Does spatial flatness forbid the turnaround epoch of collapsing structures?

Boud Roukema, Jan Ostrowski
TCfA, NCBJ, CRAL

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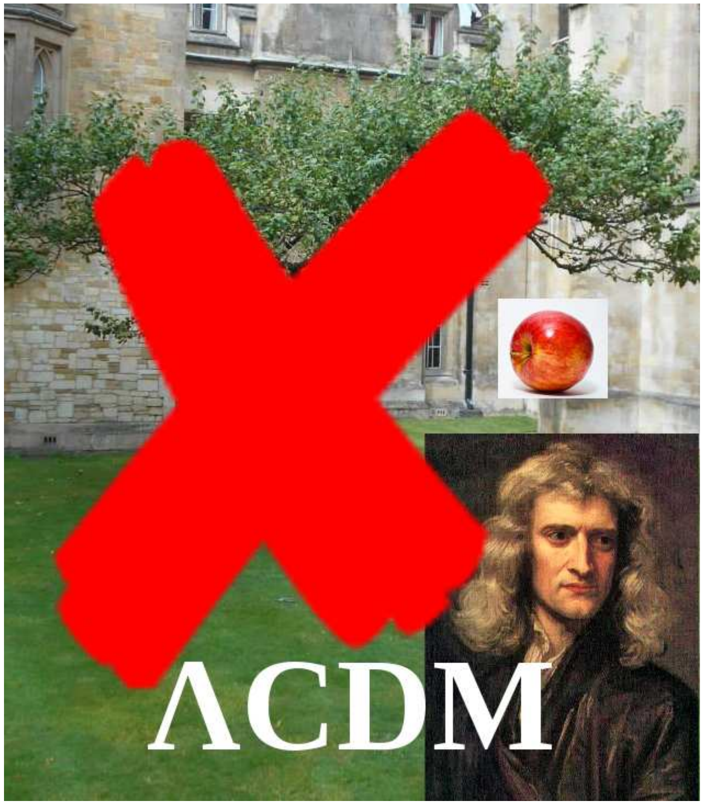
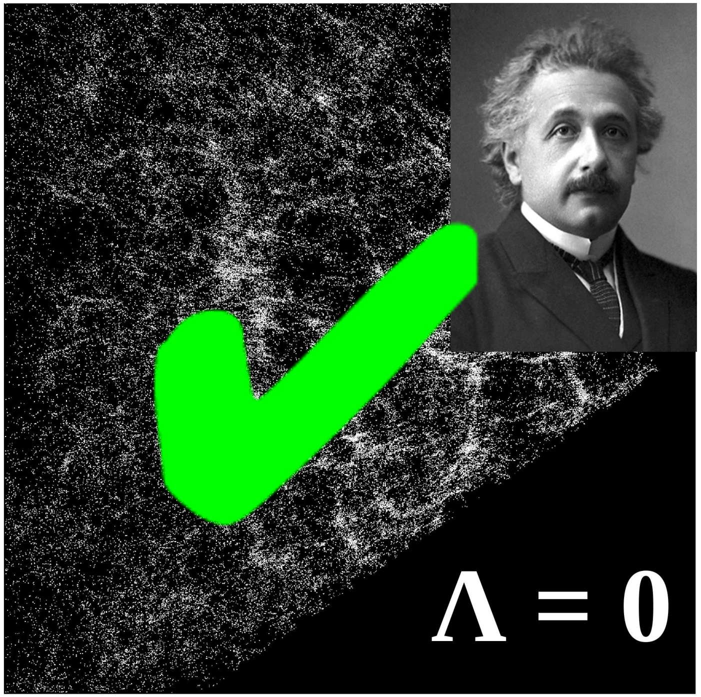
Newton vs Einstein

space–time = Universe





?

Gpc-scale galaxies/4MOST + SNe Ia/LSST ⇒ Einstein ?

local 100 Mpc/$h^{\rm eff}$ + "local" H_0 vs "global" H_0 ⇒ Einstein ?

ΛCDM: the biverse model

- ΛCDM: exactly FLRW Universe — perfectly 3-Ricci flat;
  but

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- ΛCDM: the Universe has structure: early epochs — curvature perturbations;
- Is curvature significant at late epochs?
- Does gravitational collapse require positive curvature?
- Does the curvature–expansion rate relation imply that FLRW is inaccurate at late epochs?

turnaround epoch

- rumour: “everyone knows” that curvature is associated with structure formation

turnaround epoch

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- problem: is there a simple proof with very few assumptions?
- yes: clear, elegant argument using the turnaround epoch

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$$ds^2 = a^2 \left\{ -(1+2\psi)\, d\tau^2 + (1-2\varphi) \left[dx^2 + dy^2 + dz^2 \right] \right\}$$

$\Rightarrow$

$$^3\mathcal{R} = \frac{8\,\varphi\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right) - 4\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right)}{a^2\,\left(2\,\varphi - 1\right)^3} + \frac{-6\,\left(\varphi_{,x}^2 + \varphi_{,y}^2 + \varphi_{,z}^2\right)}{a^2\,\left(2\,\varphi - 1\right)^3}$$

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$$\begin{aligned} {}^3\mathcal{R} &\approx 4\,a^{-2}\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right) \\ &= 4\,a^{-2}\,\nabla^2_{\mathbb{E}^3}\varphi \\ &= 16\pi\,G\,\delta\rho \end{aligned}$$

- positive spatial curvature tends to associate with overdensities

Hamiltonian constraint

- assume: expanding dust universe with flow-orthogonal foliation, irrotational fluid

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Einstein Eqn time–time part gives (pointwise):

$$\frac{1}{3}\Theta^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

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$$3\,H^2 = 3\,H^2\Omega_{\mathrm{m}} + 0 + 3\,H^2\Omega_{\mathrm{k}} + 3\,H^2\Omega_\Lambda \qquad \text{cf FLRW}$$

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- turnaround: $\Theta = 0 \Rightarrow \mathcal{R} > 0$
- "collapsing" domain must have $\mathcal{R} > 0$ to turn around

  arXiv:0902.09064 (RO19)

averaged case

What happens for an averaged domain?

averaged Hamiltonian constraint

averaged Hamiltonian constraint

$$\frac{1}{3}\langle\Theta\rangle_{\mathcal{D}}^2 = 8\pi G\langle\rho\rangle_{\mathcal{D}} + \left\langle\sigma^2\right\rangle_{\mathcal{D}} - \frac{1}{3}\left\langle\left(\Theta - \langle\Theta\rangle_{\mathcal{D}}\right)^2\right\rangle_{\mathcal{D}} - \frac{1}{2}\langle\mathcal{R}\rangle_{\mathcal{D}} + \Lambda$$

$$\mathcal{Q}_{\mathcal{D}} := \frac{2}{3}\left\langle\left(\Theta - \langle\Theta\rangle_{\mathcal{D}}\right)^2\right\rangle_{\mathcal{D}} - 2\left\langle\sigma^2\right\rangle_{\mathcal{D}}$$

