\nu})$$

$$(\alpha, \beta^i, \gamma_{ij}, K_{ij})$$

The boiling water: the line of sight



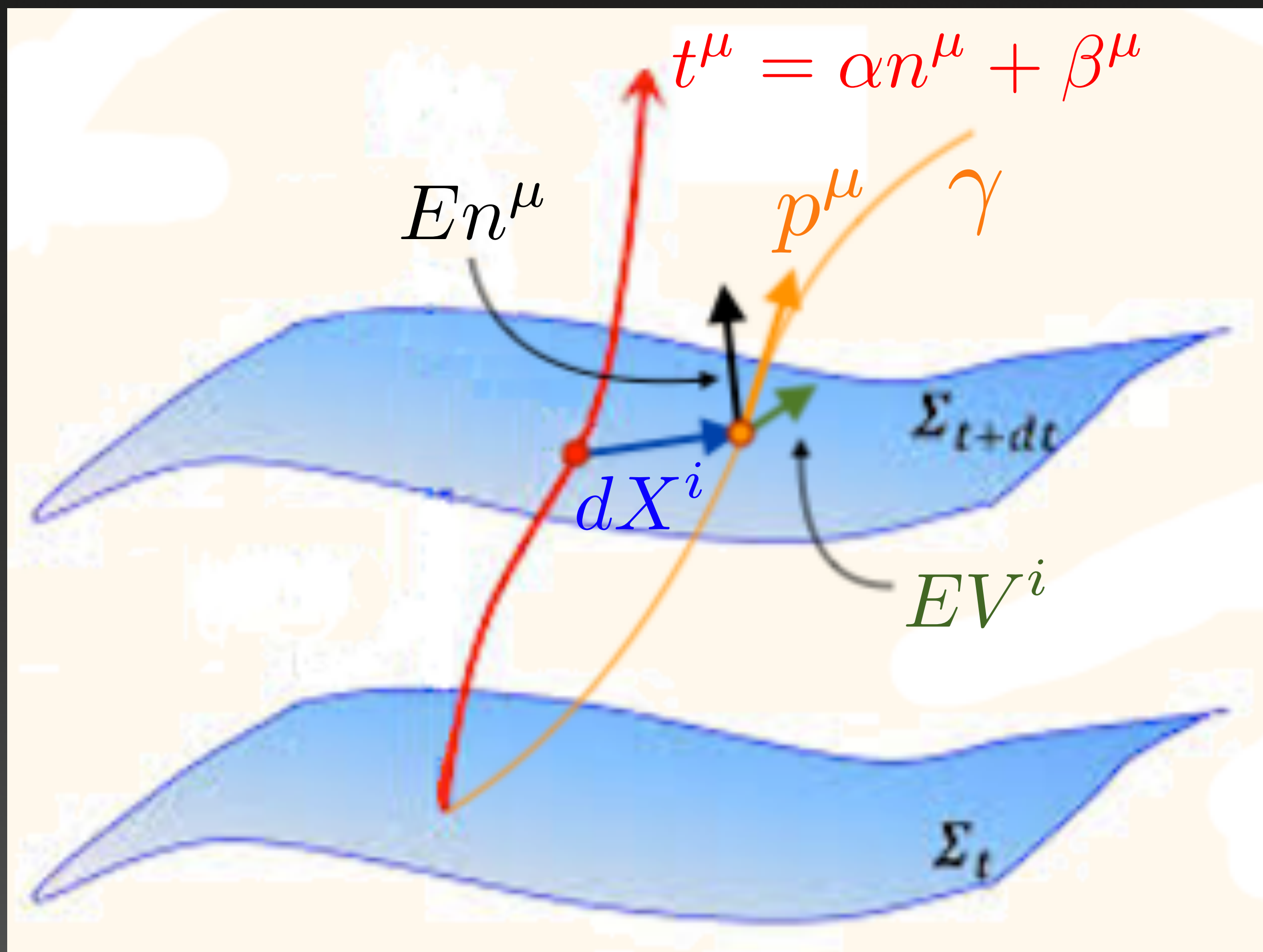
Vincent, F. H., Gourgoulhon, E., & Novak, J. (2012). CQG 29(24), 245005.

$$p^\mu = E(n^\mu + V^\mu)$$

$$p^\mu n_\mu = -E \;\; ; \;\; V^\mu n_\mu = 0$$

$$p^\mu \nabla_\mu p^\nu = 0$$

The boiling water: the line of sight



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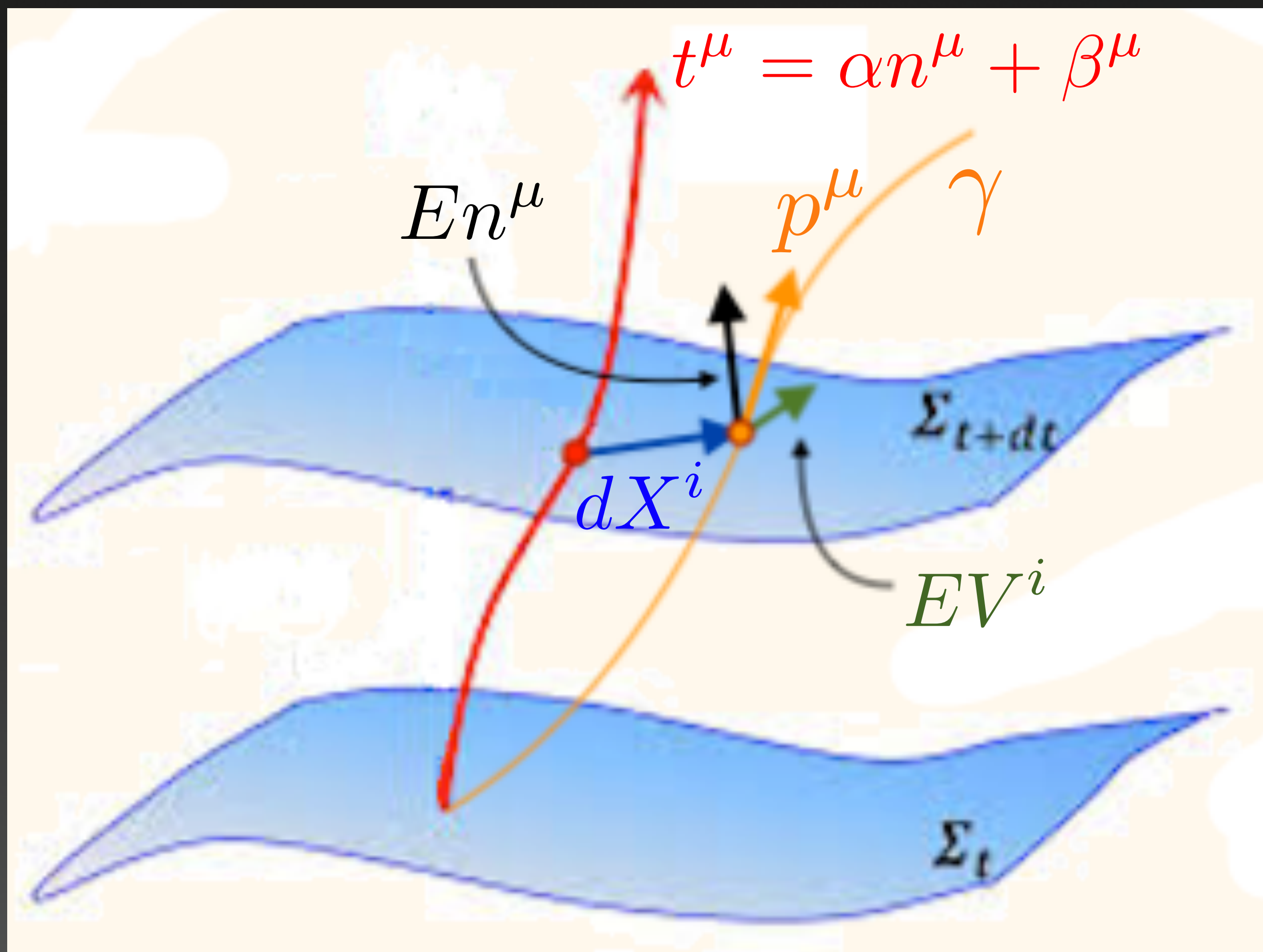

