Weighing the spacetime along the line of sight

Michele Grasso

• Mikołaj Korzyński

Julius Serbenta

Eleonora Villa

Centre for Theoretical Physics, Polish Academy of Sciences

Warsaw

Inhomogeneous Cosmologies IV

Center for Astronomy

Nicolaus Copernicus University, Toruń

July 2019

Idea

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy
- Most methods known use gravity

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy
- Most methods known use gravity
- Two possibilities

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy
- Most methods known use gravity
- Two possibilities

1. Use the motions of test bodies in the gravitational field
2. Use the impact of gravity on light propagation - gravitational light bending, lensing, Shapiro delays…

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy
- Most methods known use gravity
- Two possibilities

1. Use the motions of test bodies in the gravitational field
2. Use the impact of gravity on light propagation - gravitational light bending, lensing, Shapiro delays…

- **Idea:** new, very direct method of measuring the amount of matter along the line of sight using 2.

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy
- Most methods known use gravity
- Two possibilities

1. Use the motions of test bodies in the gravitational field
2. Use the impact of gravity on light propagation - gravitational light bending, lensing, Shapiro delays…

- **Idea:** new, very direct method of measuring the amount of matter along the line of sight using 2.
- Based on simultaneous measurement of the parallax and: either the apparent size of an object (standard ruler) or the apparent luminosity of that object (standard candle)

Idea

- Determining mass distribution in the spacetime = one of the fundamental problems of astronomy
- Most methods known use gravity
- Two possibilities

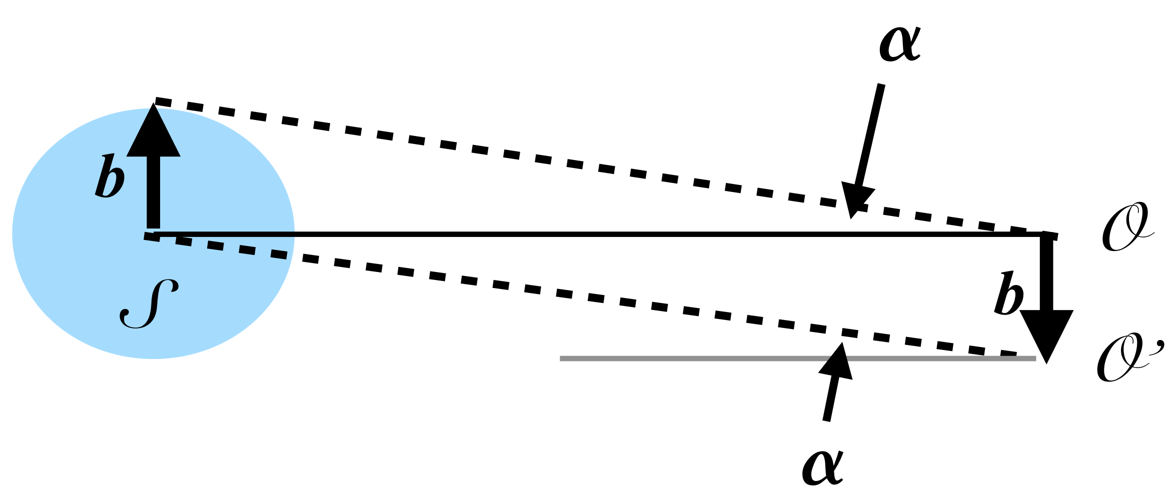
  1. Use the motions of test bodies in the gravitational field
  2. Use the impact of gravity on light propagation - gravitational light bending, lensing, Shapiro delays…

- **Idea:** new, very direct method of measuring the amount of matter along the line of sight using 2.
- Based on simultaneous measurement of the parallax and: either the apparent size of an object (standard ruler) or the apparent luminosity of that object (standard candle)
- **Possible application:** dark and ordinary matter mapping, cosmological isotropy tests, …

Hand-waving explanation

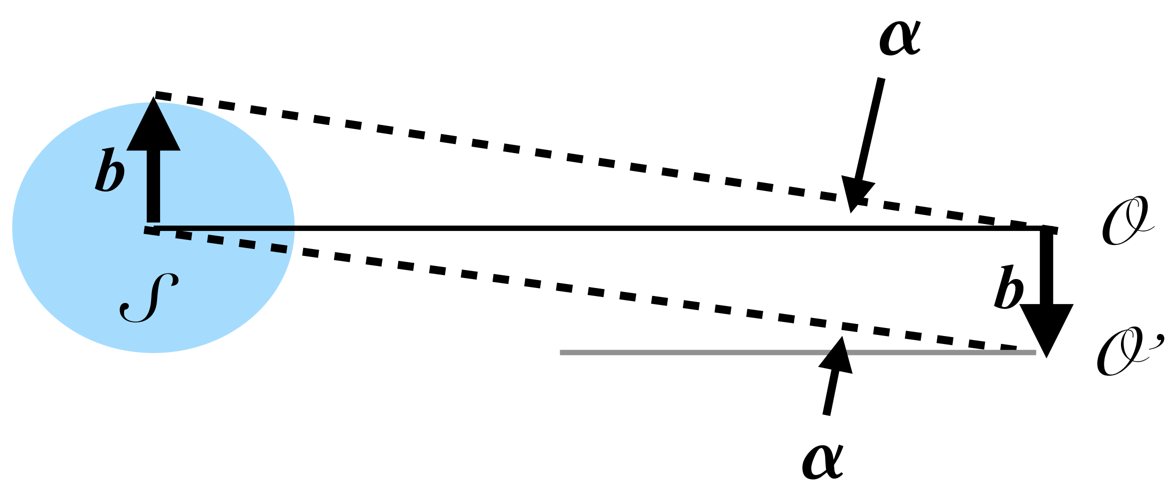
Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax ($D_{\rm par}$) and by angular size ($D_{\rm ang}$) must give the same result



Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result

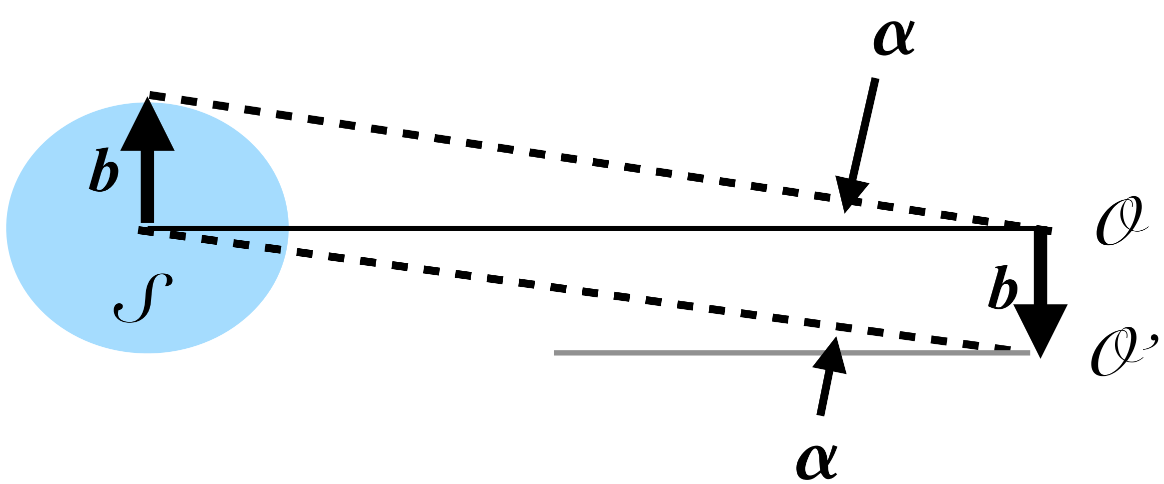


$$D_{\textbf{ang}} = \frac{b}{\alpha}$$

$$D_{\textbf{par}} = \frac{b}{\alpha}$$

Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result



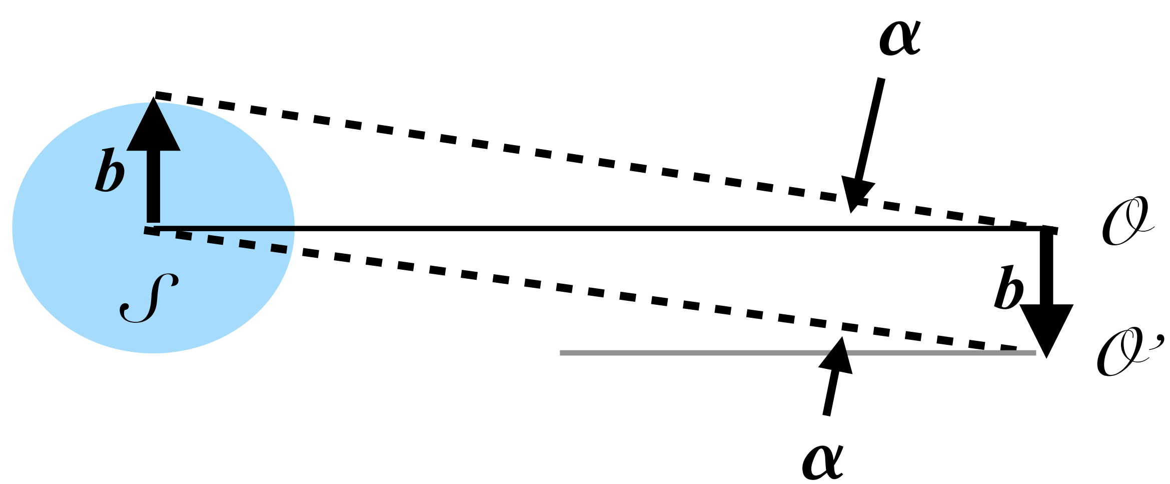
$$D_{\textbf{ang}} = \frac{b}{\alpha}$$

$$D_{\textbf{par}} = \frac{b}{\alpha}$$

$$D_{\textbf{par}} = D_{\textbf{ang}}$$

Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result

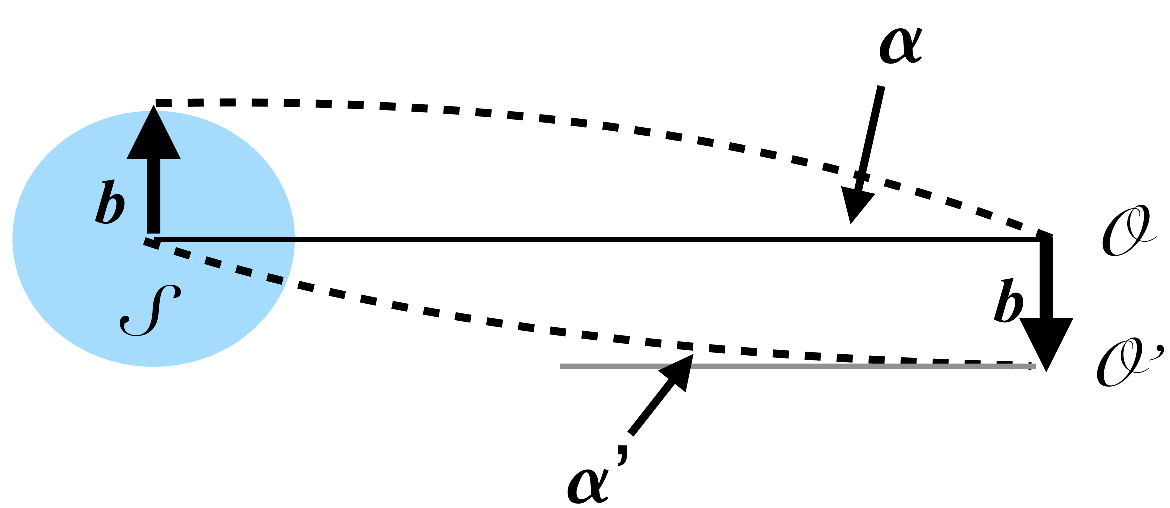


$$D_{\text{ang}} = \frac{b}{\alpha}$$

$$D_{\text{par}} = \frac{b}{\alpha}$$

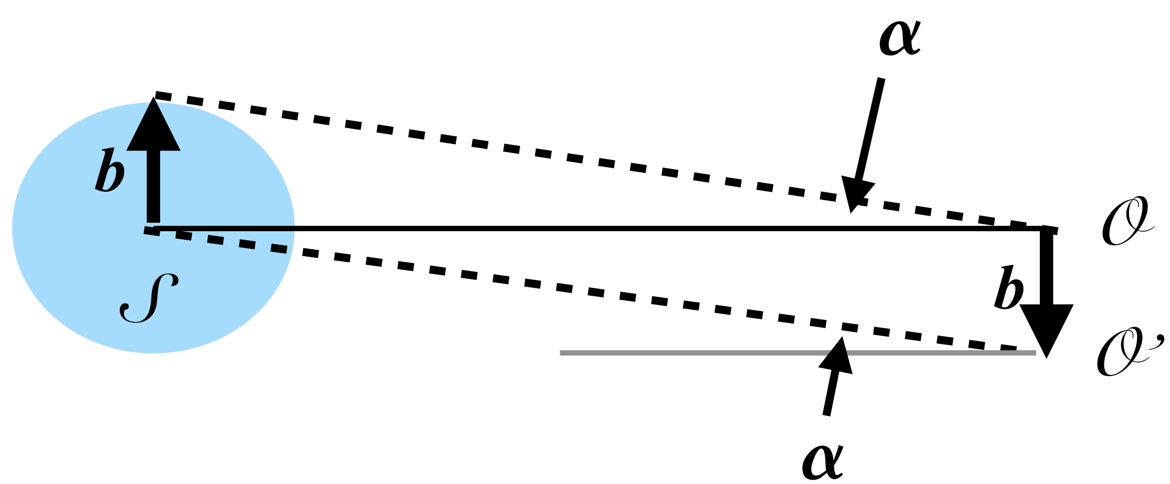
$$D_{\text{par}} = D_{\text{ang}}$$

- Matter present along the line of sight ⇒ gravitational light bending. Both distance measurements affected, but affected *differently*!



Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result

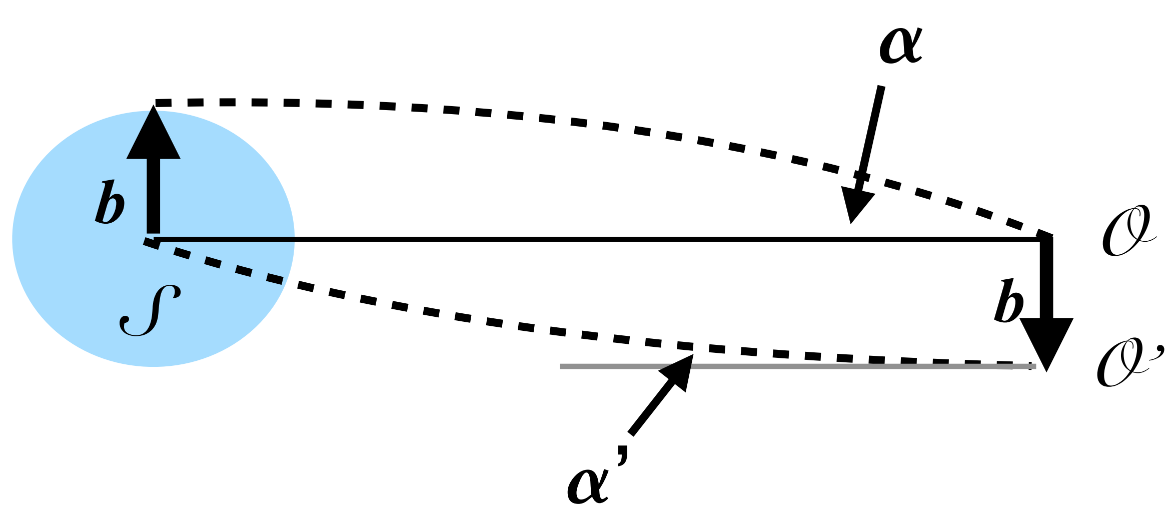


$$D_{\mathbf{ang}} = \frac{b}{\alpha}$$

$$D_{\mathbf{par}} = \frac{b}{\alpha}$$

$$D_{\mathbf{par}} = D_{\mathbf{ang}}$$

- Matter present along the line of sight ⇒ gravitational light bending. Both distance measurements affected, but affected *differently*!



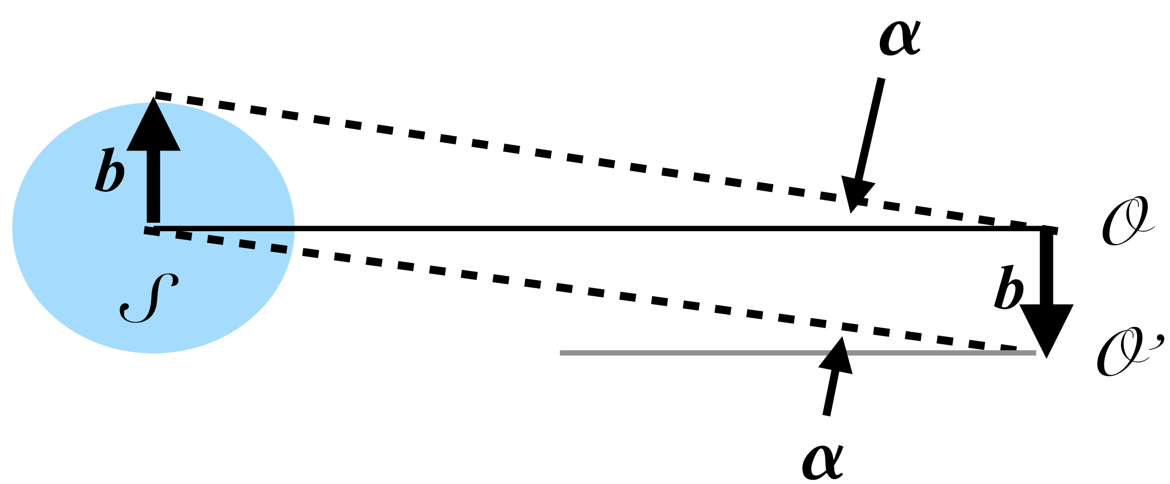
$$D_{\mathbf{ang}} = \frac{b}{\alpha}$$

$$D_{\mathbf{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \Longrightarrow D_{\mathbf{par}} > D_{\mathbf{ang}}$$

Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result

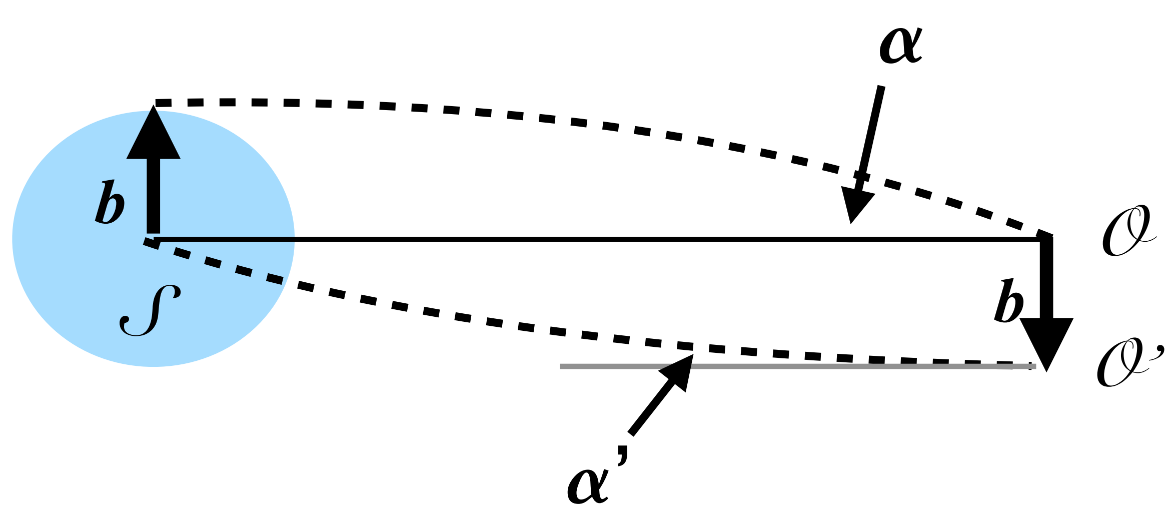


$$D_{\text{ang}} = \frac{b}{\alpha}$$

$$D_{\text{par}} = \frac{b}{\alpha}$$

$$D_{\text{par}} = D_{\text{ang}}$$

- Matter present along the line of sight ⇒ gravitational light bending. Both distance measurements affected, but affected *differently*!



$$D_{\text{ang}} = \frac{b}{\alpha}$$

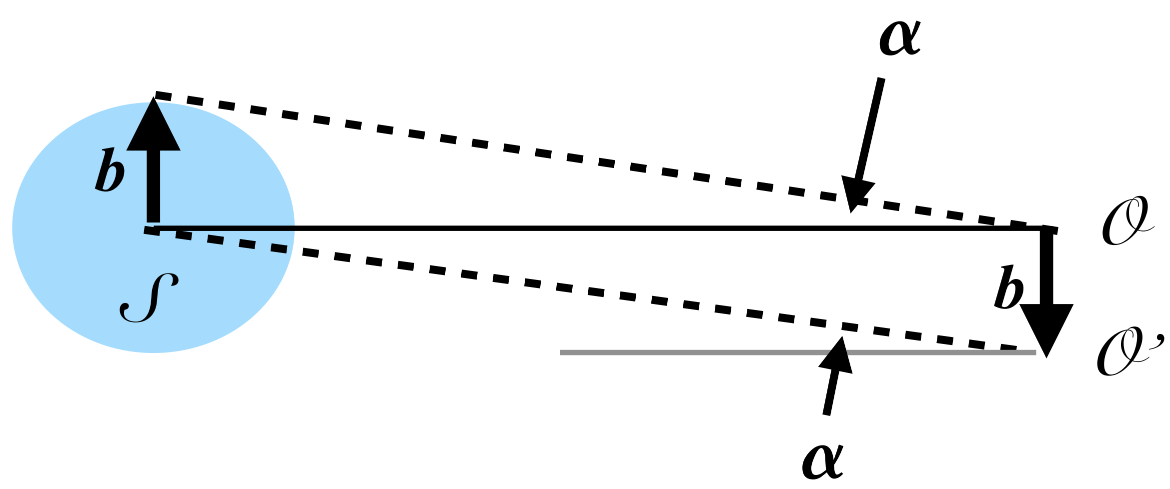
$$D_{\text{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \Longrightarrow D_{\text{par}} > D_{\text{ang}}$$

- Objects appear further away when the distance is measured by parallax than by the angular size!

Hand-waving explanation

- Flat spacetime, no matter: measurements of distance to an object by parallax (D_{par}) and by angular size (D_{ang}) must give the same result

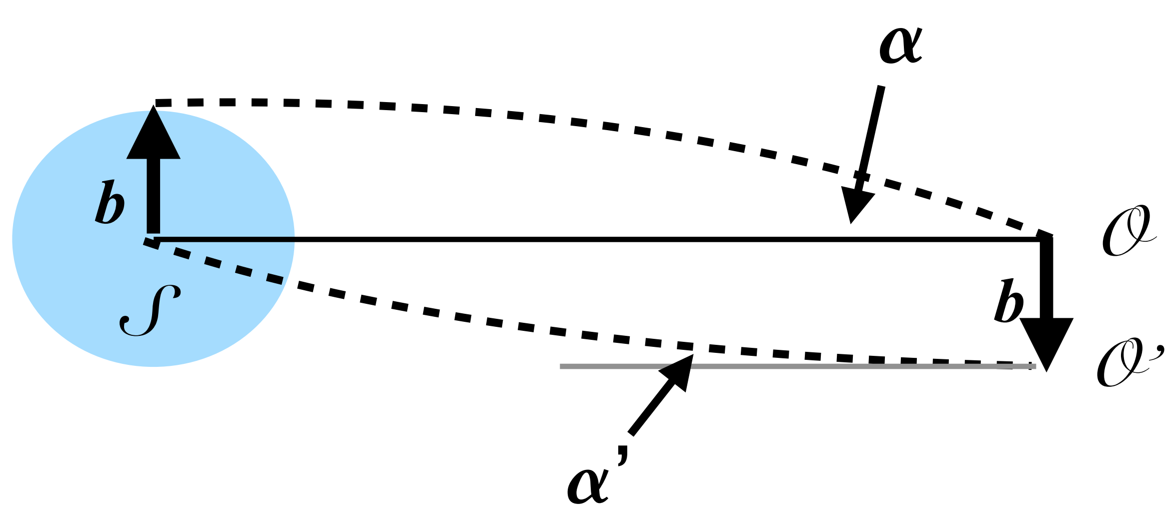


$$D_{\text{ang}} = \frac{b}{\alpha}$$

$$D_{\text{par}} = \frac{b}{\alpha}$$

$$D_{\text{par}} = D_{\text{ang}}$$

- Matter present along the line of sight ⇒ gravitational light bending. Both distance measurements affected, but affected *differently*!



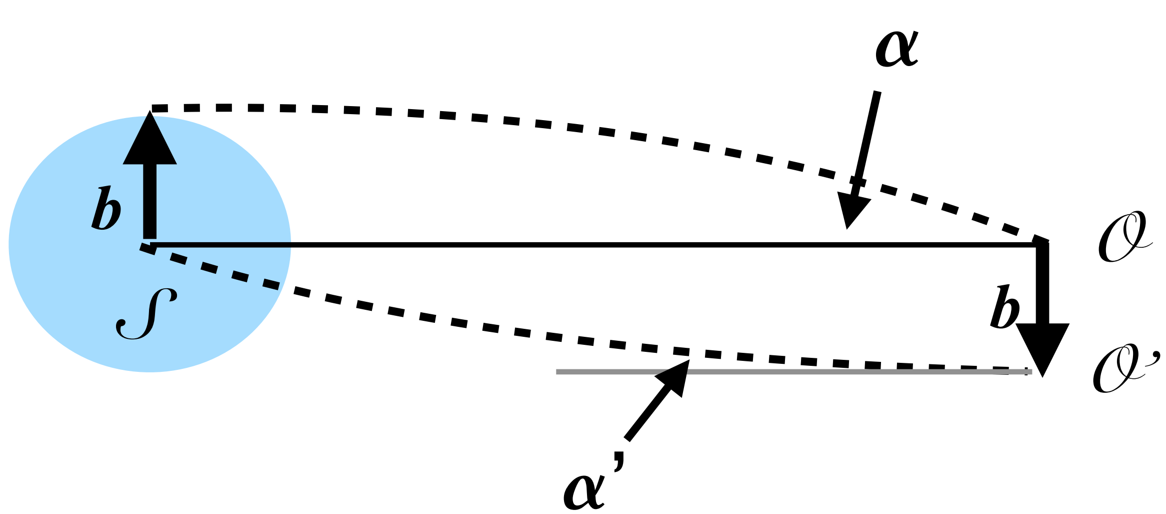
$$D_{\text{ang}} = \frac{b}{\alpha}$$

$$D_{\text{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \Longrightarrow D_{\text{par}} > D_{\text{ang}}$$

- Objects appear further away when the distance is measured by parallax than by the angular size!
- **Claim:** the difference measures the amount of matter along the LOS between $\mathcal{S}$ and $\mathcal{O}$

Hand-waving explanation

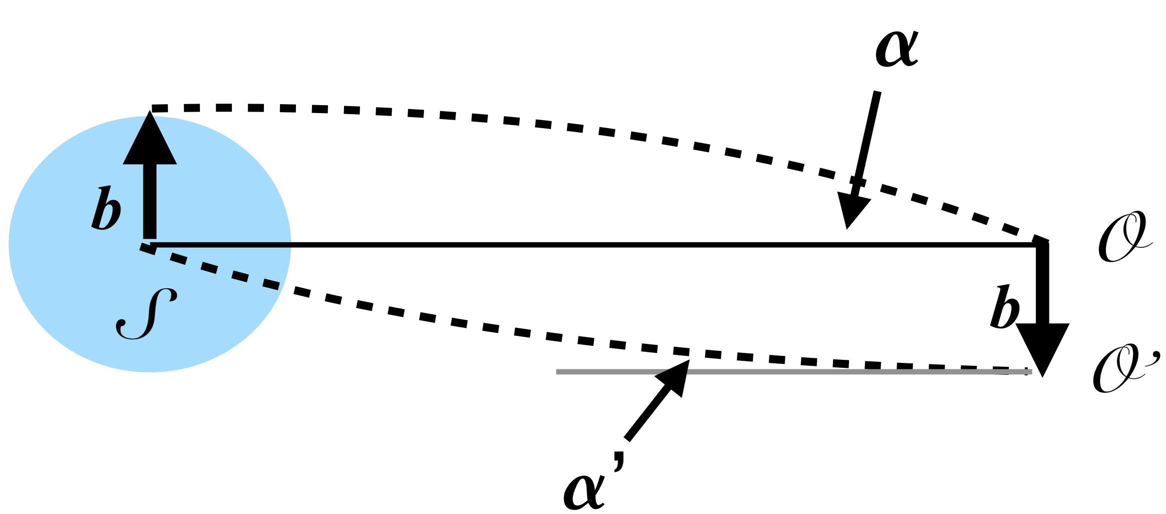


$$D_{\mathsf{ang}} = \frac{b}{\alpha}$$

$$D_{\mathsf{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \implies D_{\mathsf{par}} > D_{\mathsf{ang}}$$

Hand-waving explanation



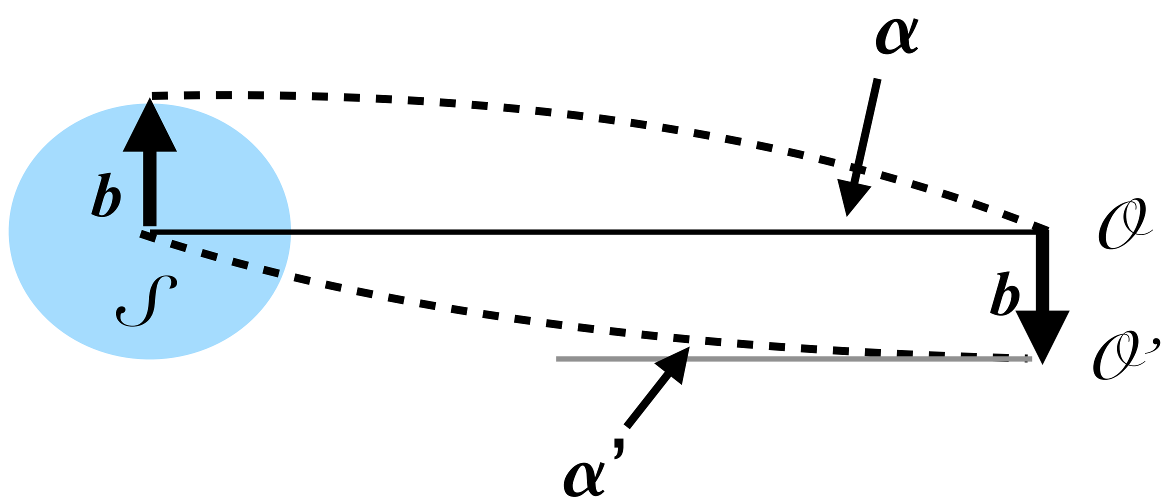
$$D_{\textbf{ang}} = \frac{b}{\alpha}$$

$$D_{\textbf{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \Longrightarrow D_{\textbf{par}} > D_{\textbf{ang}}$$

- What about light bending by matter *off* the line of sight?

Hand-waving explanation



$$D_{\textbf{ang}} = \frac{b}{\alpha}$$

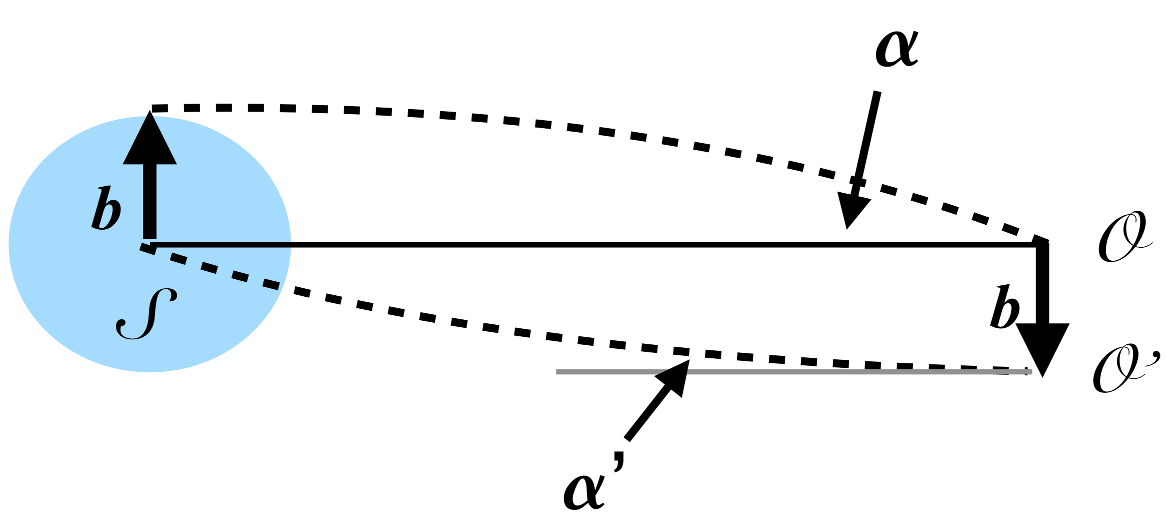
$$D_{\textbf{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \Longrightarrow D_{\textbf{par}} > D_{\textbf{ang}}$$

- What about light bending by matter *off* the line of sight?

- What about the motions of the source and the observer and the special relativistic effects (time dilation, stellar aberration…)?

Hand-waving explanation



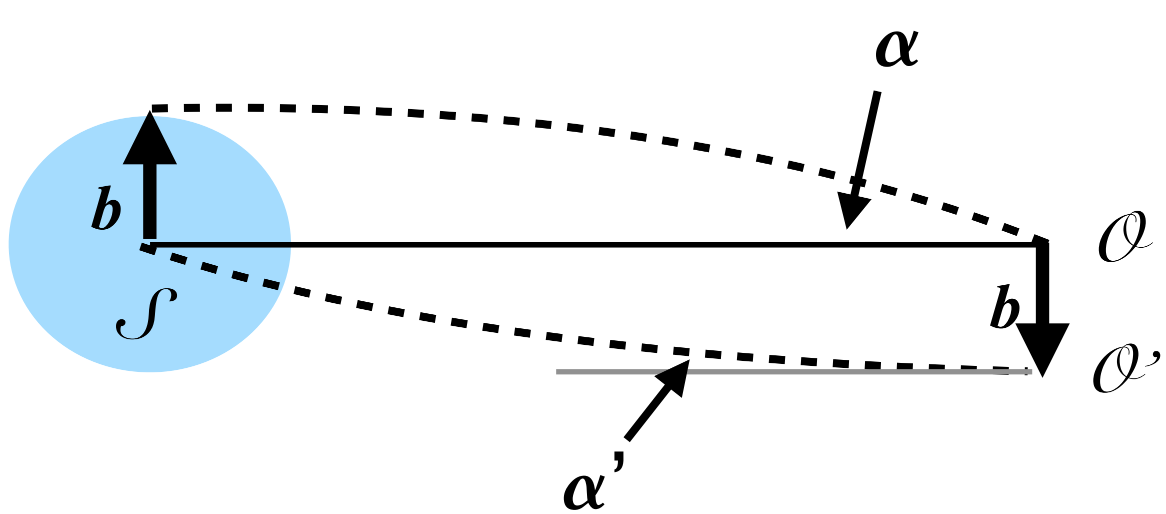
$$D_{\mathbf{ang}} = \frac{b}{\alpha}$$

$$D_{\mathbf{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \implies D_{\mathbf{par}} > D_{\mathbf{ang}}$$

- What about light bending by matter *off* the line of sight?
- What about the motions of the source and the observer and the special relativistic effects (time dilation, stellar aberration…)?
- Directional dependence of the effect: need to consider 2D angles and displacements

Hand-waving explanation



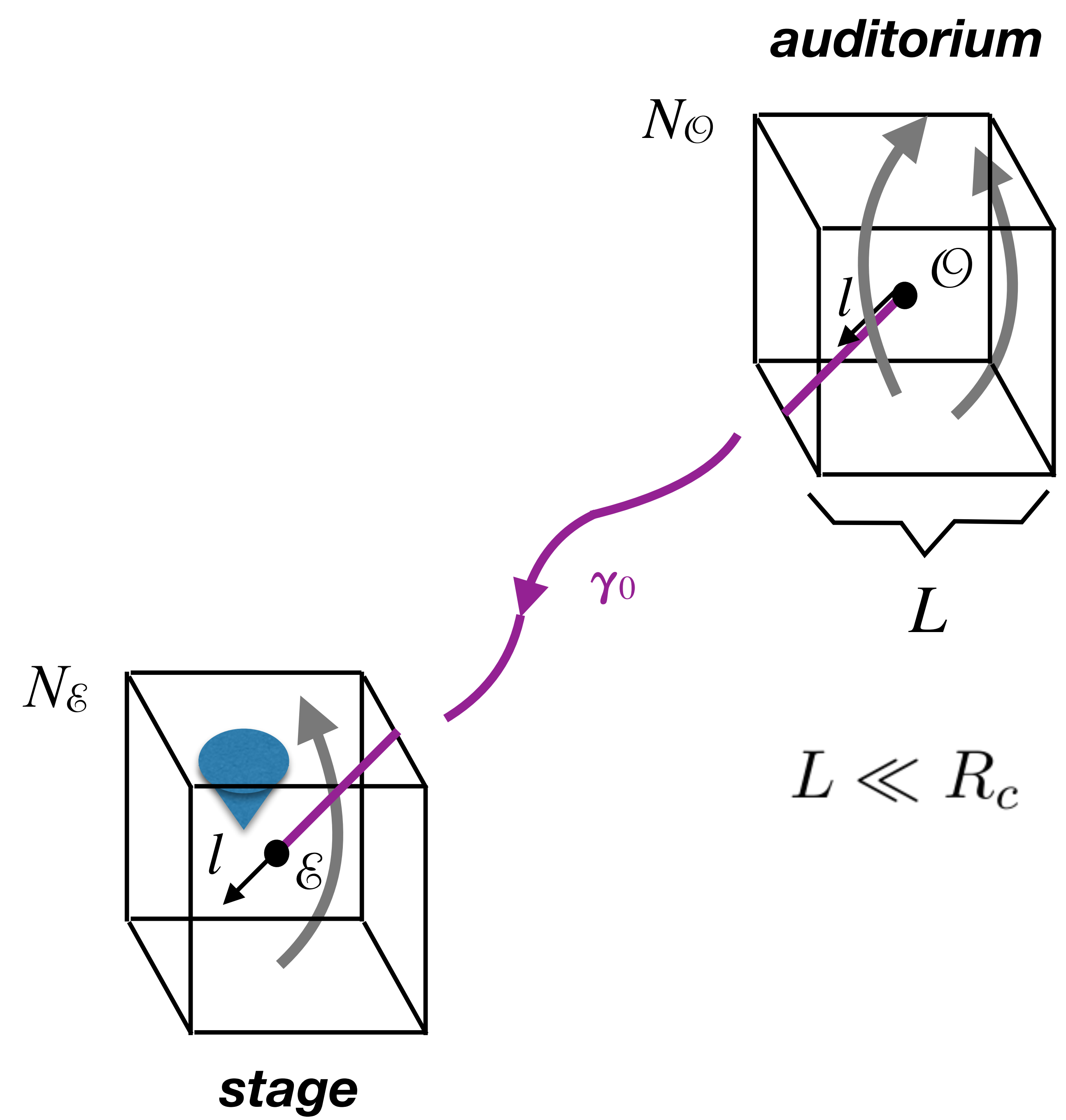
$$D_{\textbf{ang}} = \frac{b}{\alpha}$$

$$D_{\textbf{par}} = \frac{b}{\alpha'}$$

$$\alpha' < \alpha \Longrightarrow D_{\textbf{par}} > D_{\textbf{ang}}$$

- What about light bending by matter *off* the line of sight?
- What about the motions of the source and the observer and the special relativistic effects (time dilation, stellar aberration…)?
- Directional dependence of the effect: need to consider 2D angles and displacements
- ***Need of a fully relativistic theory! (all GR and SR effects)***

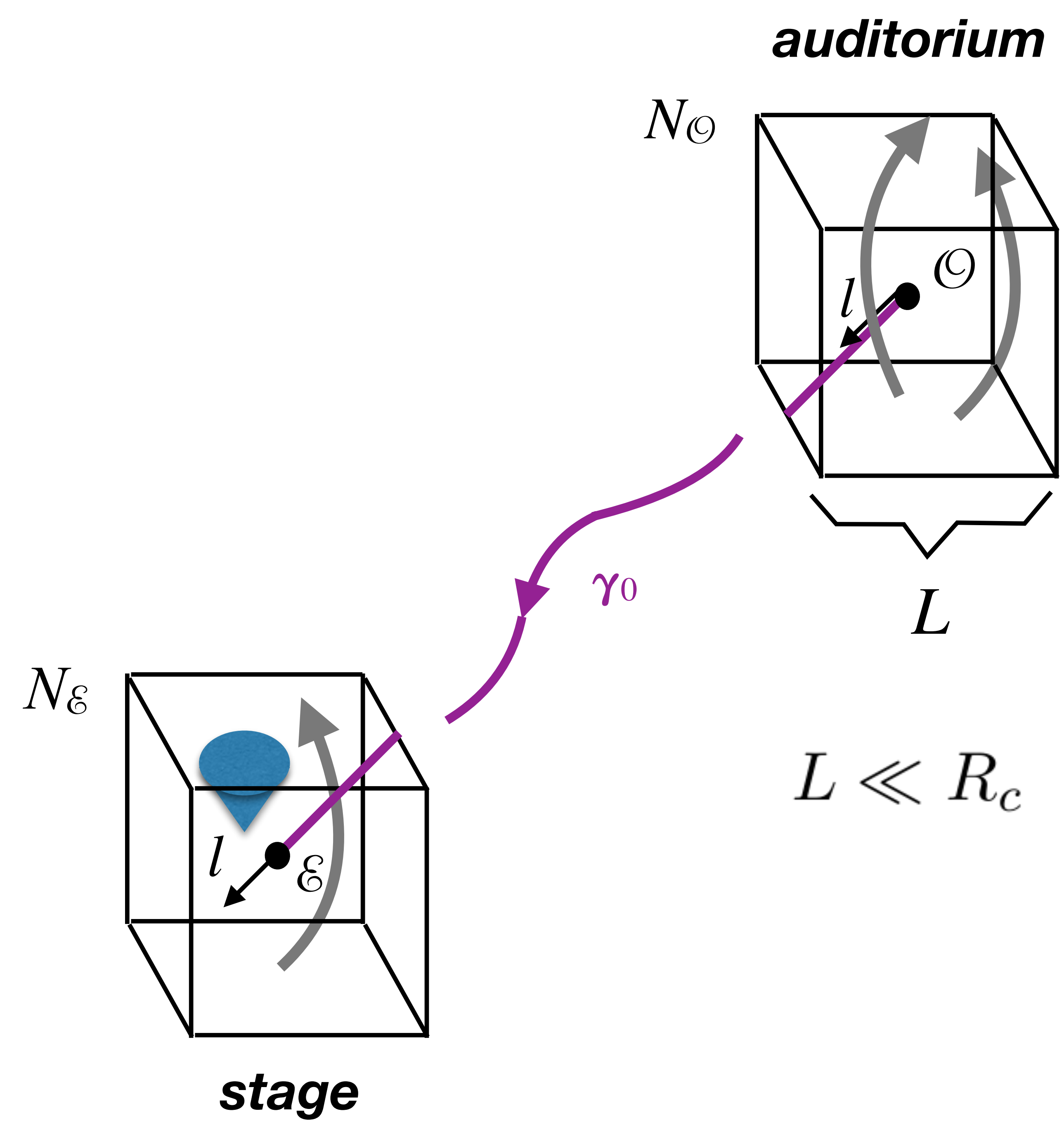
Problem



auditorium
$N_{\mathcal{O}}$
$\mathcal{O}$
l
L
γ_0
$N_{\mathcal{E}}$
l
$\mathcal{E}$
stage
$L \ll R_c$