RZA2

averaged Hamiltonian constraint

averaged Hamiltonian constraint

$$\Omega_{\mathrm{m}}^{\mathcal{D}} + \Omega_{\mathcal{Q}}^{\mathcal{D}} + \Omega_{\mathcal{R}}^{\mathcal{D}} + \Omega_{\Lambda}^{\mathcal{D}} = \frac{H_{\mathcal{D}}^2}{H_{\mathrm{eff}}^2} \,,$$

- turnaround: strong expansion variance term $\left\langle \left(\Theta - \langle\Theta\rangle_{\mathcal{D}}\right)^2 \right\rangle_{\mathcal{D}}$ might allow flat ($\mathcal{R} = 0$) turnaround ($\Theta = 0$)
- non-perturbative: $\mathcal{Q}_{\mathcal{D}}$ relativistic Zel'dovich approximation (QZA)

arXiv:0902.09064 (RO19)

Plane-symmetric case

general:

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \left[(1 + S(w,t))^2 \left(\mathrm{d}x^2 + \mathrm{d}y^2\right) + \big(1 + P(w,t)\big)^2 \,\mathrm{d}w^2\right]$$

Plane-symmetric case

spatially flat subcase (RZA2 V.A):

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- either EdS or ΛCDM reference model:
$$\lim_{t\to\infty} \Theta = 3H := 3\frac{\dot{a}}{a}$$
- $\Theta > 0$ (and $H > 0$) initially $\Rightarrow$ no pancake collapse
- RZA2 V.A: “pancake collapse possible” — assumed growing mode allowed
- fundamental difference Newtonian vs GR:
GR forbids growing mode in this case

Analytical calculation: special case

- assume

$$\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0 \; , \quad \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0 \; .$$

- define

$$\alpha := \frac{H}{H_{\mathrm{eff}}} \lesssim 1$$

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- turnaround condition:
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$\Rightarrow$
$$\Omega_{\mathcal{R}}^{\mathcal{D}} = -5\alpha^2, \;\; \Omega_{\mathcal{Q}}^{\mathcal{D}} = \alpha^2, \;\; \Omega_{\mathrm{m}}^{\mathcal{D}} = 4\alpha^2$$

- $\exists$ critical Ω parameters at turnaround

scalar averaging: Raychaudhuri eq

averaged Raychaudhuri eq [RZA2, PRD arXiv:1303.6193, (9)]:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3} \frac{M_{\mathcal{D}_{\mathbf{i}}}\, a^3_{\mathcal{D}_{\mathbf{i}}}}{V_{\mathcal{D}_{\mathbf{i}}}\, a^3_{\mathcal{D}}} + \frac{\mathcal{Q}_{\mathcal{D}}}{3} + \Lambda \,,$$

remove a free parameter by setting $\Lambda := 0$

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where kinematical backreaction [Newtonian case: BKS, arXiv:astro-ph/9912347, II.B., (5)]

$$\mathcal{Q}_{\mathcal{D}} := 2\;\langle \mathrm{II} \rangle_{\mathcal{D}} - \frac{2}{3}{\langle \mathrm{I} \rangle_{\mathcal{D}}}^2 \;,$$

scalar averaging: Raychaudhuri eq

with invariants of the peculiar expansion tensor [Newtonian case: Buchert 94, MNRAS arXiv:astro-ph/9309055]:

$$\begin{aligned}
\mathrm{I}(v^i_{\ ,j}) &:= \mathrm{tr}(v^i_{\ ,j}) = v^i_{\ ,i} = \nabla \cdot \mathbf{v} \\
\mathrm{II}(v^i_{\ ,j}) &:= \tfrac{1}{2}\left\{\left[\mathrm{tr}\left(v^i_{\ ,j}\right)\right]^2 - \mathrm{tr}\left[\left(v^i_{\ ,j}\right)^2\right]\right\} \\
&= \tfrac{1}{2}\left((v^i_{\ ,i})^2 - v^i_{\ ,j}\, v^j_{\ ,i}\right) \\
&= \tfrac{1}{2}\nabla \cdot \Big(\mathbf{v}(\nabla \cdot \mathbf{v}) - (\mathbf{v} \cdot \nabla)\mathbf{v}\Big) \\
\mathrm{III}(v^i_{\ ,j}) &:= \det(v^i_{\ ,j}).
\end{aligned}$$

scalar averaging: QZA

$$\mathcal{Q}_\mathcal{D} = \frac{\dot{\xi}^2 \left(\gamma_1 + \xi\gamma_2 + \xi^2\gamma_3\right)}{\left(1 + \xi \left\langle \mathrm{I}_\mathbf{i} \right\rangle_\mathcal{I} + \xi^2 \left\langle \mathrm{II}_\mathbf{i} \right\rangle_\mathcal{I} + \xi^3 \left\langle \mathrm{III}_\mathbf{i} \right\rangle_\mathcal{I}\right)^2}$$

$$\text{where} \begin{cases} \gamma_1 := 2\left\langle \mathrm{II}_\mathbf{i} \right\rangle_\mathcal{I} - \frac{2}{3}\left\langle \mathrm{I}_\mathbf{i} \right\rangle_\mathcal{I}^2 \\ \gamma_2 := 6\left\langle \mathrm{III}_\mathbf{i} \right\rangle_\mathcal{I} - \frac{2}{3}\left\langle \mathrm{II}_\mathbf{i} \right\rangle_\mathcal{I}\left\langle \mathrm{I}_\mathbf{i} \right\rangle_\mathcal{I} \\ \gamma_3 := 2\left\langle \mathrm{I}_\mathbf{i} \right\rangle_\mathcal{I}\left\langle \mathrm{III}_\mathbf{i} \right\rangle_\mathcal{I} - \frac{2}{3}\left\langle \mathrm{II}_\mathbf{i} \right\rangle_\mathcal{I}^2 \end{cases}$$