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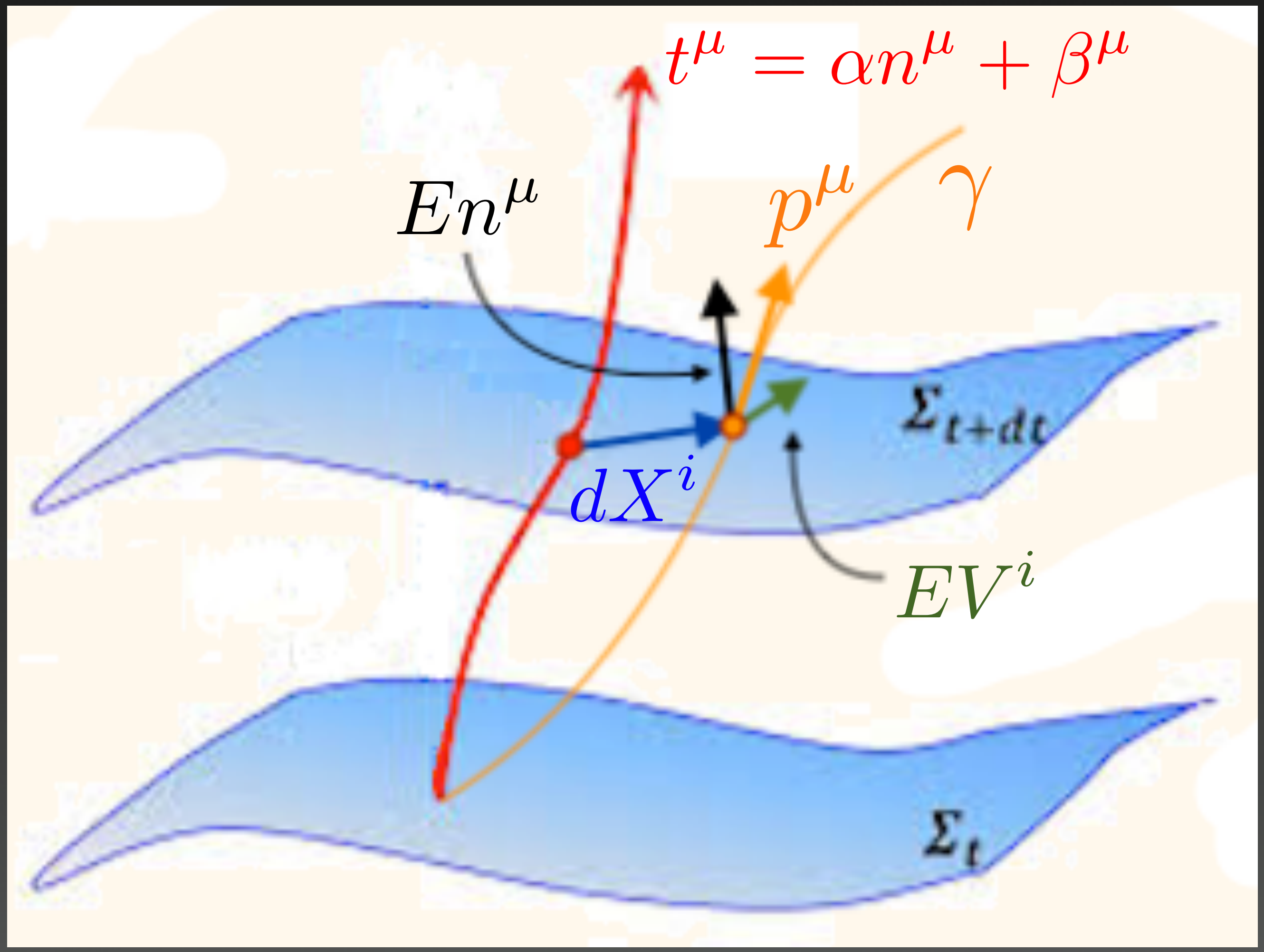
$$\frac{d\lambda}{dt} = \frac{\alpha}{E}$$

$$\frac{dE}{dt} = E(\alpha K_{jk} V^j V^k - V^j \partial_j \alpha)$$

The boiling water: the line of sight



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$$\frac{d\lambda}{dt} = \frac{\alpha}{E}$$

$$\frac{dE}{dt} = E(\alpha K_{jk} V^j V^k - V^j \partial_j \alpha)$$

$$\begin{cases} \dfrac{dX^i}{dt} = \alpha V^i - \beta^i \\ \dfrac{dV^i}{dt} = \alpha V^j [V^i(\partial_i \log\alpha - K_{jk}V^k) + 2K^i{}_j - {}^{(3)}\Gamma^i_{jk} V^k] - \gamma^{ij}\partial_j \alpha - V^j \partial_j \beta^i \end{cases}$$

The salt: Parallel transported SNF

$$\nabla_p {}^{(4)}e^\mu = 0$$

$${}^{(4)}e^\mu = C n^\mu + e^\mu$$

$$\alpha^{-1}\frac{dC}{dt} + e^i \partial_i \log\alpha - K_{ij} V^i e^j = 0$$

$$\alpha^{-1}\left(\frac{de^i}{dt} + e^j \partial_j \beta^i\right) + {}^{(3)}\Gamma^i{}_{jk} V^j e^k - K^i{}_j e^j + C\gamma^{ij}\partial_j \log\alpha - C K^i{}_j V^j = 0$$