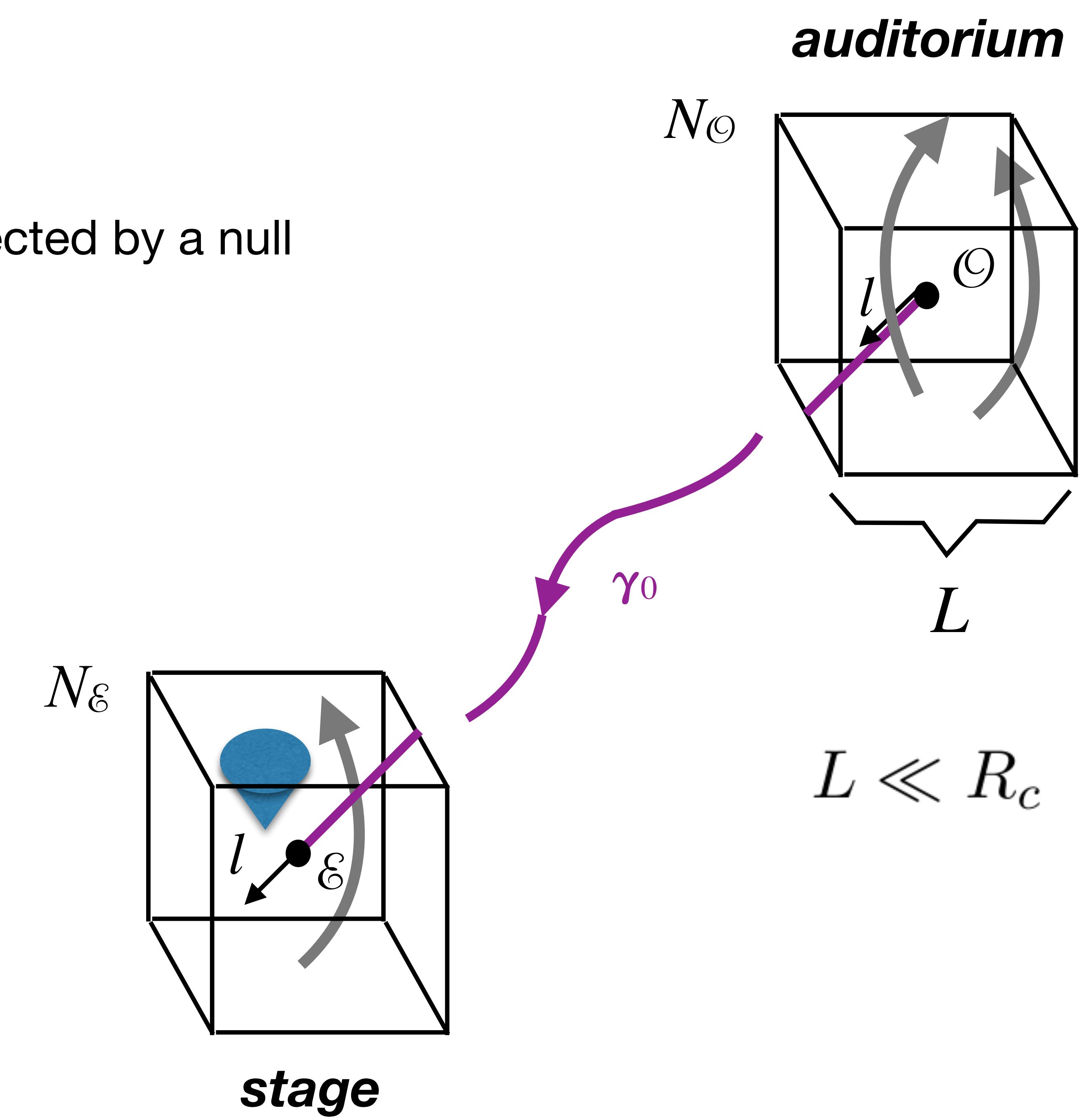
Problem

- Spacetime with *any* Lorentzian metric



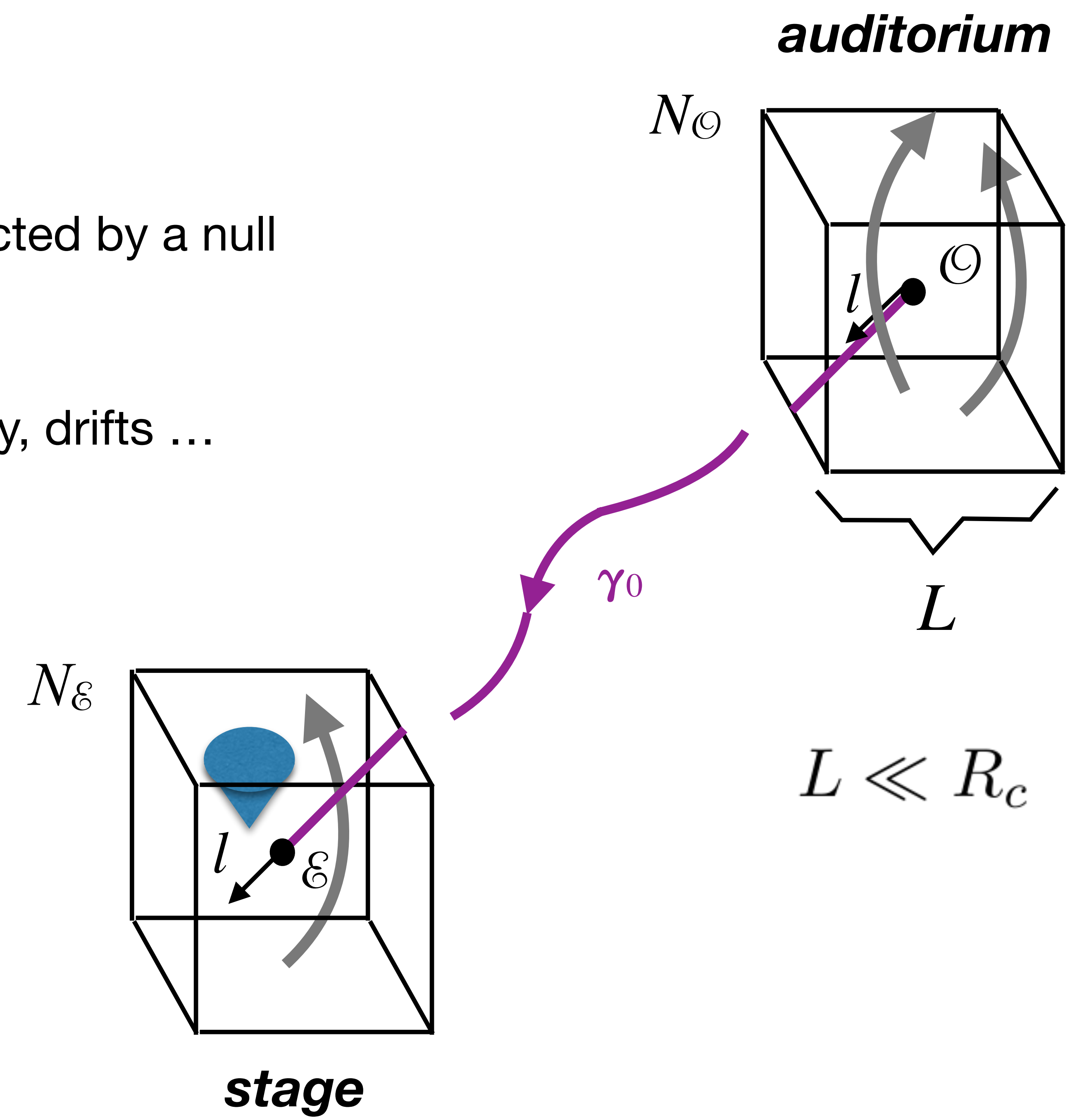
Problem

- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics



Problem

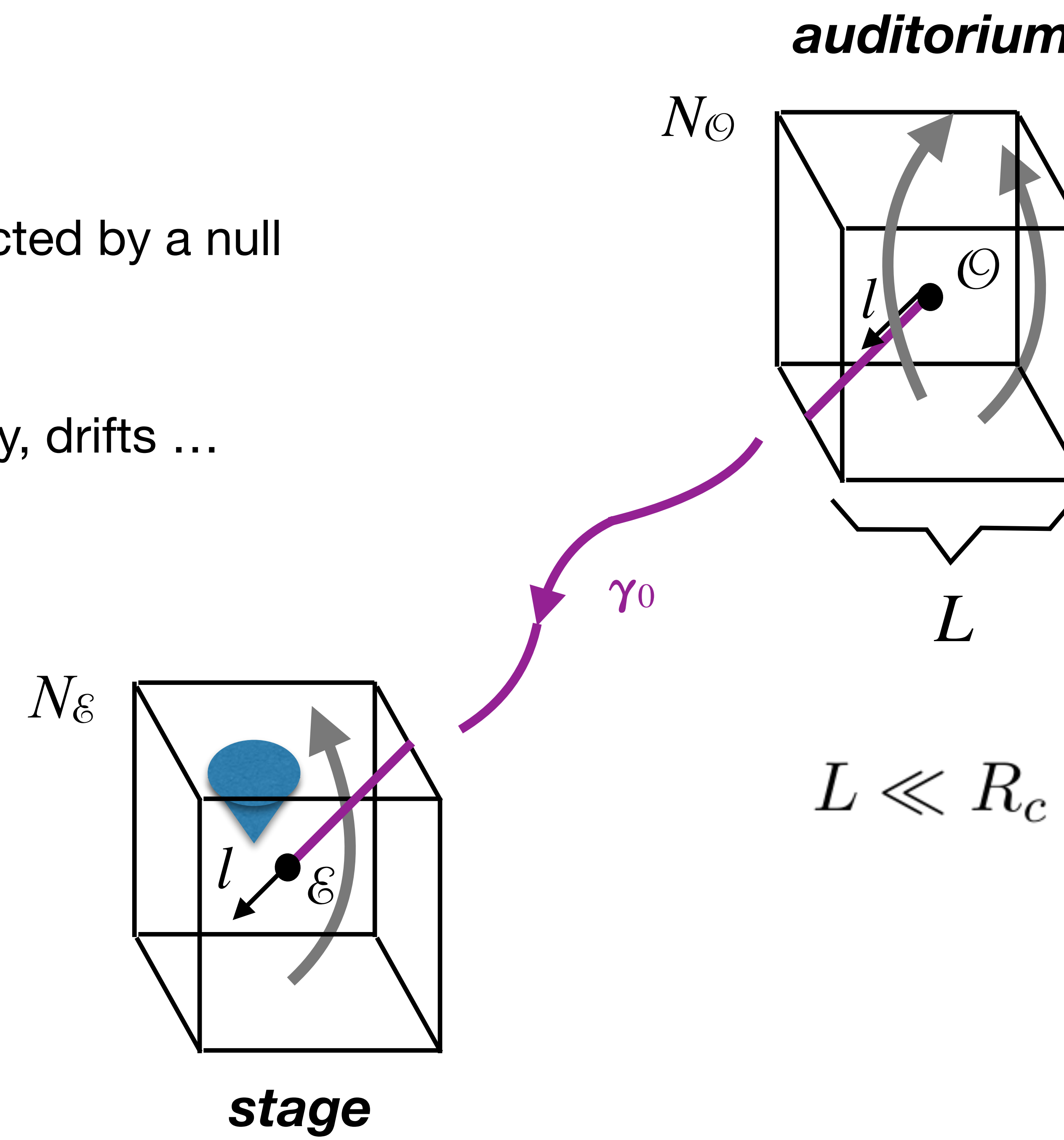
- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics
- Observers measure the TOA, position on the sky, drifts …



$L \ll R_c$

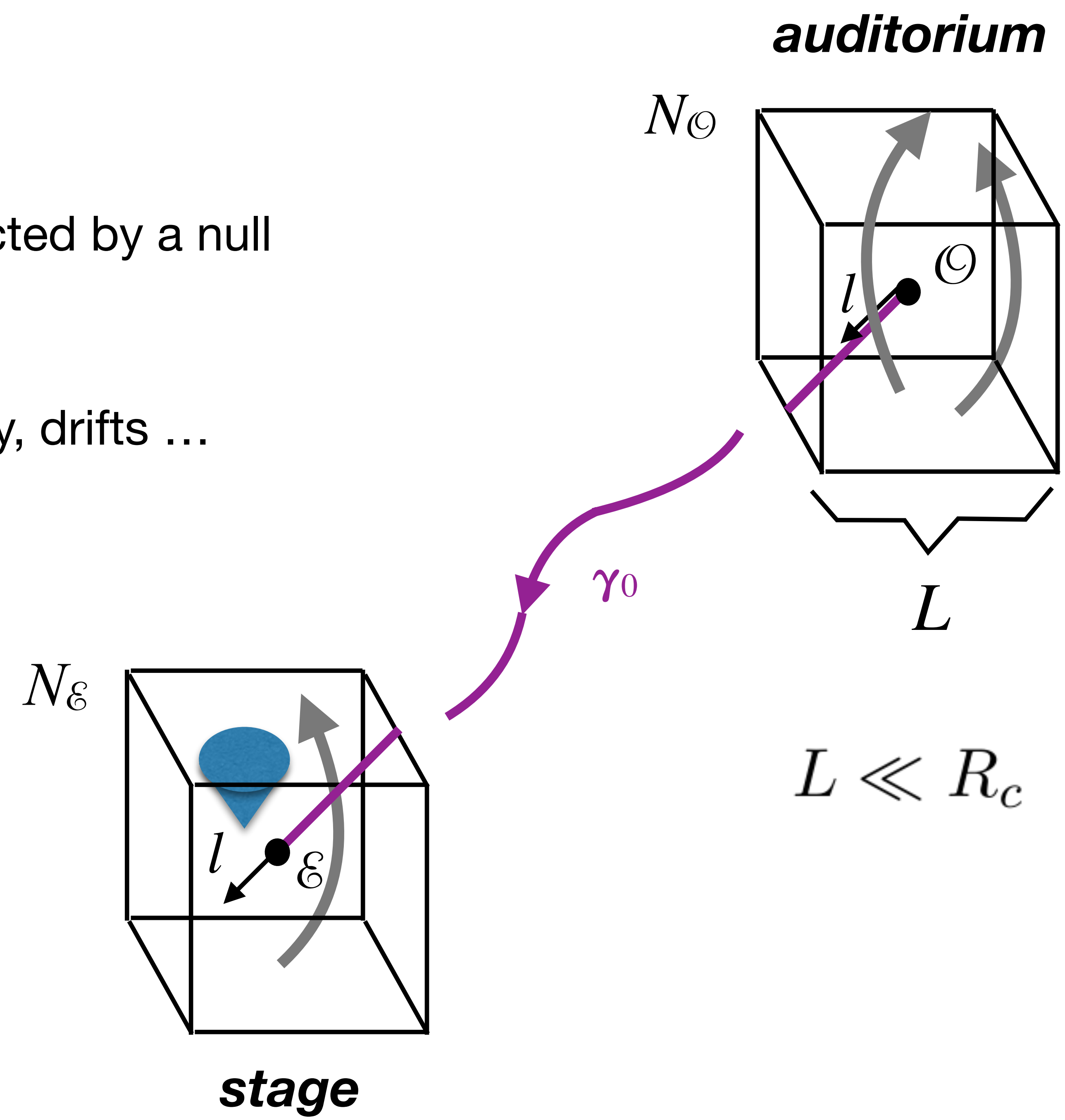
Problem

- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics
- Observers measure the TOA, position on the sky, drifts …
- Two types of effects:



Problem

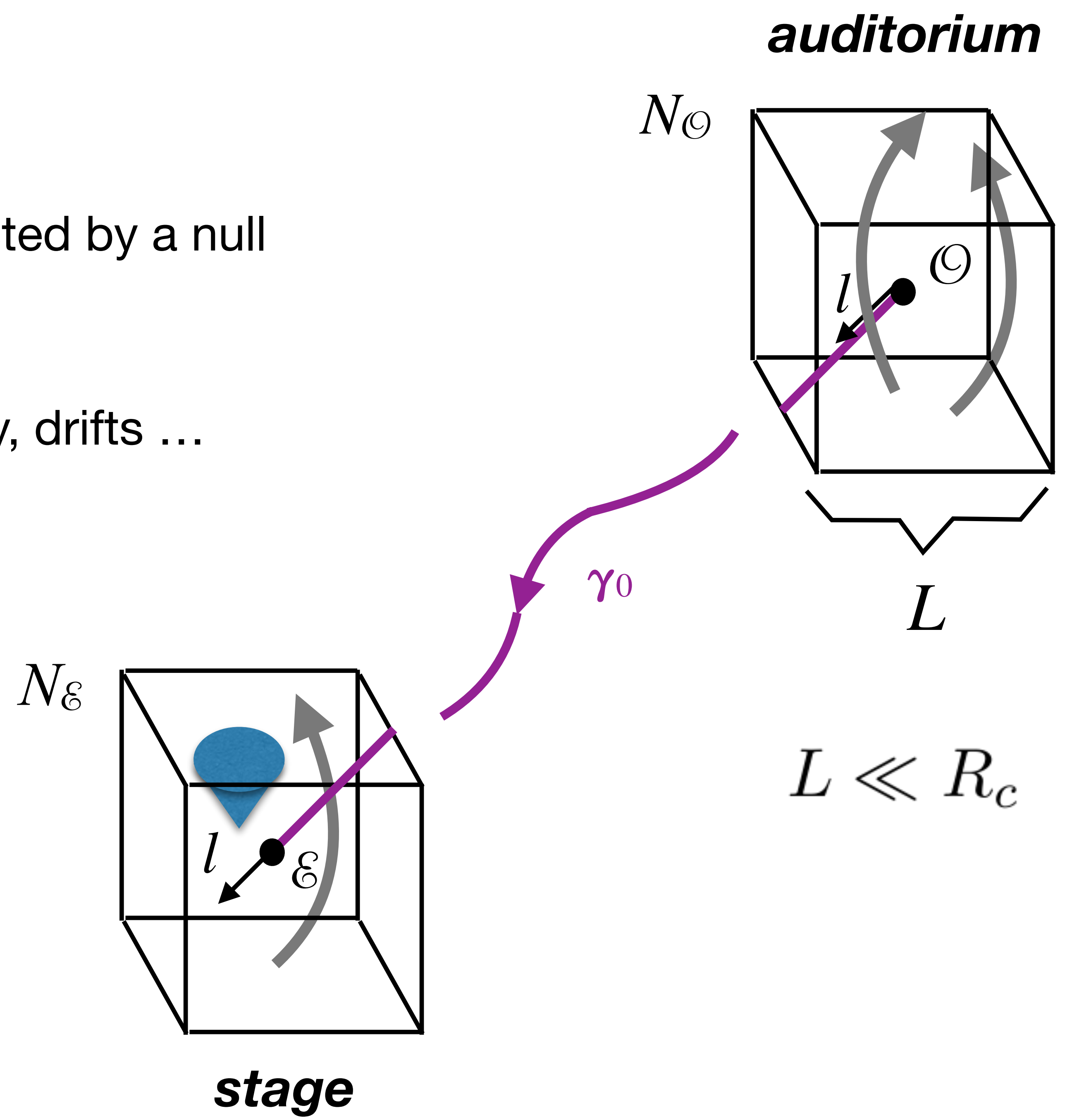
- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics
- Observers measure the TOA, position on the sky, drifts …
- Two types of effects:
  - GR light propagation effects: 1st order GDE (linearisation in the transverse coordinates)



$L \ll R_c$

Problem

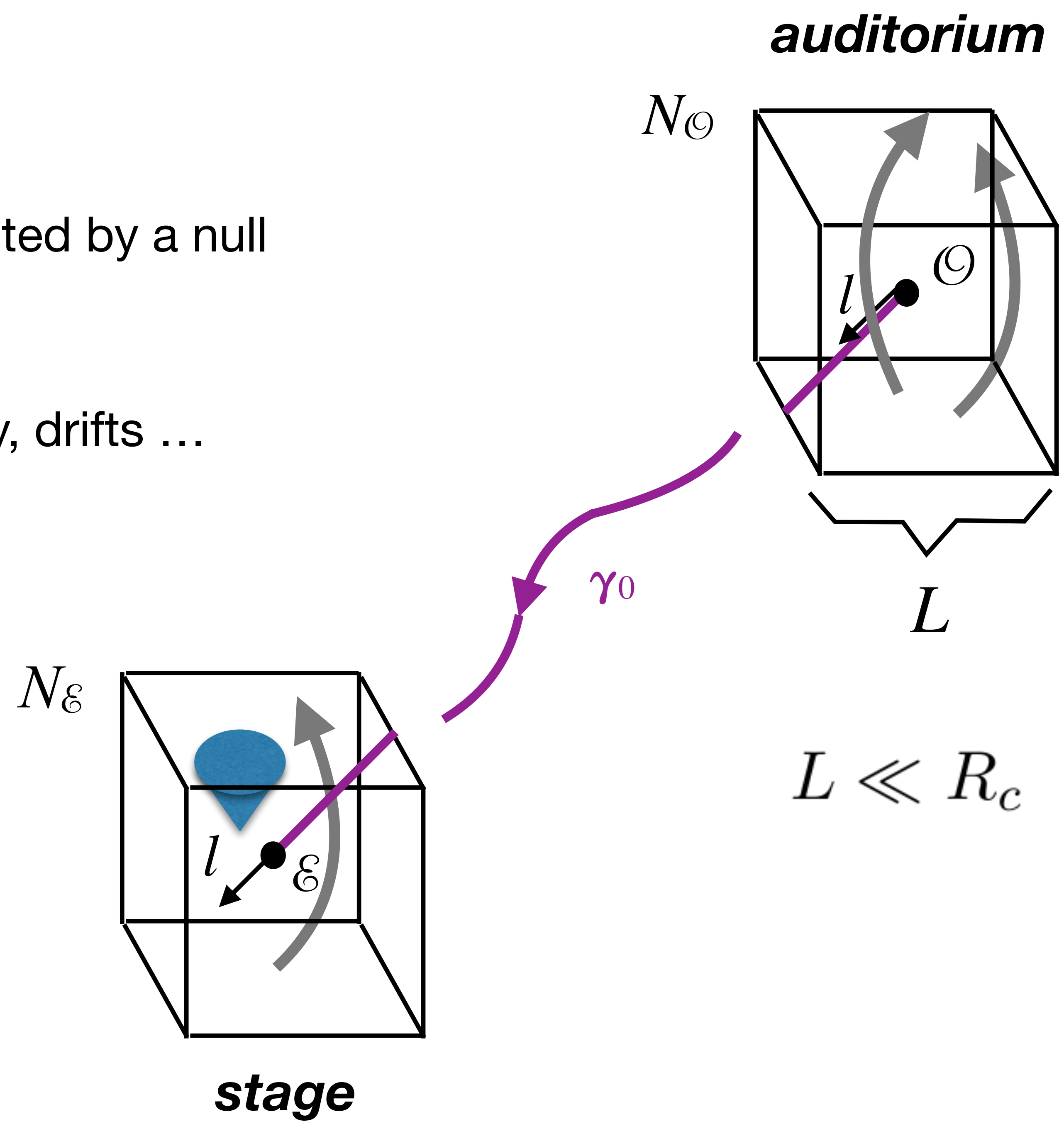
- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics
- Observers measure the TOA, position on the sky, drifts …
- Two types of effects:
  - GR light propagation effects: 1st order GDE (linearisation in the transverse coordinates)
  - SR effects (aberration, time dilation, Doppler…) given an emitter and observer



Problem

- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics
- Observers measure the TOA, position on the sky, drifts …
- Two types of effects:
  - GR light propagation effects: 1st order GDE (linearisation in the transverse coordinates)
  - SR effects (aberration, time dilation, Doppler…) given an emitter and observer

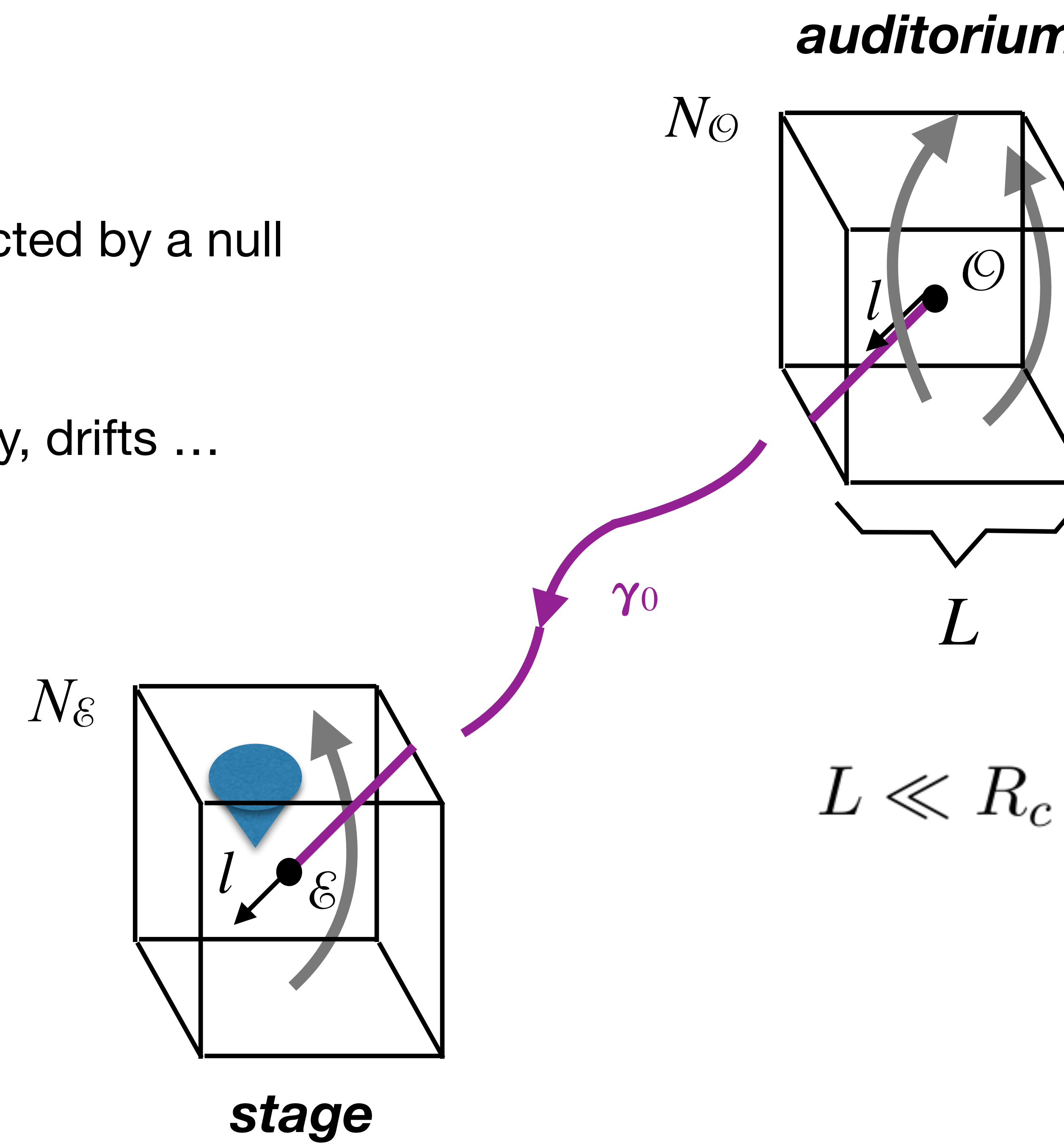
Separate the dependence in the expressions



Problem

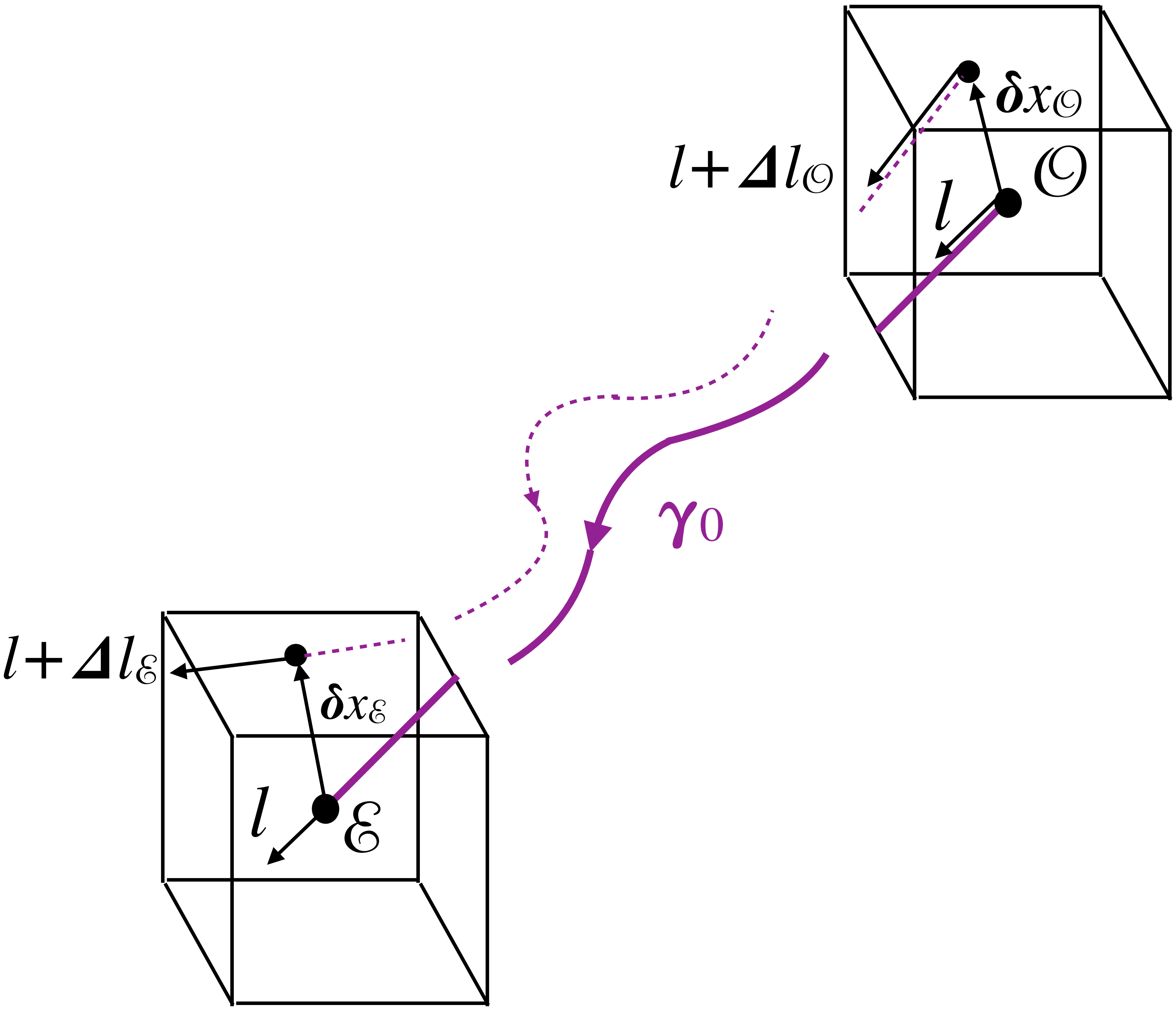
- Spacetime with *any* Lorentzian metric
- Two distant, small regions of spacetime, connected by a null geodesics
- Observers measure the TOA, position on the sky, drifts …
- Two types of effects:
  - GR light propagation effects: 1st order GDE (linearisation in the transverse coordinates)
  - SR effects (aberration, time dilation, Doppler…) given an emitter and observer

Separate the dependence in the expressions



M. Grasso, MK, J. Serbenta, *Geometric optics in general relativity using bilocal operators,* Phys. Rev. **D 99**, 064038 (2019) (Editors' suggestion)

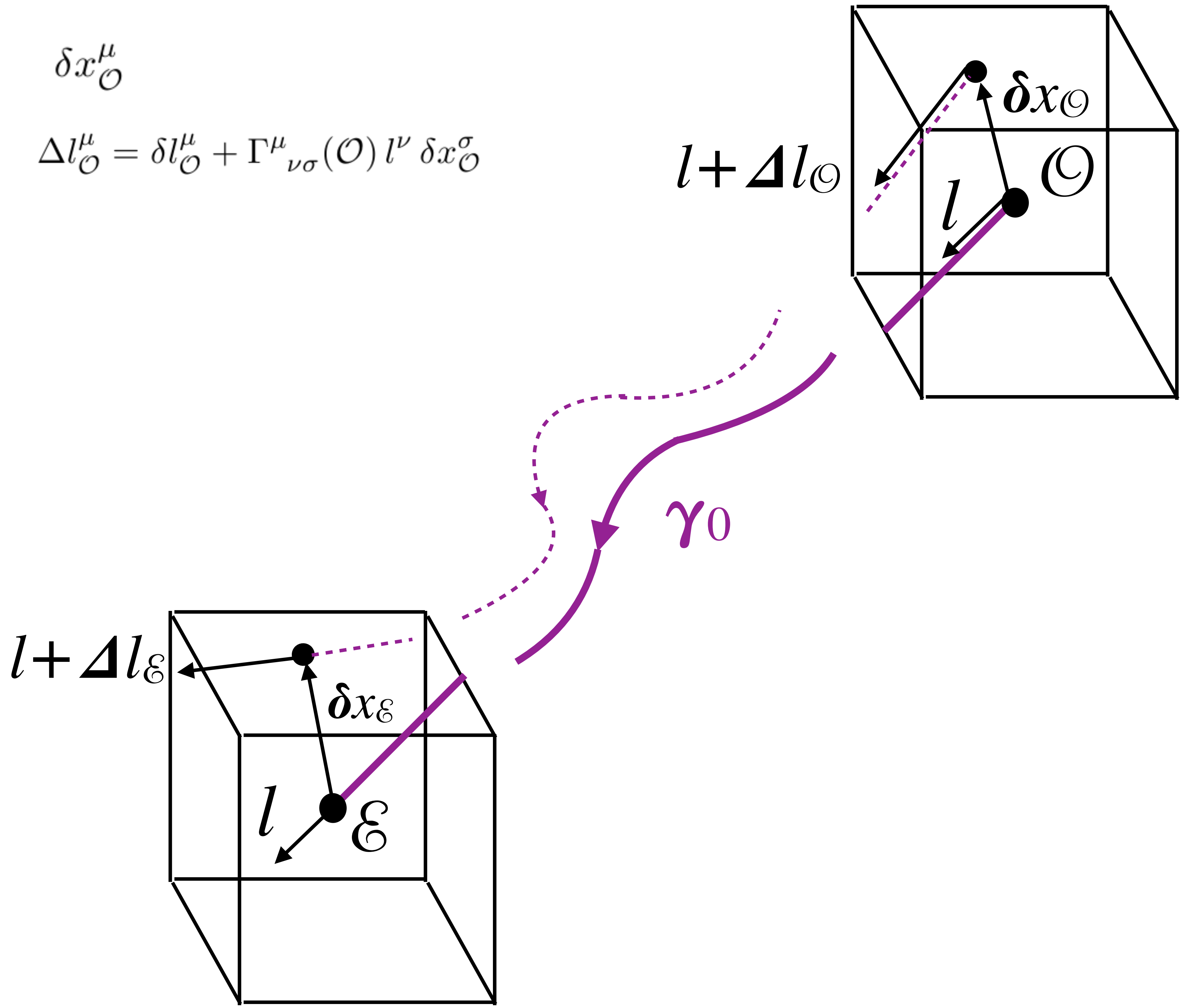
Light propagation effects



Light propagation effects

$\delta x^\mu_\mathcal{O}$

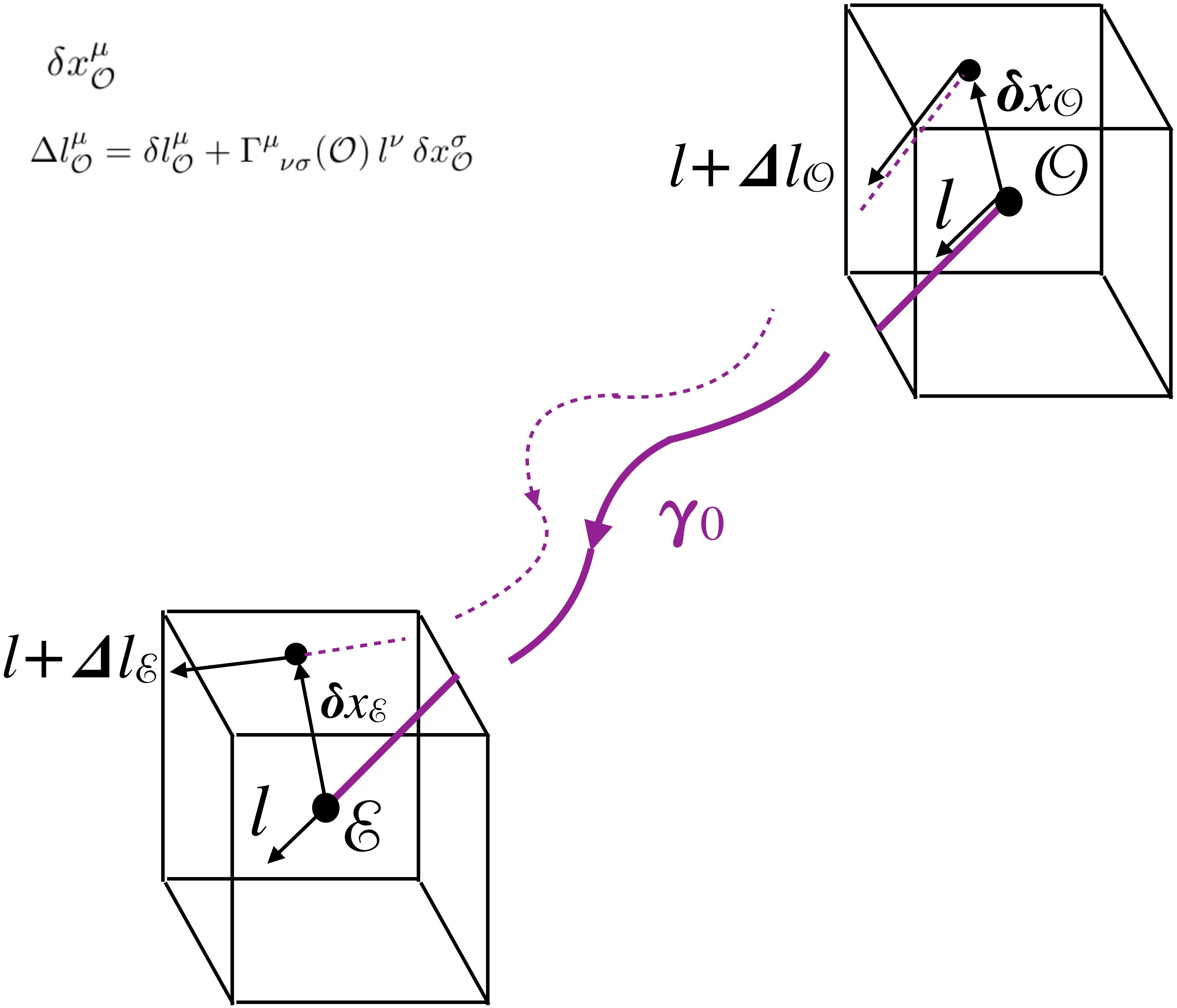
$\Delta l^\mu_\mathcal{O} = \delta l^\mu_\mathcal{O} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_\mathcal{O}$



Light propagation effects

$\delta x^\mu_\mathcal{O}$

$\Delta l^\mu_\mathcal{O} = \delta l^\mu_\mathcal{O} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_\mathcal{O}$



$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\alpha\beta\nu}\, l^\alpha\, l^\beta\, \xi^\nu = 0$

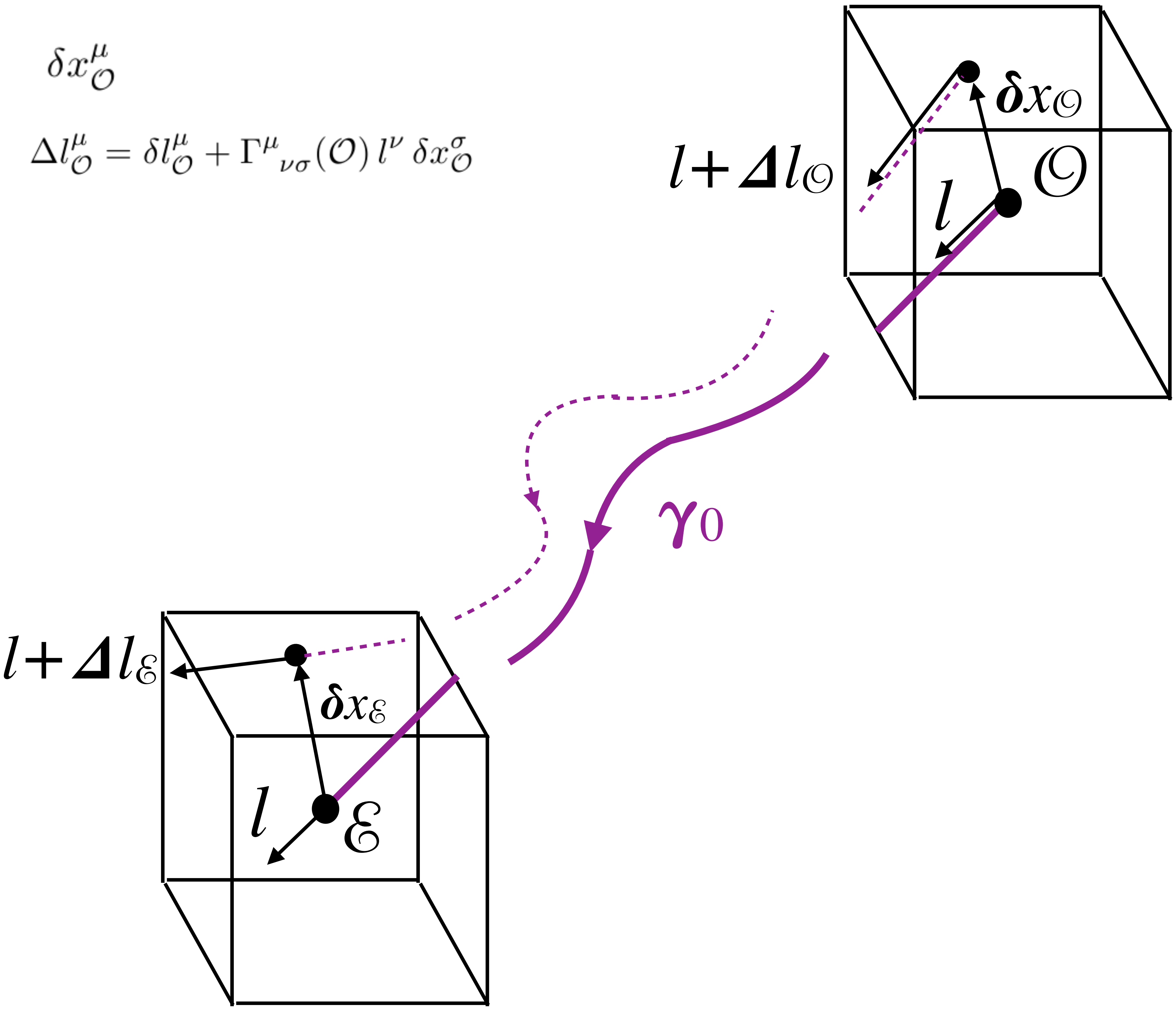
$\xi^\mu(\lambda_\mathcal{O}) = \delta x^\mu_\mathcal{O}$

$\nabla_l \xi^\mu(\lambda_\mathcal{O}) = \Delta l^\mu_\mathcal{O}$

Light propagation effects

$$\delta x^\mu_\mathcal{O}$$

$$\Delta l^\mu_\mathcal{O} = \delta l^\mu_\mathcal{O} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_\mathcal{O}$$