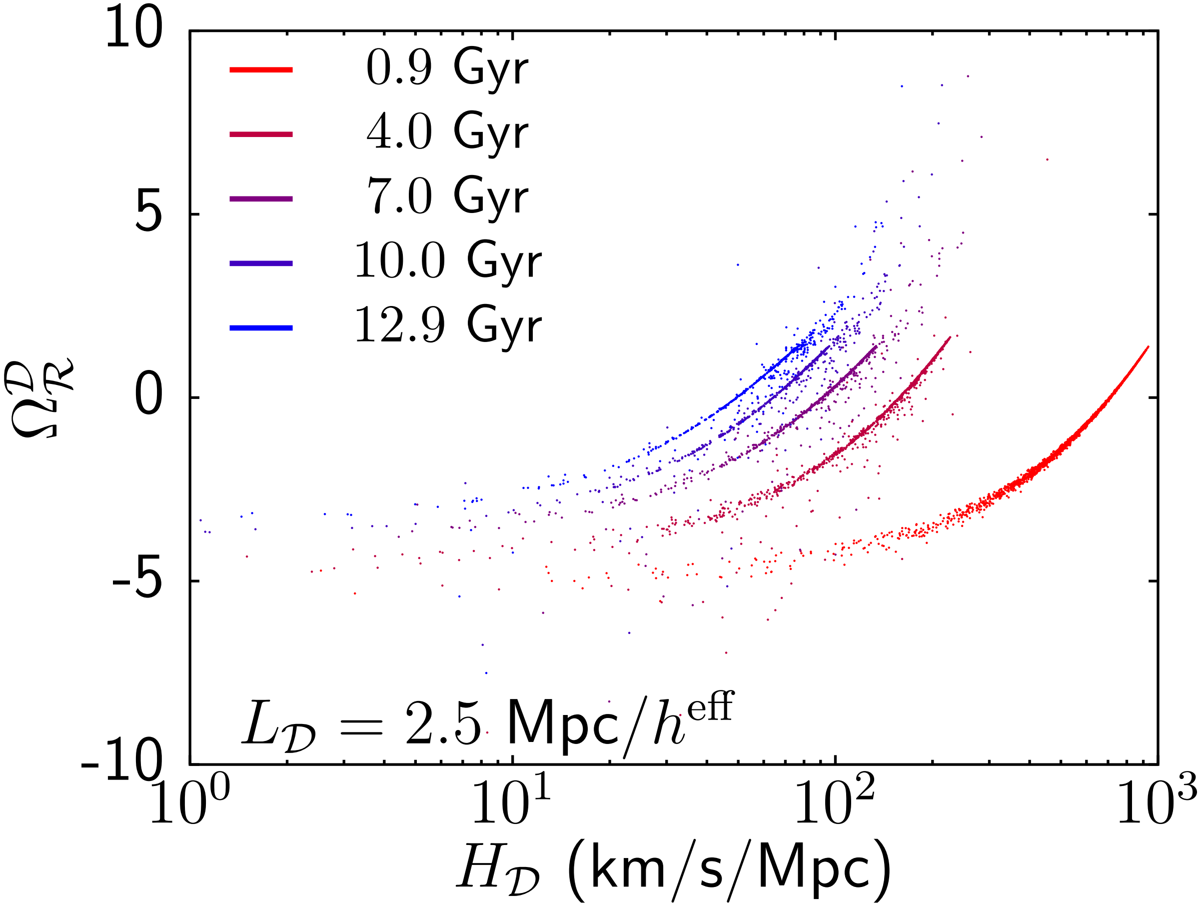
QZA $=$ $\mathcal{Q}_\mathcal{D}$ Zel'dovich approximation:

- algebraic structure same in Newtonian and GR cases
- initial invariants conceptually differ (Newt vs GR)
- initial invariants numerically approximated for zero curvature
- ξ is the reference model (EdS) linear growth rate

software

- MPGRAFIC — initial conditions (GPL, f90)
  http://tracker.debian.org/mpgrafic
- DTFE — measure I, II, III (GPL, C++)
- INHOMOG — evolve QZA; stabilise virialised domains (GPL, C)
  http://tracker.debian.org/inhomog
- RAMSES-SCALAV — extension of RAMSES as front end to the above (Cecill, f90)
- start at http://bitbucket.org/broukema/ramses-scalav
- *Debian Astro:* library dependence management; compilation and unit tests on 22 different architecture/kernel ports; LINTIAN; bug tracker;
  https://lists.debian.org/debian-astro
  https://wiki.debian.org/DebianAstro
  `irc: irc.debian.org #debian-astro`

curvature–expansion-rate relation



EdS

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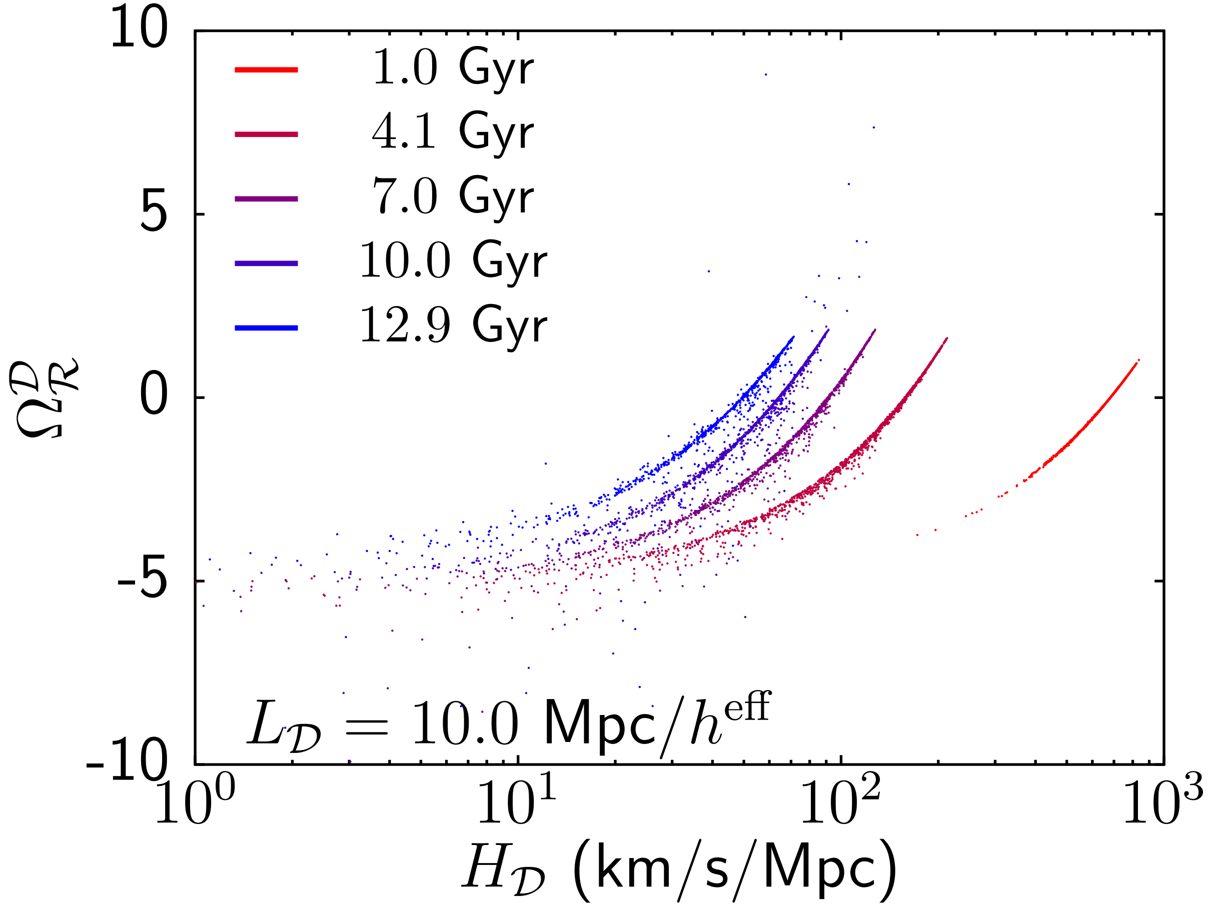