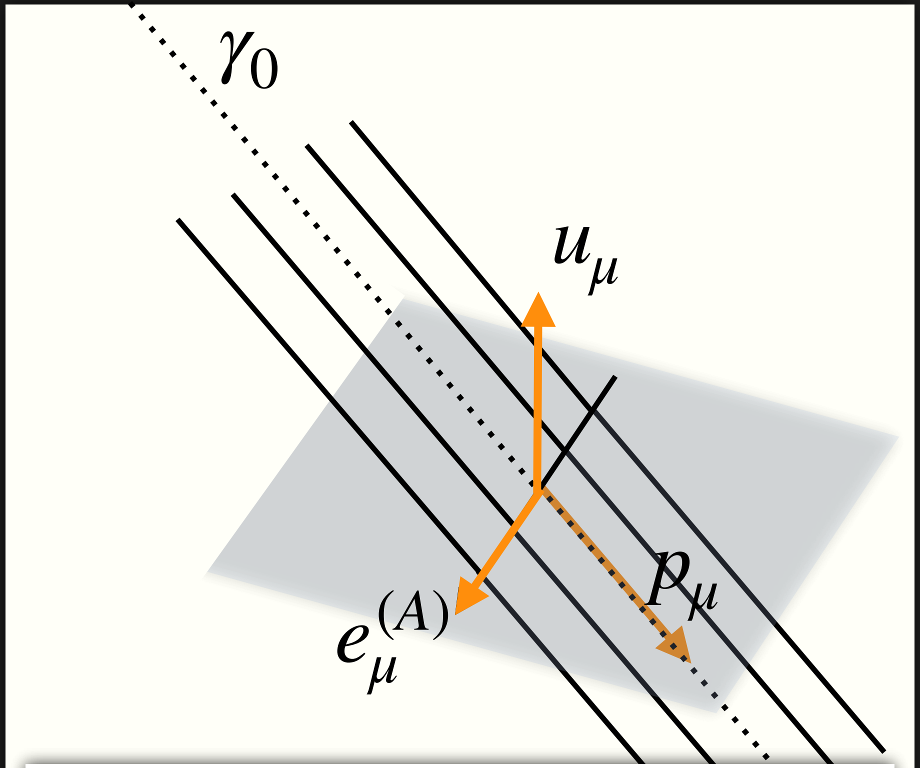
$$\mathbf{f}_{(a)} = (u^\mu, f_1^\mu, f_2^\mu, p^\mu) \qquad \mathbf{e}^{(a)} = (u_\mu, e_\mu^1, e_\mu^2, p_\mu)$$

$$u^\mu u_\mu = -1 \; ; \; f^\mu_{(A)} e^{(B)}_\mu = \delta^{(A)}_{(B)} \; ; \; p^\mu p_\mu = 0 \; ; \; p^\mu u_\mu = Q > 0$$

GDE in parallel transported frame

GDE

$$\nabla_p \nabla_p \xi^\mu - {}^{(4)}R^\mu{}_{\alpha\beta\nu} p^\alpha p^\beta \xi^\nu = 0$$



$$\mathbf{e}^{(a)} = (u_\mu, e^1_\mu, e^2_\mu, p_\mu)$$

$$\frac{d^2\xi^{(a)}}{dt^2} - {}^{(4)}R^{(a)}{}_{\alpha\,\beta\,(b)}\, p^\alpha p^\beta \xi^{(b)} + f(t)\frac{d\xi^{(a)}}{dt} = 0$$

$$\xi^{(a)} = \xi^\mu e_\mu^{(a)}$$

The raw pasta: the curvature

$$\mathscr{G}^{\mu}{}_{\alpha\beta\nu} = \gamma^{\mu}_{\rho}\,\gamma^{\phi}_{\alpha}\,\gamma^{\theta}_{\beta}\,\gamma^{\chi}_{\nu}\,{}^{(4)}R^{\rho}{}_{\phi\theta\chi} = R^{\mu}{}_{\alpha\beta\nu} + K^{\mu}{}_{\beta}K_{\alpha\nu} - K^{\mu}{}_{\alpha}K_{\beta\nu}$$

$$\mathscr{C}^{\mu}{}_{\alpha\nu} = \gamma^{\mu}{}_{\rho}\,n^{\phi}\,\gamma^{\theta}{}_{\alpha}\,\gamma^{\chi}{}_{\nu}\,{}^{(4)}R^{\rho}{}_{\phi\theta\chi} = D_{\nu}K^{\mu}{}_{\alpha} - D_{\alpha}K^{\mu}{}_{\nu}$$

$$\mathscr{R}^{\mu}{}_{\nu} = \gamma^{\mu}{}_{\rho}\,n^{\phi}\gamma^{\theta}{}_{\nu}n^{\chi}\,{}^{(4)}R^{\rho}{}_{\phi\theta\chi} = \frac{1}{\alpha}\mathscr{L}_{\alpha\mathbf{n}}K^{\mu}{}_{\nu} + \frac{1}{\alpha}\gamma^{\mu\sigma}D_{\sigma}D_{\nu}\alpha + K^{\mu}{}_{\sigma}K^{\sigma}{}_{\nu}$$

$$\begin{aligned}{}^{(4)}R^{\mu}{}_{\alpha\beta\nu} = {} & \gamma^{\mu}{}_{\rho}\gamma^{\delta}{}_{\alpha}\gamma^{\eta}{}_{\beta}\gamma^{\sigma}{}_{\nu}\,{}^{(4)}R^{\rho}{}_{\delta\eta\sigma} - 2\gamma^{\mu}{}_{\rho}\gamma^{\delta}{}_{\alpha}\gamma^{\eta}{}_{[\beta}n_{\nu]}n^{\sigma}\,{}^{(4)}R^{\rho}{}_{\delta\eta\sigma} \\ & -2\gamma_{\beta\rho}\gamma^{\delta}{}_{\nu}\gamma^{\eta[\mu}n_{\alpha]}n^{\sigma}\,{}^{(4)}R^{\rho}{}_{\delta\eta\sigma} + 2\gamma^{\mu}{}_{\rho}\gamma^{\eta}{}_{[\beta}n_{\nu]}n_{\alpha}n^{\delta}n^{\sigma}\,{}^{(4)}R^{\rho}{}_{\delta\eta\sigma} \\ & -2\gamma_{\alpha\rho}\gamma^{\eta}{}_{[\beta}n_{\nu]}n^{\mu}n^{\delta}n^{\sigma}\,{}^{(4)}R^{\rho}{}_{\delta\eta\sigma}\end{aligned}$$

APPLICATIONS: Schwarzschild metric as toy model

Initial conditions:

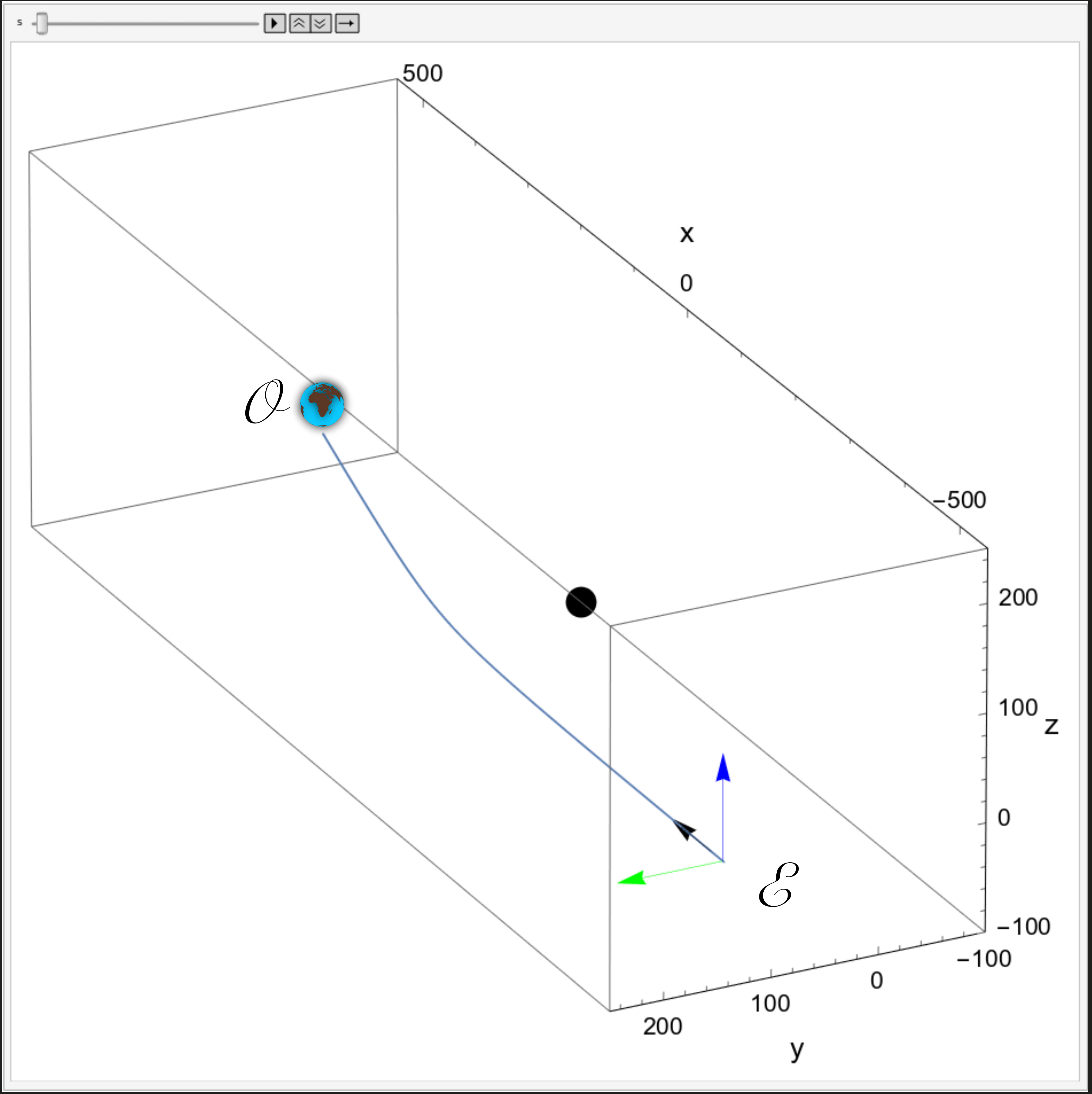
(i) $x_{in} = 40\, r_S$

(ii) $y_{in} = 10\, r_S$

(iii) $z_{in} = 0$

(iv) $k^{\mu}_{in} = \{1,1,0,0\}$

(v) Static emitter and observer



APPLICATIONS: Schwarzschild metric as toy model

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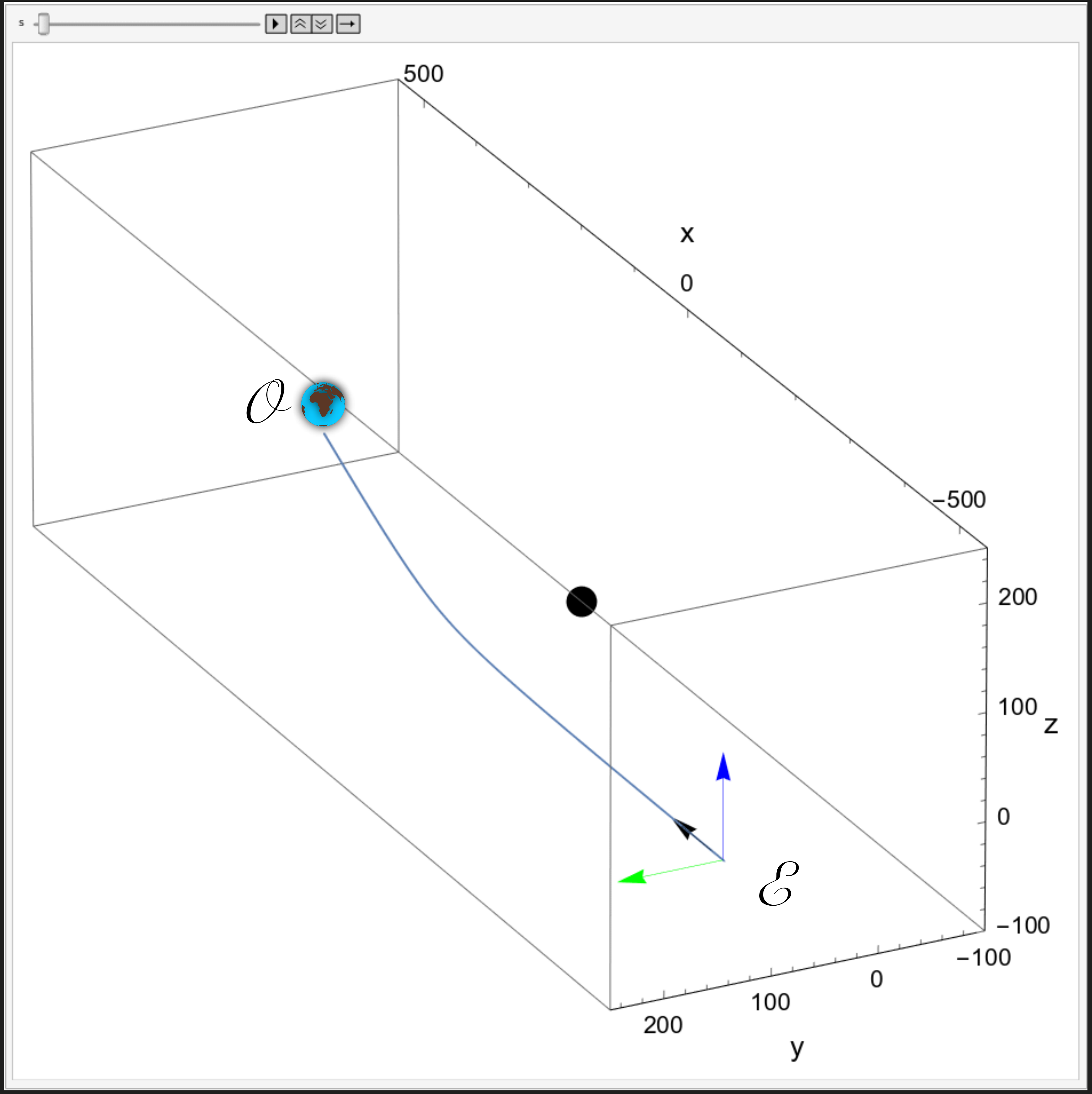
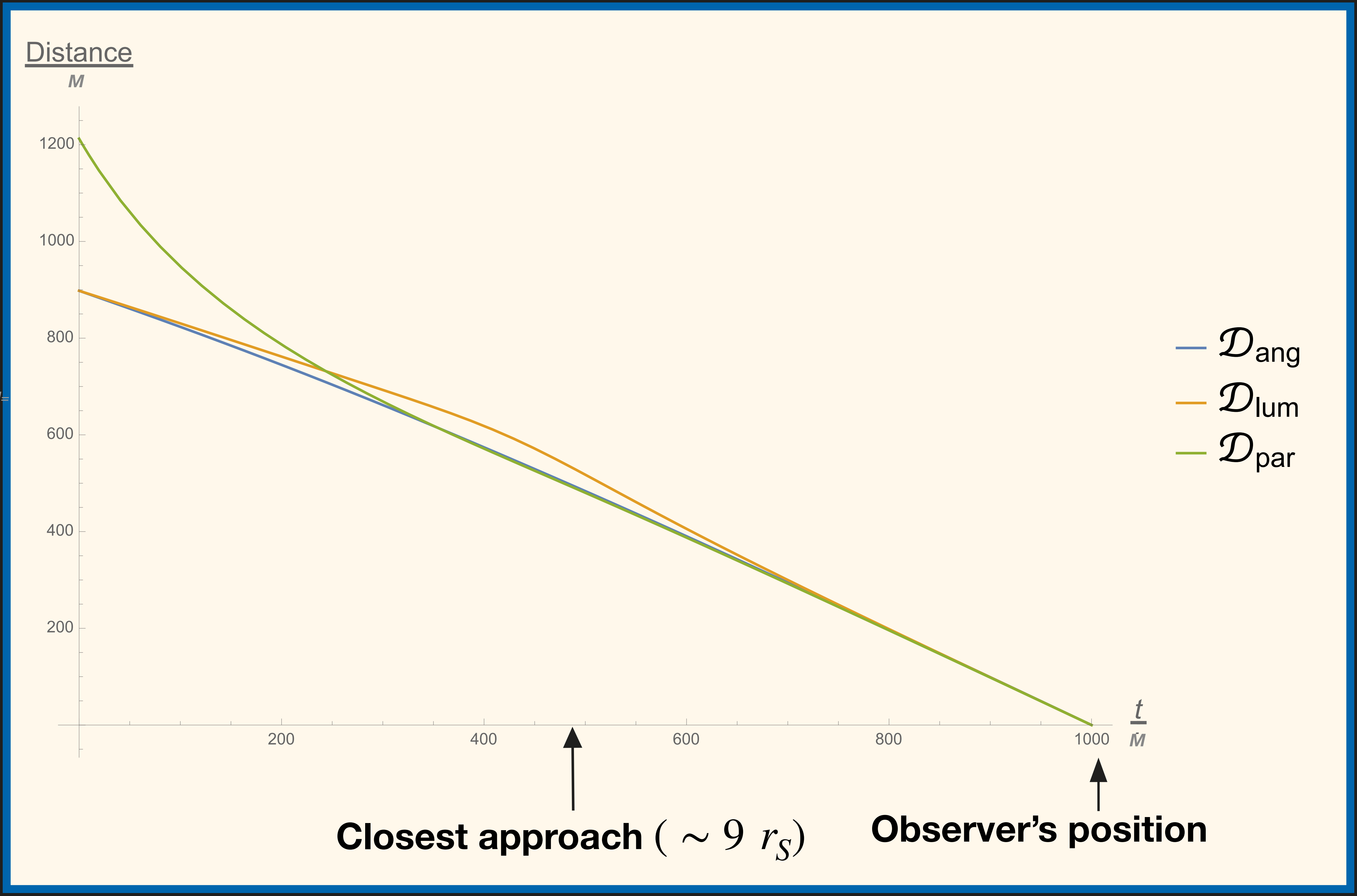


From the BGO we get $\mathscr{D}_{\rm ang}$ and $\mathscr{D}_{\rm par}$

From the BGO + redshift we get $\mathscr{D}_{\rm lum}$



APPLICATIONS: isolating non linearities in light propagation in inhomogeneous cosmology

Our toy-model: plane-parallel dynamics in the PN approximation

- Perturbations depend on x and time only
- Leading order in the $1/c^{2n}$ expansion of Einstein equations

$$ds^2 = a^2(\eta)\left\{-d\eta^2 + \left[\gamma_{11}^{\mathrm{Nwt}}(\eta,x) + \gamma_{11}^{\mathrm{PN}}(\eta,x)\right] dx^2 + \left[1 + \gamma_{22}^{\mathrm{PN}}(\eta,x)\right](dy^2 + dz^2)\right\}$$

Villa, E., Matarrese, S., & Maino, D. (2011). ***JCAP***, *2011*(08), 024.