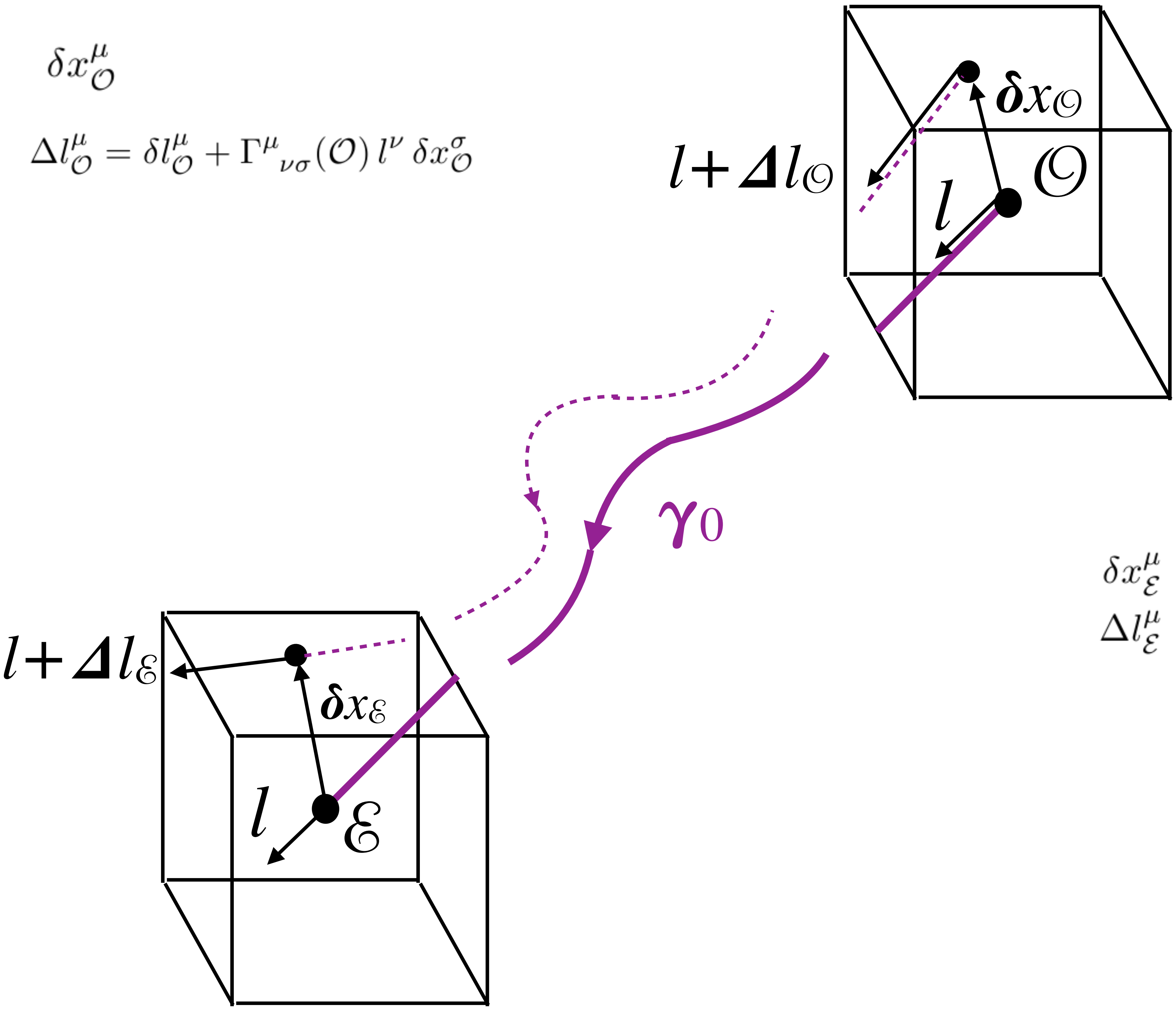
$$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\alpha\beta\nu}\, l^\alpha\, l^\beta\, \xi^\nu = 0$$
$$\xi^\mu(\lambda_\mathcal{O}) = \delta x^\mu_\mathcal{O}$$
$$\nabla_l \xi^\mu(\lambda_\mathcal{O}) = \Delta l^\mu_\mathcal{O}$$

$$\delta x^\mu_\mathcal{E} = \xi^\mu(\lambda_\mathcal{E})$$
$$\Delta l^\mu_\mathcal{E} = \nabla_l \xi^\mu(\lambda_\mathcal{E})$$

Light propagation effects

$\delta x^{\mu}_{\mathcal{O}}$

$$\Delta l^{\mu}_{\mathcal{O}} = \delta l^{\mu}_{\mathcal{O}} + \Gamma^{\mu}{}_{\nu\sigma}(\mathcal{O})\, l^{\nu}\, \delta x^{\sigma}_{\mathcal{O}}$$



$$\nabla_l \nabla_l \xi^{\mu} - R^{\mu}{}_{\alpha\beta\nu}\, l^{\alpha}\, l^{\beta}\, \xi^{\nu} = 0$$
$$\xi^{\mu}(\lambda_{\mathcal{O}}) = \delta x^{\mu}_{\mathcal{O}}$$
$$\nabla_l \xi^{\mu}(\lambda_{\mathcal{O}}) = \Delta l^{\mu}_{\mathcal{O}}$$

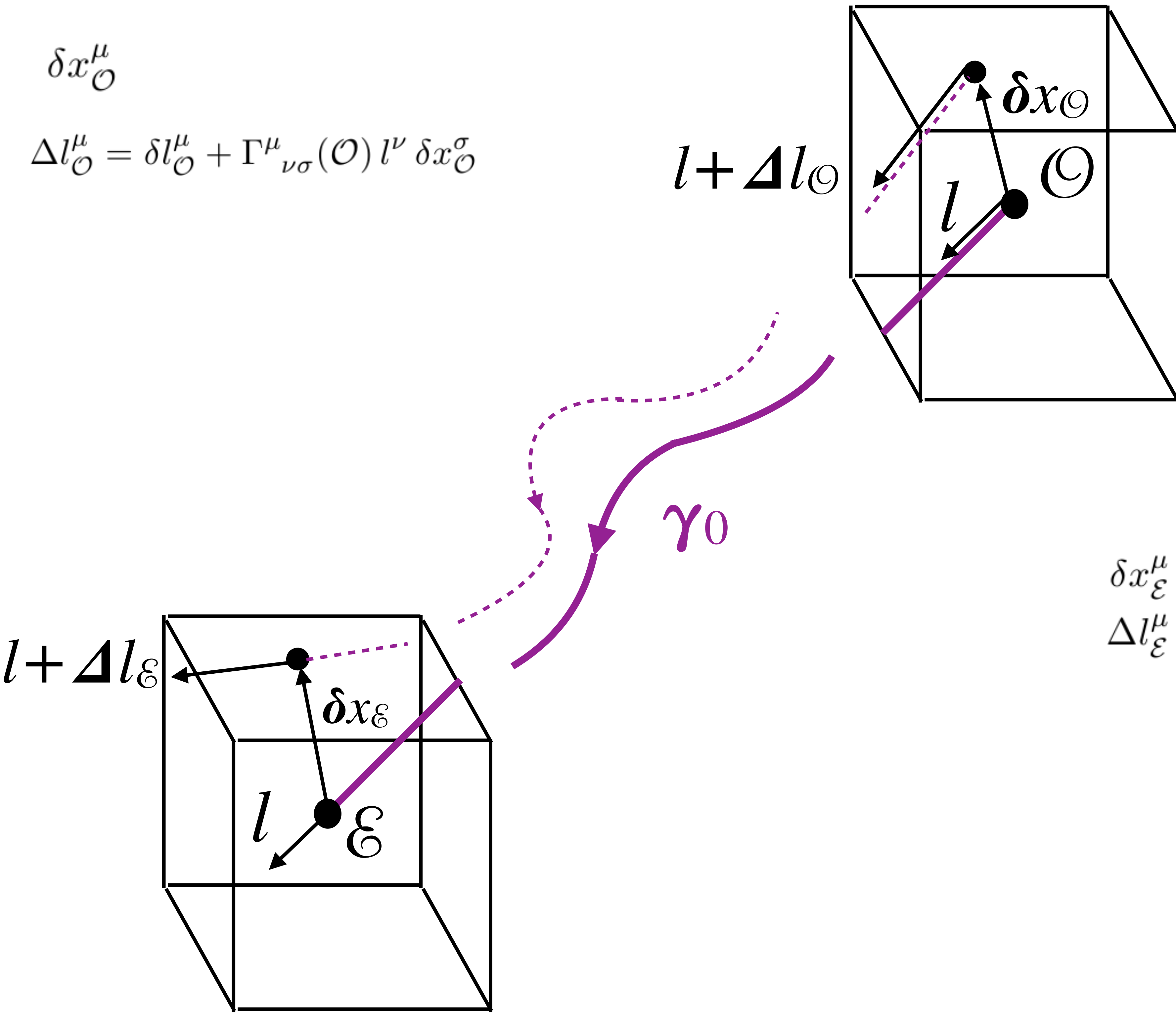
$$\delta x^{\mu}_{\mathcal{E}} = \xi^{\mu}(\lambda_{\mathcal{E}})$$
$$\Delta l^{\mu}_{\mathcal{E}} = \nabla_l \xi^{\mu}(\lambda_{\mathcal{E}})$$

$$\begin{aligned} \delta x^{\mu}_{\mathcal{E}} &= W_{xx}{}^{\mu}{}_{\nu}\, \delta x^{\nu}_{\mathcal{O}} + W_{xl}{}^{\mu}{}_{\nu}\, \Delta l^{\nu}_{\mathcal{O}} \\ \Delta l^{\mu}_{\mathcal{E}} &= W_{lx}{}^{\mu}{}_{\nu}\, \delta x^{\nu}_{\mathcal{O}} + W_{ll}{}^{\mu}{}_{\nu}\, \Delta l^{\nu}_{\mathcal{O}} \end{aligned}$$

Light propagation effects

$\delta x^\mu_\mathcal{O}$

$\Delta l^\mu_\mathcal{O} = \delta l^\mu_\mathcal{O} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_\mathcal{O}$



$$\nabla_l \nabla_l \xi^\mu - R^\mu{}_{\alpha\beta\nu}\, l^\alpha\, l^\beta\, \xi^\nu = 0$$
$$\xi^\mu(\lambda_\mathcal{O}) = \delta x^\mu_\mathcal{O}$$
$$\nabla_l \xi^\mu(\lambda_\mathcal{O}) = \Delta l^\mu_\mathcal{O}$$

$$\delta x^\mu_\mathcal{E} = \xi^\mu(\lambda_\mathcal{E})$$
$$\Delta l^\mu_\mathcal{E} = \nabla_l \xi^\mu(\lambda_\mathcal{E})$$

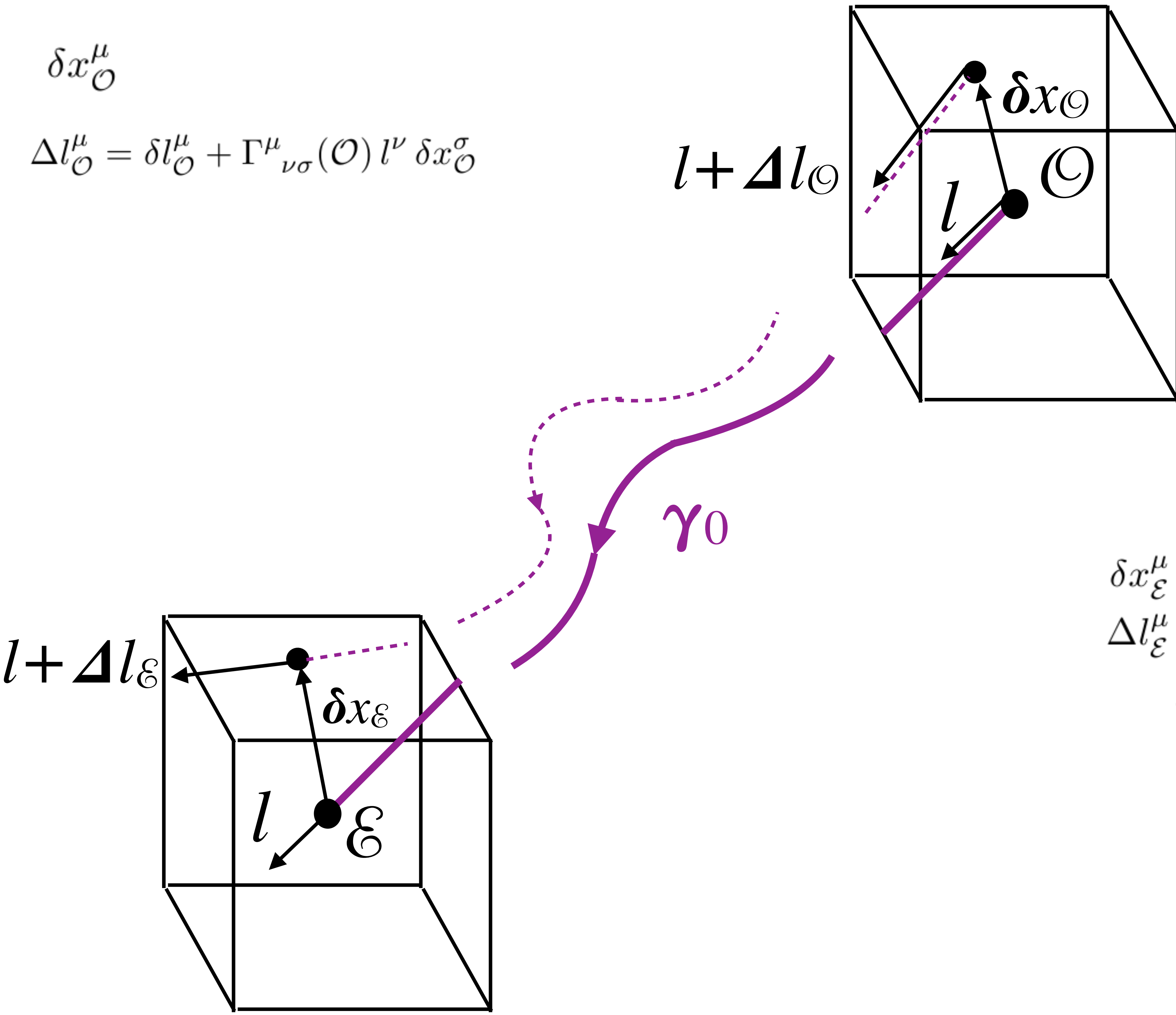
$$\begin{aligned} \delta x^\mu_\mathcal{E} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_\mathcal{O} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_\mathcal{O} \\ \Delta l^\mu_\mathcal{E} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_\mathcal{O} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_\mathcal{O} \end{aligned}$$

$$W_{**} : T_\mathcal{O} M \mapsto T_\mathcal{E} M$$

Light propagation effects

$\delta x^\mu_{\mathcal{O}}$

$\Delta l^\mu_{\mathcal{O}} = \delta l^\mu_{\mathcal{O}} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_{\mathcal{O}}$



$$\ddot{A}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, A^\sigma{}_\nu = 0$$
$$A^\mu{}_\nu(\lambda_{\mathcal{O}}) = \delta^\mu{}_\nu$$
$$\dot{A}^\mu{}_\nu(\lambda_{\mathcal{O}}) = 0$$

$$W_{xx}{}^\mu{}_\nu = A^\mu{}_\nu(\lambda_{\mathcal{E}})$$
$$W_{lx}{}^\mu{}_\nu = \dot{A}^\mu{}_\nu(\lambda_{\mathcal{E}})$$

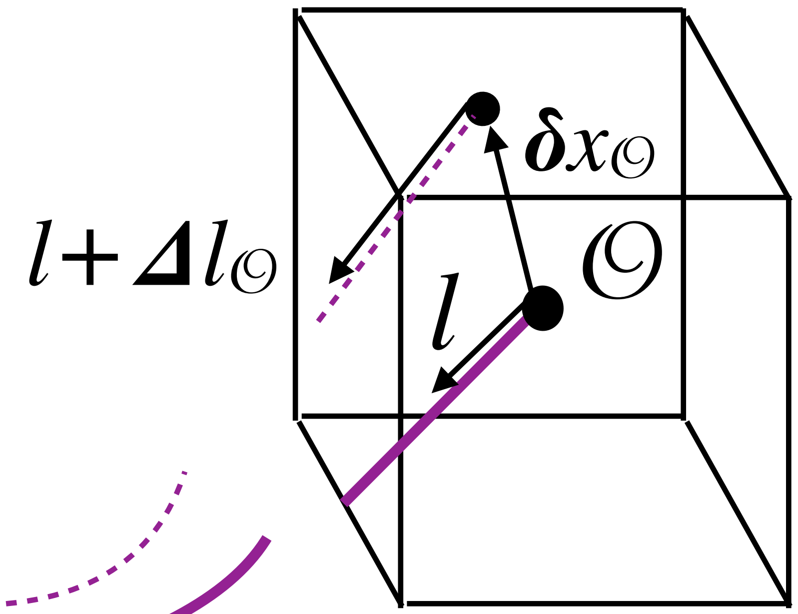
$$\begin{aligned} \delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \end{aligned}$$

$$W_{**} : T_{\mathcal{O}} M \mapsto T_{\mathcal{E}} M$$

Light propagation effects

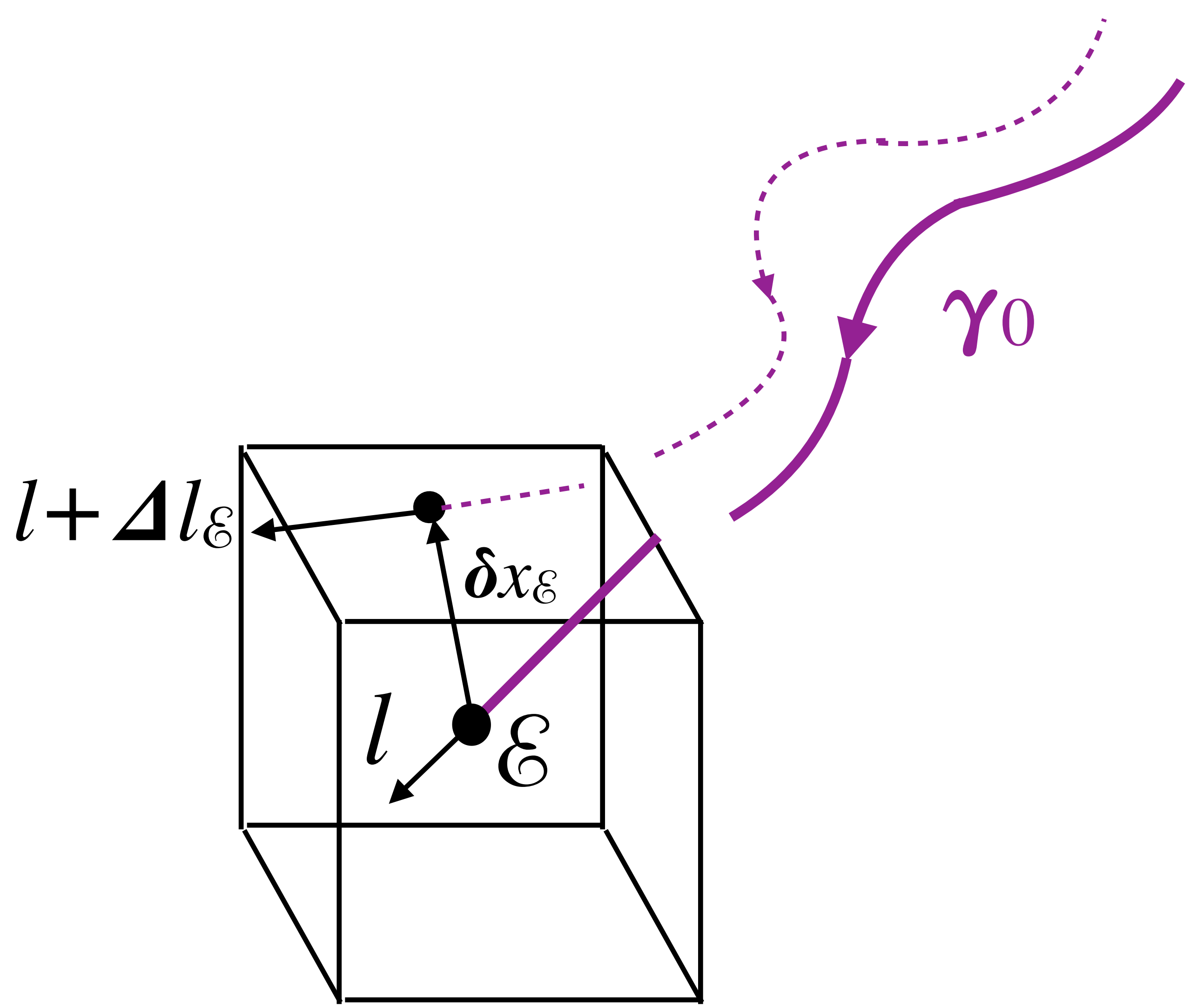
$\delta x^\mu_{\mathcal{O}}$

$\Delta l^\mu_{\mathcal{O}} = \delta l^\mu_{\mathcal{O}} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_{\mathcal{O}}$



$$\ddot{B}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, B^\sigma{}_\nu = 0$$
$$B^\mu{}_\nu(\lambda_{\mathcal{O}}) = 0$$
$$\dot{B}^\mu{}_\nu(\lambda_{\mathcal{O}}) = \delta^\mu{}_\nu$$

$$W_{lx}{}^\mu{}_\nu = B^\mu{}_\nu(\lambda_{\mathcal{E}})$$
$$W_{ll}{}^\mu{}_\nu = \dot{B}^\mu{}_\nu(\lambda_{\mathcal{E}})$$



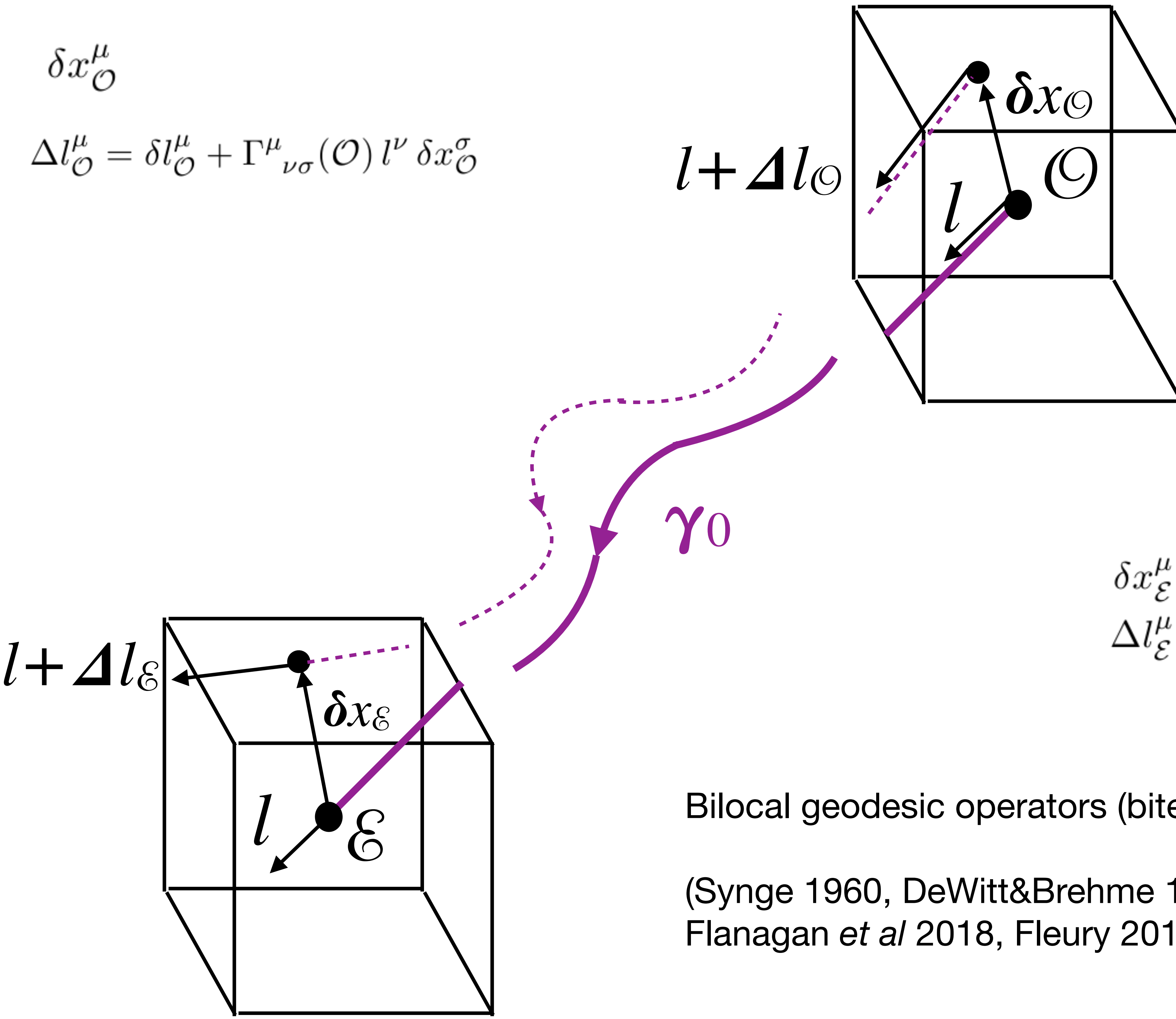
$$\begin{aligned} \delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \end{aligned}$$

$$W_{**} : T_{\mathcal{O}} M \mapsto T_{\mathcal{E}} M$$

Light propagation effects

$\delta x^\mu_\mathcal{O}$

$\Delta l^\mu_\mathcal{O} = \delta l^\mu_\mathcal{O} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_\mathcal{O}$



$$\ddot{B}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, B^\sigma{}_\nu = 0$$
$$B^\mu{}_\nu(\lambda_\mathcal{O}) = 0$$
$$\dot{B}^\mu{}_\nu(\lambda_\mathcal{O}) = \delta^\mu{}_\nu$$

$$W_{lx}{}^\mu{}_\nu = B^\mu{}_\nu(\lambda_\mathcal{E})$$
$$W_{ll}{}^\mu{}_\nu = \dot{B}^\mu{}_\nu(\lambda_\mathcal{E})$$

$$\begin{aligned} \delta x^\mu_\mathcal{E} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_\mathcal{O} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_\mathcal{O} \\ \Delta l^\mu_\mathcal{E} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_\mathcal{O} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_\mathcal{O} \end{aligned}$$

$$W_{**} : T_\mathcal{O}M \mapsto T_\mathcal{E}M$$

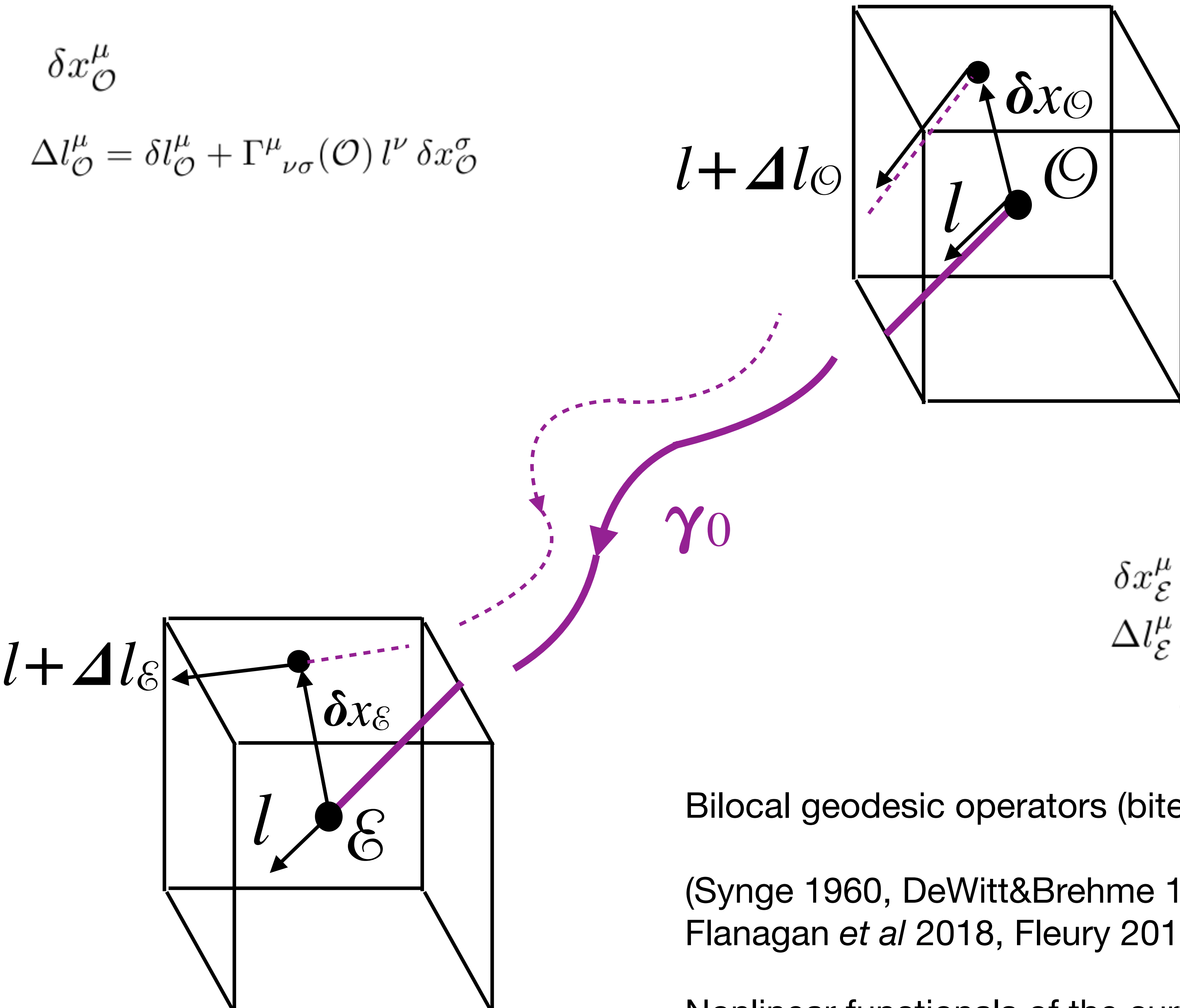
Bilocal geodesic operators (bitensors)

(Synge 1960, DeWitt&Brehme 1960, Dixon 1970, Vines 2015, Flanagan *et al* 2018, Fleury 2014, Uzun 2018…)

Light propagation effects

$\delta x^\mu_{\mathcal{O}}$

$\Delta l^\mu_{\mathcal{O}} = \delta l^\mu_{\mathcal{O}} + \Gamma^\mu{}_{\nu\sigma}(\mathcal{O})\, l^\nu\, \delta x^\sigma_{\mathcal{O}}$



$$\ddot{B}^\mu{}_\nu - R^\mu{}_{\alpha\beta\sigma}\, l^\alpha\, l^\beta\, B^\sigma{}_\nu = 0$$
$$B^\mu{}_\nu(\lambda_{\mathcal{O}}) = 0$$
$$\dot{B}^\mu{}_\nu(\lambda_{\mathcal{O}}) = \delta^\mu{}_\nu$$

$$W_{lx}{}^\mu{}_\nu = B^\mu{}_\nu(\lambda_{\mathcal{E}})$$
$$W_{ll}{}^\mu{}_\nu = \dot{B}^\mu{}_\nu(\lambda_{\mathcal{E}})$$

$$\begin{aligned} \delta x^\mu_{\mathcal{E}} &= W_{xx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{xl}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \\ \Delta l^\mu_{\mathcal{E}} &= W_{lx}{}^\mu{}_\nu\, \delta x^\nu_{\mathcal{O}} + W_{ll}{}^\mu{}_\nu\, \Delta l^\nu_{\mathcal{O}} \end{aligned}$$

$$W_{**} : T_{\mathcal{O}}M \mapsto T_{\mathcal{E}}M$$

Bilocal geodesic operators (bitensors)

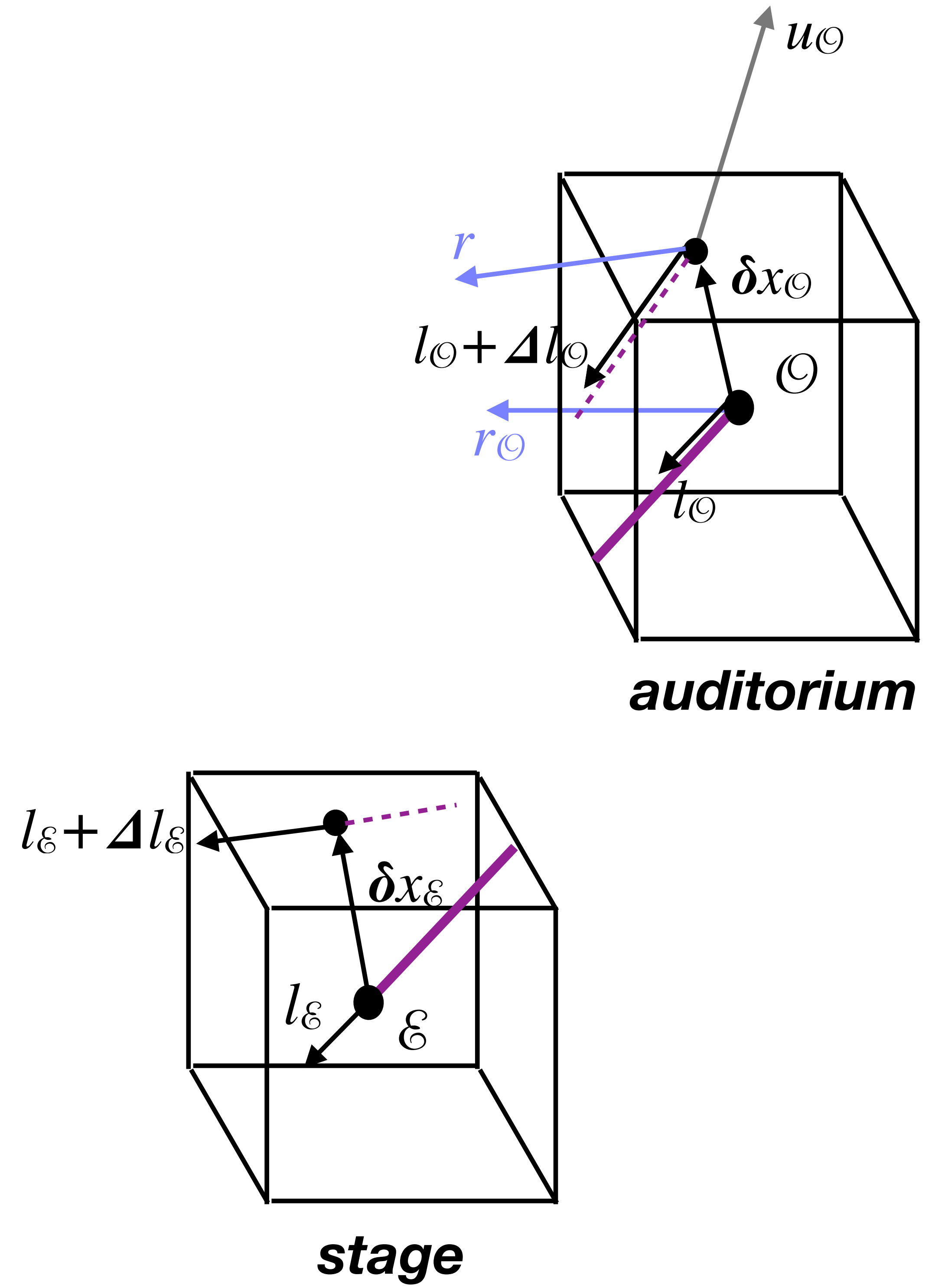
(Synge 1960, DeWitt&Brehme 1960, Dixon 1970, Vines 2015, Flanagan *et al* 2018, Fleury 2014, Uzun 2018…)

Nonlinear functionals of the curvature tensor

Observables

Apparent position on the observer's sky

γ_0 as the reference null geodesic



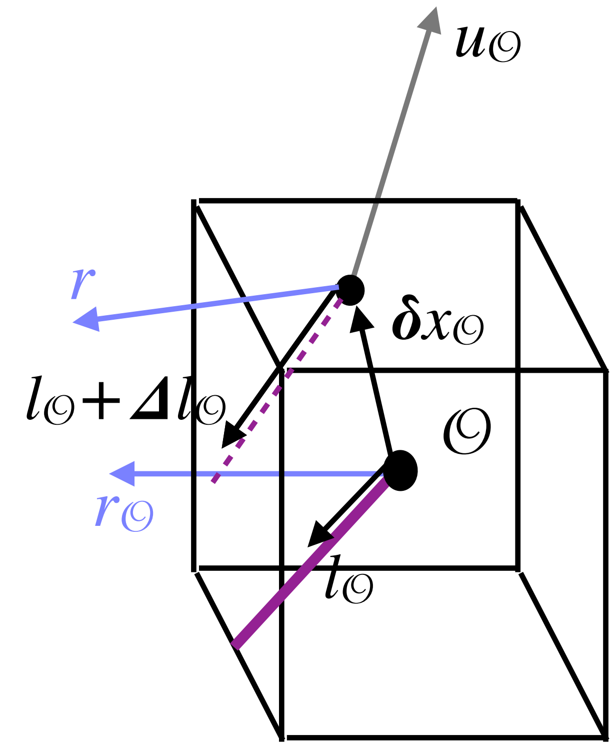
Observables

Apparent position on the observer's sky

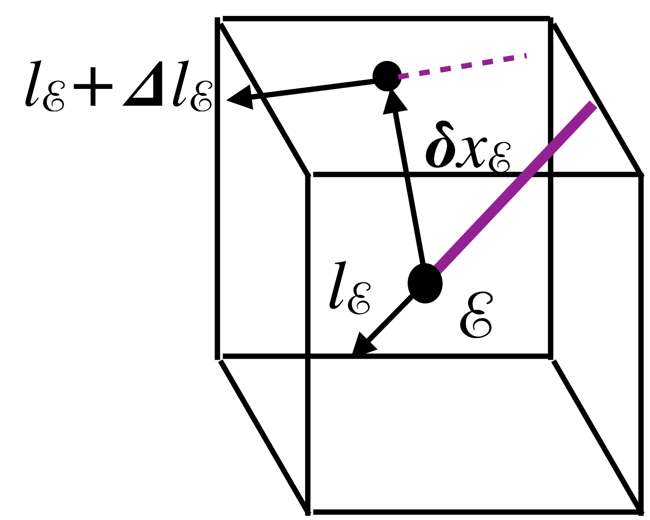
γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_\mathcal{O} + \Delta l^\mu_\mathcal{O}}{(l^\sigma_\mathcal{O} + \Delta l^\sigma_\mathcal{O})\, u_{\mathcal{O}_\sigma}} + u^\mu_\mathcal{O}$$

$$\delta r^A = \frac{\Delta l^A_\mathcal{O}}{(l^\sigma_\mathcal{O} + \Delta l^\sigma_\mathcal{O})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_\mathcal{O}}{l^\sigma_\mathcal{O}\, u_{\mathcal{O}_\sigma}} + O(\Delta l^2_\mathcal{O})$$



auditorium



stage

Observables

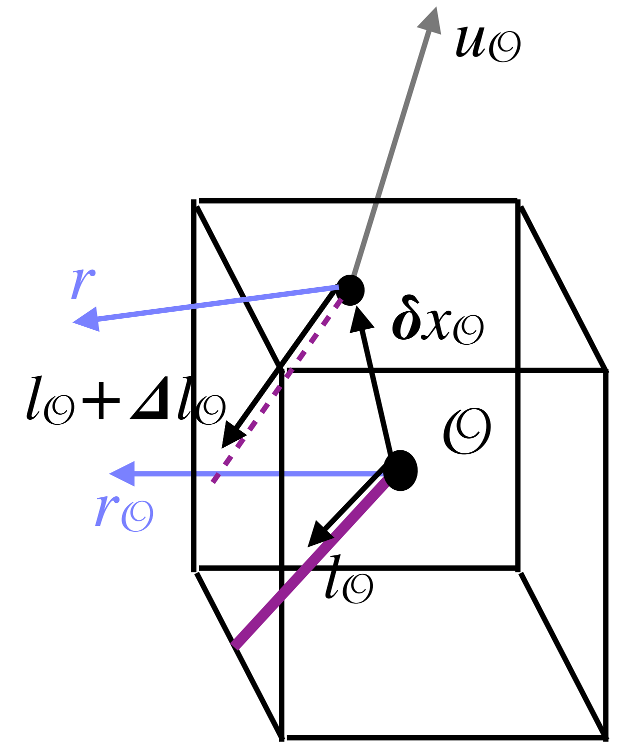
Apparent position on the observer's sky

γ_0 as the reference null geodesic

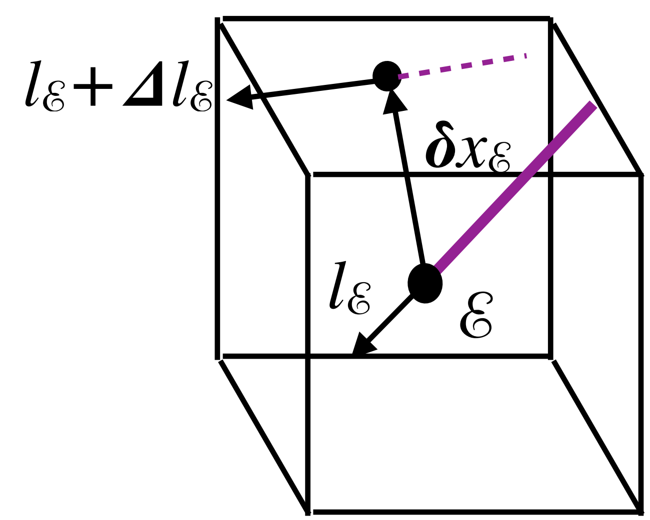
- **position the sky** $$r^\mu = \frac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} + \cancel{O(\Delta l^2_{\mathcal{O}})}$$

Parallel rays approximation (PRA)



auditorium



stage

Observables

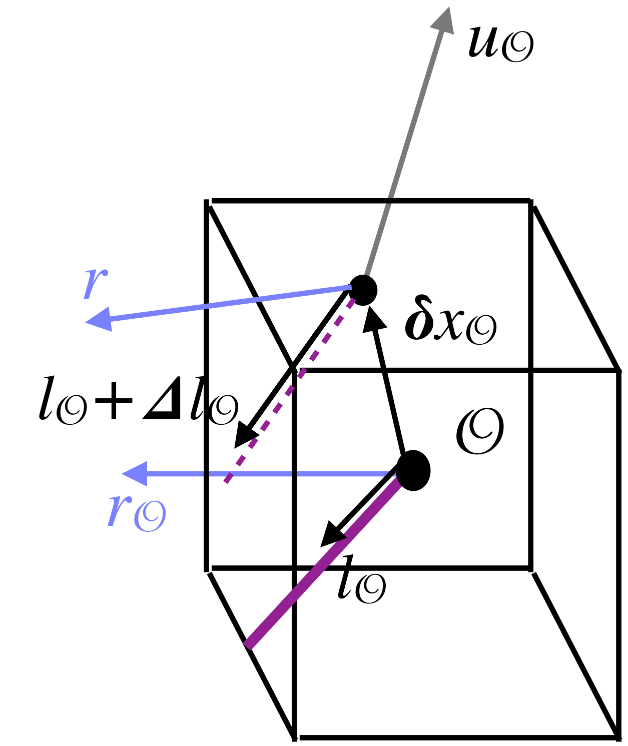
Apparent position on the observer's sky

γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_\mathcal{O} + \Delta l^\mu_\mathcal{O}}{(l^\sigma_\mathcal{O} + \Delta l^\sigma_\mathcal{O})\, u_{\mathcal{O}_\sigma}} + u^\mu_\mathcal{O}$$

$$\delta r^A = \frac{\Delta l^A_\mathcal{O}}{(l^\sigma_\mathcal{O} + \Delta l^\sigma_\mathcal{O})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_\mathcal{O}}{l^\sigma_\mathcal{O}\, u_{\mathcal{O}_\sigma}} + \xcancel{O(\Delta l^2_\mathcal{O})}$$