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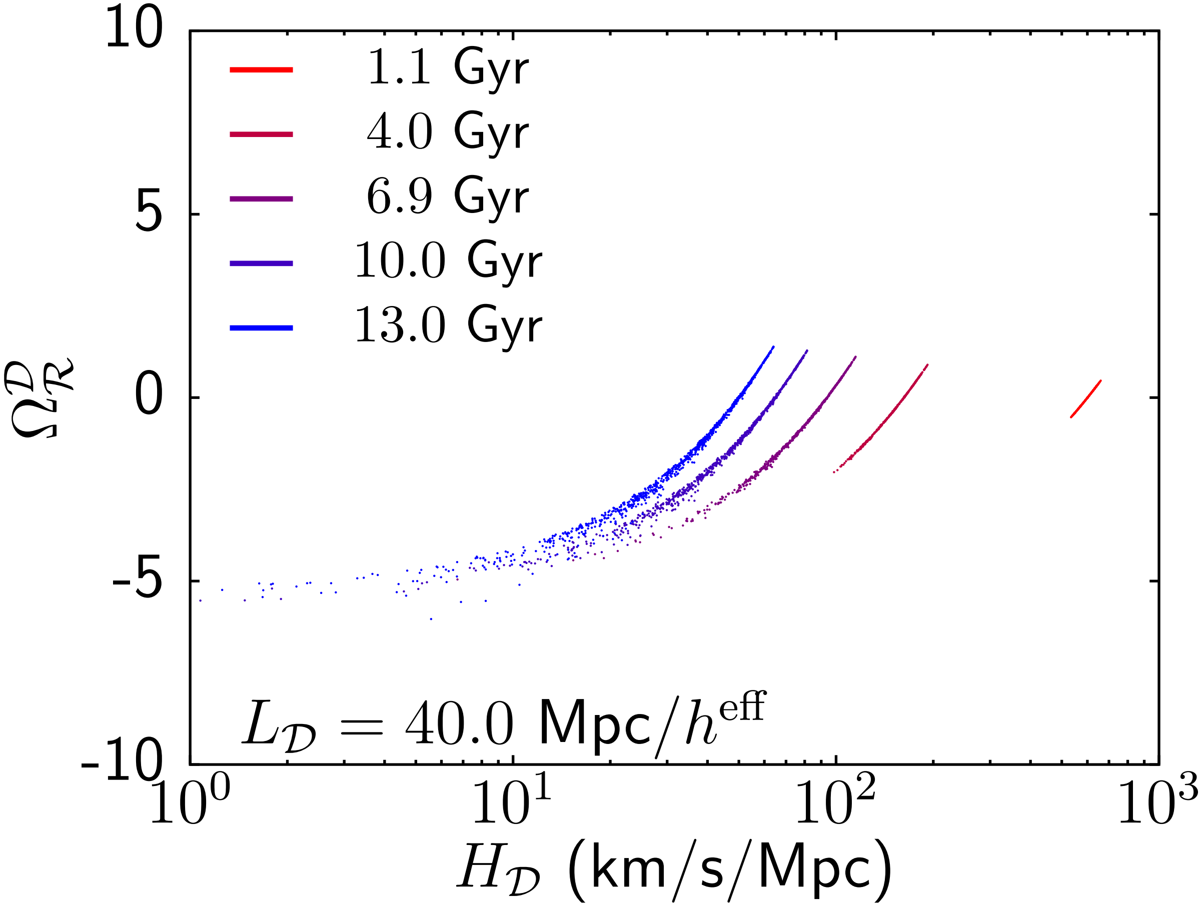

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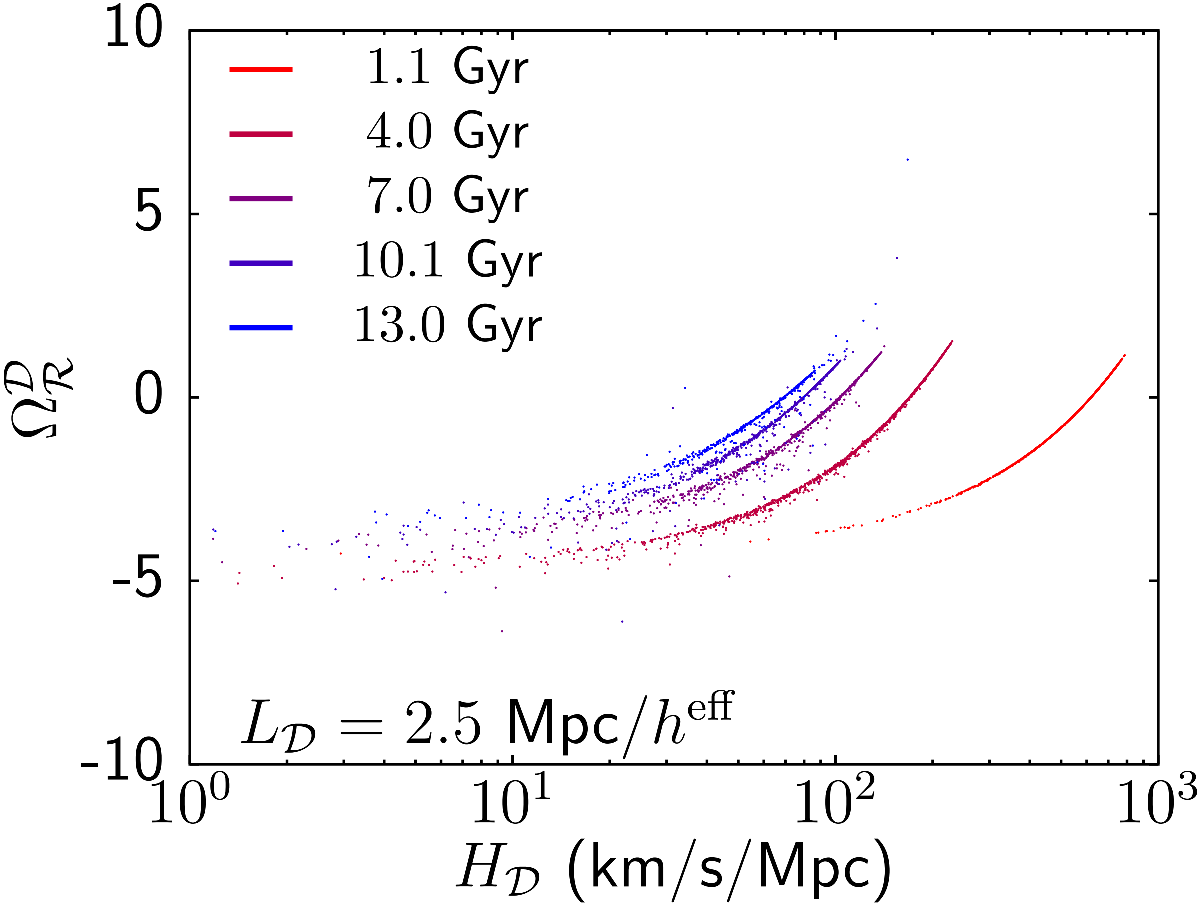