APPLICATIONS: isolating non linearities in light propagation in inhomogeneous cosmology

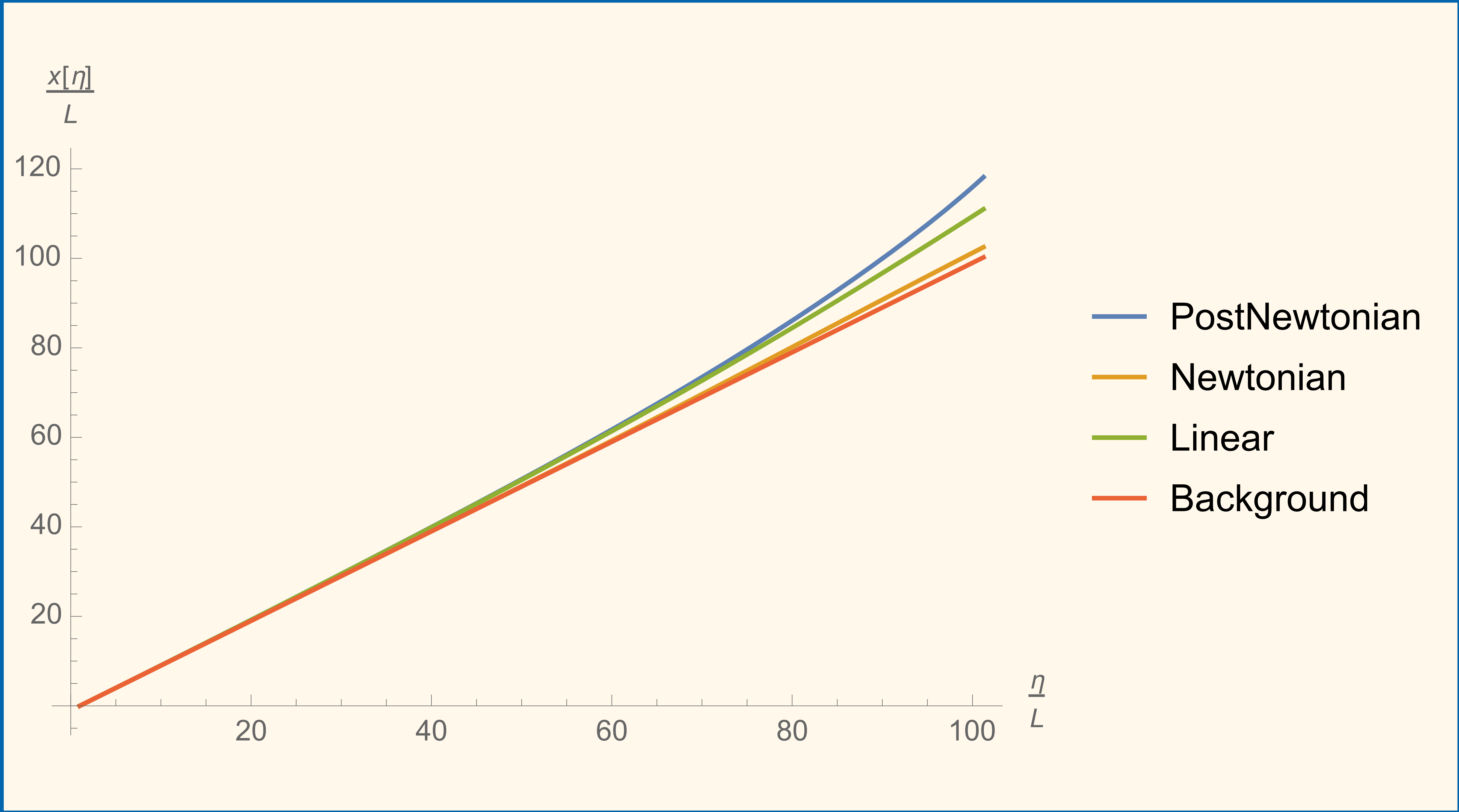
Comparison of cosmological observables in three cases:

(I) **Newtonian** dynamics + **exact** relativistic light propagation

(II) **PN** dynamics + **exact** relativistic light propagation

(III) **Standard PT at first order** for both

APPLICATIONS: isolating non linearities in light propagation in inhomogeneous cosmology