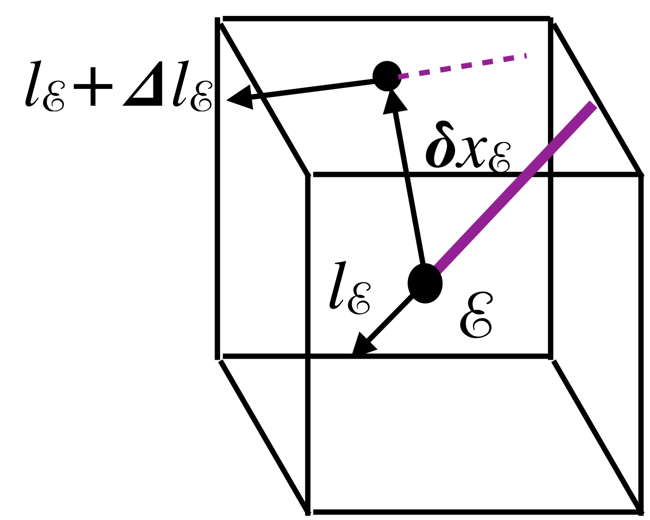
Parallel rays approximation (PRA)



auditorium

- **time of arrival** $$g_{\mu\nu}\,(l^\mu_\mathcal{O} + \Delta l^\mu_\mathcal{O})\,(l^\nu_\mathcal{O} + \Delta l^\nu_\mathcal{O}) = 0$$

$$g_{\mu\nu}\, l^\mu_\mathcal{O}\, \Delta l^\mu_\mathcal{O} + O(\Delta l^2_\mathcal{O}) = 0$$



stage

Observables

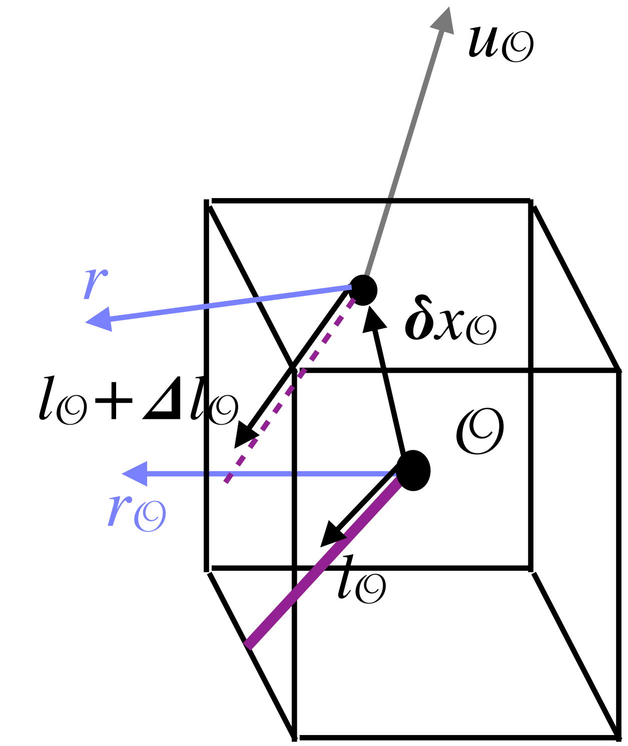
Apparent position on the observer's sky

γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} + u^\mu_{\mathcal{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}_\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}_\sigma}} \cancel{+ O(\Delta l^2_{\mathcal{O}})}$$

Parallel rays approximation (PRA)

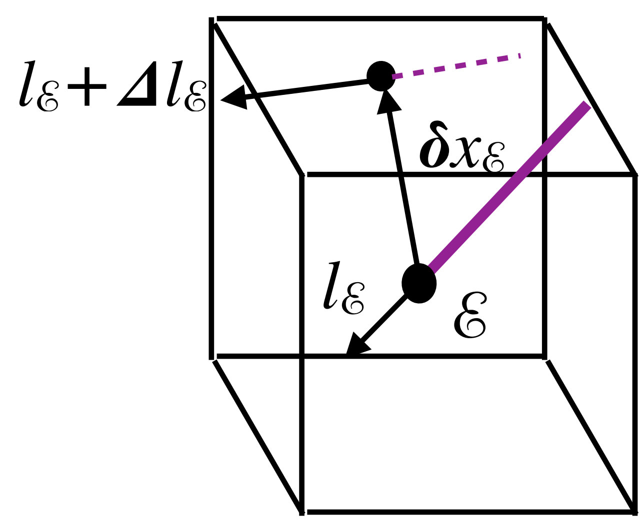


auditorium

- **time of arrival** $$g_{\mu\nu}\,(l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}})\,(l^\nu_{\mathcal{O}} + \Delta l^\nu_{\mathcal{O}}) = 0$$

$$g_{\mu\nu}\, l^\mu_{\mathcal{O}}\, \Delta l^\mu_{\mathcal{O}} \cancel{+ O(\Delta l^2_{\mathcal{O}})} = 0$$

Flat light cones approximation (FLA)



stage

Observables

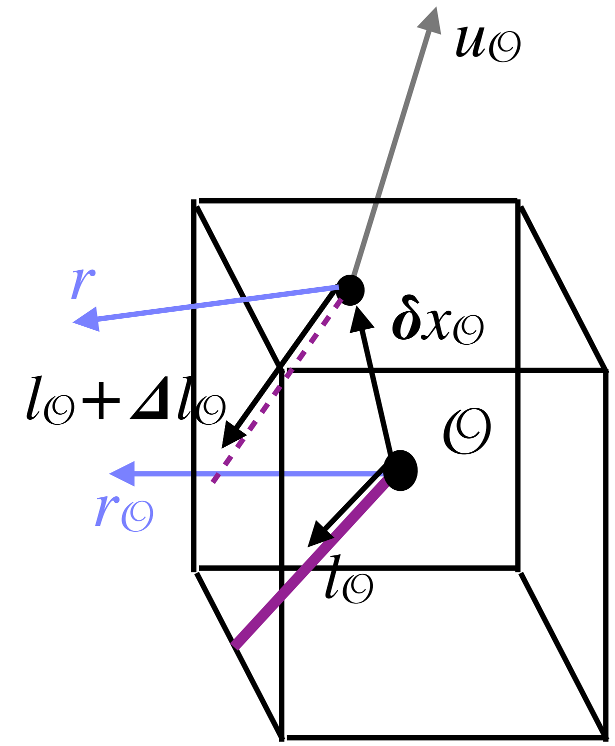
Apparent position on the observer's sky

γ_0 as the reference null geodesic

- **position the sky** $$r^\mu = \frac{l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}\,\sigma}} + u^\mu_{\mathcal{O}}$$

$$\delta r^A = \frac{\Delta l^A_{\mathcal{O}}}{(l^\sigma_{\mathcal{O}} + \Delta l^\sigma_{\mathcal{O}})\, u_{\mathcal{O}\,\sigma}} = \frac{\Delta l^A_{\mathcal{O}}}{l^\sigma_{\mathcal{O}}\, u_{\mathcal{O}\,\sigma}} + O(\Delta l^2_{\mathcal{O}})$$

Parallel rays approximation (PRA)



auditorium

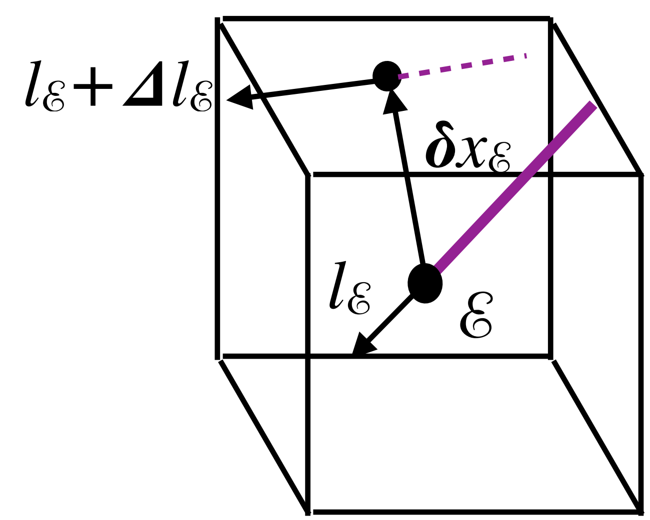
- **time of arrival** $$g_{\mu\nu}\,(l^\mu_{\mathcal{O}} + \Delta l^\mu_{\mathcal{O}})\,(l^\nu_{\mathcal{O}} + \Delta l^\nu_{\mathcal{O}}) = 0$$

$$g_{\mu\nu}\, l^\mu_{\mathcal{O}}\, \Delta l^\mu_{\mathcal{O}} + O(\Delta l^2_{\mathcal{O}}) = 0$$

Flat light cones approximation (FLA)

⇕ geodesic deviation equation

$$\delta x^\mu_{\mathcal{O}}\, l_{\mathcal{O}\,\mu} = \delta x^\mu_{\mathcal{E}}\, l_{\mathcal{E}\,\mu}$$



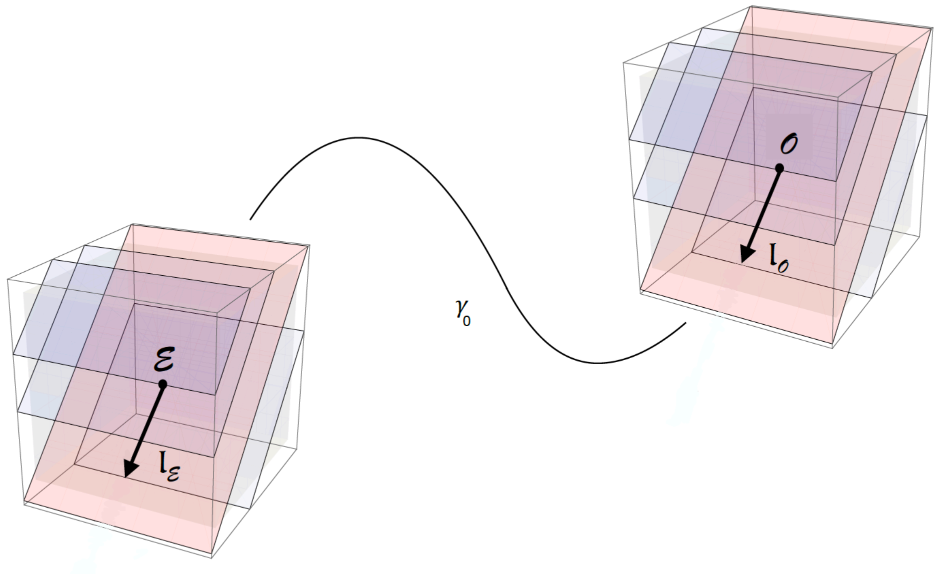
stage

Approximations

Flat light cones approximation

$$\delta x^{\mu}_{\mathscr{O}}\, l_{\mathscr{O}\,\mu} = \delta x^{\mu}_{\mathscr{E}}\, l_{\mathscr{E}\,\mu}$$

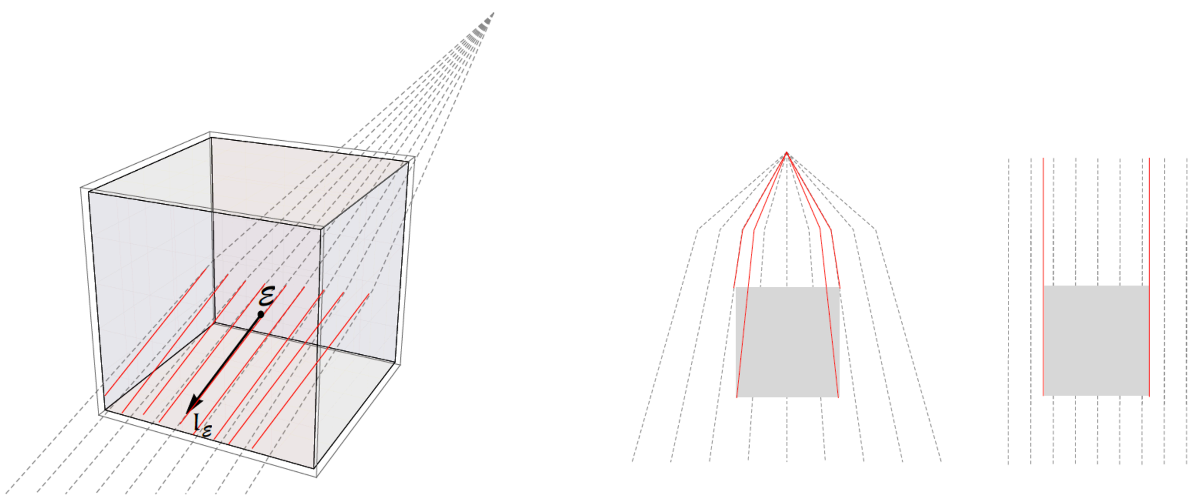
No transverse Rømer delays



Parallel rays approximation

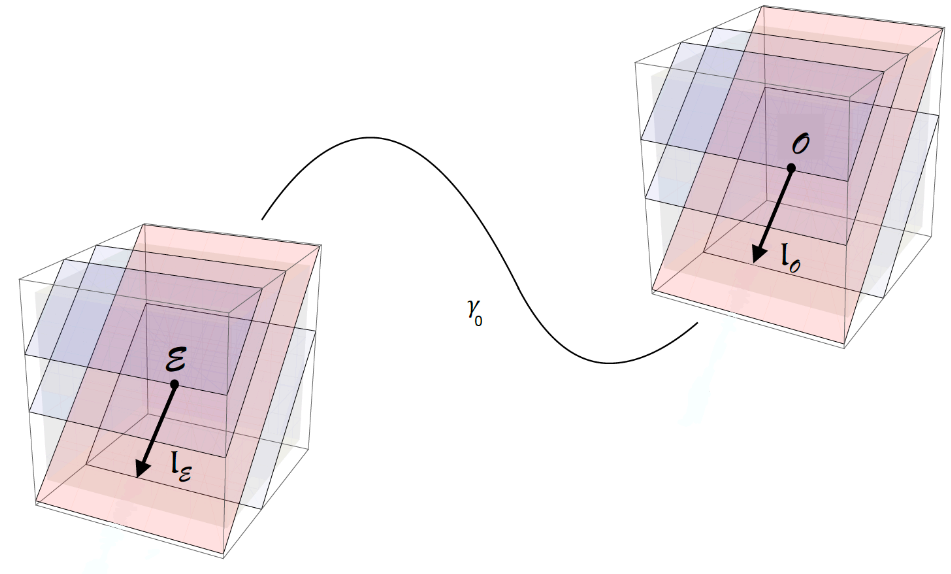
$$\delta r^{A} = \frac{\Delta l^{A}_{\mathscr{O}}}{l^{\sigma}_{\mathscr{O}}\, u_{\mathscr{O}_{\sigma}}}$$

No perspective distortions



Approximations

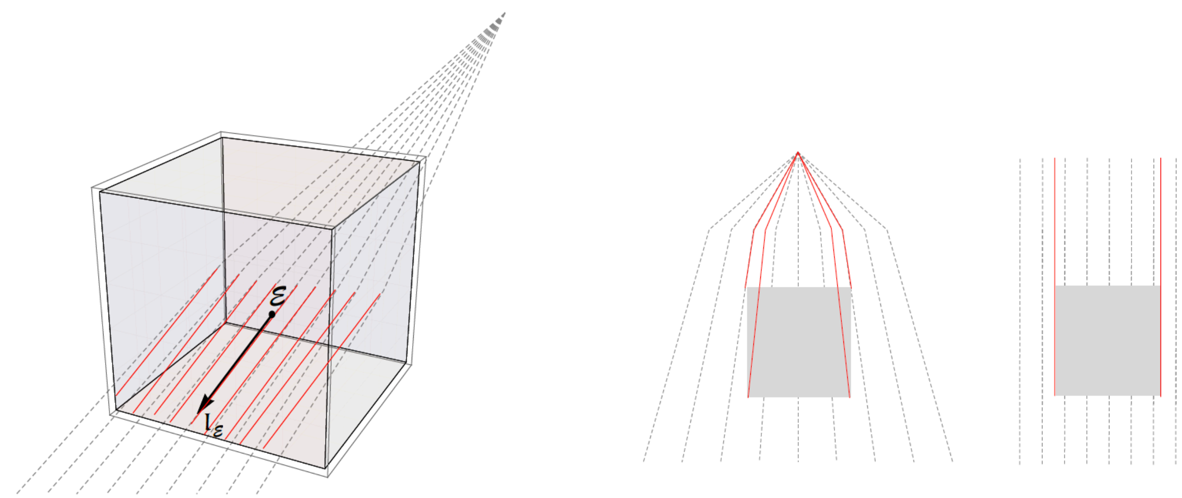
Flat light cones approximation



$$\delta x^{\mu}_{\mathscr{O}}\, l_{\mathscr{O}\,\mu} = \delta x^{\mu}_{\mathscr{E}}\, l_{\mathscr{E}\,\mu}$$

No transverse Rømer delays

Parallel rays approximation



$$\delta r^{A} = \frac{\Delta l^{A}_{\mathscr{O}}}{l^{\sigma}_{\mathscr{O}}\, u_{\mathscr{O}_{\sigma}}}$$

No perspective distortions

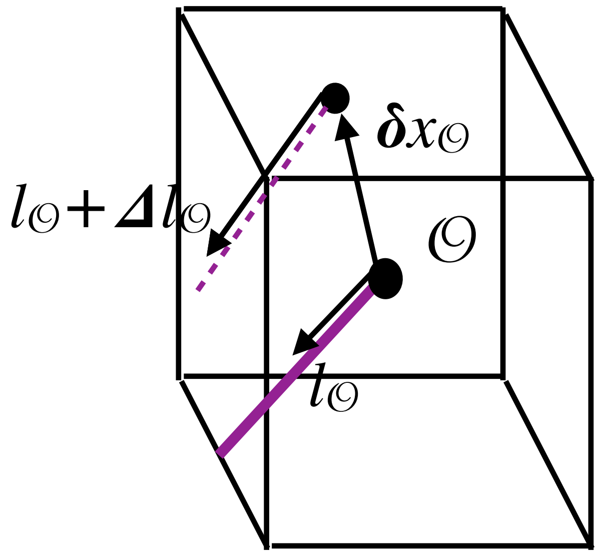
Applicability

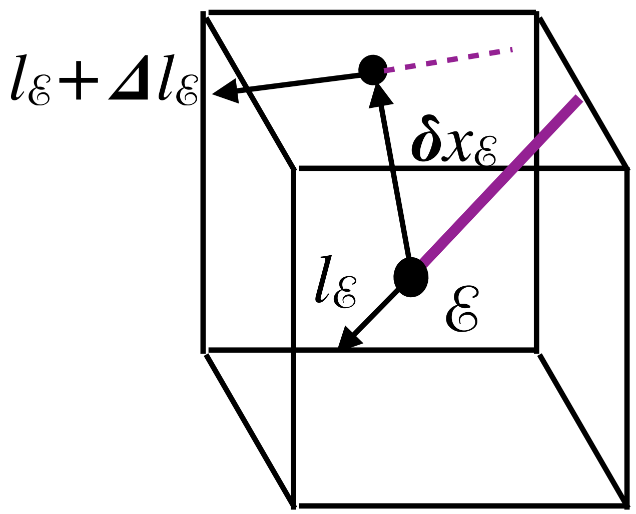
almost flat: keep $\frac{L}{R}$, disregard $\left(\frac{L}{R}\right)^2$

Displacement formulas

$$\delta x^{\mu}_{\mathscr{E}} = W_{XX}{}^{\mu}{}_{\nu}\,\delta x^{\nu}_{\mathscr{O}} + W_{XL}{}^{\mu}{}_{\nu}\,\Delta l^{\nu}_{\mathscr{O}}$$

$$\delta x^{\mu}_{\mathscr{O}}\, l_{\mathscr{O}\,\mu} = \delta x^{\mu}_{\mathscr{E}}\, l_{\mathscr{E}\,\mu}$$

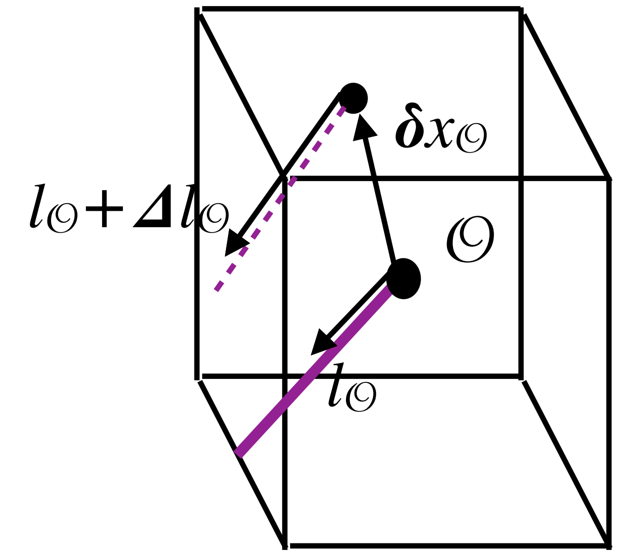


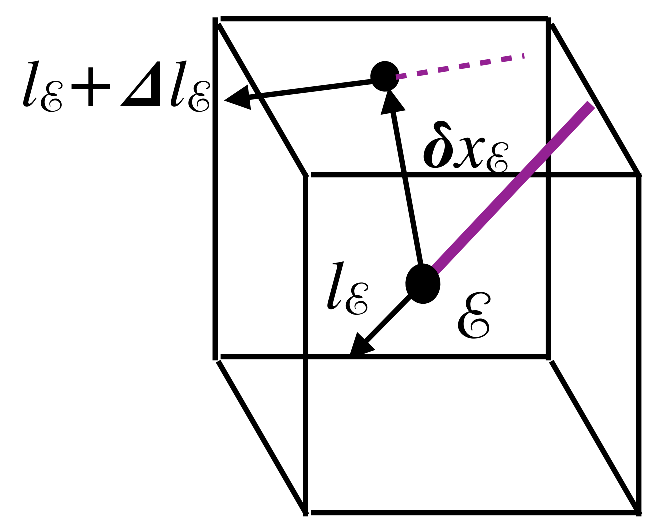


Displacement formulas

$$\mathscr{D}^A{}_B\,\Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta\hat{x}^A_{\mathscr{O}} - m^A{}_\mu\,\delta x^\mu_{\mathscr{O}}$$

$$\delta x^\mu_{\mathscr{O}}\, l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\, l_{\mathscr{E}\mu}$$





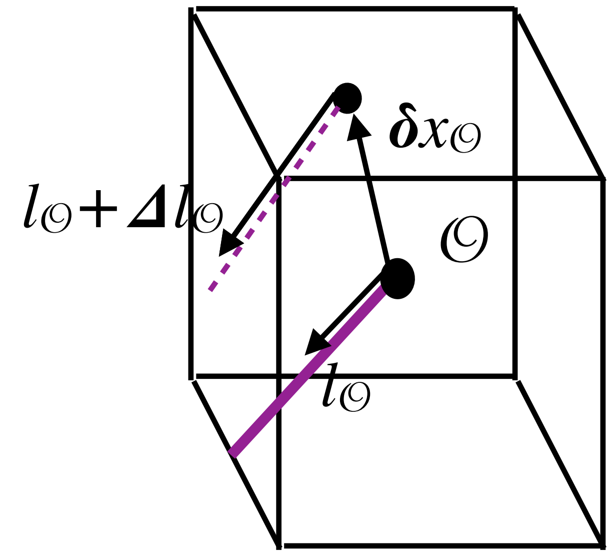
Displacement formulas

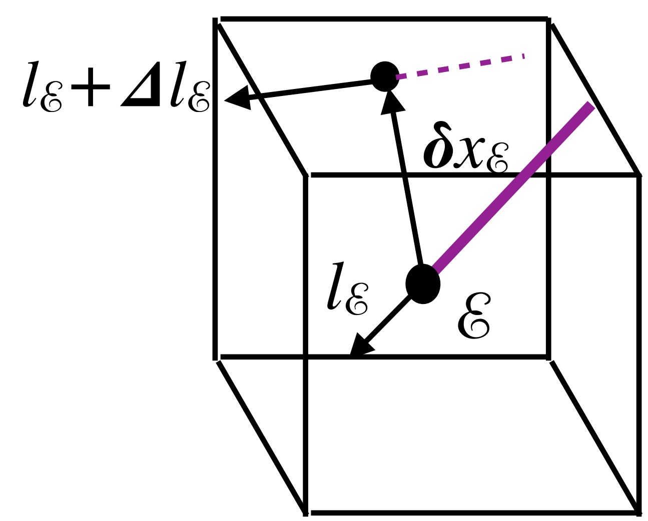
$$\mathscr{D}^{A}{}_{B}\,\Delta l^{B}_{\mathscr{O}} = \delta x^{A}_{\mathscr{E}} - \delta\hat{x}^{A}_{\mathscr{O}} - m^{A}{}_{\mu}\,\delta x^{\mu}_{\mathscr{O}}$$

$$\delta x^{\mu}_{\mathscr{O}}\, l_{\mathscr{O}\mu} = \delta x^{\mu}_{\mathscr{E}}\, l_{\mathscr{E}\mu}$$

where

^ = parallel transport

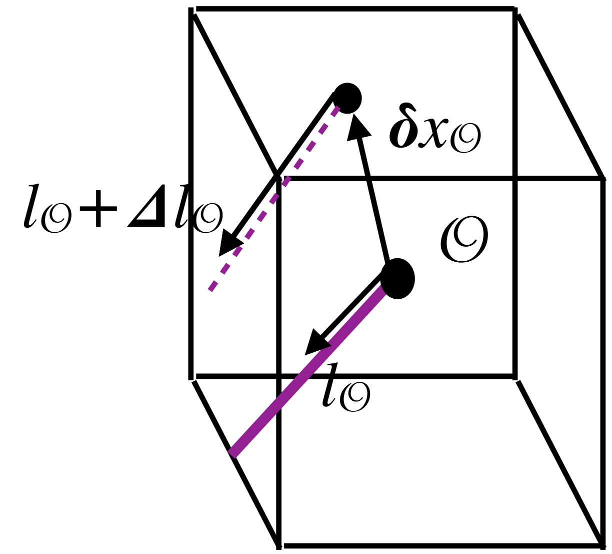




Displacement formulas

$$\mathscr{D}^A{}_B\,\Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta\hat{x}^A_{\mathscr{O}} - m^A{}_\mu\,\delta x^\mu_{\mathscr{O}}$$

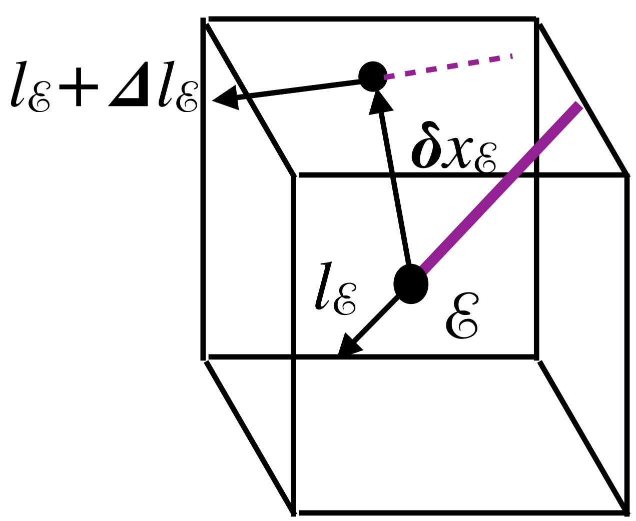
$$\delta x^\mu_{\mathscr{O}}\, l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\, l_{\mathscr{E}\mu}$$



where

^ = parallel transport

$\mathscr{D} : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$ Jacobi operator

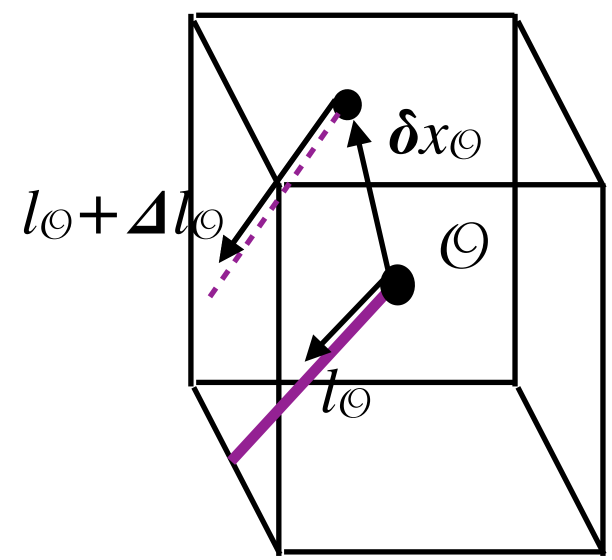


$$\ddot{\mathcal{D}}^A{}_B - R^A{}_{\mu\nu C}\, l^\mu\, l^\nu\, \mathcal{D}^C{}_B = 0$$
$$\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = 0$$
$$\dot{\mathcal{D}}^A{}_B(\lambda_{\mathcal{O}}) = \delta^A{}_B$$

Displacement formulas

$$\mathscr{D}^A{}_B\,\Delta l^B_{\mathscr{O}} = \delta x^A_{\mathscr{E}} - \delta\hat{x}^A_{\mathscr{O}} - m^A{}_\mu\,\delta x^\mu_{\mathscr{O}}$$

$$\delta x^\mu_{\mathscr{O}}\,l_{\mathscr{O}\mu} = \delta x^\mu_{\mathscr{E}}\,l_{\mathscr{E}\mu}$$

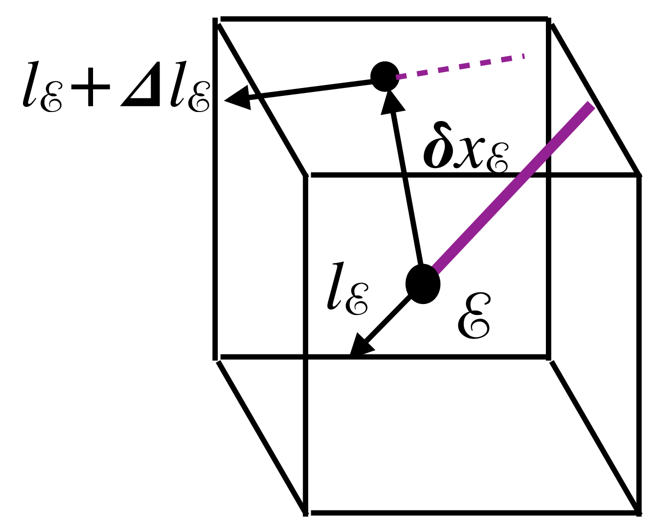


where

^ = parallel transport

$\mathscr{D} : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$ Jacobi operator

$$\ddot{\mathcal{D}}^A{}_B - R^A{}_{\mu\nu C}\,l^\mu\,l^\nu\,\mathcal{D}^C{}_B = 0$$
$$\mathcal{D}^A{}_B(\lambda_{\mathcal{O}}) = 0$$
$$\dot{\mathcal{D}}^A{}_B(\lambda_{\mathcal{O}}) = \delta^A{}_B$$



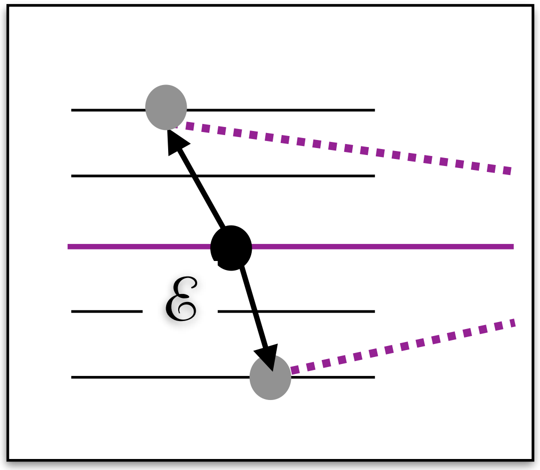
$m : \mathscr{Q}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$ E/O asymmetry operator

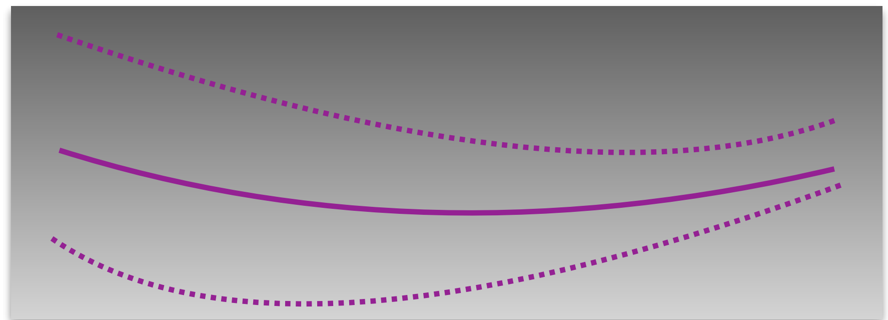
$$\ddot{m}^A{}_\sigma - R^A{}_{\mu\nu C}\,l^\mu\,l^\nu\,m^C{}_\sigma = R^A{}_{\mu\nu\sigma}\,l^\mu\,l^\nu$$
$$m^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$
$$\dot{m}^A{}_\mu(\lambda_{\mathcal{O}}) = 0$$

vanishes in a flat space!

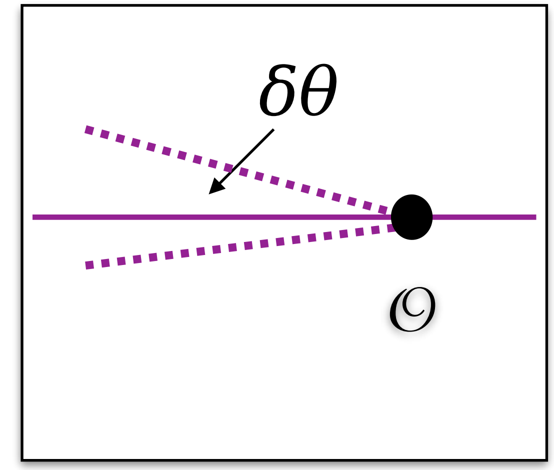
$$m|_{\mathscr{P}_{\mathscr{O}}} \equiv m_\perp : \mathscr{P}_{\mathscr{O}} \to \mathscr{P}_{\mathscr{E}}$$

Linear image distortion

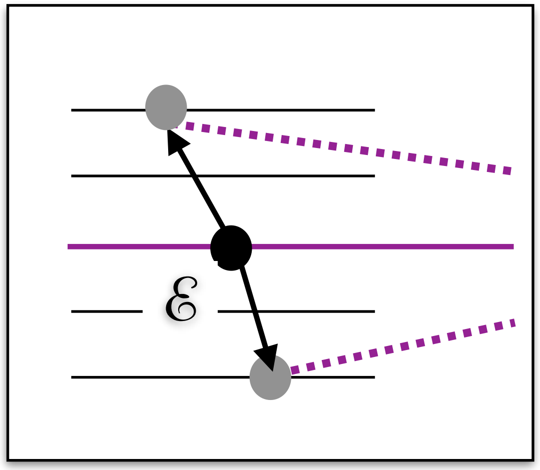


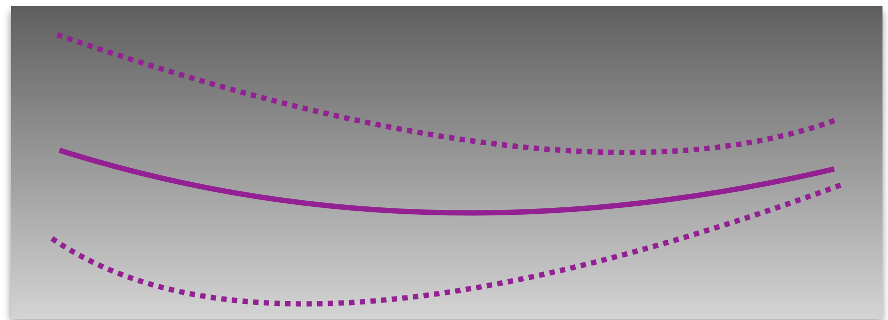


$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

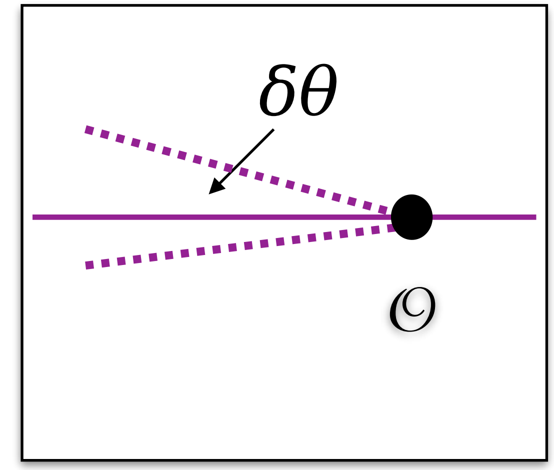


Linear image distortion



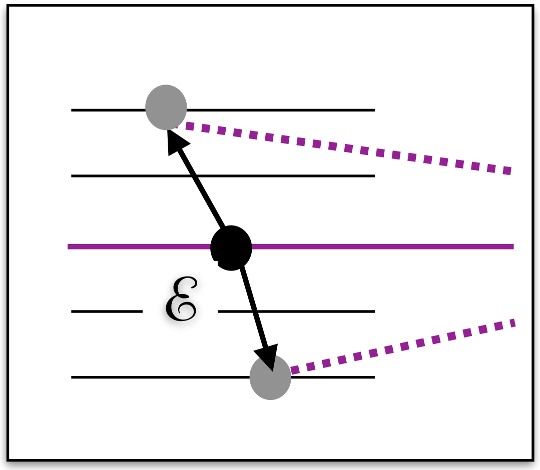


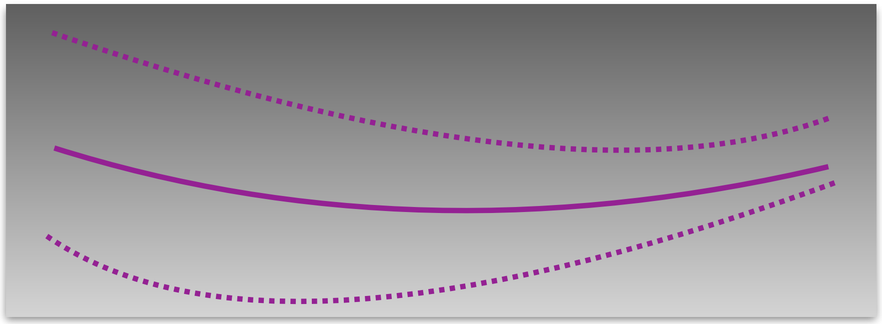
$$\mathcal{D}^{A}{}_{B}\,\Delta l^{B}_{\mathcal{O}} = \delta x^{A}_{\mathcal{E}}$$



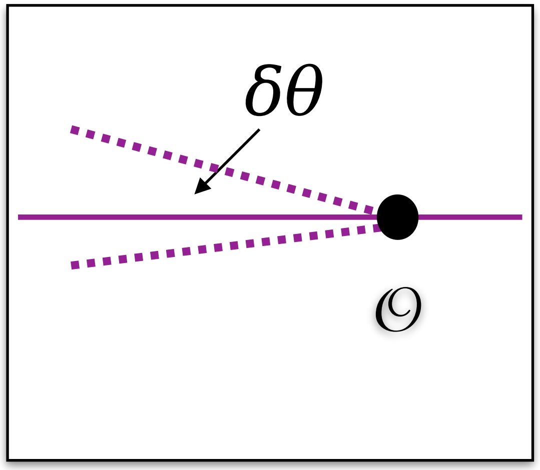
$$\delta\theta^{A} \approx \delta r^{A} = \frac{1}{u^{\sigma}_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}}\, \mathcal{D}^{-1\,A}{}_{B}\, \delta x^{B}_{\mathcal{E}}$$

Linear image distortion





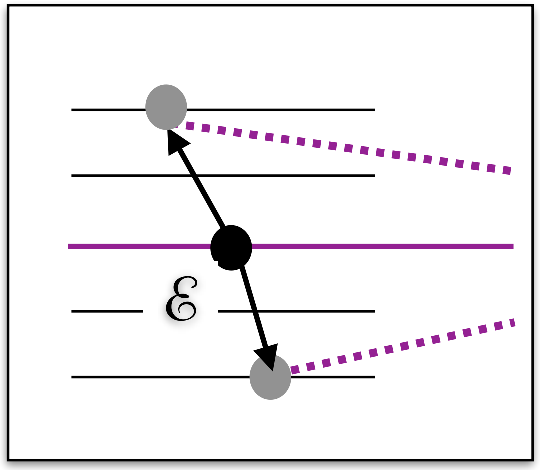
$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

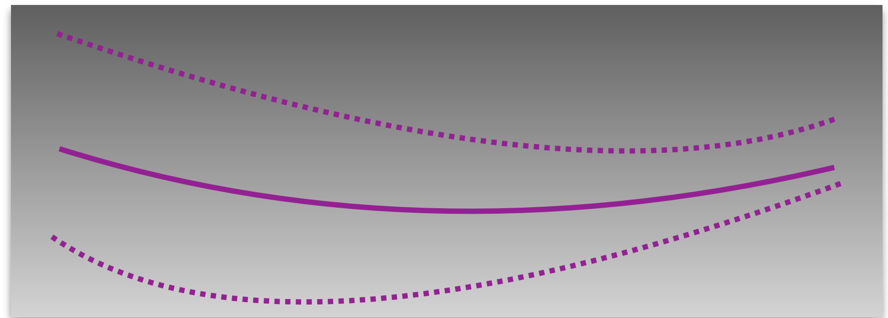


$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \mathcal{D}^{-1\,A}{}_B \, \delta x^B_{\mathcal{E}}$$

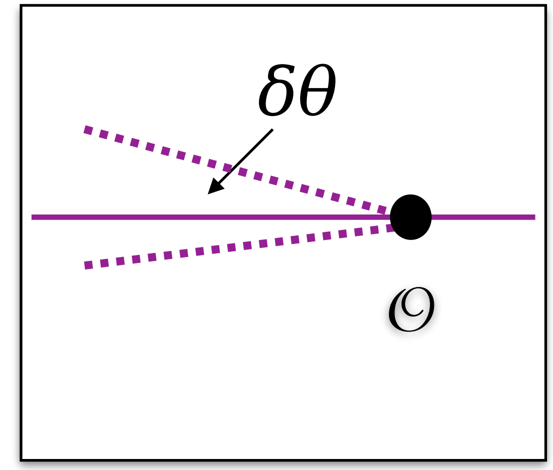
magnification matrix $M^A{}_B$

Linear image distortion



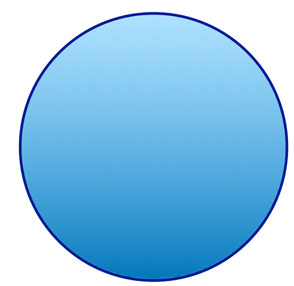


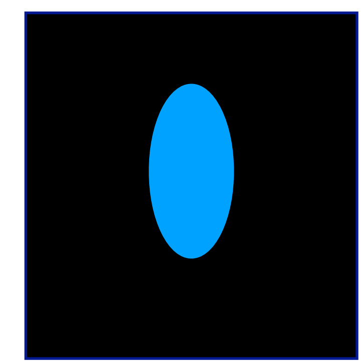
$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$



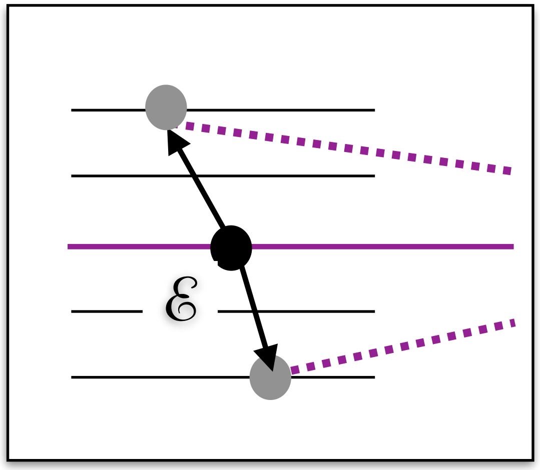
$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \mathcal{D}^{-1\,A}{}_B \, \delta x^B_{\mathcal{E}}$$