ΛCDM

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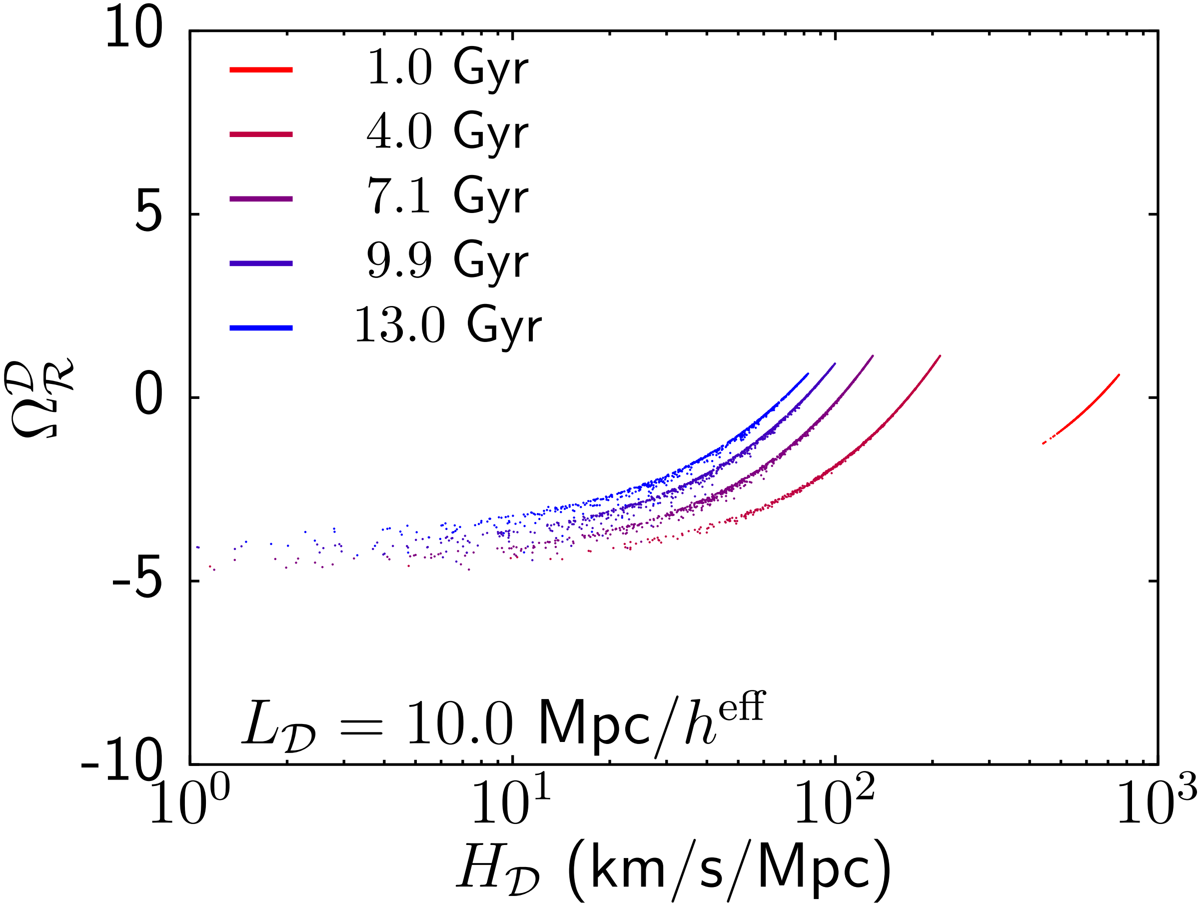

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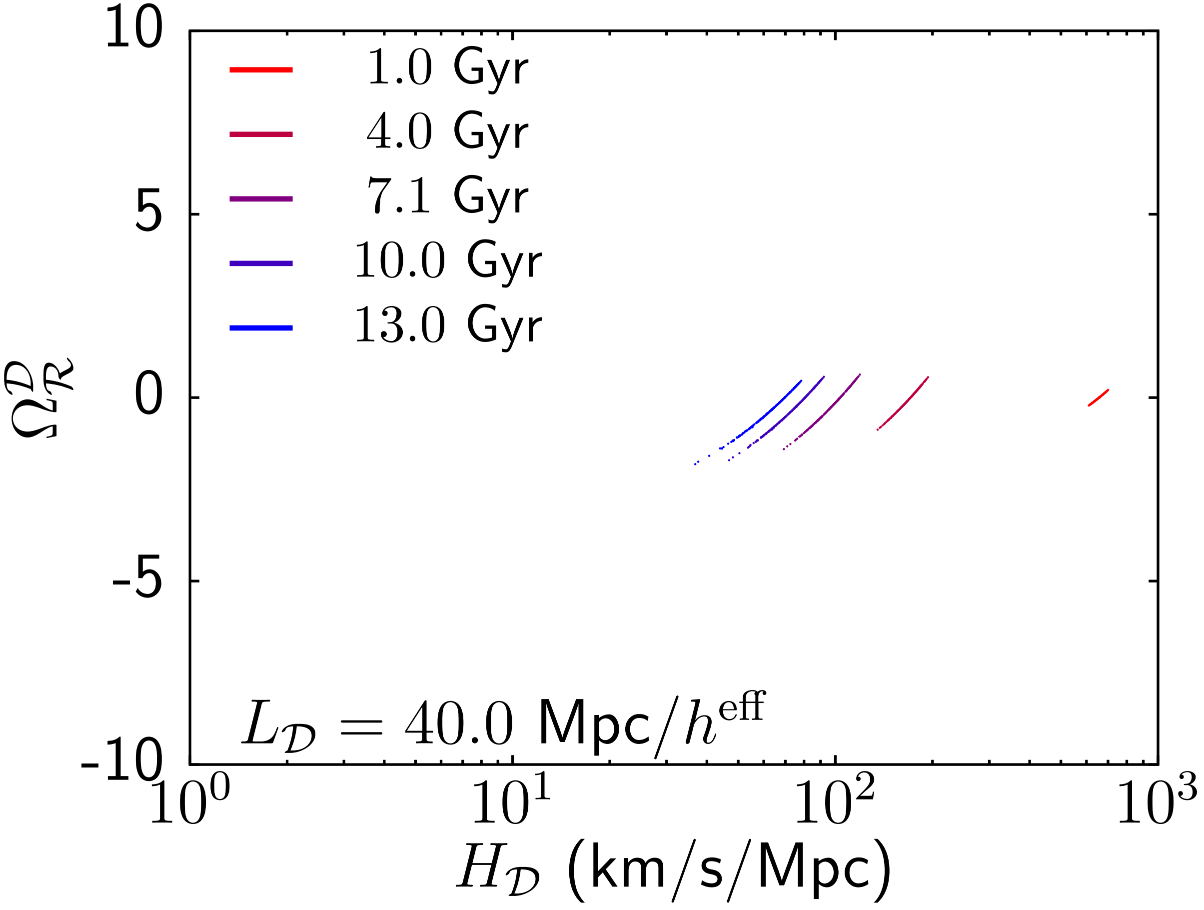
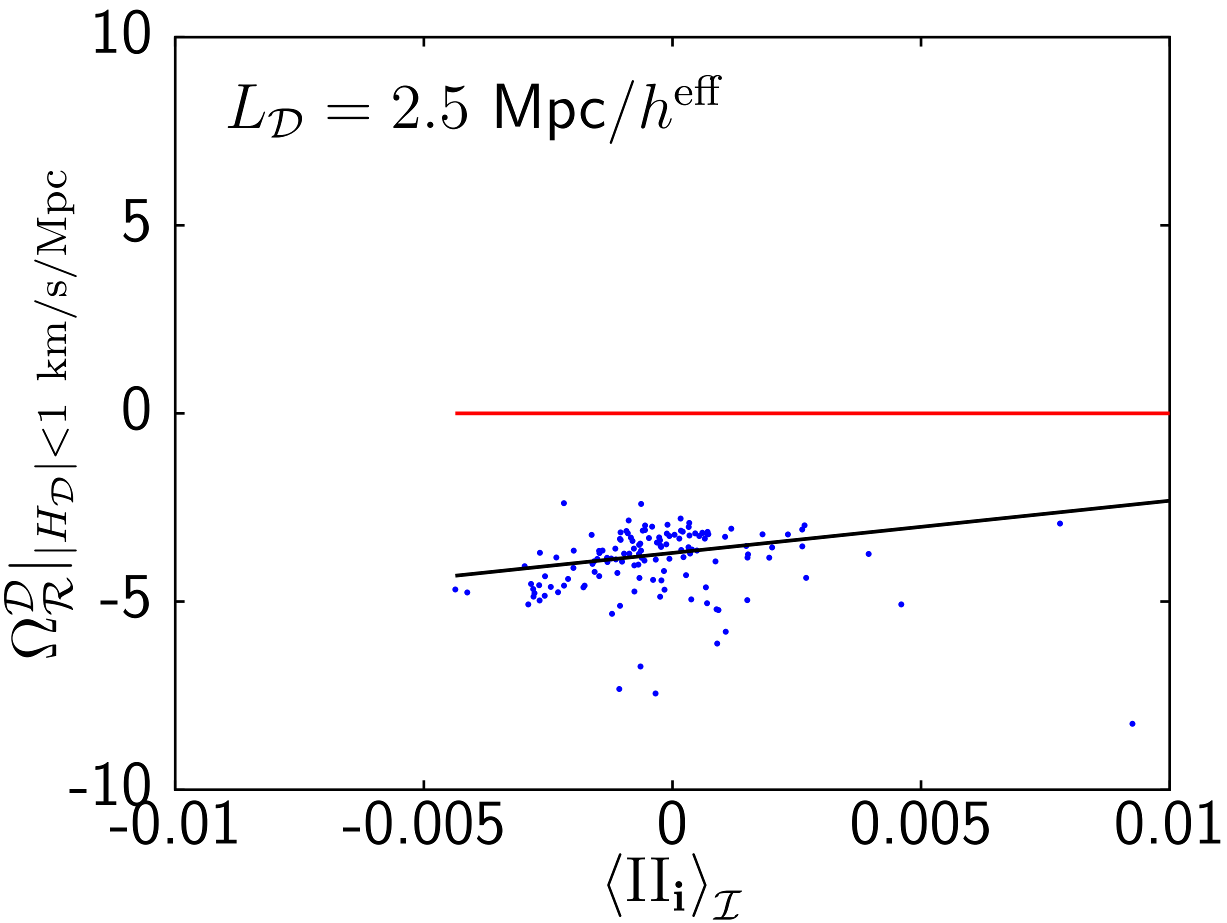