Grasso, M., Villa, E., & Korzyński, M., in preparation

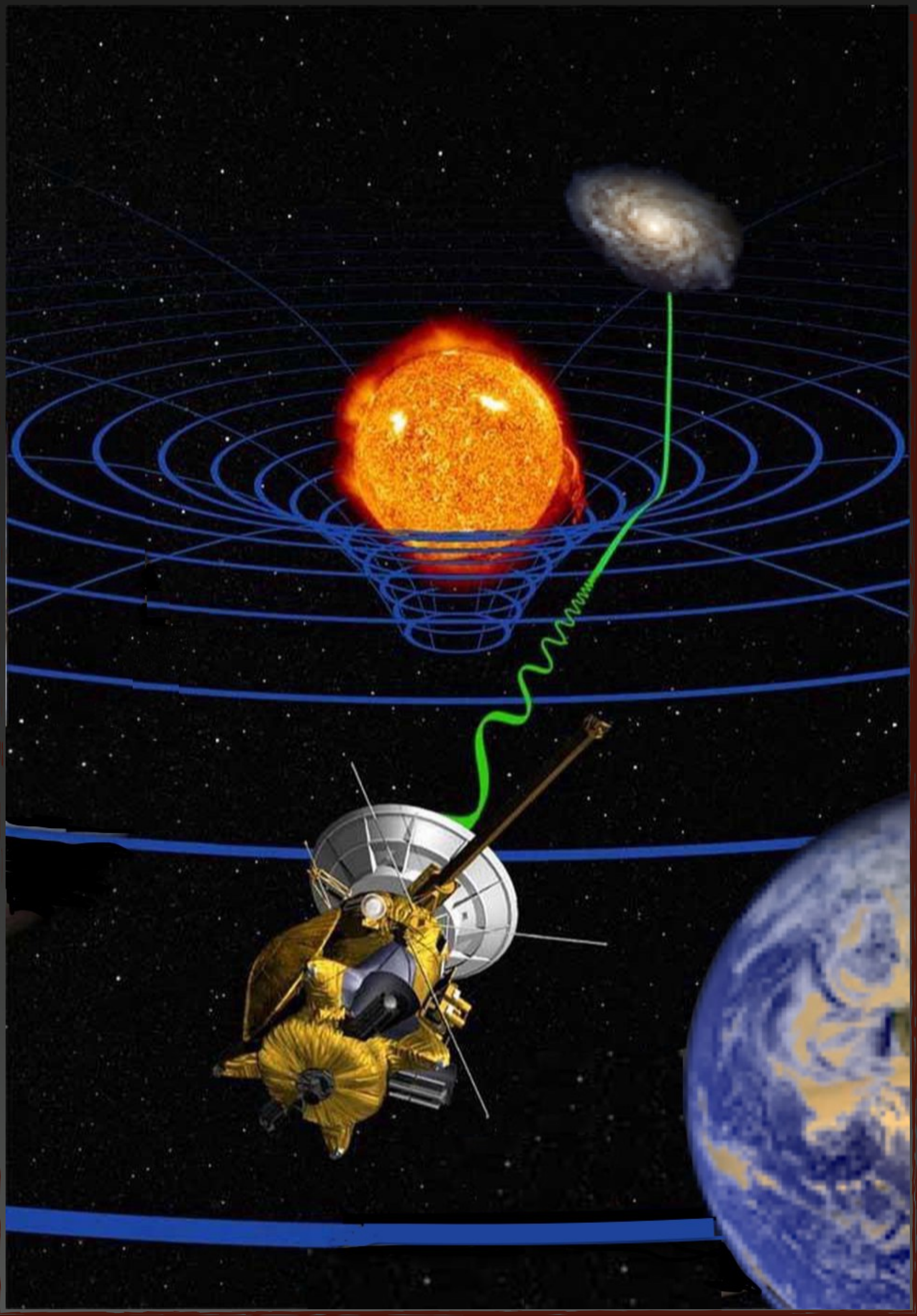
Summary

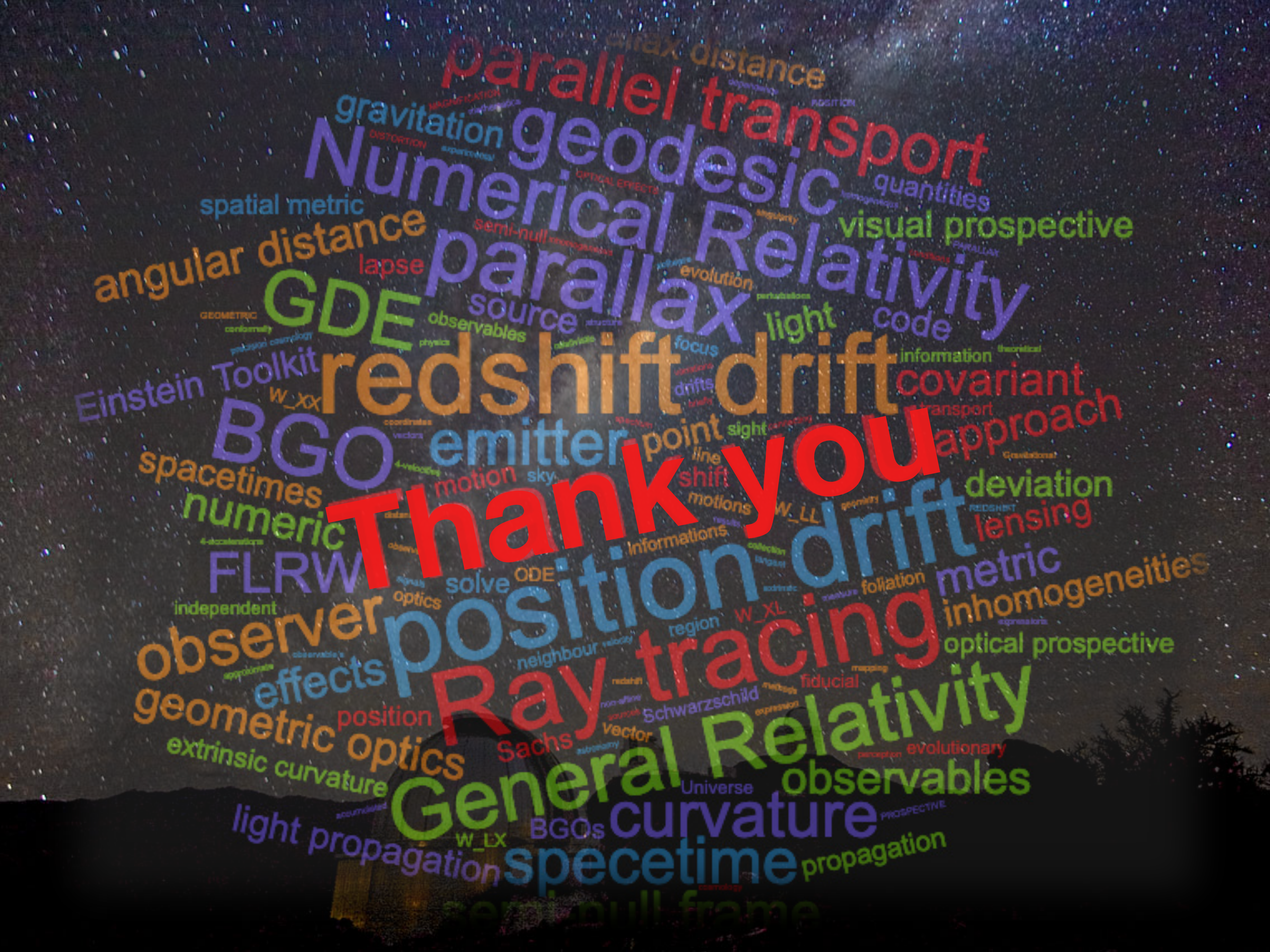
Take-home message:

- BGOs as unified framework for **all** the observables and **all** scales

Work in progress:

- investigating non-linearities
- the full numerical tool for $\mathcal{W}$ within the EINSTEIN TOOLKIT




Thank you
parallel transport
parallax distance
gravitation
geodesic
Numerical Relativity
quantities
spatial metric
visual prospective
angular distance
lapse
parallax
evolution
GDE
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light
code
observables
redshift drift
covariant
Einstein Toolkit
approach
BGO
emitter
point
sight
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motion
sky
shift
line
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motions
lensing
FLRW
informations
position drift
metric
solve
ODE
foliation
inhomogeneities
independent
optics
observer
region
optical prospective
neighbour
effects
Ray tracing
fiducial
geometric optics
position
Schwarzschild
General Relativity
Sachs
vector
evolutionary
extrinsic curvature
observables
Universe
light propagation
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curvature
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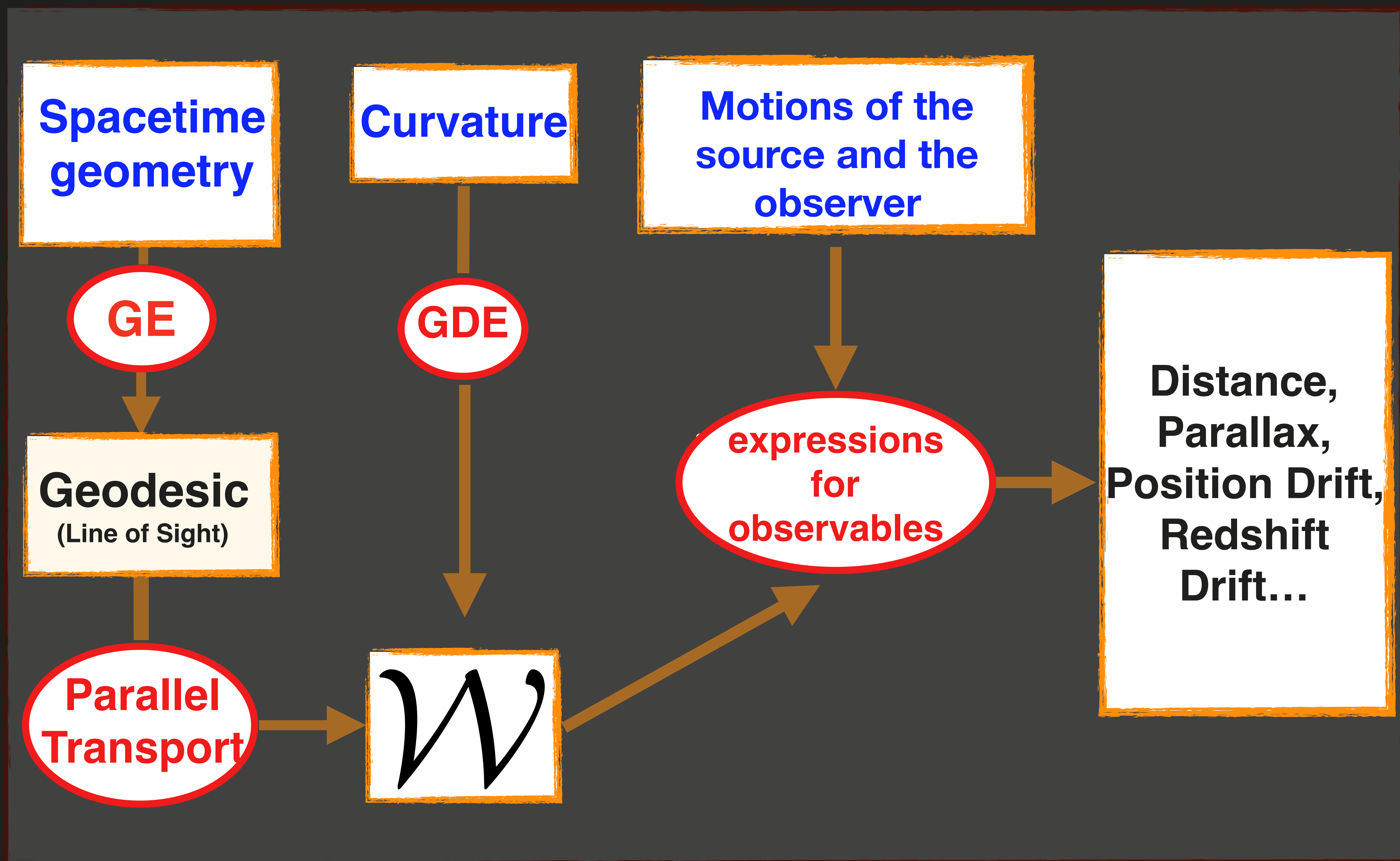
propagation