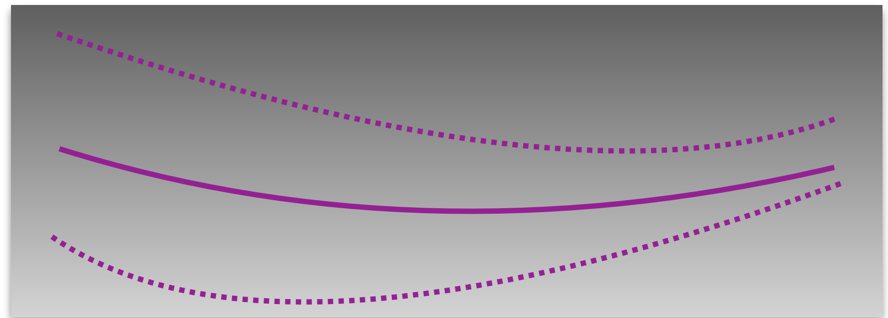
magnification matrix $M^A{}_B$



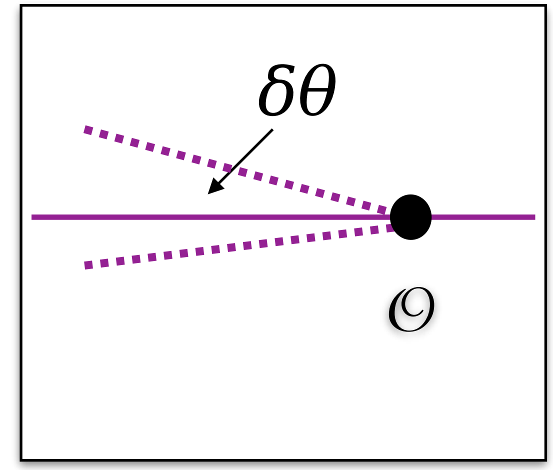


Linear image distortion



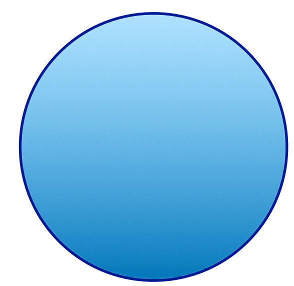


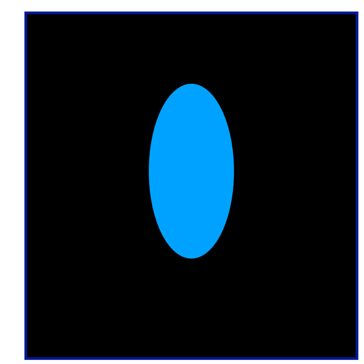
$$\mathcal{D}^A{}_B \, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$



$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \mathcal{D}^{-1\,A}{}_B \, \delta x^B_{\mathcal{E}}$$

magnification matrix $M^A{}_B$

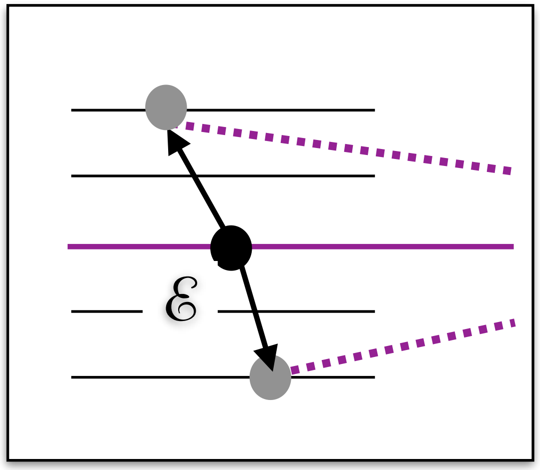


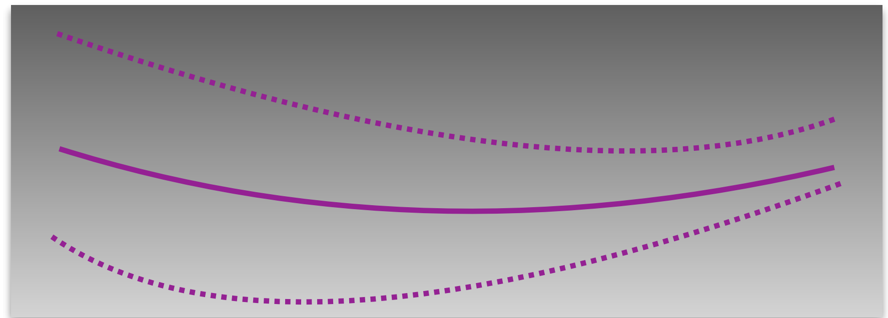


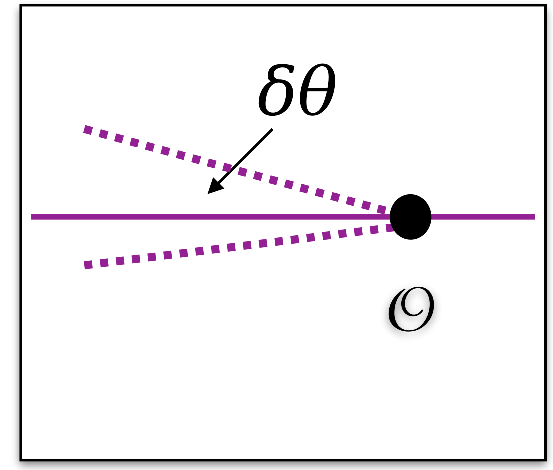
angular diameter distance

$$D_{ang} = u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma} \left| \det \mathcal{D}^A{}_B \right|^{1/2}$$

Linear image distortion



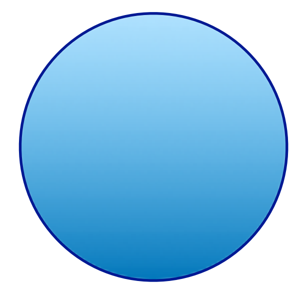


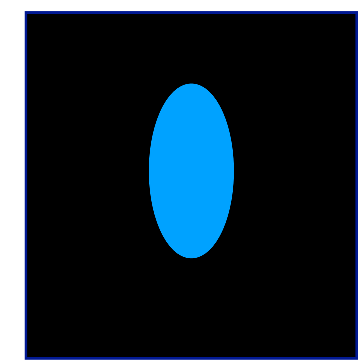


$$\mathcal{D}^A{}_B\, \Delta l^B_{\mathcal{O}} = \delta x^A_{\mathcal{E}}$$

$$\delta\theta^A \approx \delta r^A = \frac{1}{u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}}\, \mathcal{D}^{-1\,A}{}_B\, \delta x^B_{\mathcal{E}}$$

magnification matrix $M^A{}_B$





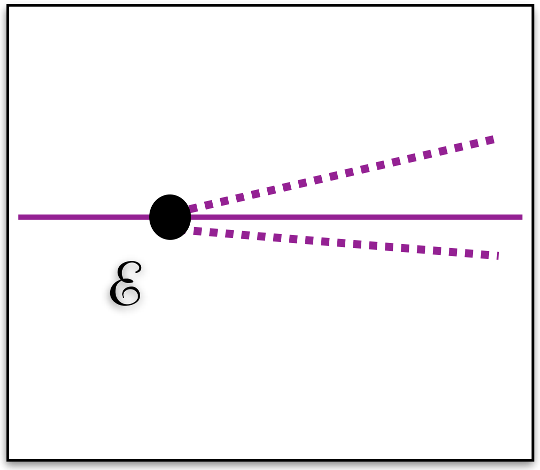
angular diameter distance

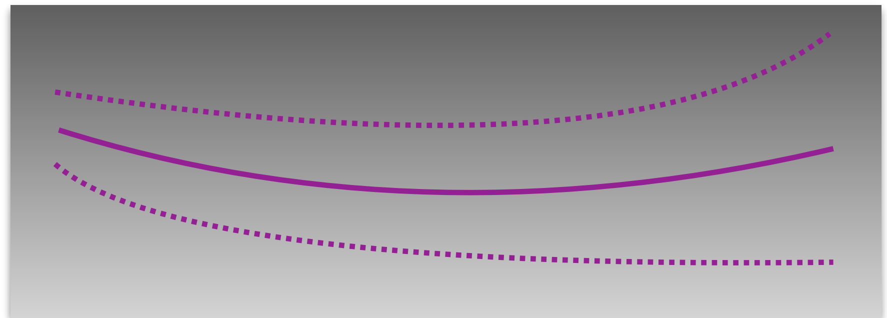
$$D_{ang} = u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma} \left|\det \mathcal{D}^A{}_B\right|^{1/2}$$

$$M^A{}_B \equiv M^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

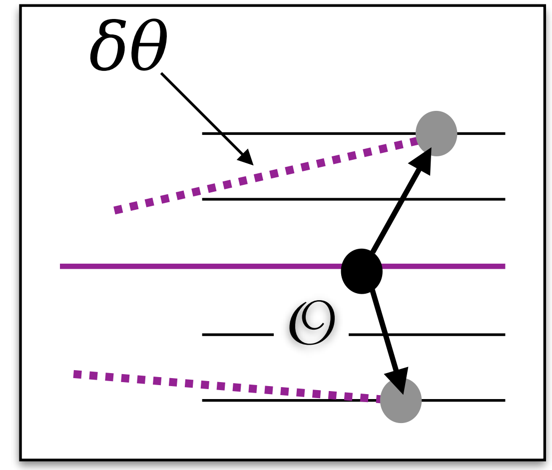
$$D_{ang} \equiv D_{ang}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

Parallax

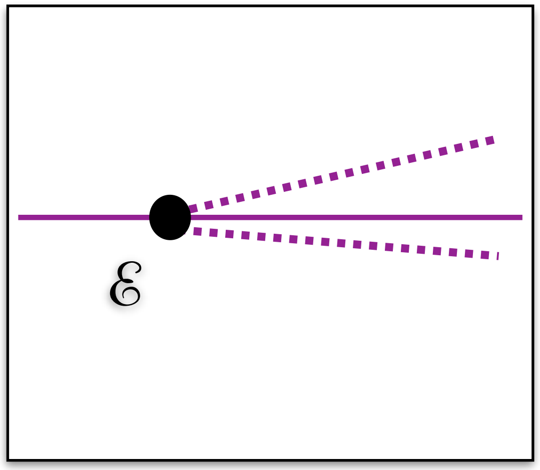


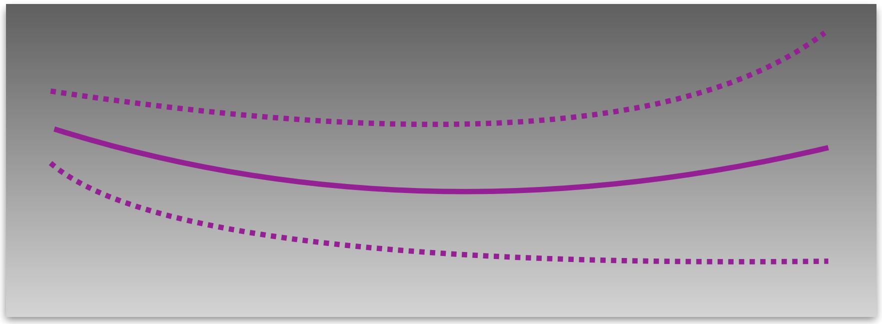


$$\mathscr{D}^A{}_B \Delta l^B = -\delta\hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B \delta x^B_{\mathcal{O}}$$

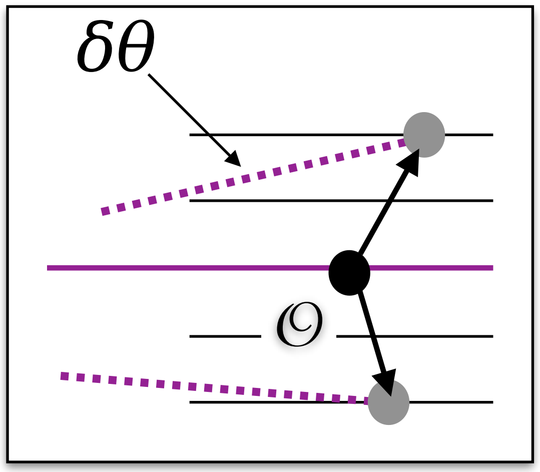


Parallax



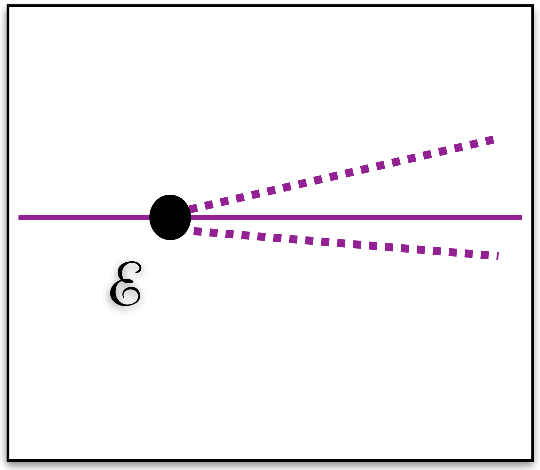


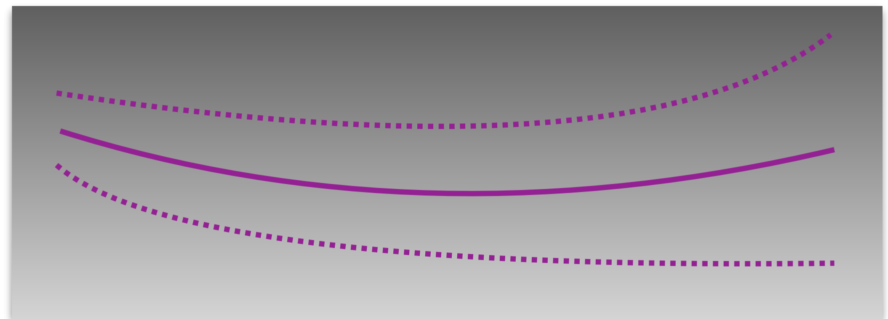
$$\mathscr{D}^A{}_B \, \Delta l^B = -\,\delta \hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathcal{O}}$$

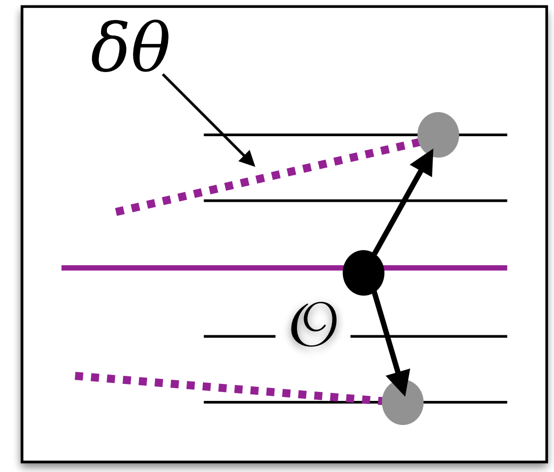


$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathcal{O}}$$

Parallax





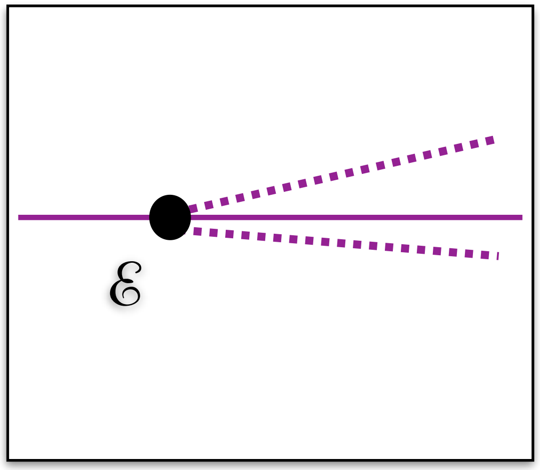


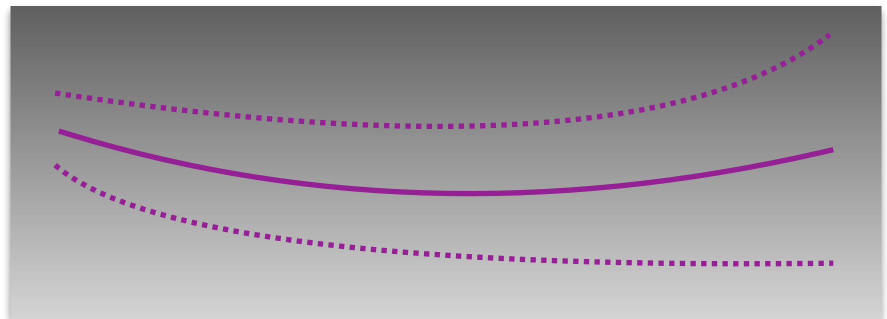
$$\mathscr{D}^A{}_B \,\Delta l^B = -\,\delta\hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B\, \delta x^B_{\mathscr{O}}$$

$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}}\, \mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathscr{O}}$$

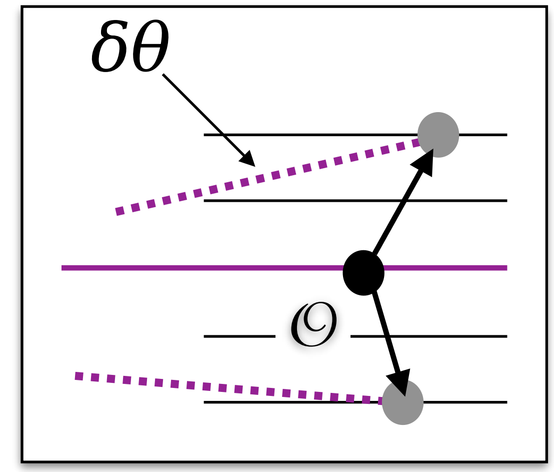
parallax matrix $\Pi^A{}_B$

Parallax



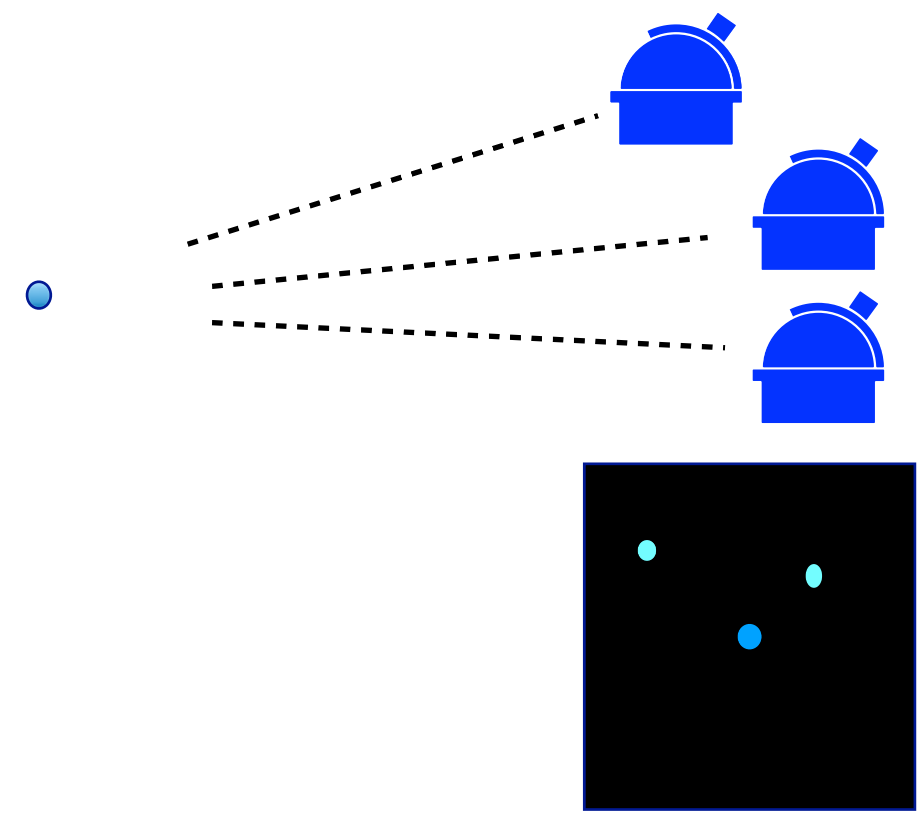


$$\mathscr{D}^A{}_B \, \Delta l^B = -\,\delta\hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathscr{O}}$$

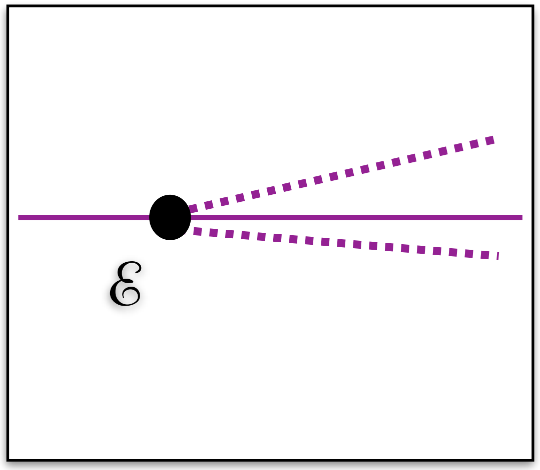


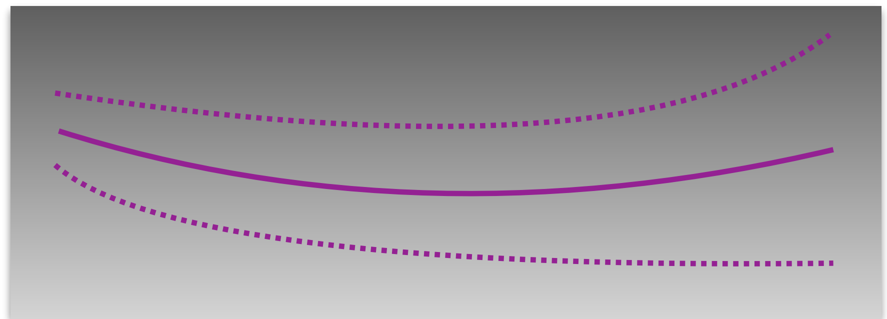
$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma}} \mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathscr{O}}$$

parallax matrix $\Pi^A{}_B$

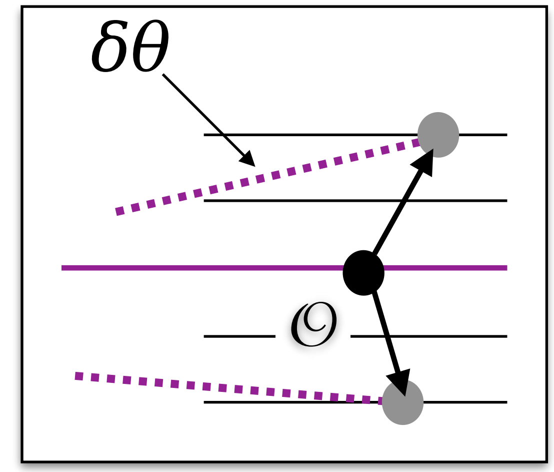


Parallax





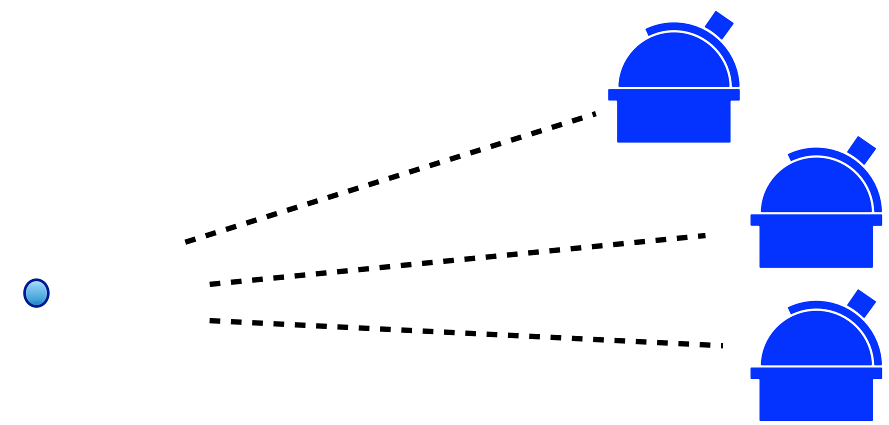
$$\mathscr{D}^A{}_B \, \Delta l^B = -\, \delta \hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathscr{O}}$$

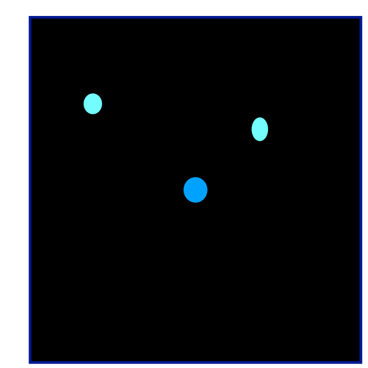


$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathscr{O}} \, l_{\mathscr{O}\,\sigma}} \mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B \right) \delta x^B_{\mathscr{O}}$$

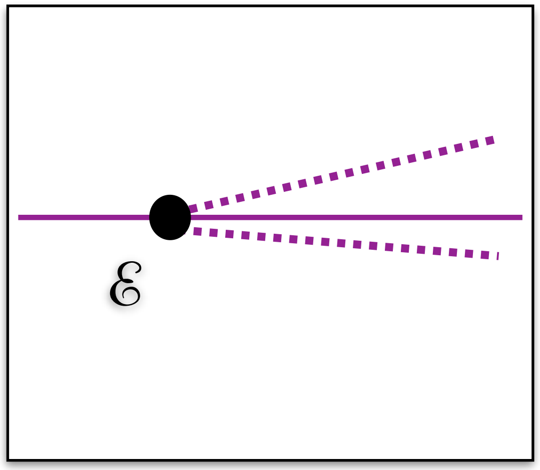
parallax matrix $\Pi^A{}_B$

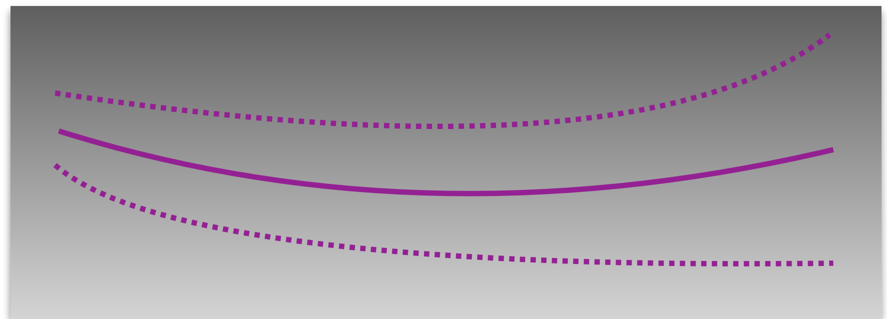
$$\Pi_{AB} = \Pi_{BA}$$



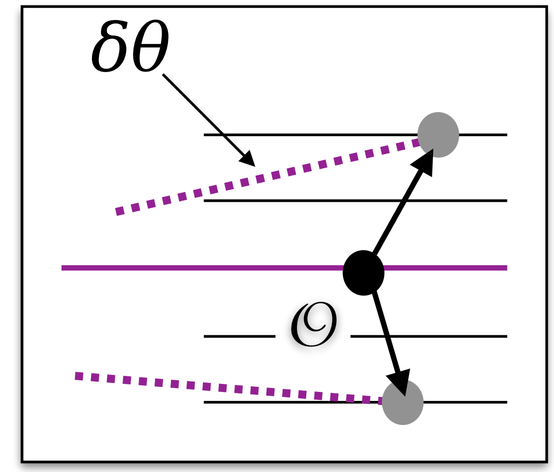


Parallax





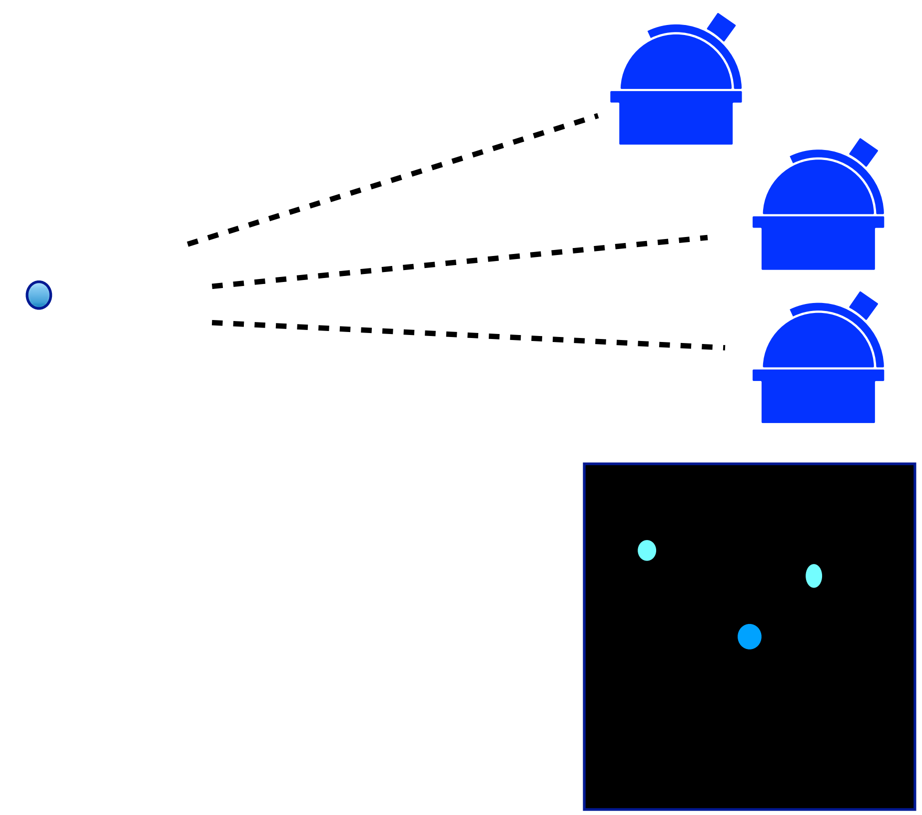
$$\mathscr{D}^A{}_B \, \Delta l^B = -\,\delta \hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B \, \delta x^B_{\mathcal{O}}$$



$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma}} \mathscr{D}^{-1\,A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right) \delta x^B_{\mathcal{O}}$$

parallax matrix $\Pi^A{}_B$

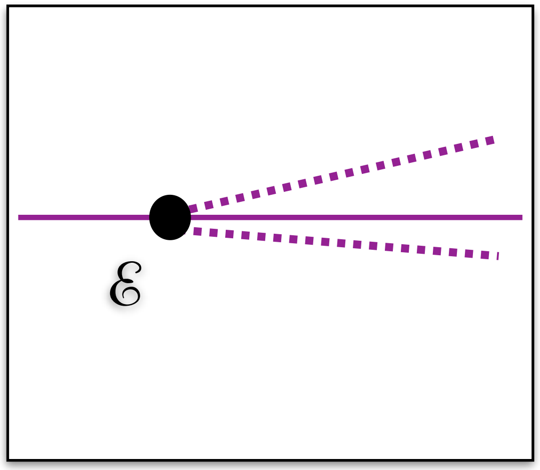
$$\Pi_{AB} = \Pi_{BA}$$

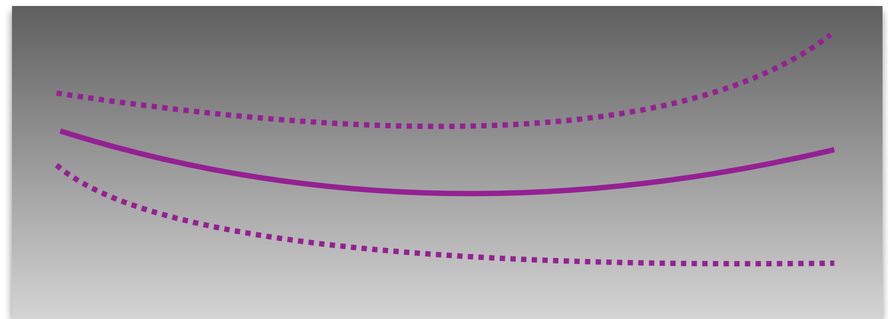


parallax distance

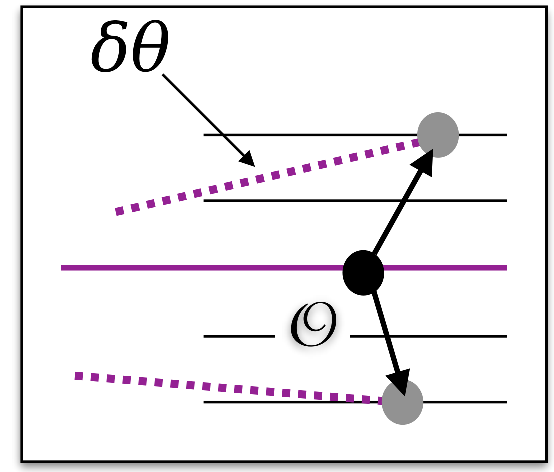
$$D_{par} = u^\sigma_{\mathcal{O}} \, l_{\mathcal{O}\,\sigma} \left| \det \mathscr{D}^A{}_B \right|^{1/2} \left| \det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)\right|^{-1/2}$$

Parallax

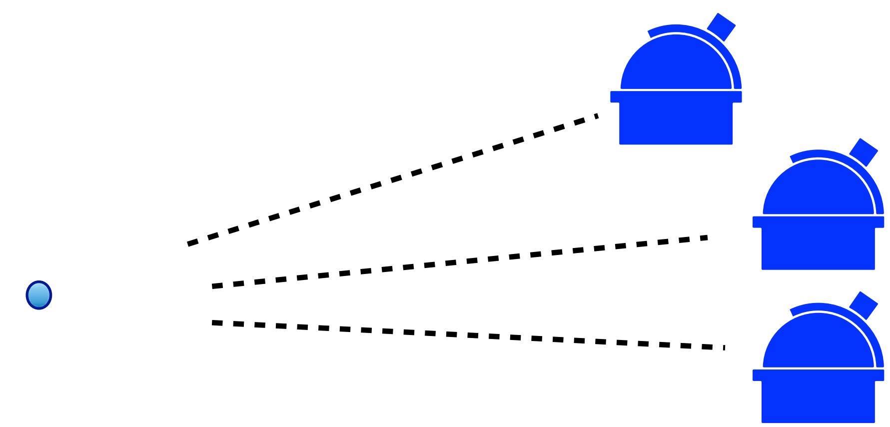




$$\mathscr{D}^A{}_B\,\Delta l^B = -\,\delta\hat{x}^A_{\mathscr{O}} - m_\perp{}^A{}_B\,\delta x^B_{\mathscr{O}}$$

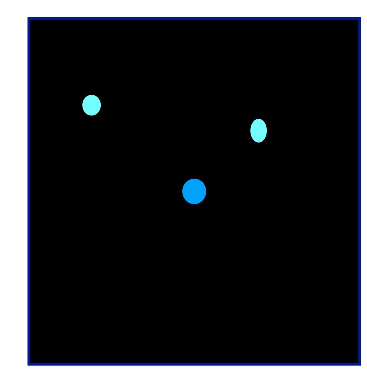


$$\delta\theta^A \approx \delta r^A = -\,\frac{1}{u^\sigma_{\mathscr{O}}\,l_{\mathscr{O}\,\sigma}}\,\mathscr{D}^{-1\,A}{}_C\left(\delta^C{}_B + m_\perp{}^C{}_B\right)\delta x^B_{\mathscr{O}}$$



parallax matrix $\Pi^A{}_B$

$$\Pi_{AB} = \Pi_{BA}$$



parallax distance

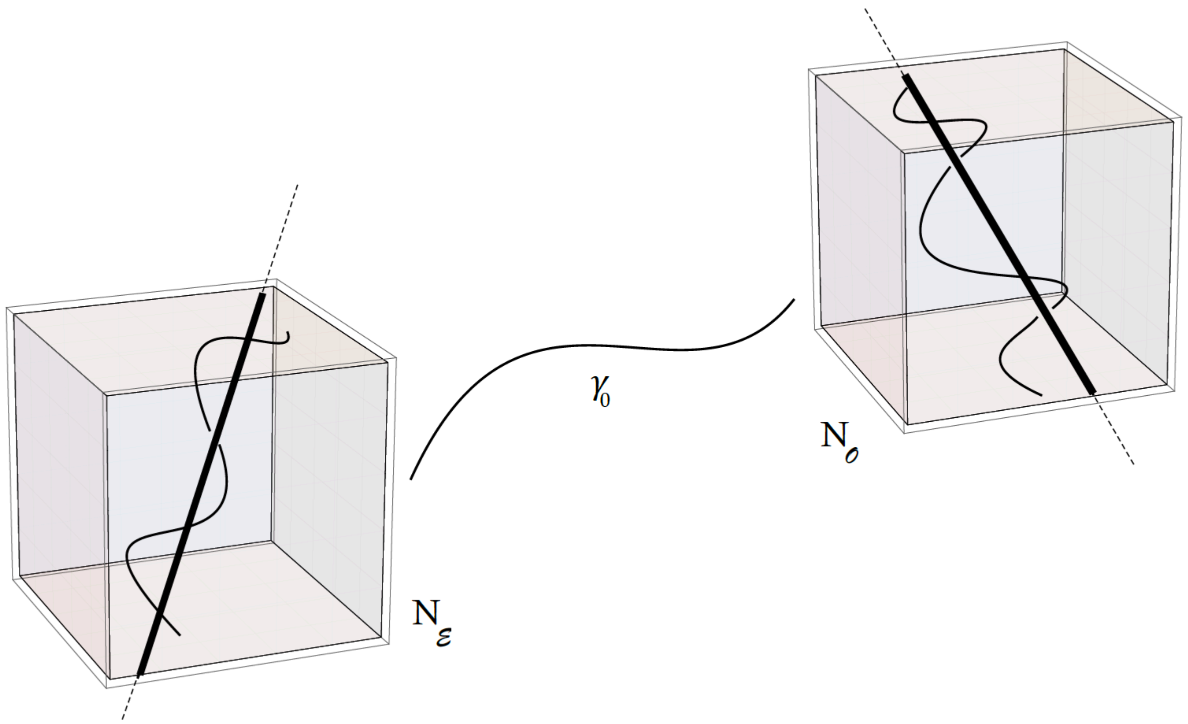
$$D_{par} = u^\sigma_{\mathscr{O}}\,l_{\mathscr{O}\,\sigma}\,\left|\det \mathscr{D}^A{}_B\right|^{1/2}\,\left|\det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)\right|^{-1/2}$$

$$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0},\,u^\mu_{\mathscr{O}}\right)$$

$$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0},\,u^\mu_{\mathscr{O}}\right)$$

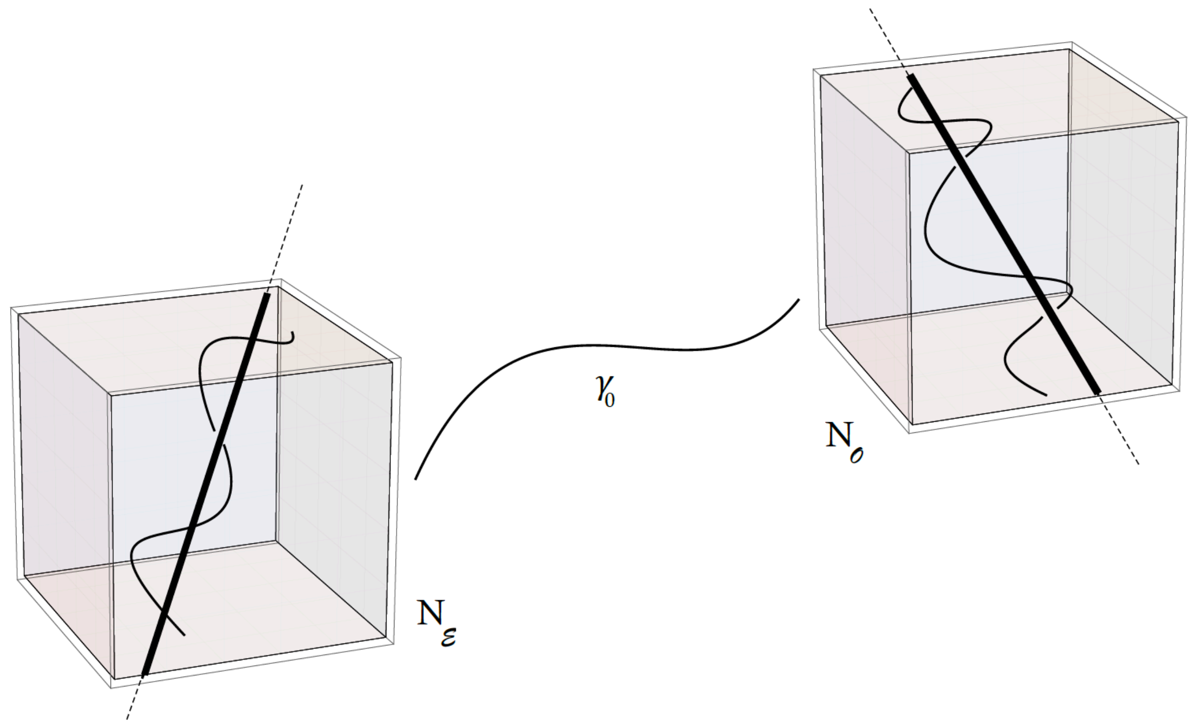
Parallax in a general situation

Parallax in a general situation



Both observer and emitter in bound systems

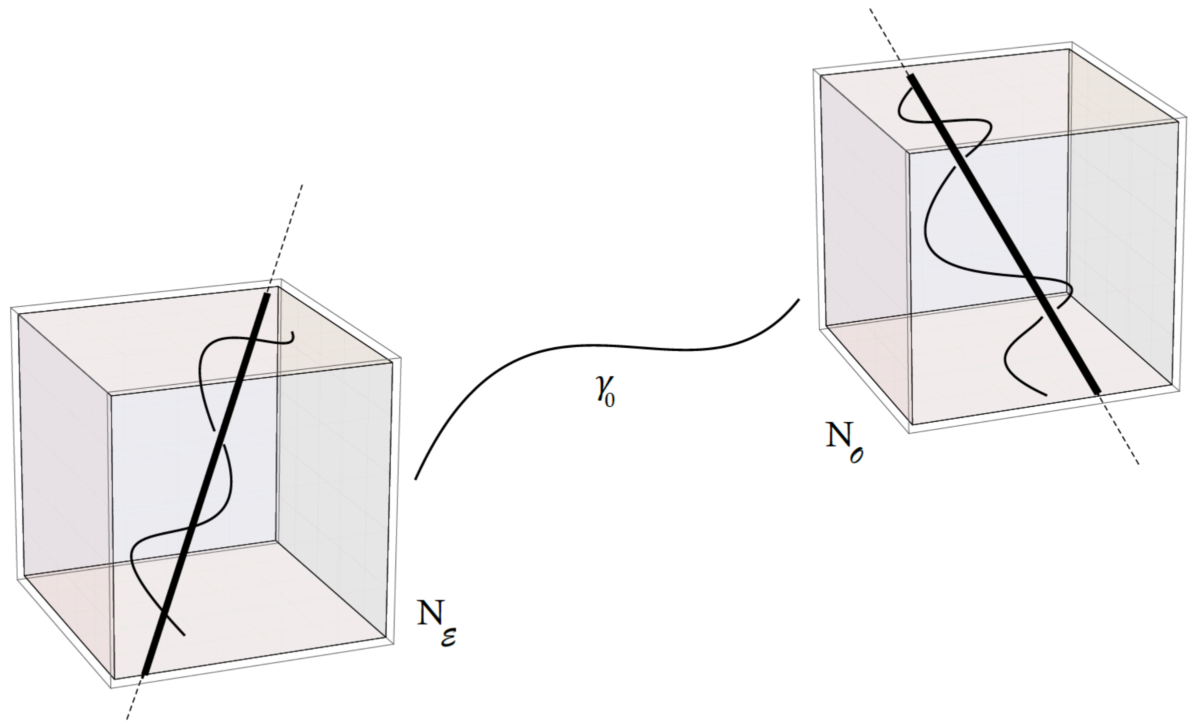
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Parallax in a general situation