ΛCDM

critical $\Omega_{\mathcal{R}}^{\mathcal{D}}$ value at turnaround



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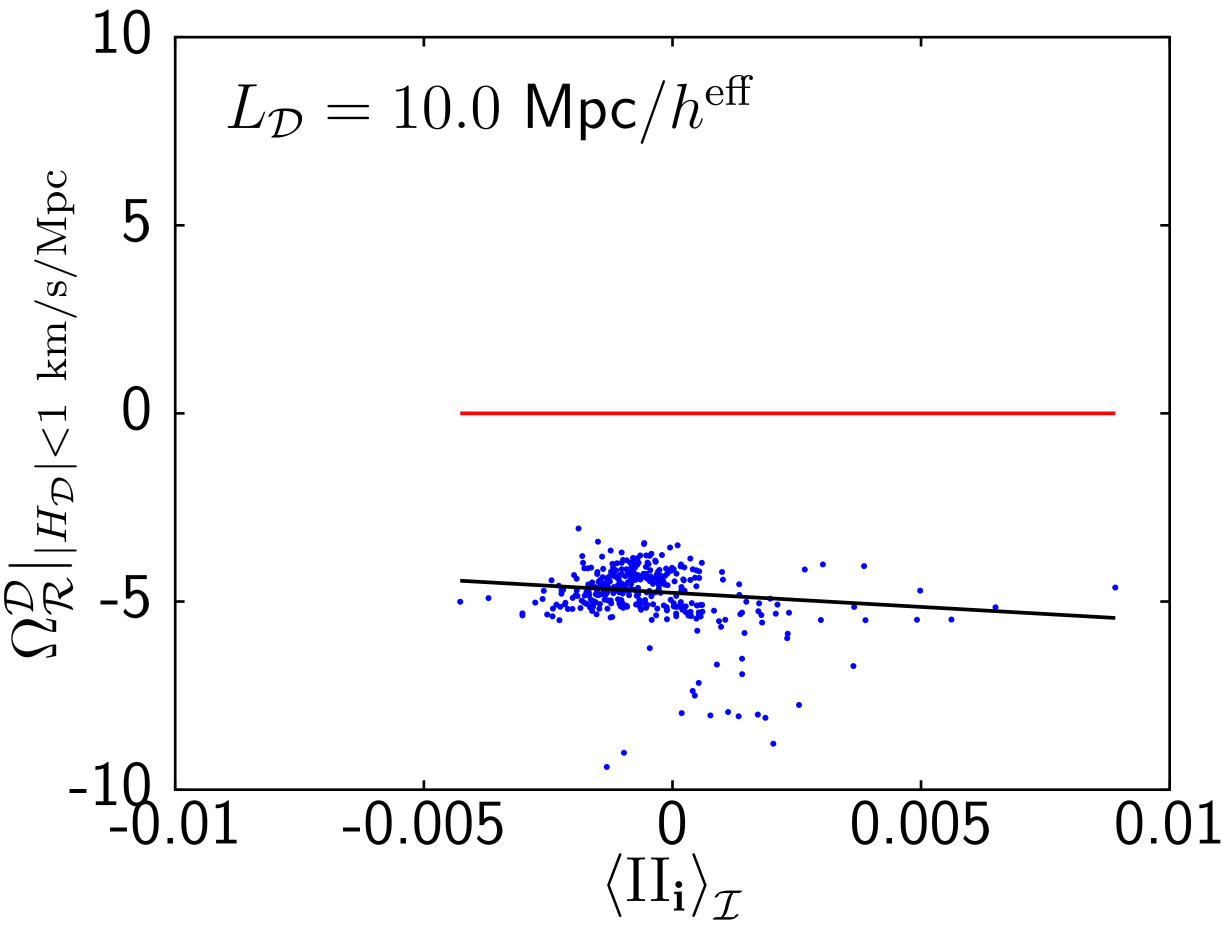