APPENDIX: Recipe for computing observables using BGO’s



APPENDIX: Relation between BGO and standard geometric optics

$$\begin{pmatrix} \delta x_{\mathcal{E}} \\ \Delta l_{\mathcal{E}} \end{pmatrix} = \mathcal{W} \begin{pmatrix} \delta x_{\mathcal{O}} \\ \Delta l_{\mathcal{O}} \end{pmatrix}$$

$$\delta x^{\mu}_{\mathscr{E}} = (W_{\mathrm{XX}})^{\mu}{}_{\nu} \delta x^{\nu}_{\mathscr{O}} + (W_{\mathrm{XL}})^{\mu}{}_{\nu} \Delta l^{\nu}_{\mathscr{O}}$$

$$\Delta l^{\mu}_{\mathscr{E}} = (W_{\mathrm{LX}})^{\mu}{}_{\nu} \delta x^{\nu}_{\mathscr{O}} + (W_{\mathrm{LL}})^{\mu}{}_{\nu} \Delta l^{\nu}_{\mathscr{O}}$$

$$(W_{\mathrm{XL}})^{(a)}{}_{(b)} = \begin{pmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \multicolumn{2}{c}{D^{A}{}_{B}} & \mathrm{x} \\ \mathrm{x} & & & \mathrm{x} \\ \mathrm{x} & x & x & \mathrm{x} \end{pmatrix} \qquad (W_{\mathrm{XX}})^{(a)}{}_{(b)} = \begin{pmatrix} \mathrm{x} & \mathrm{x} & \mathrm{x} & \mathrm{x} \\ \mathrm{x} & \multicolumn{2}{c}{(\delta + m)^{A}{}_{B}} & \mathrm{x} \\ \mathrm{x} & & & \mathrm{x} \\ \mathrm{x} & x & x & \mathrm{x} \end{pmatrix}$$

APPENDIX: Relation between BGO and different notions of distances

$$\mathscr{D}_{\text{ang}} = (p_{\mathscr{O}\,\mu}\, u^{\mu}_{\mathscr{O}}) \left| \det D^{(A)}_{\ \ (B)} \right|^{\frac{1}{2}}$$

$$\mathscr{D}_{\text{lum}} = (1+z)^2 \mathscr{D}_{\text{ang}}$$

$$\mathscr{D}_{\text{par}} = (p_{\mathscr{O}\,\mu}\, u^{\mu}_{\mathscr{O}}) \left| \det D^{(A)}_{\ \ (B)} \right|^{\frac{1}{2}} \left| \det \left(\delta^{(A)}_{\ \ (B)} + m^{(A)}_{\ \ (B)} \right) \right|^{-\frac{1}{2}}$$

APPENDIX: Plane-parallel metric

$$ds^2 = a^2(\eta)\left\{-c^2 d\eta^2 + \gamma_{11}(\eta,x)dx^2 + \gamma_{22}(\eta,x)dy^2 + \gamma_{33}(\eta,x)dz^2\right\}$$

- $$\gamma_{11} = \left(1 - \frac{2}{3}\frac{D\partial_1^2\phi_0}{\mathcal{H}_0^2\Omega_{\mathrm{m0}}}\right)^2 + \frac{1}{c^2}\left[-\frac{10}{3}\phi_0 + (4a_{\mathrm{nl}} - 5)\frac{10}{9}\frac{D(\partial_1\phi_0)^2}{\mathcal{H}_0^2\Omega_{\mathrm{m0}}} + (a_{\mathrm{nl}} - 1)\frac{40}{9}\frac{D\phi_0\partial_1^2\phi_0}{\mathcal{H}_0^2\Omega_{\mathrm{m0}}}\right.$$
$$\left.-\left(32a_{\mathrm{nl}} - 49\right)\frac{5}{54}\frac{D^2(\partial_1\phi_0)^2\partial_1^2\phi_0}{(\mathcal{H}_0^2\Omega_{\mathrm{m0}})^2} + \left(1 - \frac{2}{3}a_{\mathrm{nl}}\right)\frac{40}{9}\frac{D^2\phi_0(\partial_1^2\phi_0)^2}{(\mathcal{H}_0^2\Omega_{\mathrm{m0}})^2} - \frac{5}{9}\frac{D^3(\partial_1\phi_0)^2(\partial_1^2\phi_0)^2}{(\mathcal{H}_0^2\Omega_{\mathrm{m0}})^3}\right]$$

- $$\gamma_{22} = \gamma_{33} = 1 + \frac{1}{c^2}\left[\frac{10}{9}\left(\frac{D(\partial_1\phi_0)^2}{\mathcal{H}_0^2\Omega_{\mathrm{m0}}} - 3\phi_0\right)\right].$$