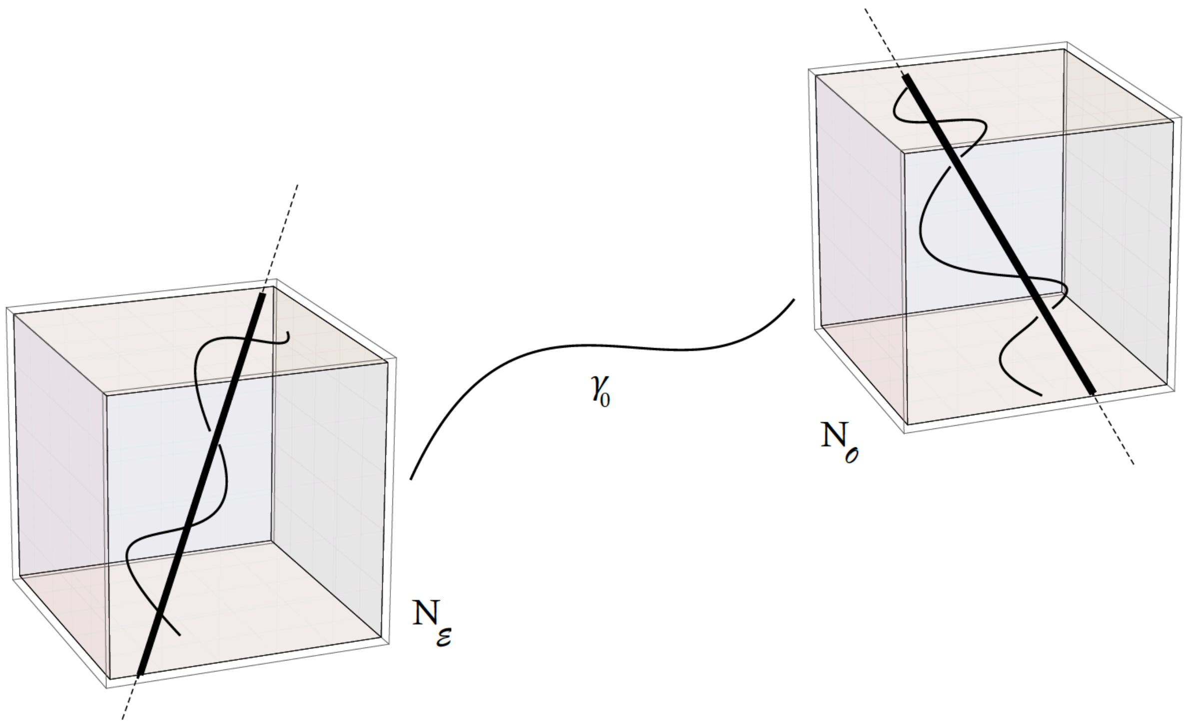


Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects

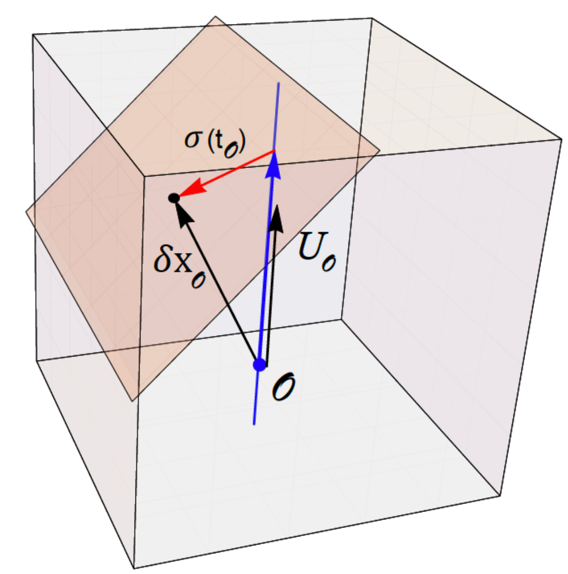
Parallax in a general situation



Both observer and emitter in bound systems

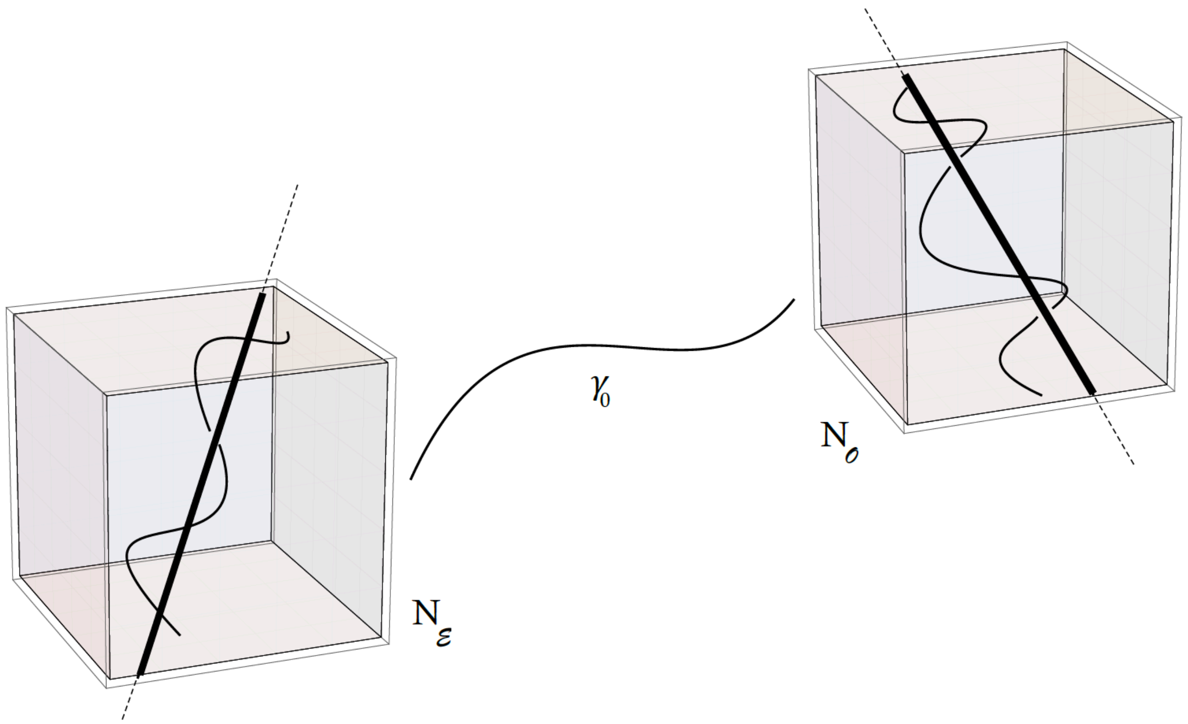
Barycenters in free fall

Question: parallax without the aberration effects



$$\delta x^\mu_{\mathcal{O}} = U^\mu_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^\mu(t_{\mathcal{O}}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

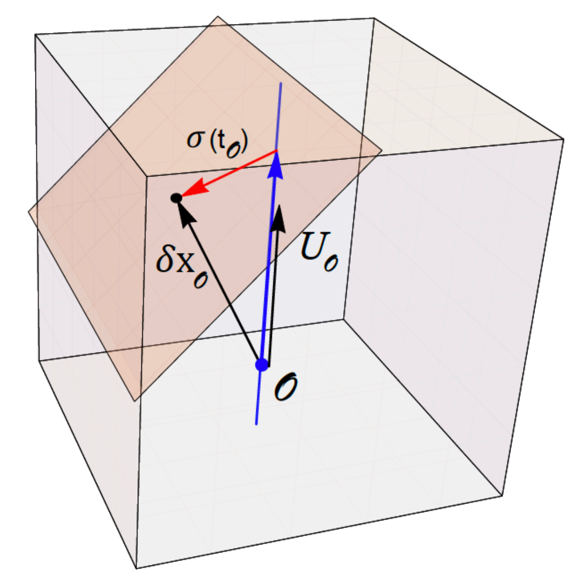
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

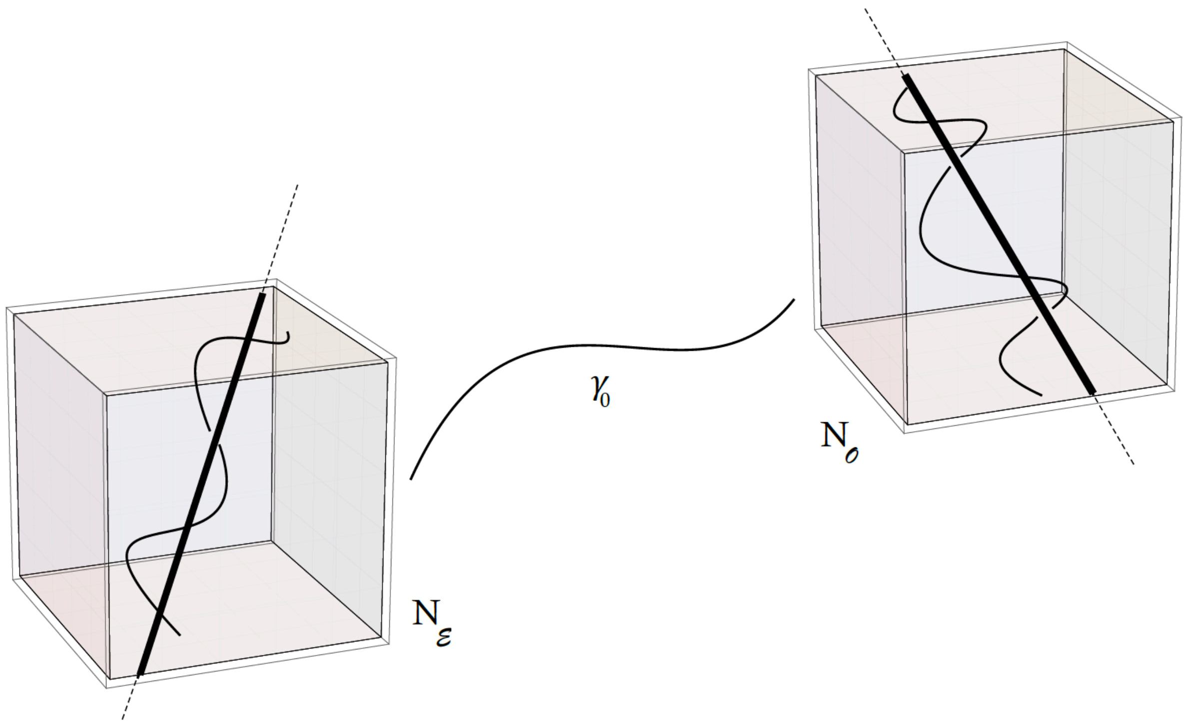
Question: parallax without the aberration effects



$$\delta x^\mu_{\mathscr{O}} = U^\mu_{\mathscr{O}}\, t_{\mathscr{O}} + \sigma^\mu(t_{\mathscr{O}}) \qquad \sigma^\mu\, l_{\mathscr{O}\,\mu} = 0$$

$$\delta x^\mu_{\mathscr{E}} = U^\mu_{\mathscr{E}}\, t_{\mathscr{E}} + \rho^\mu(t_{\mathscr{E}}) \qquad \rho^\mu\, l_{\mathscr{E}\,\mu} = 0$$

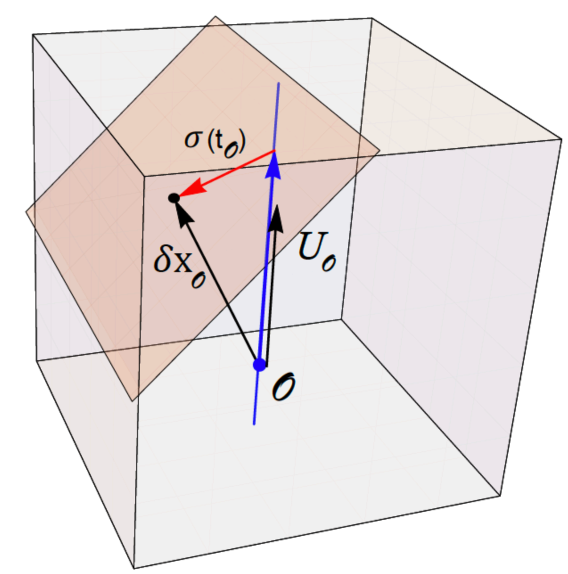
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects

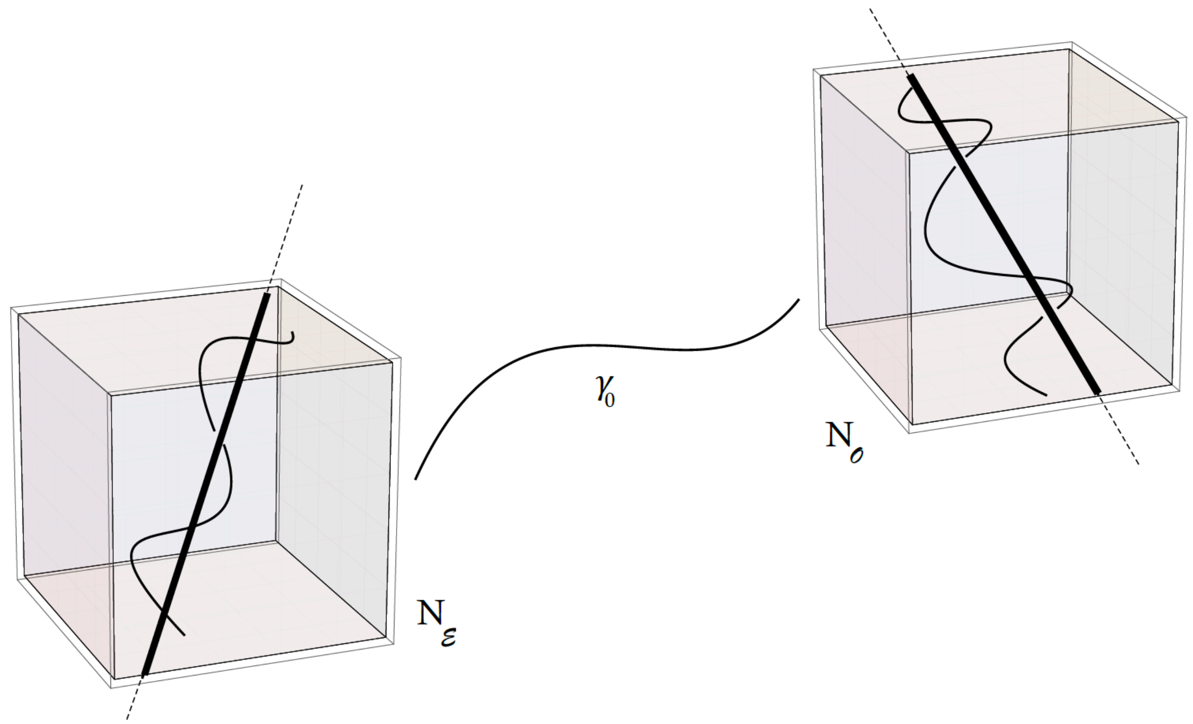


$$\delta x^\mu_{\mathcal{O}} = U^\mu_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^\mu(t_{\mathcal{O}}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^\mu_{\mathcal{E}} = U^\mu_{\mathcal{E}}\, t_{\mathcal{E}} + \rho^\mu(t_{\mathcal{E}}) \qquad \rho^\mu\, l_{\mathcal{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_{\mathcal{O}} r^A\, t_{\mathcal{O}} + M^A{}_B\, \rho^B\left((1+z)^{-1}\, t_{\mathcal{O}}\right) - \Pi^A{}_B\, \sigma^B(t_{\mathcal{O}})$$

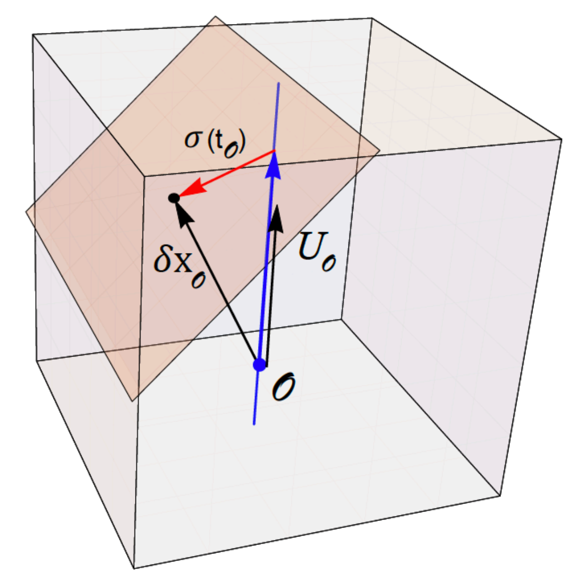
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects



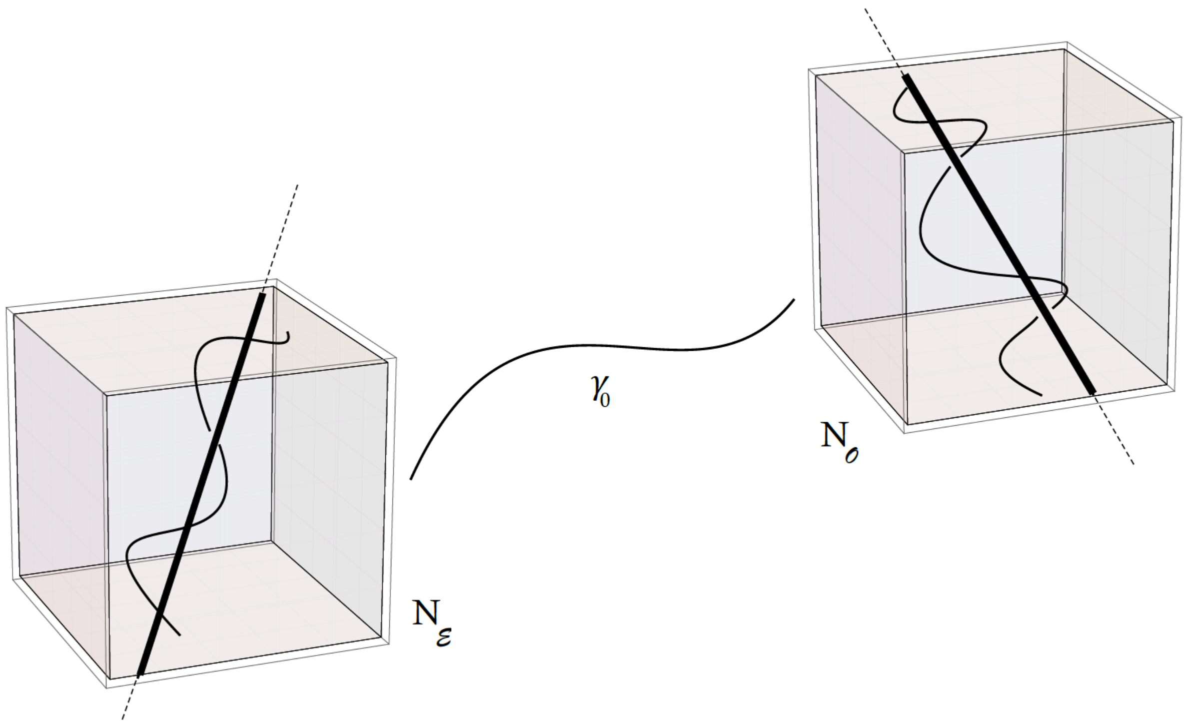
$$\delta x^\mu_{\mathcal{O}} = U^\mu_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^\mu(t_{\mathcal{O}}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^\mu_{\mathcal{E}} = U^\mu_{\mathcal{E}}\, t_{\mathcal{E}} + \rho^\mu(t_{\mathcal{E}}) \qquad \rho^\mu\, l_{\mathcal{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_{\mathcal{O}} r^A\, t_{\mathcal{O}} + M^A{}_B\, \rho^B\left((1+z)^{-1}\, t_{\mathcal{O}}\right) - \Pi^A{}_B\, \sigma^B(t_{\mathcal{O}})$$

barycenter drift (linear)

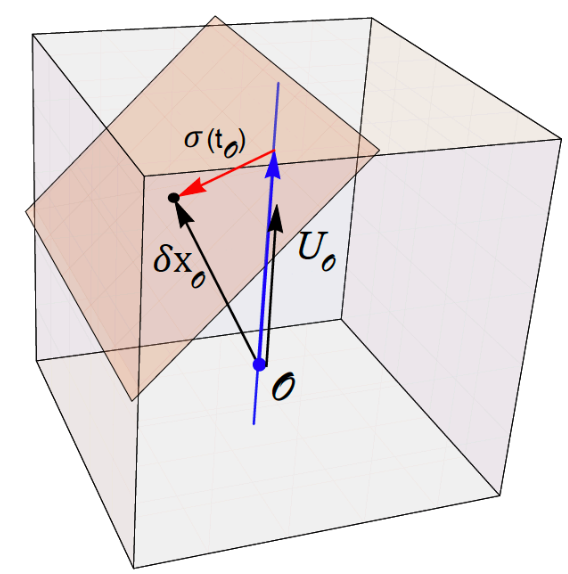
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the aberration effects



$$\delta x^\mu_{\mathcal{O}} = U^\mu_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^\mu(t_{\mathcal{O}}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^\mu_{\mathcal{E}} = U^\mu_{\mathcal{E}}\, t_{\mathcal{E}} + \rho^\mu(t_{\mathcal{E}}) \qquad \rho^\mu\, l_{\mathcal{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_{\mathcal{O}} r^A\, t_{\mathcal{O}} + M^A{}_B\, \rho^B\left((1+z)^{-1}\, t_{\mathcal{O}}\right) - \Pi^A{}_B\, \sigma^B(t_{\mathcal{O}})$$

barycenter drift (linear)

parallax (periodic)

Motions-independent observables

$$z \equiv z(u^\mu_\mathscr{O}, u^\mu_\mathscr{E})$$

$$r^\mu \equiv r^\mu(u^\mu_\mathscr{O})$$

$$M^A{}_B \equiv M^A{}_B \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O} \right)$$

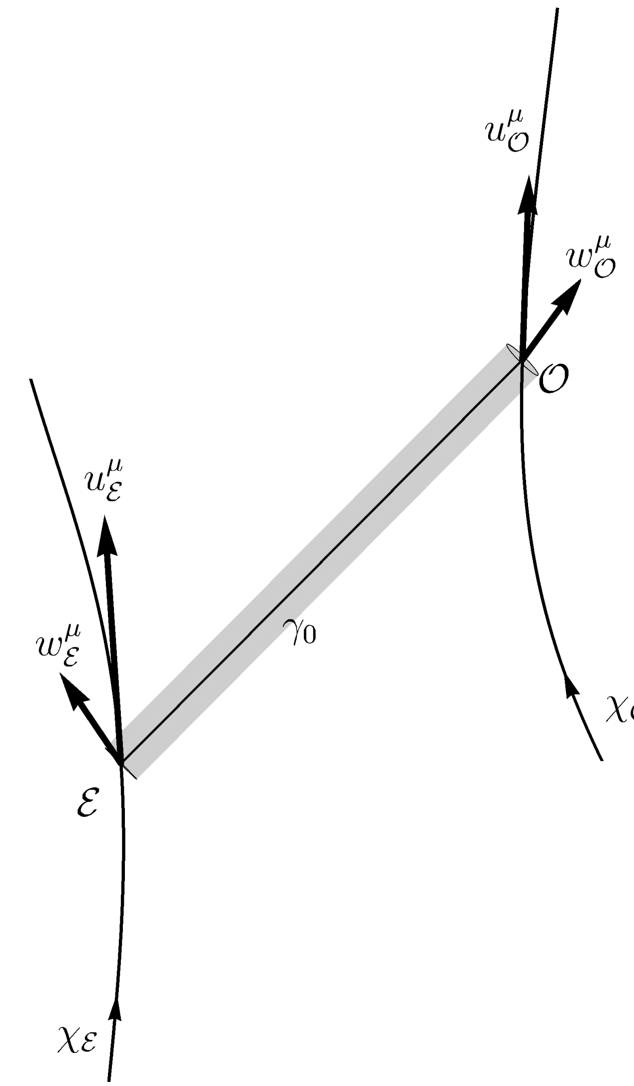
$$\Pi^A{}_B \equiv \Pi^A{}_B \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O} \right)$$

$$D_{ang} \equiv D_{ang} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O} \right)$$

$$D_{par} \equiv D_{par} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O} \right)$$

$$\delta_\mathscr{O} r^A \equiv \delta_\mathscr{O} r^A \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}, u^\mu_\mathscr{E}, w^\mu_\mathscr{O} \right)$$

$$\frac{dz}{d\tau_\mathscr{O}} \equiv \frac{dz}{d\tau_\mathscr{O}} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}, u^\mu_\mathscr{E}, w^\mu_\mathscr{O}, w^\mu_\mathscr{E} \right)$$



Motions-independent observables

$z \equiv z(u^\mu_\mathcal{O}, u^\mu_\mathcal{E})$

$r^\mu \equiv r^\mu(u^\mu_\mathcal{O})$

$M^A{}_B \equiv M^A{}_B \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O} \right)$

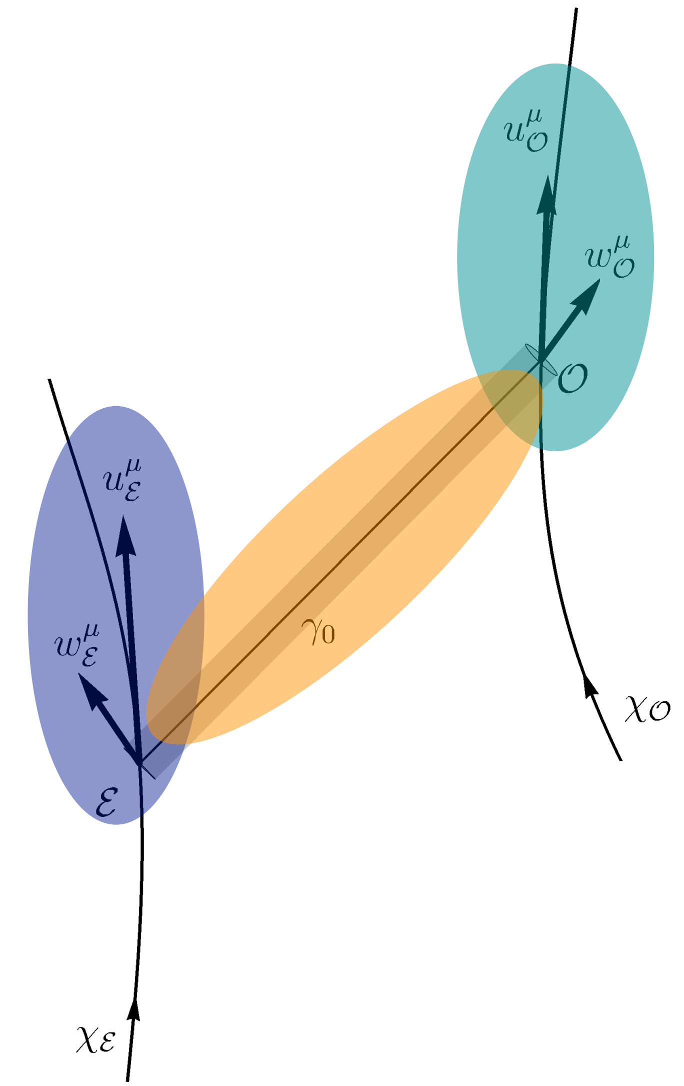
$\Pi^A{}_B \equiv \Pi^A{}_B \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O} \right)$

$D_{ang} \equiv D_{ang} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O} \right)$

$D_{par} \equiv D_{par} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O} \right)$

$\delta_\mathcal{O} r^A \equiv \delta_\mathcal{O} r^A \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O}, u^\mu_\mathcal{E}, w^\mu_\mathcal{O} \right)$

$\frac{dz}{d\tau_\mathcal{O}} \equiv \frac{dz}{d\tau_\mathcal{O}} \left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathcal{O}, u^\mu_\mathcal{E}, w^\mu_\mathcal{O}, w^\mu_\mathcal{E} \right)$



Motions-independent observables

$z \equiv z(u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}})$

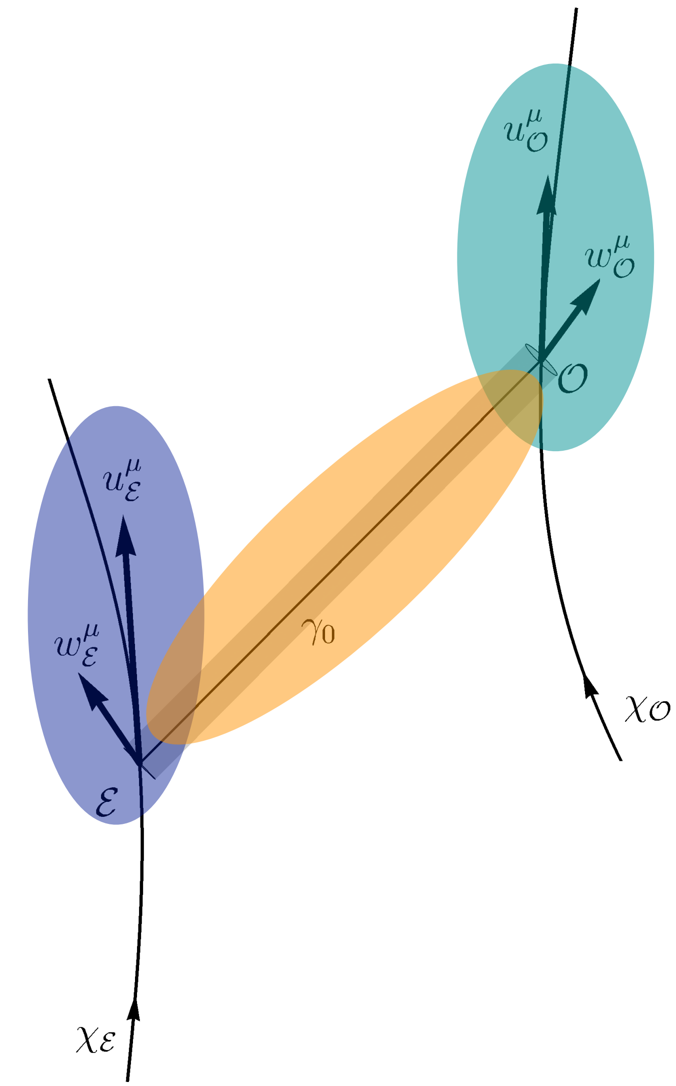
$r^\mu \equiv r^\mu(u^\mu_{\mathcal{O}})$

$M^A{}_B \equiv M^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$

$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$

$D_{ang} \equiv D_{ang}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$

$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$



$\delta_{\mathcal{O}} r^A \equiv \delta_{\mathcal{O}} r^A\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}\right)$

$w_\perp{}^A{}_B = M^{-1}{}^A{}_C\,\Pi^C{}_B$

$\dfrac{dz}{d\tau_{\mathcal{O}}} \equiv \dfrac{dz}{d\tau_{\mathcal{O}}}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}, w^\mu_{\mathcal{E}}\right)$

Motions-independent observables

$$z \equiv z(u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}})$$

$$r^\mu \equiv r^\mu(u^\mu_{\mathcal{O}})$$

$$M^A{}_B \equiv M^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

$$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

$$D_{ang} \equiv D_{ang}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

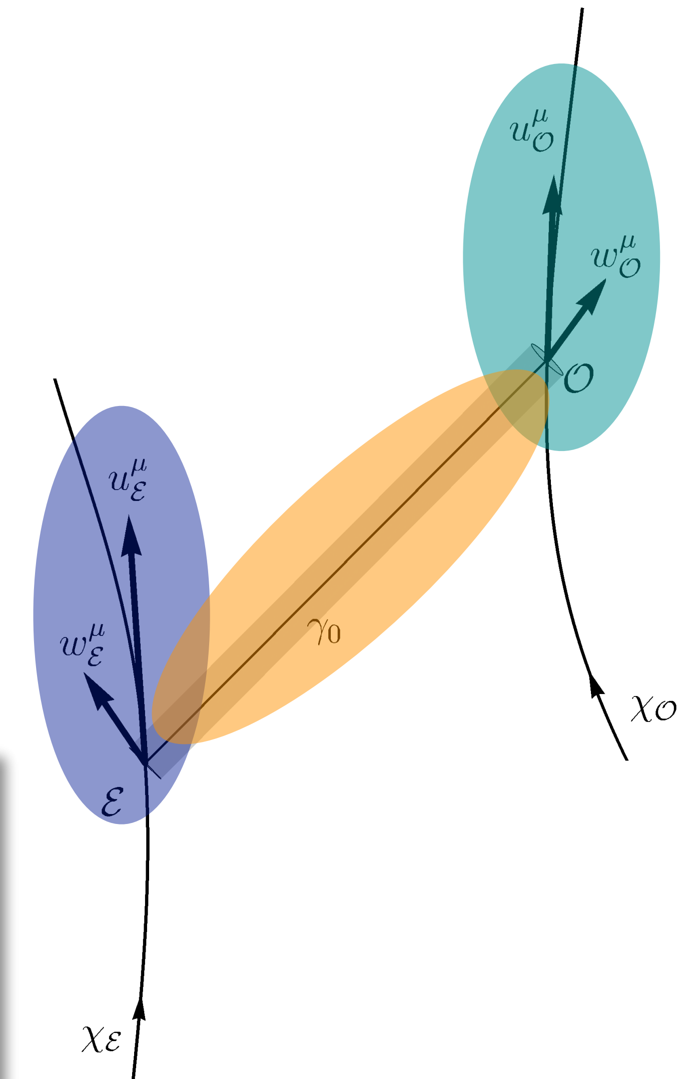
$$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

$$\delta_{\mathcal{O}} r^A \equiv \delta_{\mathcal{O}} r^A\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}\right)$$

$$\frac{dz}{d\tau_{\mathcal{O}}} \equiv \frac{dz}{d\tau_{\mathcal{O}}}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}, w^\mu_{\mathcal{E}}\right)$$

$$M^A{}_B = \frac{1}{u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}}\, \mathcal{D}^{-1\,A}{}_B$$

$$\Pi^A{}_B = \frac{1}{u^\sigma_{\mathcal{O}}\, l_{\mathcal{O}\,\sigma}}\, \mathcal{D}^{-1\,A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right)$$



$$w_\perp{}^A{}_B = M^{-1\,A}{}_C\, \Pi^C{}_B$$

Motions-independent observables

$z \equiv z(u^\mu_\mathscr{O}, u^\mu_\mathscr{E})$

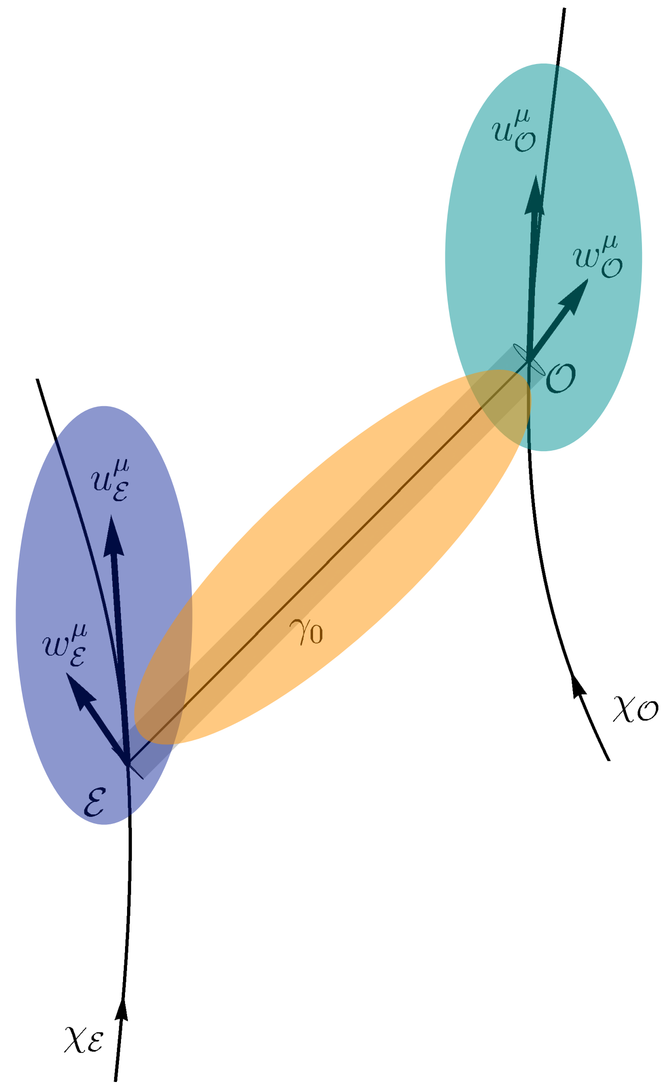
$r^\mu \equiv r^\mu(u^\mu_\mathscr{O})$

$M^A{}_B \equiv M^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}\right)$

$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}\right)$

$D_{ang} \equiv D_{ang}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}\right)$

$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}\right)$



$\delta_\mathscr{O} r^A \equiv \delta_\mathscr{O} r^A\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}, u^\mu_\mathscr{E}, w^\mu_\mathscr{O}\right)$

$w_\perp{}^A{}_B = M^{-1}{}^A{}_C\,\Pi^C{}_B$

$\frac{dz}{d\tau_\mathscr{O}} \equiv \frac{dz}{d\tau_\mathscr{O}}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_\mathscr{O}, u^\mu_\mathscr{E}, w^\mu_\mathscr{O}, w^\mu_\mathscr{E}\right)$

Motions-independent observables

$$z \equiv z(u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}})$$

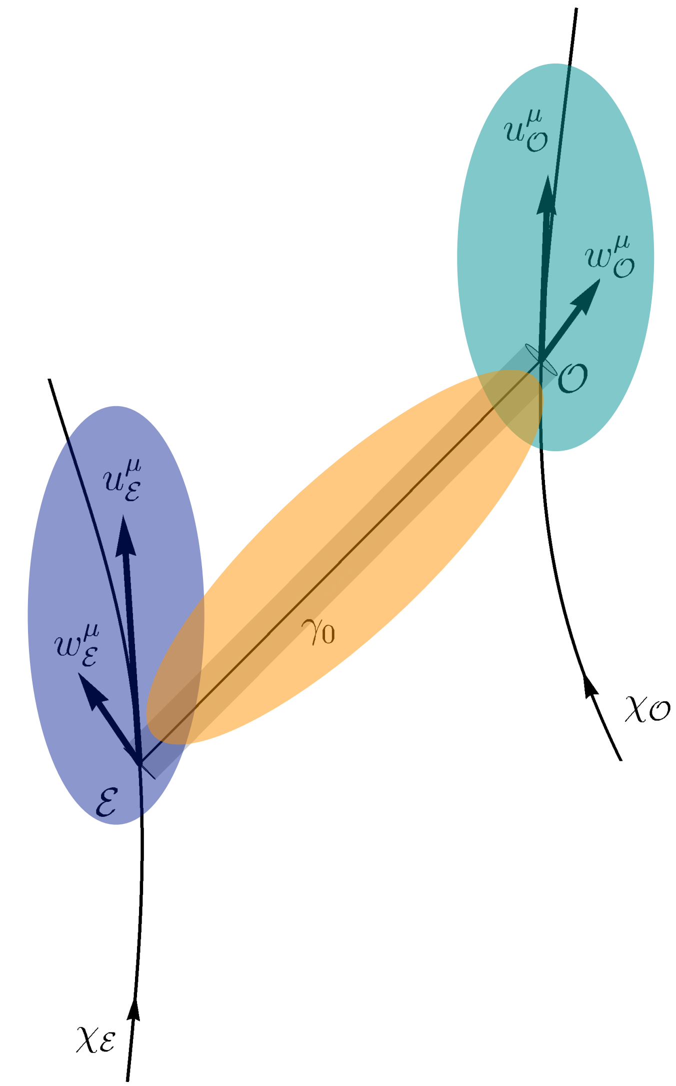
$$r^\mu \equiv r^\mu(u^\mu_{\mathcal{O}})$$

$$M^A{}_B \equiv M^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}} \right)$$

$$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}} \right)$$

$$D_{ang} \equiv D_{ang}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}} \right)$$

$$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}} \right)$$



$$\delta_{\mathcal{O}} r^A \equiv \delta_{\mathcal{O}} r^A\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}} \right)$$

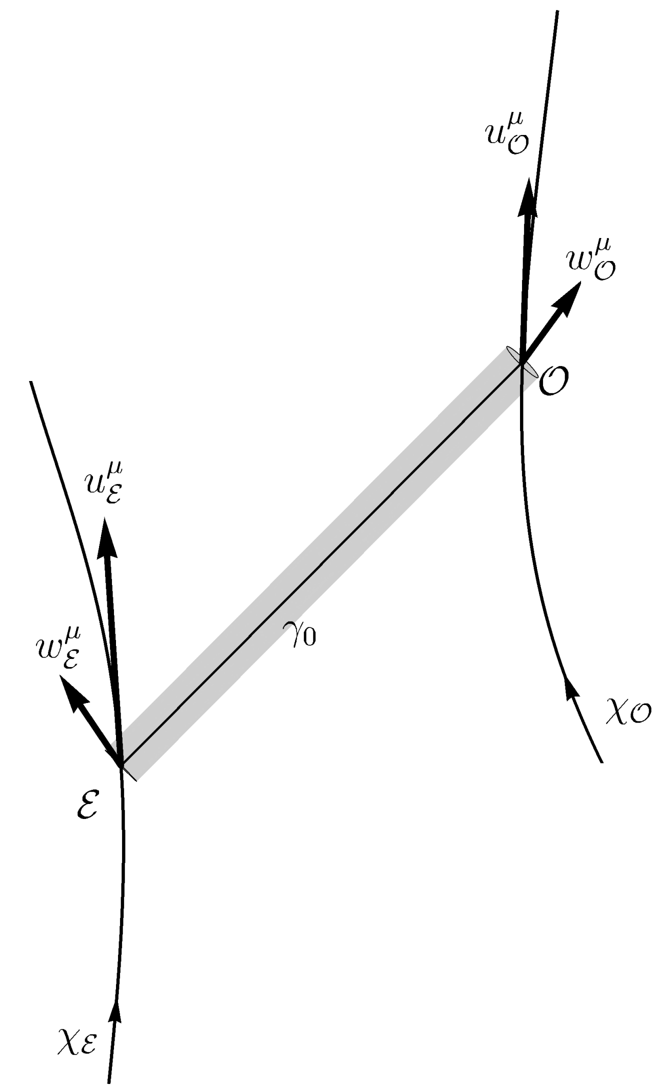
$$\frac{dz}{d\tau_{\mathcal{O}}} \equiv \frac{dz}{d\tau_{\mathcal{O}}}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}, w^\mu_{\mathcal{E}} \right)$$

$$w_\perp{}^A{}_B = M^{-1\,A}{}_C\,\Pi^C{}_B$$

$$w_\perp{}^A{}_B \equiv w_\perp{}^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0} \right) = \delta^A{}_B + m_\perp{}^A{}_B$$

Parameter μ

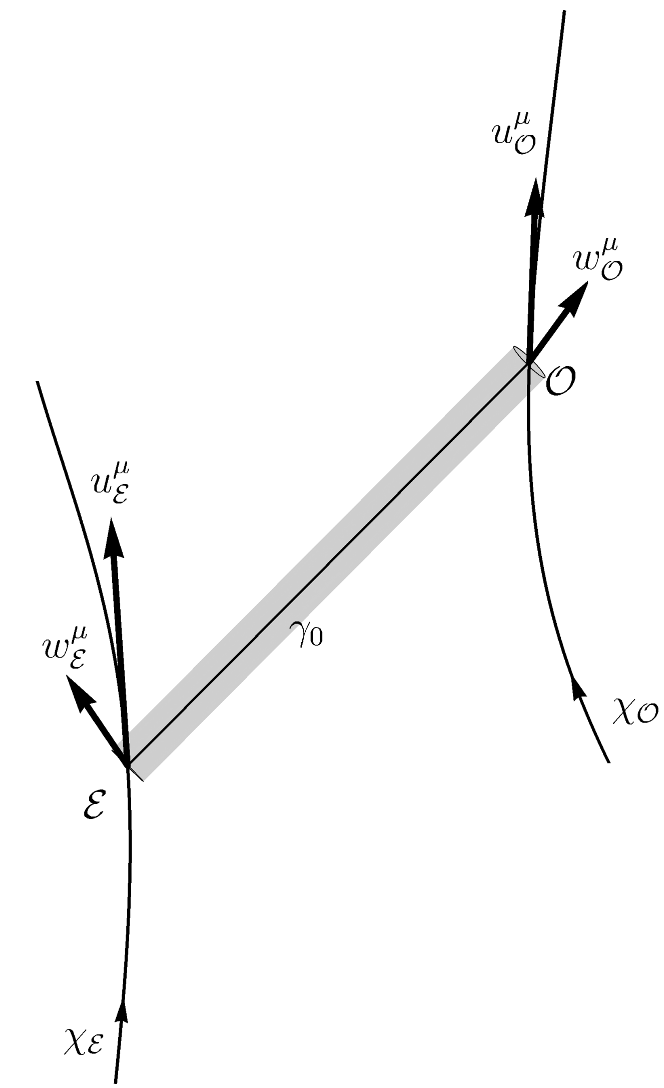
$$\mu = 1 - \det w_\perp{}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$