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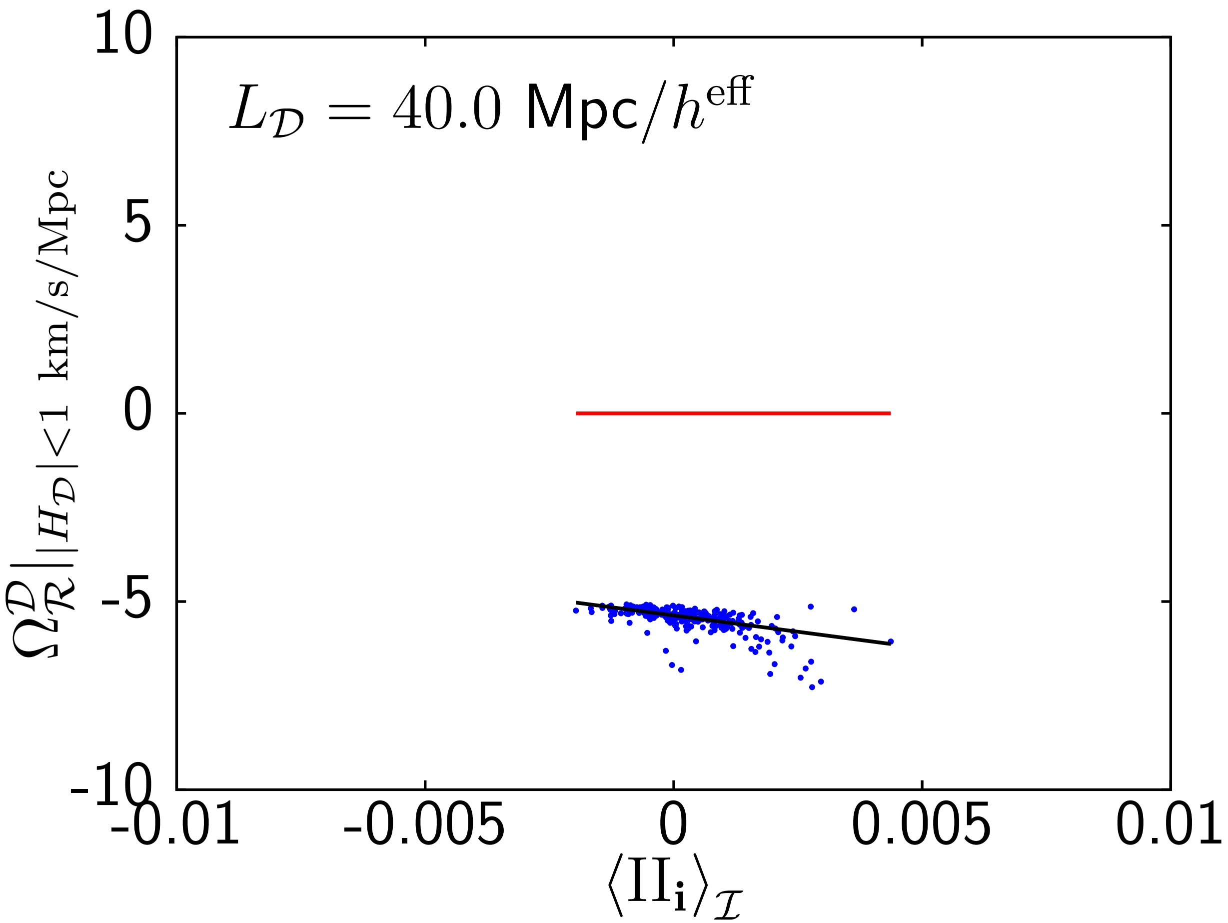

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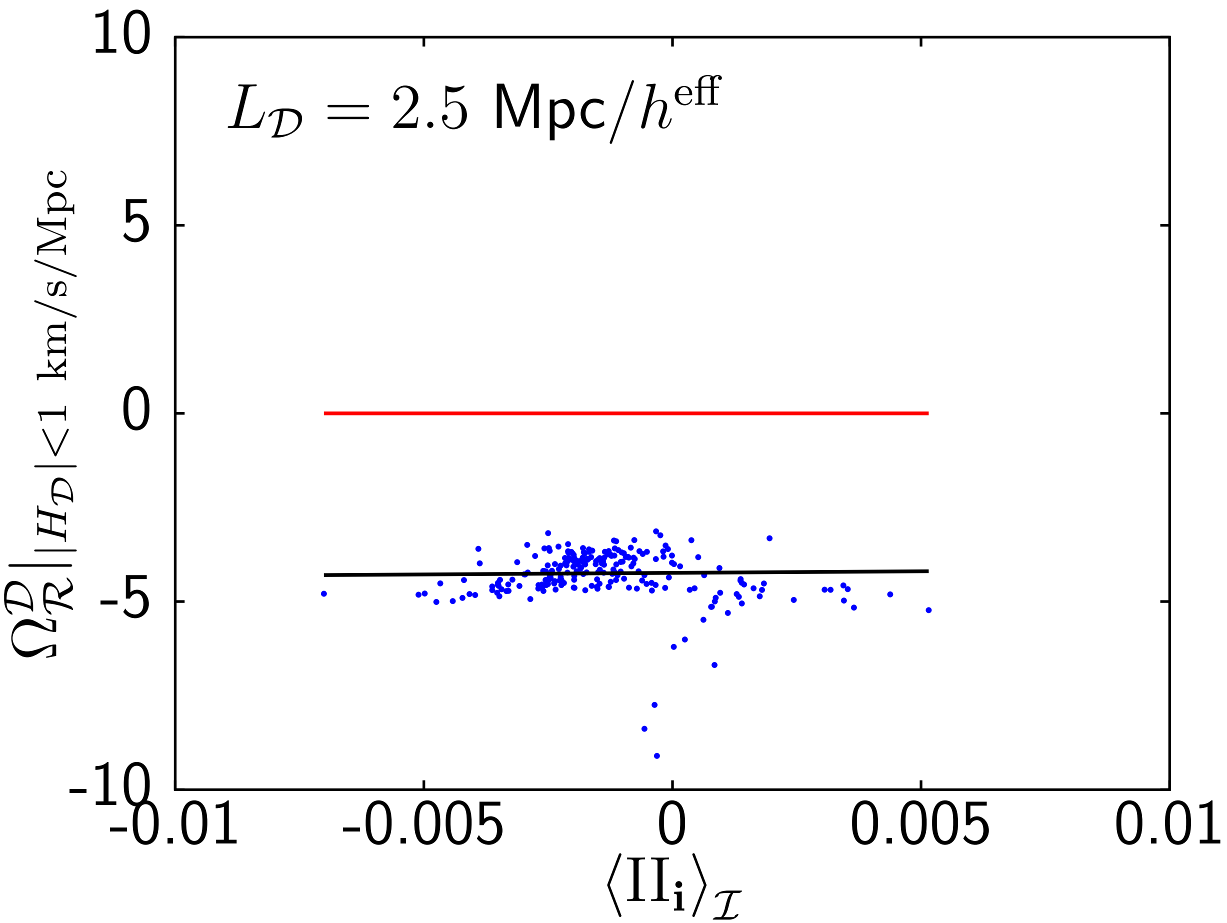