Parameter μ

$$\mu = 1 - \det w_\perp{}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$

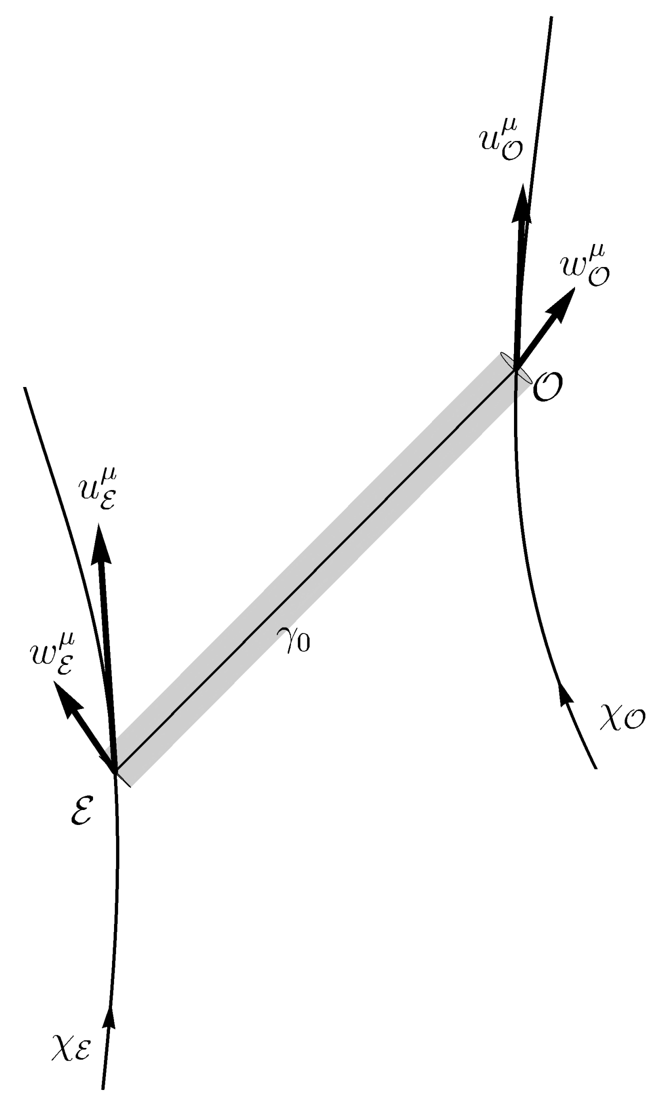
- scalar, dimensionless



Parameter μ

$$\mu = 1 - \det {w_\perp}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$

- scalar, dimensionless
- momentary motions-independent

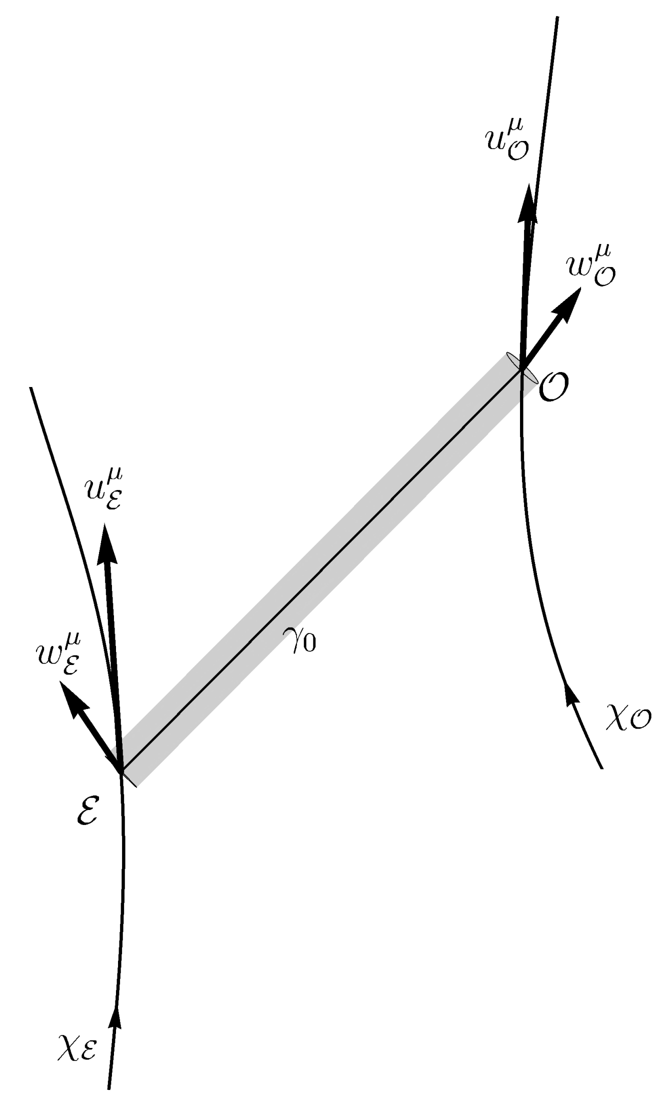


Parameter μ

$$\mu = 1 - \det w_\perp{}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$

- scalar, dimensionless
- momentary motions-independent
- (theoretically) simple to measure

$$\mu = 1 \mp \frac{D_{ang}^2}{D_{par}^2}$$

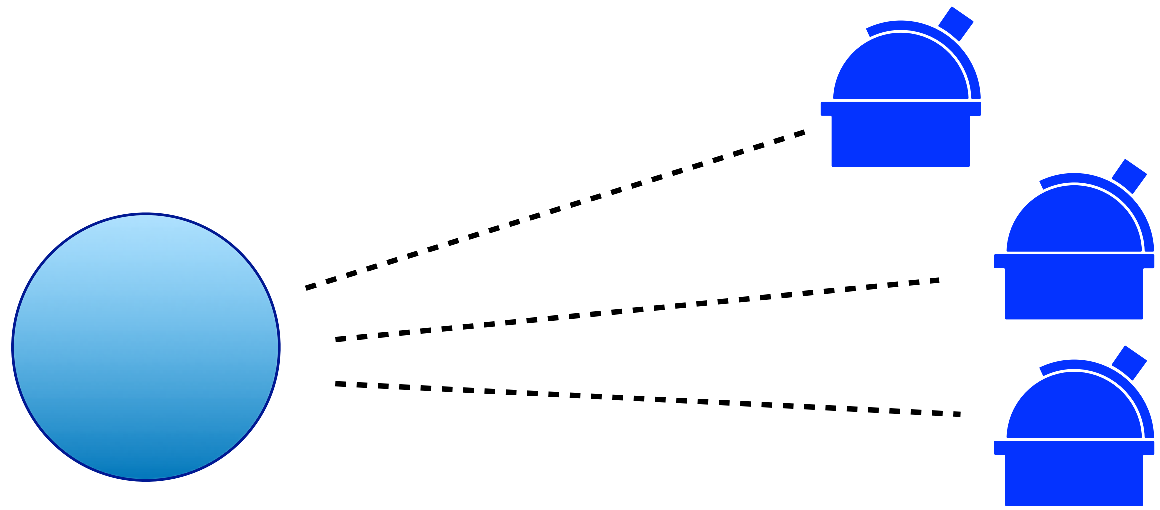


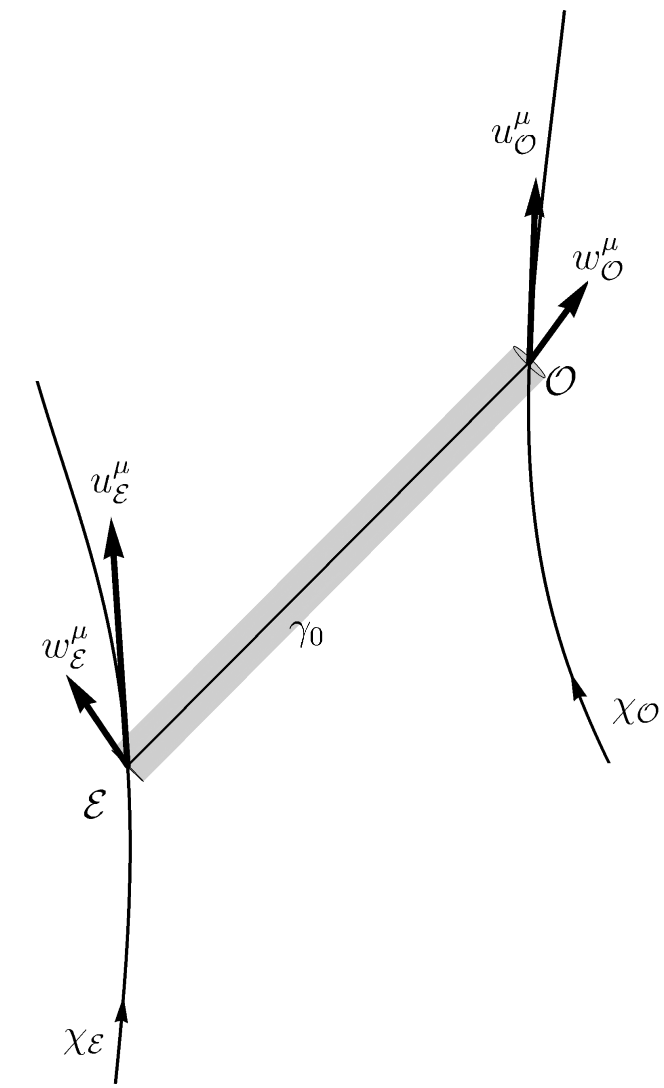
Parameter μ

$$\mu = 1 - \det {w_\perp}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$

- scalar, dimensionless
- momentary motions-independent
- (theoretically) simple to measure

$$\mu = 1 \mp \frac{D^2_{ang}}{D^2_{par}}$$



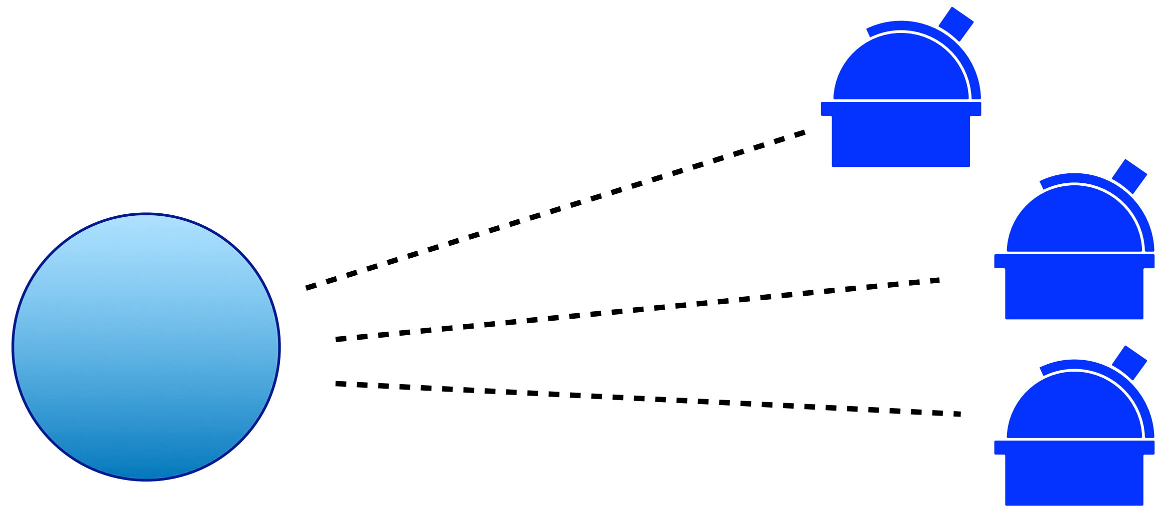


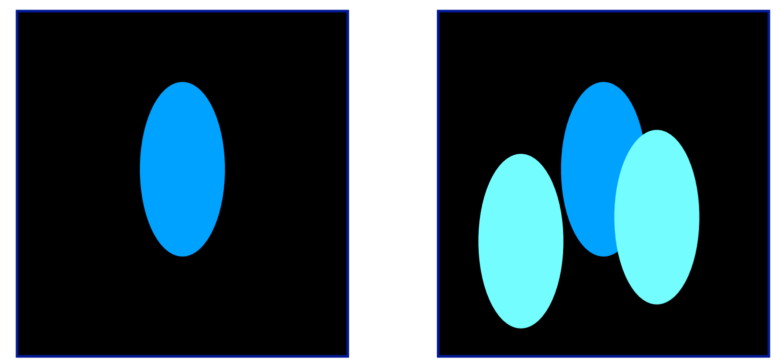
Parameter μ

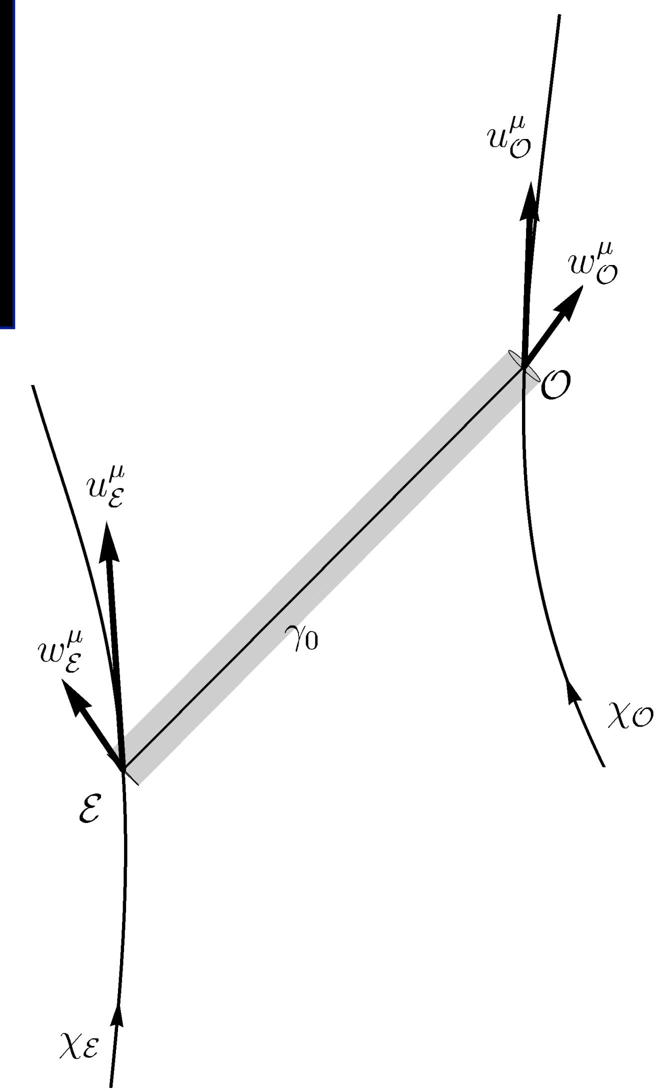
$$\mu = 1 - \det {w_\perp}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$

- scalar, dimensionless
- momentary motions-independent
- (theoretically) simple to measure

$$\mu = 1 \mp \frac{D^2_{ang}}{D^2_{par}}$$

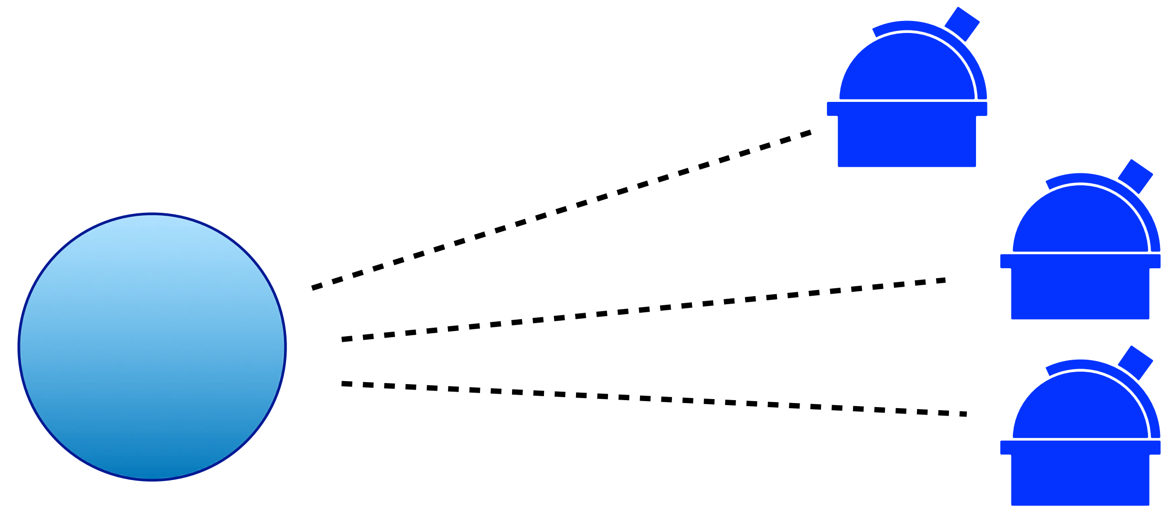






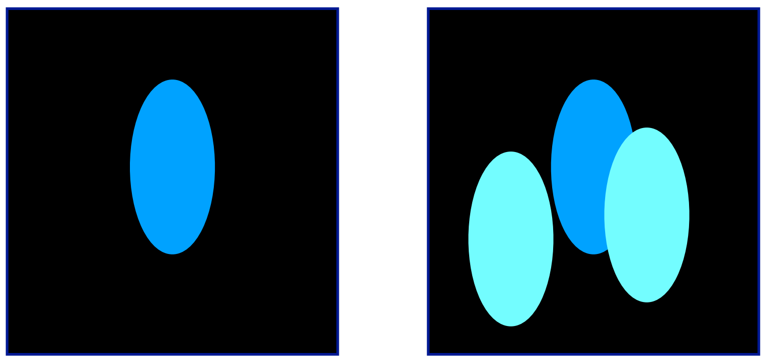
Parameter μ

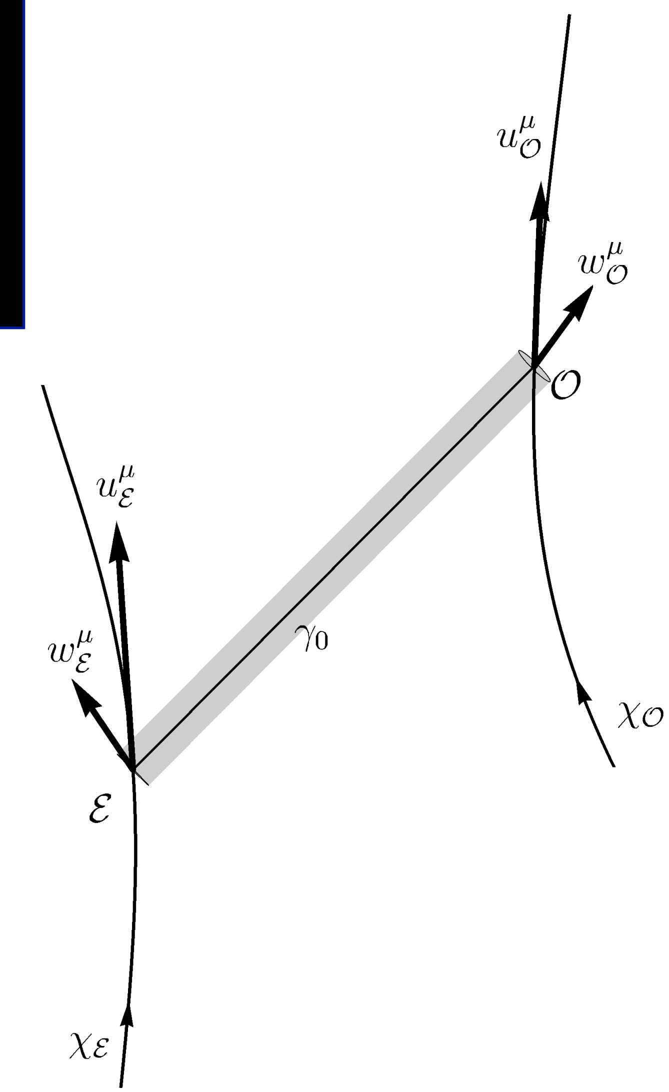
$$\mu = 1 - \det {w_\perp}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$



- scalar, dimensionless
- momentary motions-independent
- (theoretically) simple to measure

$$\mu = 1 \mp \frac{D^2_{ang}}{D^2_{par}}$$





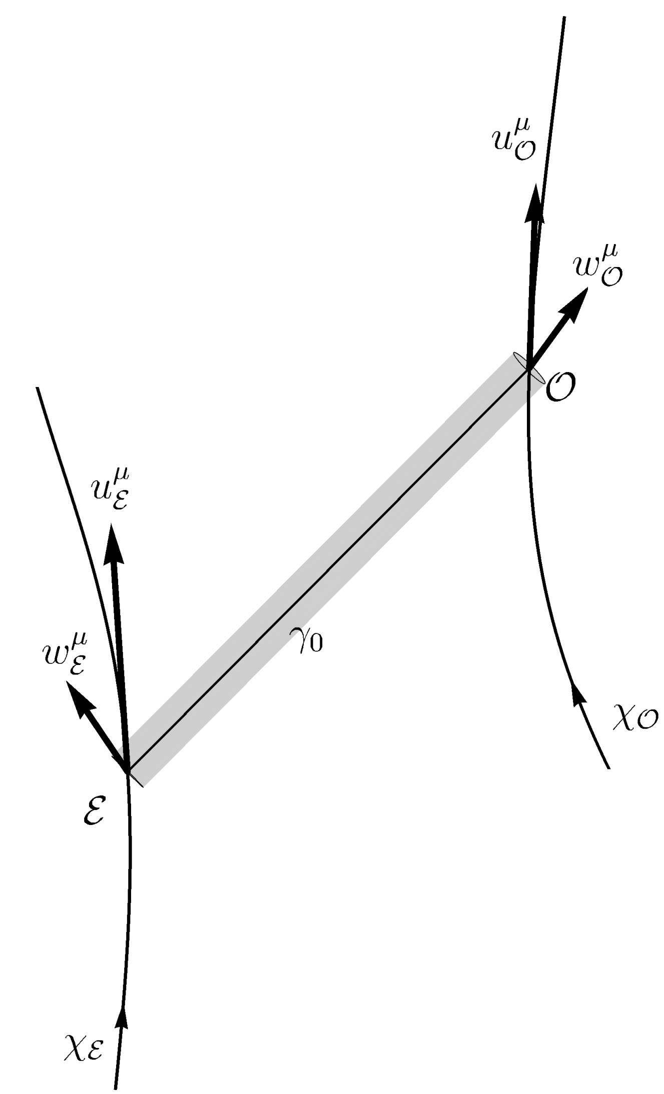
- no need to measure the parallel transport independently

$${m_\perp}^A{}_B = {w_\perp}^A{}_B - \delta^A{}_B$$

parallel transport of perpendicular vectors

Parameter μ

$$\mu = 1 - \det w_{\perp}{}^{A}{}_{B} = 1 - \frac{\det \Pi^{A}{}_{B}}{\det M^{A}{}_{B}} = 1 \mp \frac{D_{ang}^{2}}{D_{par}^{2}}$$



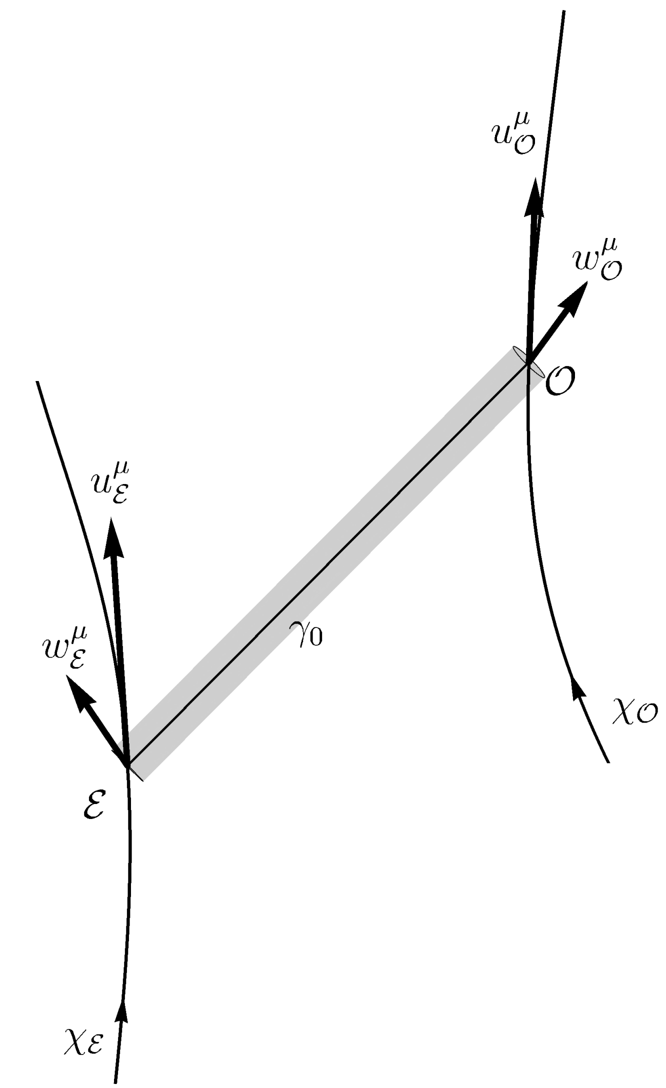
Parameter μ

$$\mu = 1 - \det w_{\perp}{}^{A}{}_{B} = 1 - \frac{\det \Pi^{A}{}_{B}}{\det M^{A}{}_{B}} = 1 \mp \frac{D^2_{ang}}{D^2_{par}}$$

- curvature detector

$$\mu = 1 - \det\left(\delta^{A}{}_{B} + m_{\perp}{}^{A}{}_{B}\right)$$

$$R^{\mu}{}_{\nu\alpha\beta}\Big|_{\gamma_0}$$



Parameter μ

$$\mu = 1 - \det {w_\perp}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B} = 1 \mp \frac{D_{ang}^2}{D_{par}^2}$$

- curvature detector

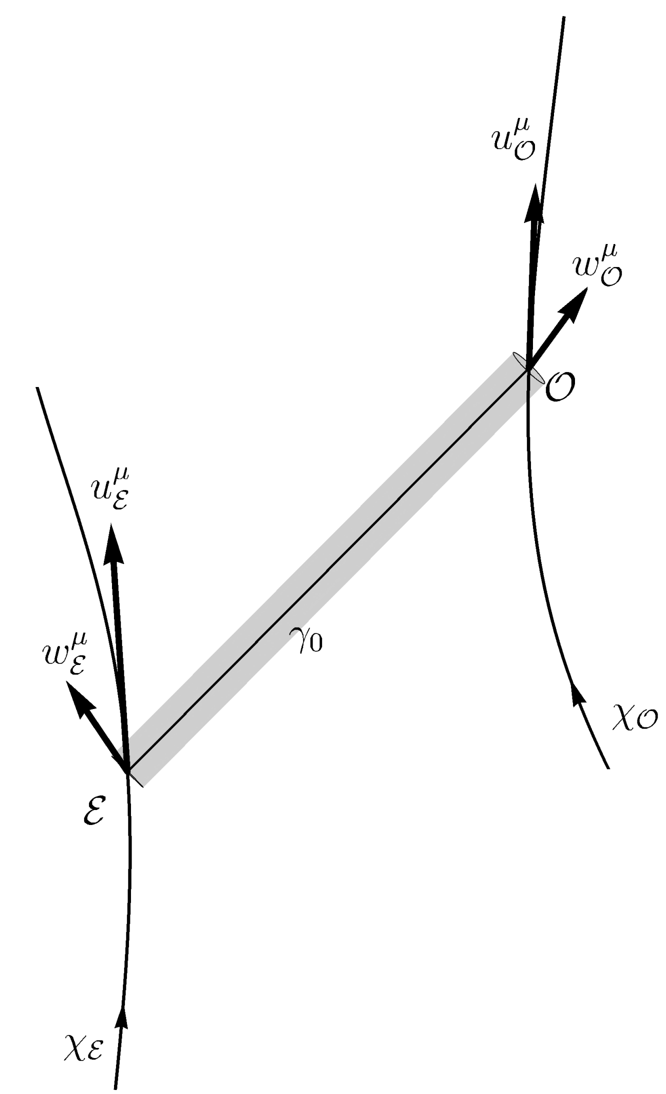
$$\mu = 1 - \det\left(\delta^A{}_B + {m_\perp}^A{}_B\right)$$

$$R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}$$

flat space:

$$D_{ang} = D_{par} = D_{\mathcal{O}}$$

$$\mu = 0$$



Parameter μ

$$\mu = 1 - \det w_\perp{}^A{}_B = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B} = 1 \mp \frac{D_{ang}^2}{D_{par}^2}$$

- curvature detector

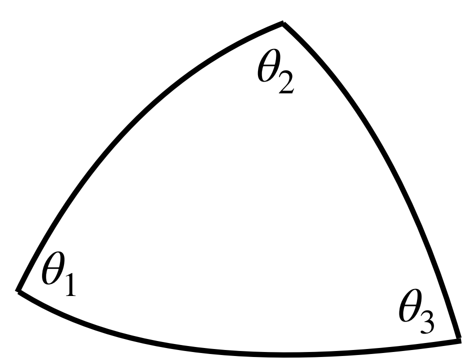
$$\mu = 1 - \det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)$$

$$R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}$$

flat space:

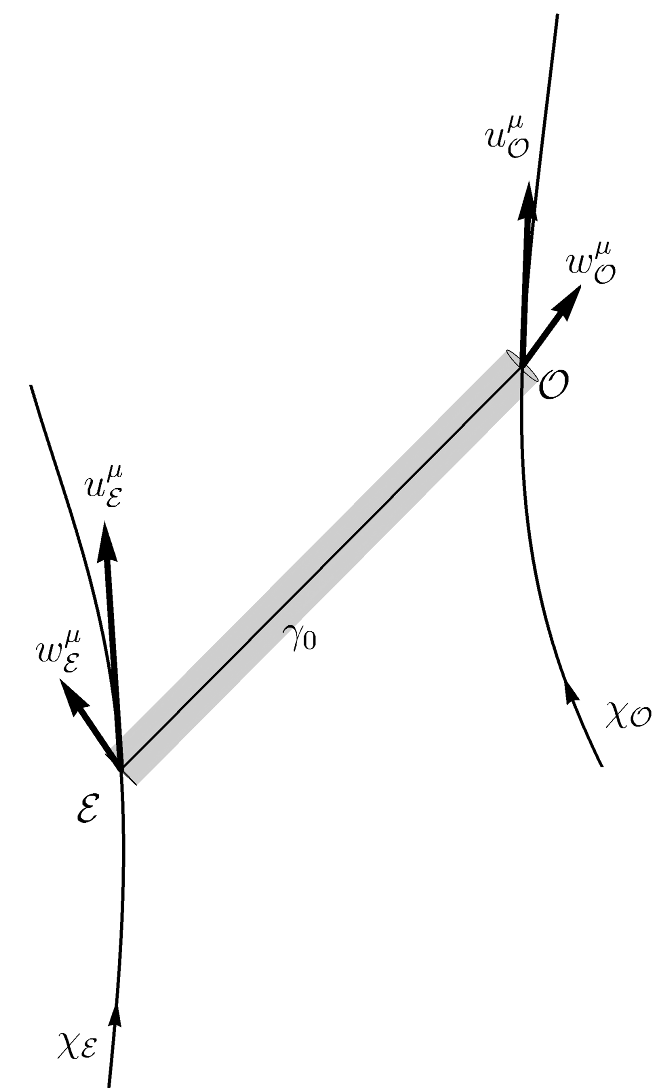
$$D_{ang} = D_{par} = D_{\mathcal{O}}$$

$$\mu = 0$$



compare the angular deficit formula

$$\theta_1 + \theta_2 + \theta_3 - \pi = \int K \, d^2A$$



Parameter μ

spacetime geometry $g_{\mu\nu}(x), \Gamma^{\mu}{}_{\alpha\beta}(x)$

observation and emission points along a null geodesic $\gamma_0^{\mu}(\lambda), \mathcal{O}, \mathcal{E}$

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

(null) geodesic deviation equation

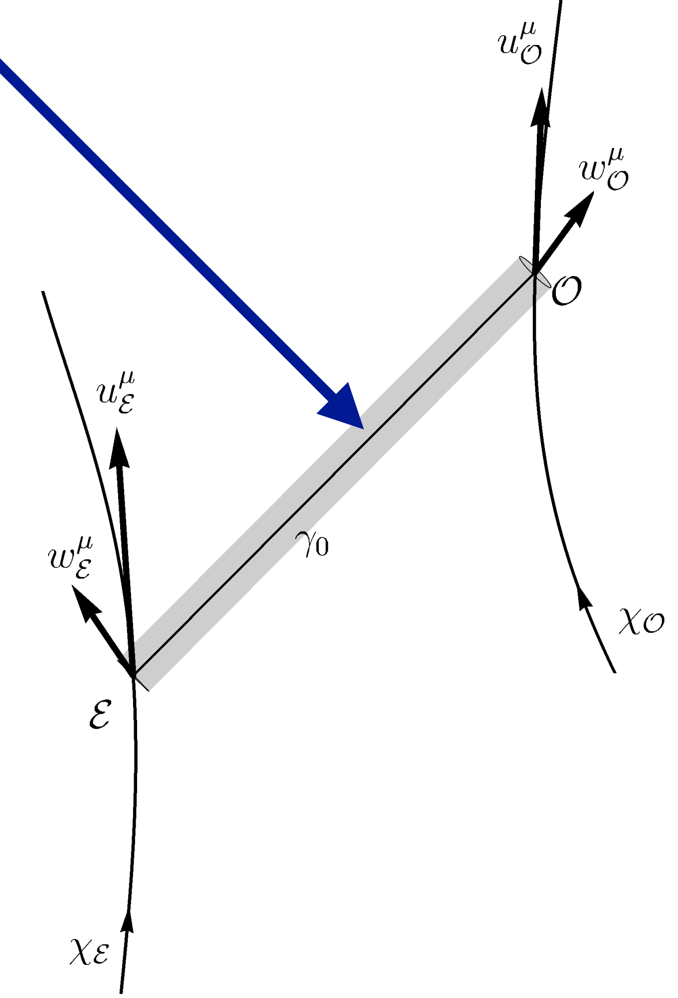
bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}$

covariant expressions for observables

momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathcal{O}}, \delta x^{\mu}_{\mathcal{E}}, u^{\mu}_{\mathcal{O}}, u^{\mu}_{\mathcal{E}}, w^{\mu}_{\mathcal{O}}, w^{\mu}_{\mathcal{E}}$

observables $r^{\mu}, z, \delta_{\mathcal{O}} r^{A}, \delta_{\mathcal{O}} z, \ldots$

$g_{\mu\nu}$ = **[flat] + [curvature corrections]** $\left(R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}\right) + h.o.t.$



Parameter μ

spacetime geometry $g_{\mu\nu}(x), \Gamma^{\mu}{}_{\alpha\beta}(x)$

↓

observation and emission points along a null geodesic $\gamma_0^{\mu}(\lambda), \mathscr{O}, \mathscr{E}$

↓

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

↓ *(null) geodesic deviation equation*

bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}$

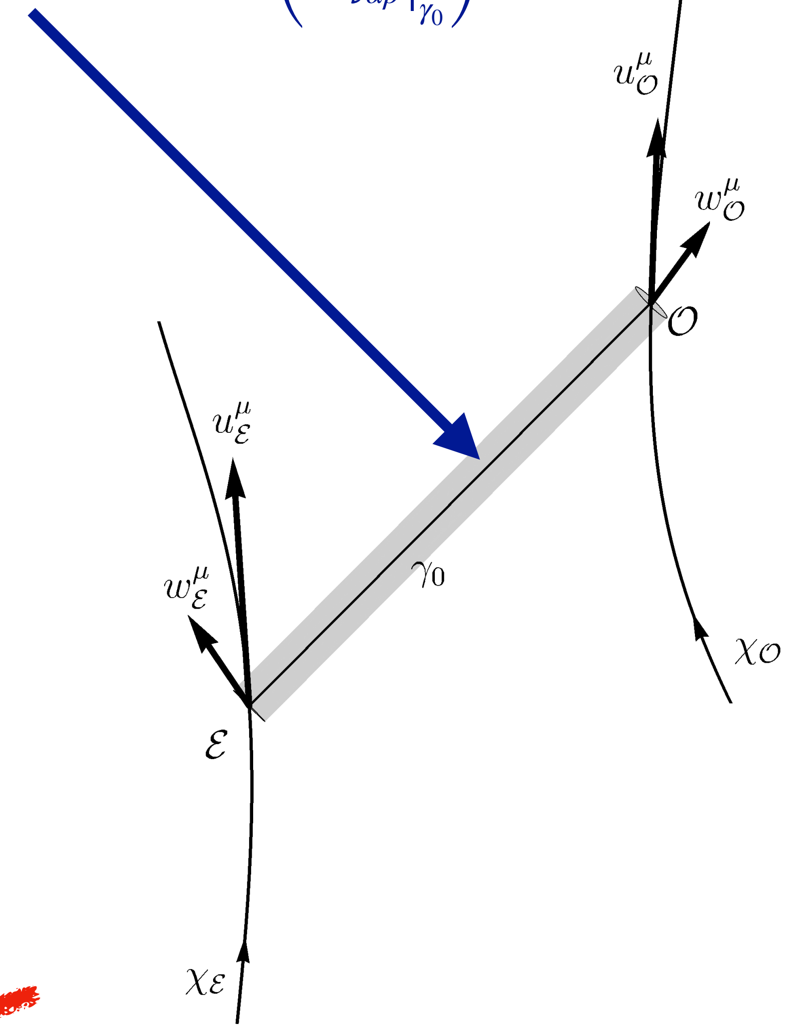
↓ *covariant expressions for observables*

~~momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathscr{O}}, \delta x^{\mu}_{\mathscr{E}}, u^{\mu}_{\mathscr{O}}, u^{\mu}_{\mathscr{E}}, w^{\mu}_{\mathscr{O}}, w^{\mu}_{\mathscr{E}}$~~

↓

observables $r^{\mu}, z, \delta_{\mathscr{O}} r^{A}, \delta_{\mathscr{O}} z, \dots$

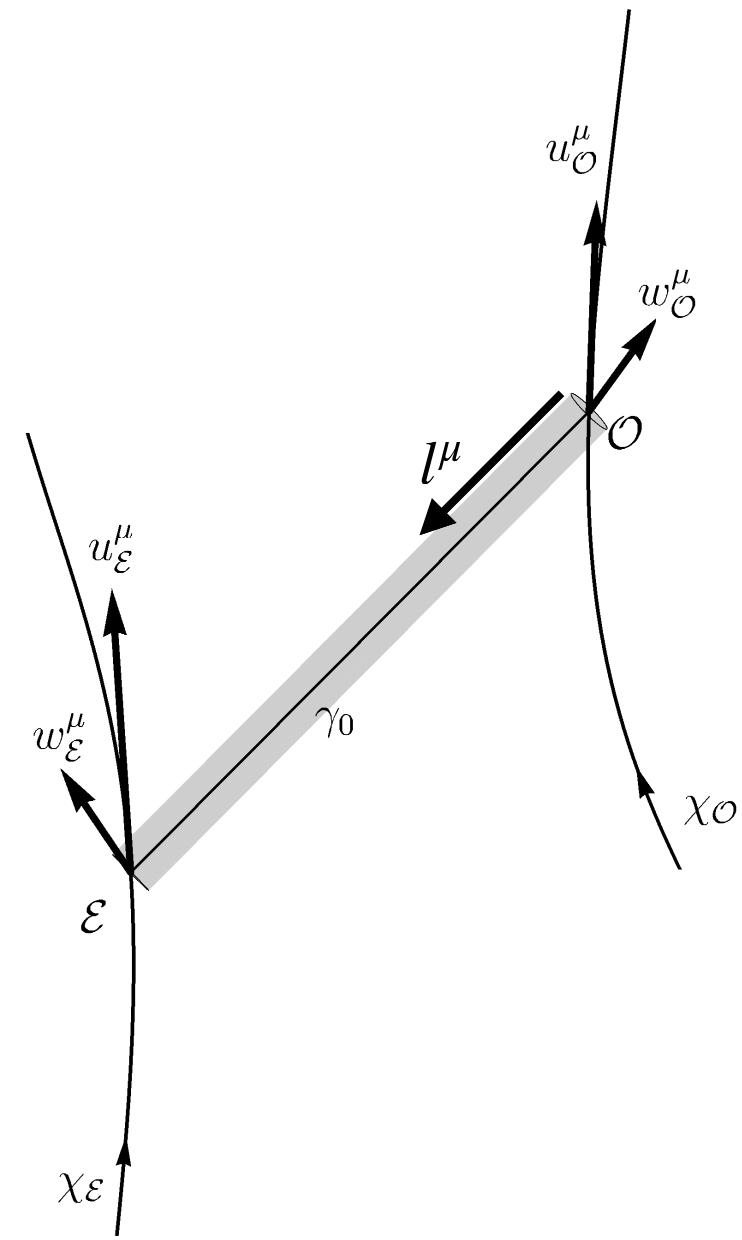
$g_{\mu\nu}$ = **[flat]** + **[curvature corrections]** $\left(R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}\right) + h.o.t.$



Parameter μ

Approximate formula for short distances/small curvature

Linearisation in the curvature of all expressions…

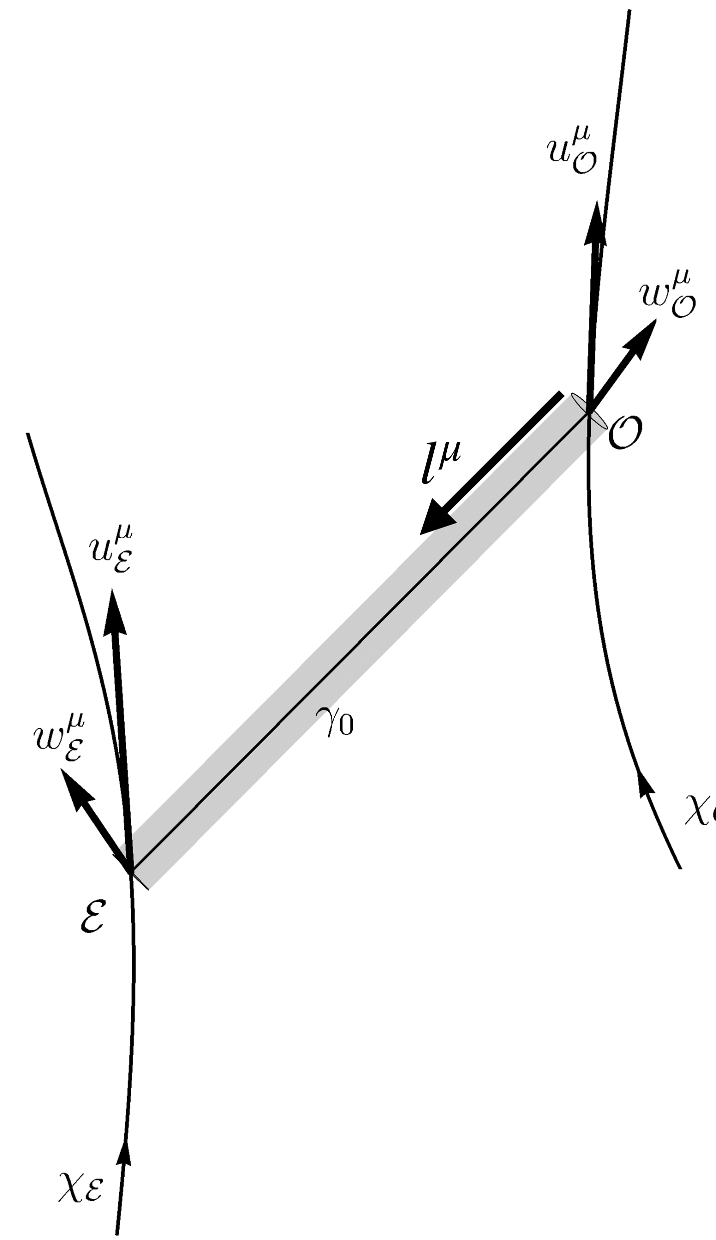


Parameter μ

Approximate formula for short distances/small curvature

Linearisation in the curvature of all expressions…

$$\mu \approx -\, {m_\perp}^{A}{}_{A} \approx -\int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} R^{A}{}_{llA}(\lambda)\,(\lambda_{\mathcal{E}}-\lambda)\,d\lambda$$



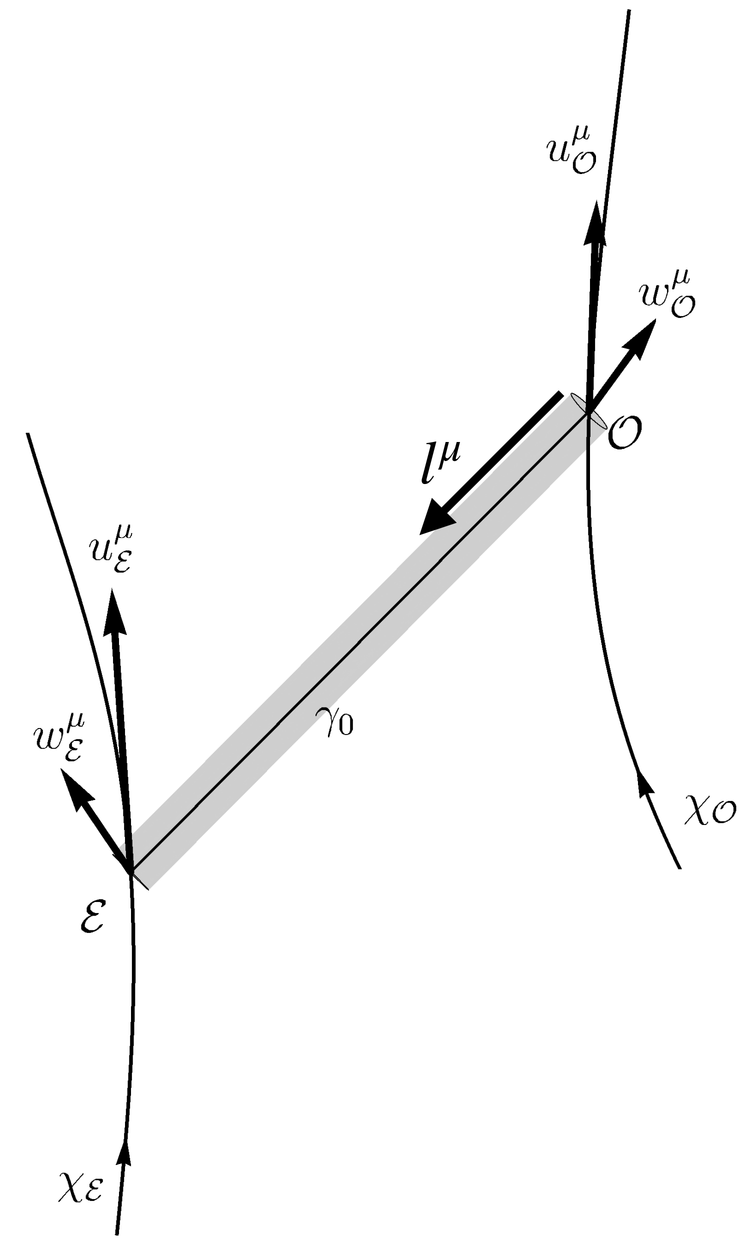
Parameter μ

Approximate formula for short distances/small curvature

Linearisation in the curvature of all expressions…

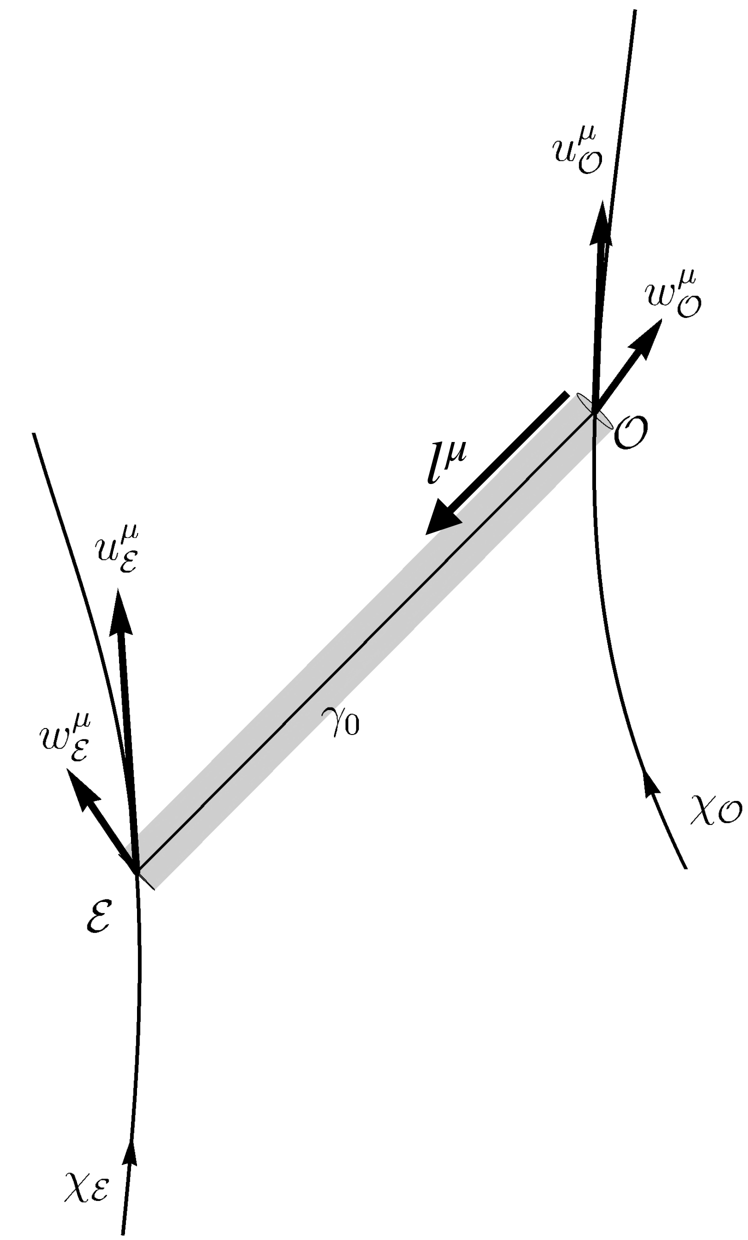
$$\mu \approx -{m_\perp}^A{}_A \approx -\int_{\lambda_\mathcal{O}}^{\lambda_\mathcal{E}} R^A{}_{llA}(\lambda)\,(\lambda_\mathcal{E} - \lambda)\,d\lambda$$

a bit of tensor manipulation…



Parameter μ

Approximate formula for short distances/small curvature



Linearisation in the curvature of all expressions…

$$\mu \approx -\, {m_\perp}^{A}{}_{A} \approx -\int_{\lambda_\mathcal{O}}^{\lambda_\mathcal{E}} R^{A}{}_{llA}(\lambda)\,(\lambda_\mathcal{E} - \lambda)\, d\lambda$$

a bit of tensor manipulation…

$$\mu \approx \frac{8\pi G}{c^4} \int_{\lambda_\mathcal{O}}^{\lambda_\mathcal{E}} T_{ll}(\lambda)\,(\lambda_\mathcal{E} - \lambda)\, d\lambda$$

Weighing spacetime along the line of sight

tidal forces, gravitational waves (non-local)

↓

Weyl tensor $C^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

matter content along the line of sight (local) $T_{\mu\nu}\big|_{\gamma_0}$

↓ *Einstein equations*

Ricci tensor $R_{\mu\nu}\big|_{\gamma_0}$

cosmological constant Λ → Ricci tensor

Ricci decomposition

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

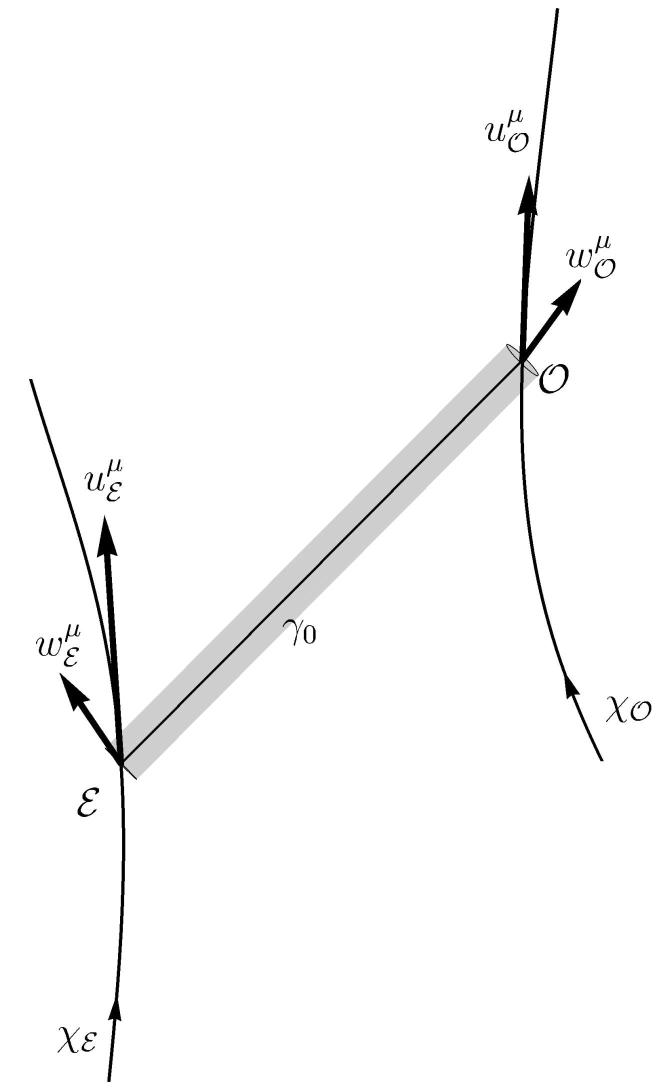
↓ *(null) geodesic deviation equation*

bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}$

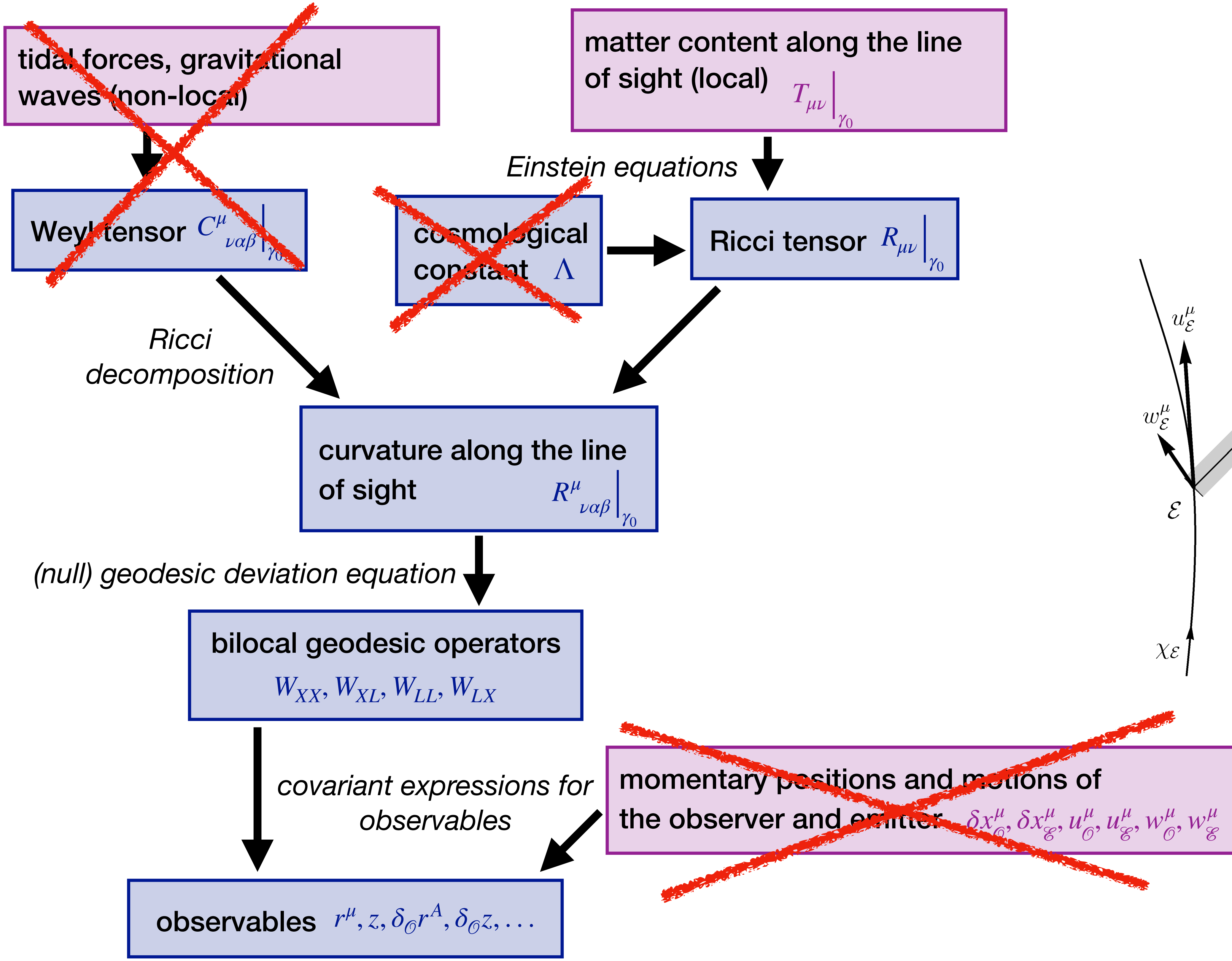
↓ *covariant expressions for observables*

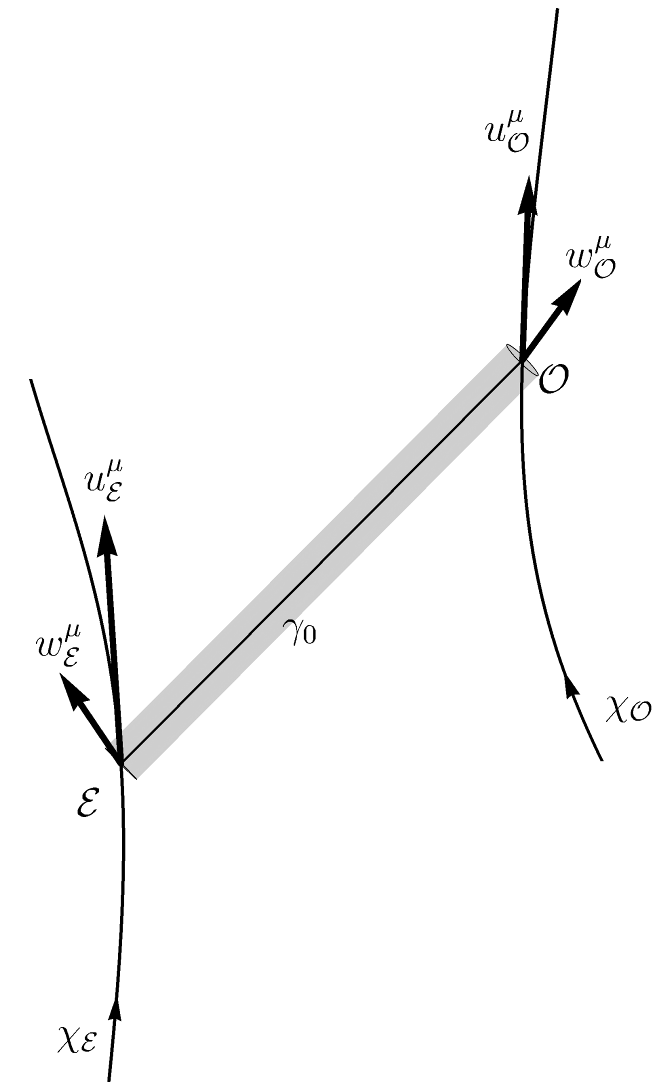
momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathcal{O}}, \delta x^{\mu}_{\mathcal{E}}, u^{\mu}_{\mathcal{O}}, u^{\mu}_{\mathcal{E}}, w^{\mu}_{\mathcal{O}}, w^{\mu}_{\mathcal{E}}$

observables $r^{\mu}, z, \delta_{\mathcal{O}} r^{A}, \delta_{\mathcal{O}} z, \ldots$



Weighing spacetime along the line of sight





Weighing spacetime along the line of sight

tidal forces, gravitational waves (non-local)

Weyl tensor $C^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

matter content along the line of sight (local) $T_{\mu\nu}\big|_{\gamma_0}$

Einstein equations

cosmological constant Λ

Ricci tensor $R_{\mu\nu}\big|_{\gamma_0}$

Ricci decomposition

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

(null) geodesic deviation equation

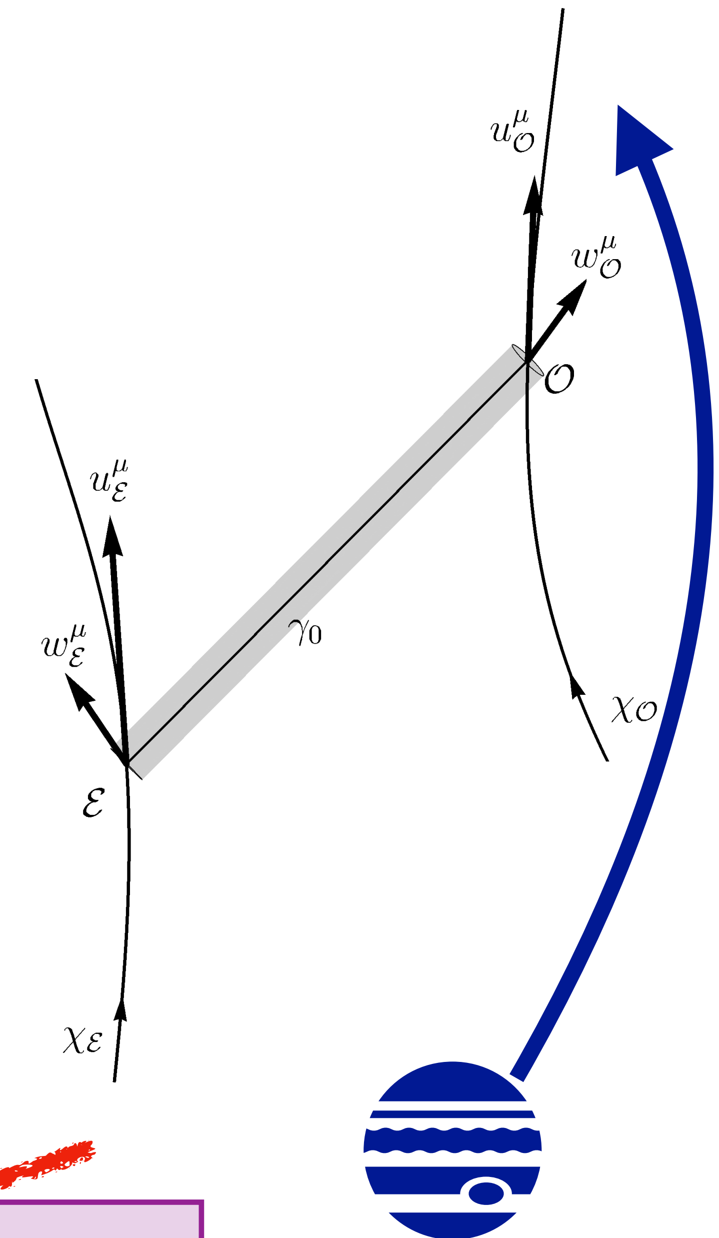
bilocal geodesic operators

$W_{XX}, W_{XL}, W_{LL}, W_{LX}$

covariant expressions for observables

momentary positions and motions of the observer and emitter $\delta x^{\mu}_{\mathcal{O}}, \delta x^{\mu}_{\mathcal{E}}, u^{\mu}_{\mathcal{O}}, u^{\mu}_{\mathcal{E}}, w^{\mu}_{\mathcal{O}}, w^{\mu}_{\mathcal{E}}$

observables $r^{\mu}, z, \delta_{\mathcal{O}} r^{A}, \delta_{\mathcal{O}} z, \ldots$



Weighing spacetime along the line of sight

tidal forces, gravitational waves (non-local)

Weyl tensor $C^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

Ricci decomposition

matter content along the line of sight (local) $T_{\mu\nu}\big|_{\gamma_0}$

Einstein equations

cosmological constant Λ

Ricci tensor $R_{\mu\nu}\big|_{\gamma_0}$

curvature along the line of sight $R^{\mu}{}_{\nu\alpha\beta}\big|_{\gamma_0}$

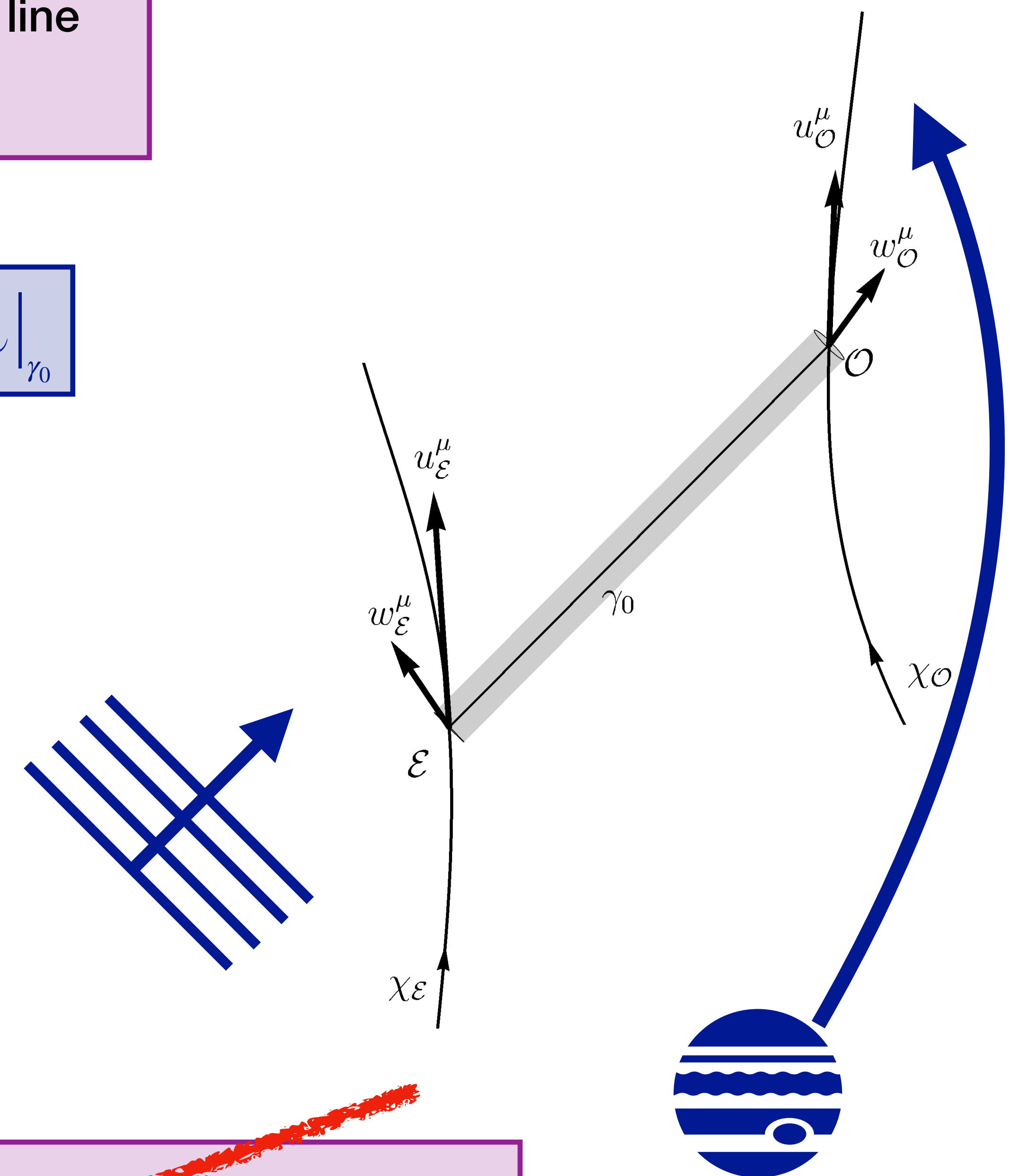
(null) geodesic deviation equation

bilocal geodesic operators $W_{XX}, W_{XL}, W_{LL}, W_{LX}$

covariant expressions for observables

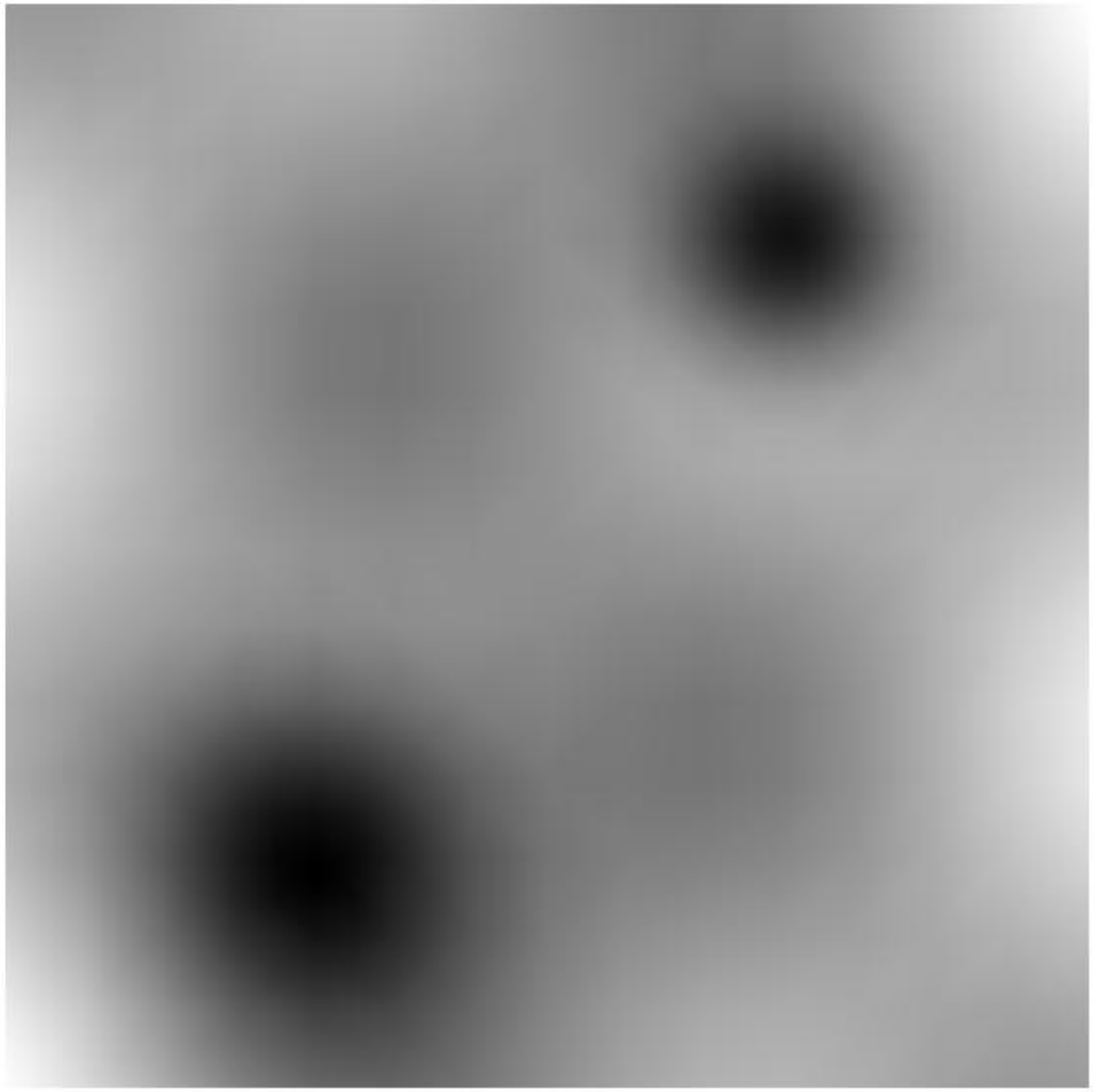
momentary positions and motions of the observer and emitter $\delta x^\mu_{\mathcal{O}}, \delta x^\mu_{\mathcal{E}}, u^\mu_{\mathcal{O}}, u^\mu_{\mathcal{E}}, w^\mu_{\mathcal{O}}, w^\mu_{\mathcal{E}}$

observables $r^\mu, z, \delta_{\mathcal{O}} r^A, \delta_{\mathcal{O}} z, \ldots$



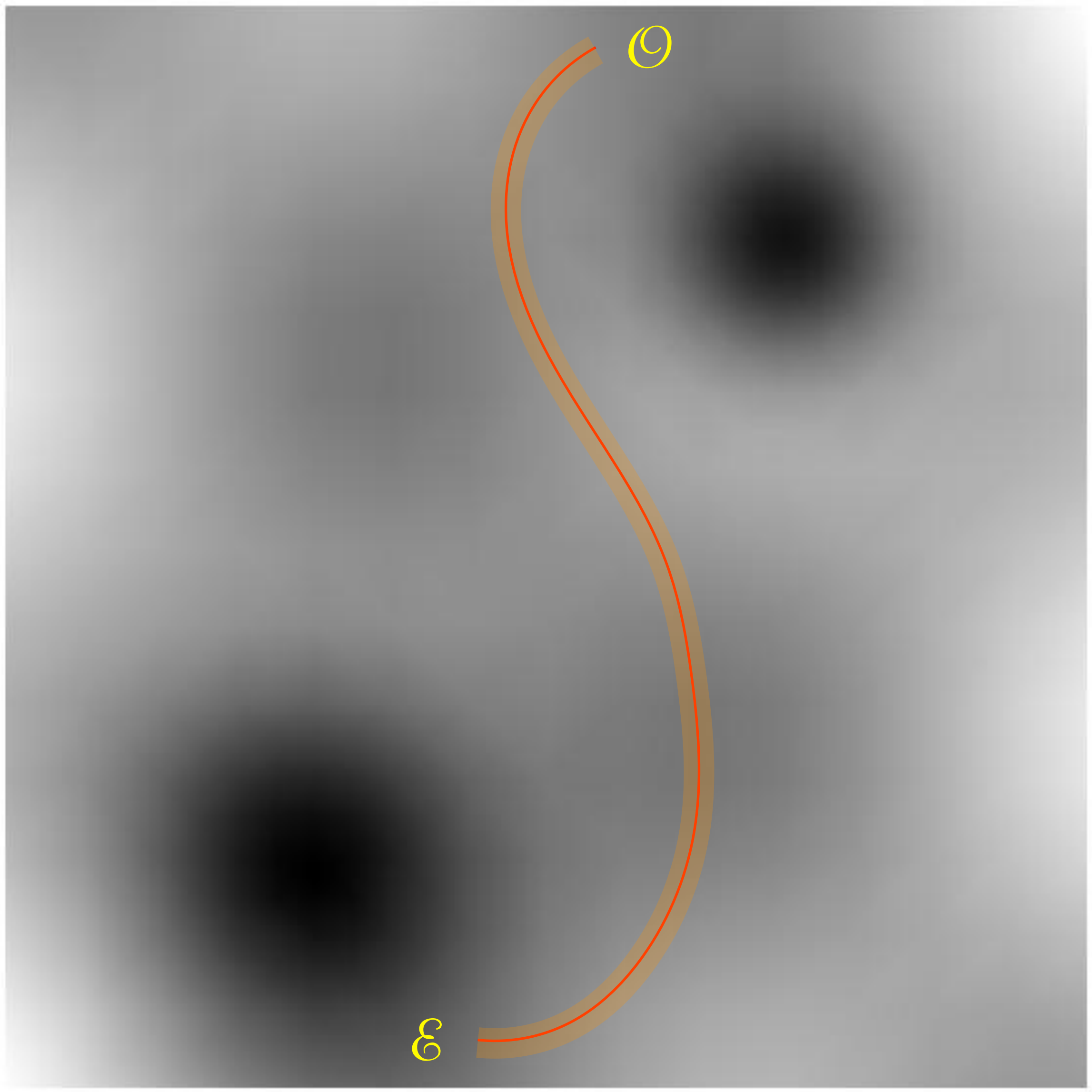
Weighing spacetime along the line of sight

- tomography-like properties



Weighing spacetime along the line of sight

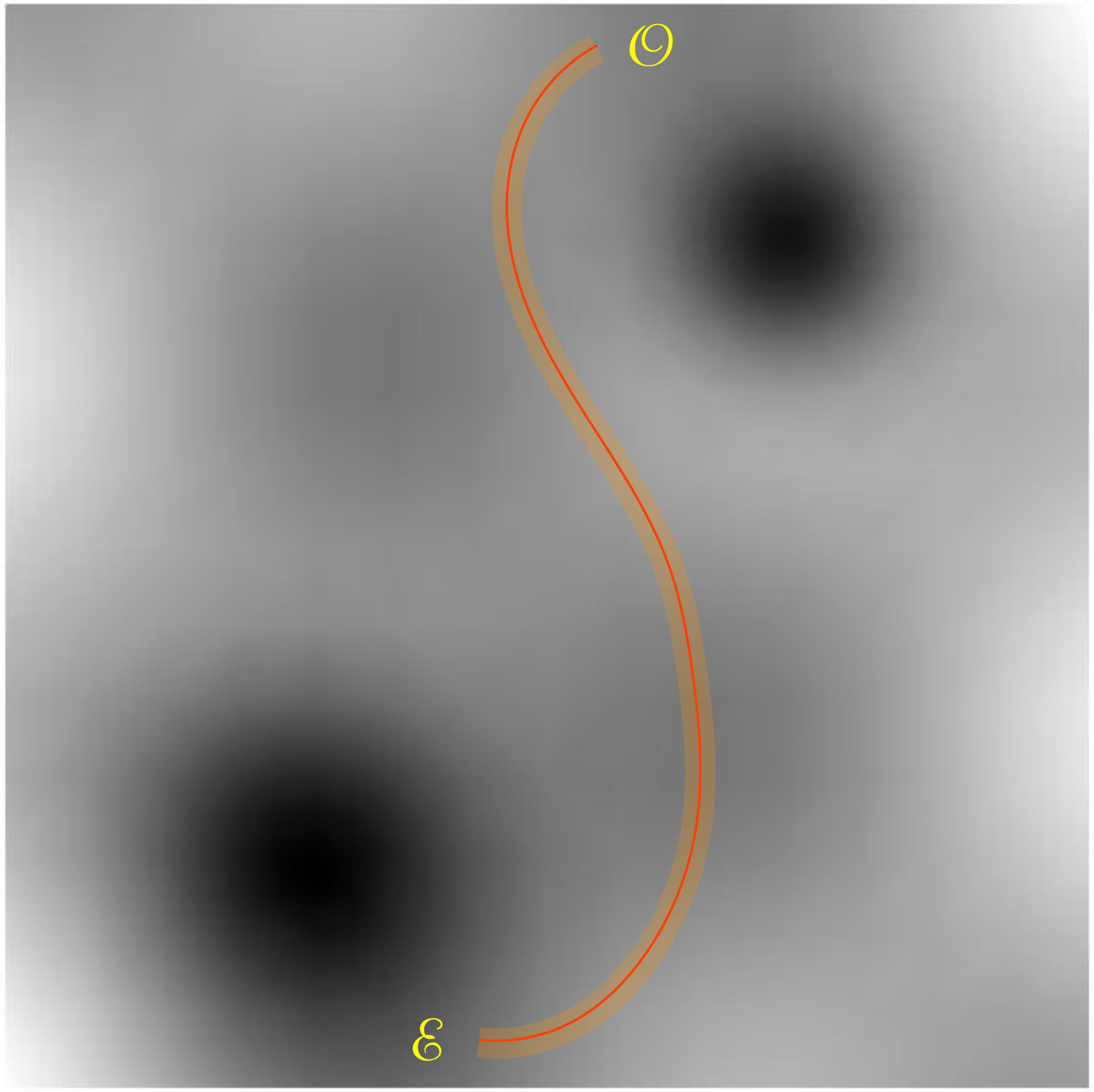
- tomography-like properties



Weighing spacetime along the line of sight

- tomography-like properties

$$\mu \approx \frac{8\pi G}{c^4} \int_{\lambda_{\mathcal{O}}}^{\lambda_{\mathcal{E}}} T_{ll}(\lambda)\,(\lambda_{\mathcal{E}} - \lambda)\,d\lambda$$



Parameter μ

Useful?

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

On the galactic scale (D = 30 kpc, $\boldsymbol{\rho}$ = 10^{-2} $M_\odot$ pc^{-3} - 1 $M_\odot$ pc^{-3}) μ =10^{-6} -10^{-4}

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

On the galactic scale (D = 30 kpc, $\boldsymbol{\rho} = 10^{-2}\ M_\odot\ pc^{-3} - 1\ M_\odot\ pc^{-3}$) $\mu = 10^{-6} - 10^{-4}$

On cosmological scales (MK, E. Villa): $\mu = \frac{3}{2}\Omega_{\mathbf{m}_0} z^2 + O(z^3)$

$$\mu(z=1) = 0.22 \qquad (\Lambda CDM \quad \Omega_{\Lambda_0} = 0.69, \quad \Omega_{\mathbf{m}_0} = 0.31, \quad \Omega_{\mathbf{k}_0} = 0)$$

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

On the galactic scale (D = 30 kpc, $\boldsymbol{\rho}$ = 10^{-2} $M_\odot$ pc^{-3} - 1 $M_\odot$ pc^{-3}) $\mu = 10^{-6} - 10^{-4}$

On cosmological scales (MK, E. Villa): $\mu = \frac{3}{2}\Omega_{\mathbf{m}_0} z^2 + O(z^3)$

$$\mu(z=1) = 0.22 \qquad (\Lambda CDM \quad \Omega_{\Lambda_0} = 0.69, \quad \Omega_{\mathbf{m}_0} = 0.31, \quad \Omega_{\mathbf{k}_0} = 0)$$

Requires simultaneous measurement of parallax as well as D_{ang} to an object

Either a standard ruler or a standard candle

$$D_{ang} = D_{lum}(1+z)^{-2}$$

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

On the galactic scale (D = 30 kpc, $\rho = 10^{-2}\ M_\odot\ pc^{-3}$ - $1\ M_\odot\ pc^{-3}$) $\mu = 10^{-6}$ - 10^{-4}

On cosmological scales (MK, E. Villa): $\mu = \frac{3}{2}\Omega_{\mathbf{m}_0} z^2 + O(z^3)$

$$\mu(z=1) = 0.22 \qquad (\Lambda CDM \quad \Omega_{\Lambda_0} = 0.69, \quad \Omega_{\mathbf{m}_0} = 0.31, \quad \Omega_{\mathbf{k}_0} = 0)$$

Requires simultaneous measurement of parallax as well as D_{ang} to an object

Either a standard ruler or a standard candle

$$D_{ang} = D_{lum}(1+z)^{-2}$$

Requires *many sources* to increase the S/N

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

On the galactic scale (D = 30 kpc, $\boldsymbol{\rho}$ = 10^{-2} $M_\odot$ pc^{-3} - 1 $M_\odot$ pc^{-3}) μ = 10^{-6} - 10^{-4}

On cosmological scales (MK, E. Villa): $\mu = \frac{3}{2}\Omega_{\mathbf{m}_0} z^2 + O(z^3)$

$$\mu(z=1) = 0.22 \qquad (\Lambda CDM \quad \Omega_{\Lambda_0} = 0.69, \quad \Omega_{\mathbf{m}_0} = 0.31, \quad \Omega_{\mathbf{k}_0} = 0)$$

Requires simultaneous measurement of parallax as well as D_{ang} to an object

Either a standard ruler or a standard candle

$$D_{ang} = D_{lum}(1+z)^{-2}$$

Requires *many sources* to increase the S/N

Measurements of the annual parallax on cosmological scales impossible today

Parameter μ

Useful?

Very small effects (unless for cosmological distances) (E. Villa, MK)

On the galactic scale (D = 30 kpc, $\boldsymbol{\rho}$ = 10^{-2} $M_\odot$ pc^{-3} - 1 $M_\odot$ pc^{-3}) μ = 10^{-6} - 10^{-4}

On cosmological scales (MK, E. Villa): $\mu = \frac{3}{2}\,\Omega_{\mathbf{m}_0}\, z^2 + O(z^3)$

$$\mu(z=1) = 0.22 \qquad (\Lambda CDM \quad \Omega_{\Lambda_0} = 0.69, \quad \Omega_{\mathbf{m}_0} = 0.31, \quad \Omega_{\mathbf{k}_0} = 0)$$

Requires simultaneous measurement of parallax as well as D_{ang} to an object

Either a standard ruler or a standard candle

$$D_{ang} = D_{lum}\,(1+z)^{-2}$$

Requires *many sources* to increase the S/N

Measurements of the annual parallax on cosmological scales impossible today

…but we may use the motion of the LC wrt CMB frame in the future [Kardashev 1986, Räsänen 2014, Quercellini *et al* 2012, Marcori *et al* 2018], effects borderline visible

Parameter μ

Parameter μ

Still:

Conceptually simple

The only observable so insensitive to external gravitational perturbations
and peculiar motions (meaning: no systematics due to tidal distortions or peculiar motions!)

Tomography-like measurement

Parameter μ

Still:

Conceptually simple

The only observable so insensitive to external gravitational perturbations
and peculiar motions (meaning: no systematics due to tidal distortions or peculiar motions!)

Tomography-like measurement

Similar ideas before:

McCrea 1935 - parallax distance in FLRW metric carries additional information

Weinberg 1970 - parallax distance in FLRW metric determines k = 0,1,-1

Kasai 1988, Rosquist 1988 - parallax distance in FLRW (+ perturbations)

Räsänen 2014 - parallax distance vs. luminosity distance as consistency test of FLRW

Future plans and projects

Future plans and projects

Cosmological applications of $\boldsymbol{\mu}$ (with E. Villa)

Tests of spacetime isotropy

Local dark matter mapping

Determination of cosmological parameters

Future plans and projects

Cosmological applications of μ (with E. Villa)

Tests of spacetime isotropy

Local dark matter mapping

Determination of cosmological parameters

Parallax and drifts in the Schwarzschild (and Kerr?) spacetime: observations of free-falling objects in those spacetimes (J. Serbenta)

Future plans and projects

Cosmological applications of $\boldsymbol{\mu}$ (with E. Villa)

Tests of spacetime isotropy

Local dark matter mapping

Determination of cosmological parameters

Parallax and drifts in the Schwarzschild (and Kerr?) spacetime: observations of free-falling objects in those spacetimes (J. Serbenta)

Numerical applications: ray-tracing and $\mathscr{W}$ in numerical spacetimes (M. Grasso)

Future plans and projects

Cosmological applications of $\boldsymbol{\mu}$ (with E. Villa)

Tests of spacetime isotropy

Local dark matter mapping

Determination of cosmological parameters

Parallax and drifts in the Schwarzschild (and Kerr?) spacetime: observations of free-falling objects in those spacetimes (J. Serbenta)

Numerical applications: ray-tracing and $\mathscr{W}$ in numerical spacetimes (M. Grasso)

Redshift-based observables, generalised reciprocity relations

Future plans and projects

Cosmological applications of $\boldsymbol{\mu}$ (with E. Villa)

- Tests of spacetime isotropy
- Local dark matter mapping
- Determination of cosmological parameters

Parallax and drifts in the Schwarzschild (and Kerr?) spacetime: observations of free-falling objects in those spacetimes (J. Serbenta)

Numerical applications: ray-tracing and $\mathscr{W}$ in numerical spacetimes (M. Grasso)

Redshift-based observables, generalised reciprocity relations

Thank you!