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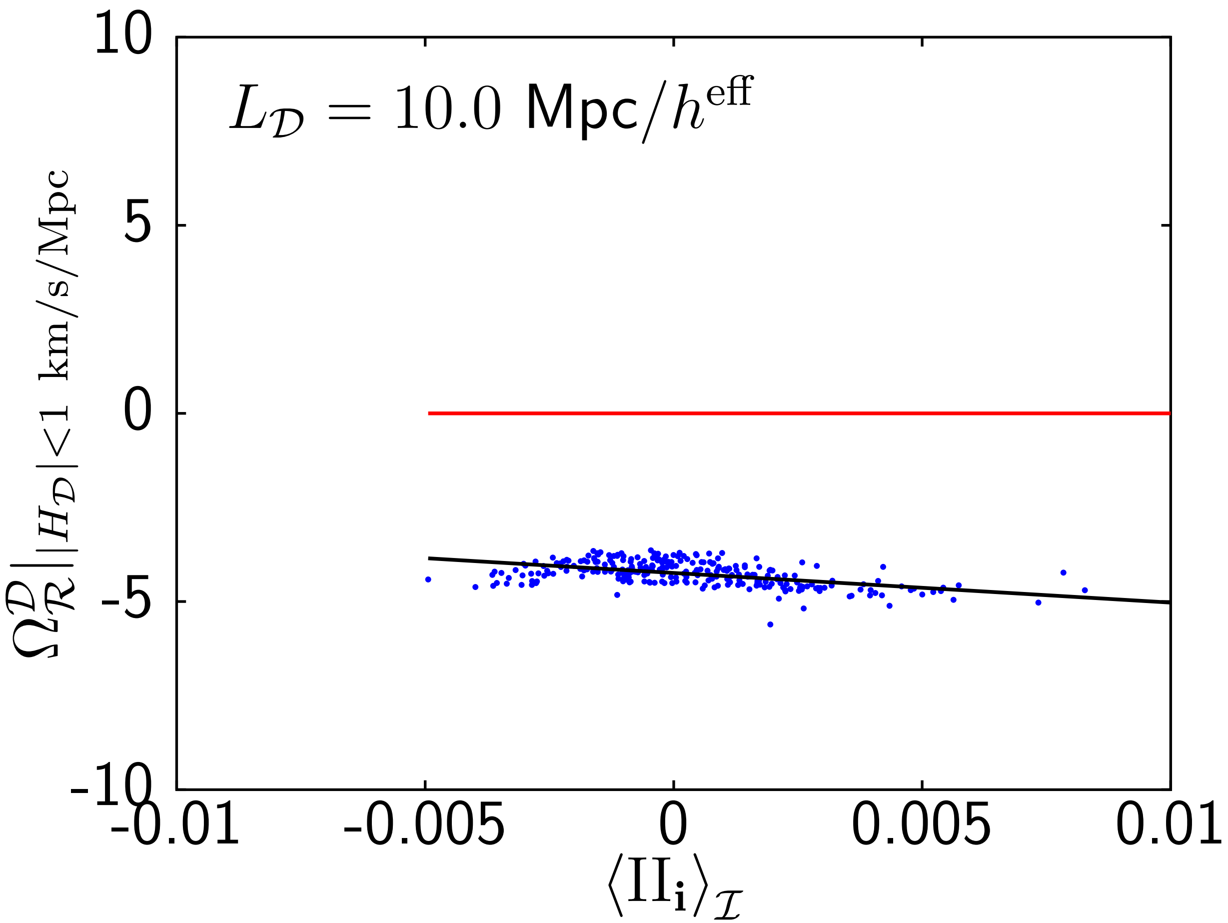
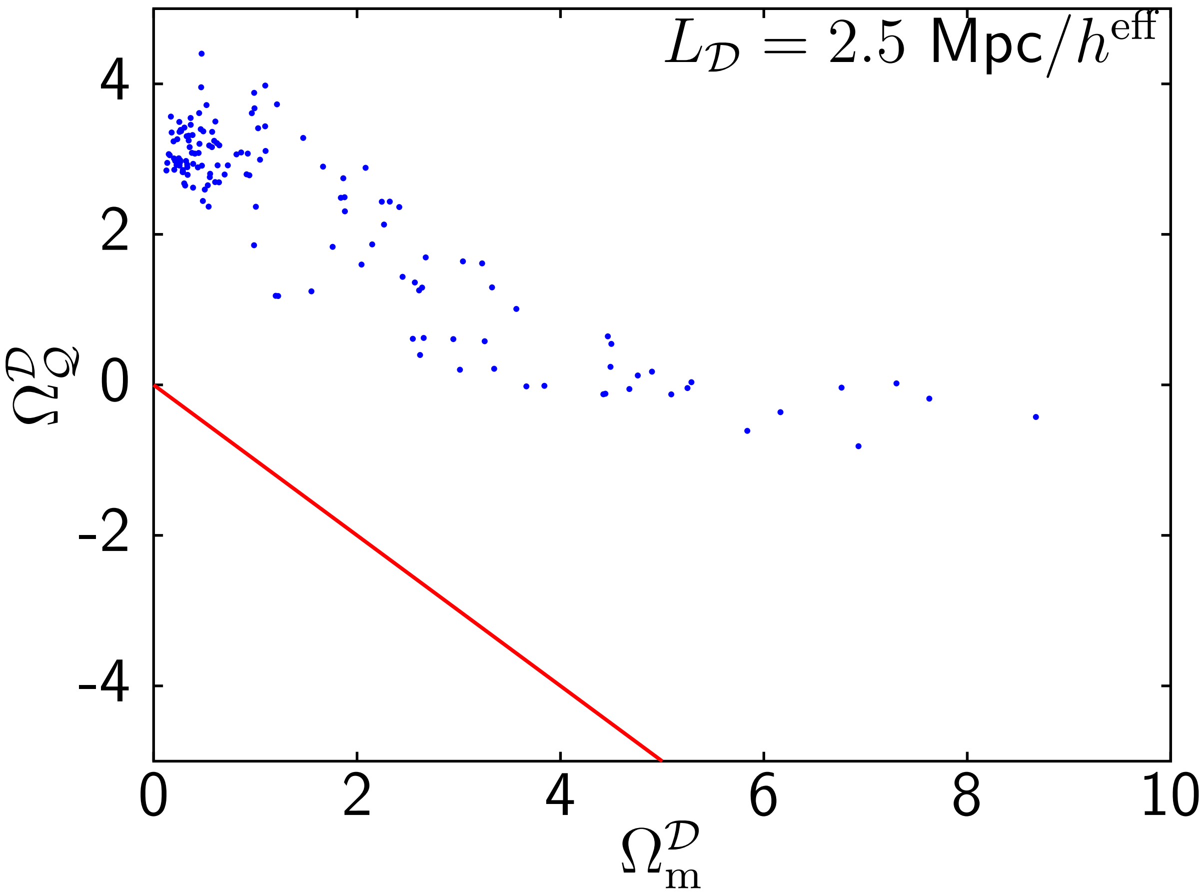


ΛCDM

Does $\Omega_{\mathcal{Q}}^{\mathcal{D}}$ allow negative ${}^{3}\mathcal{R}$ at TA?



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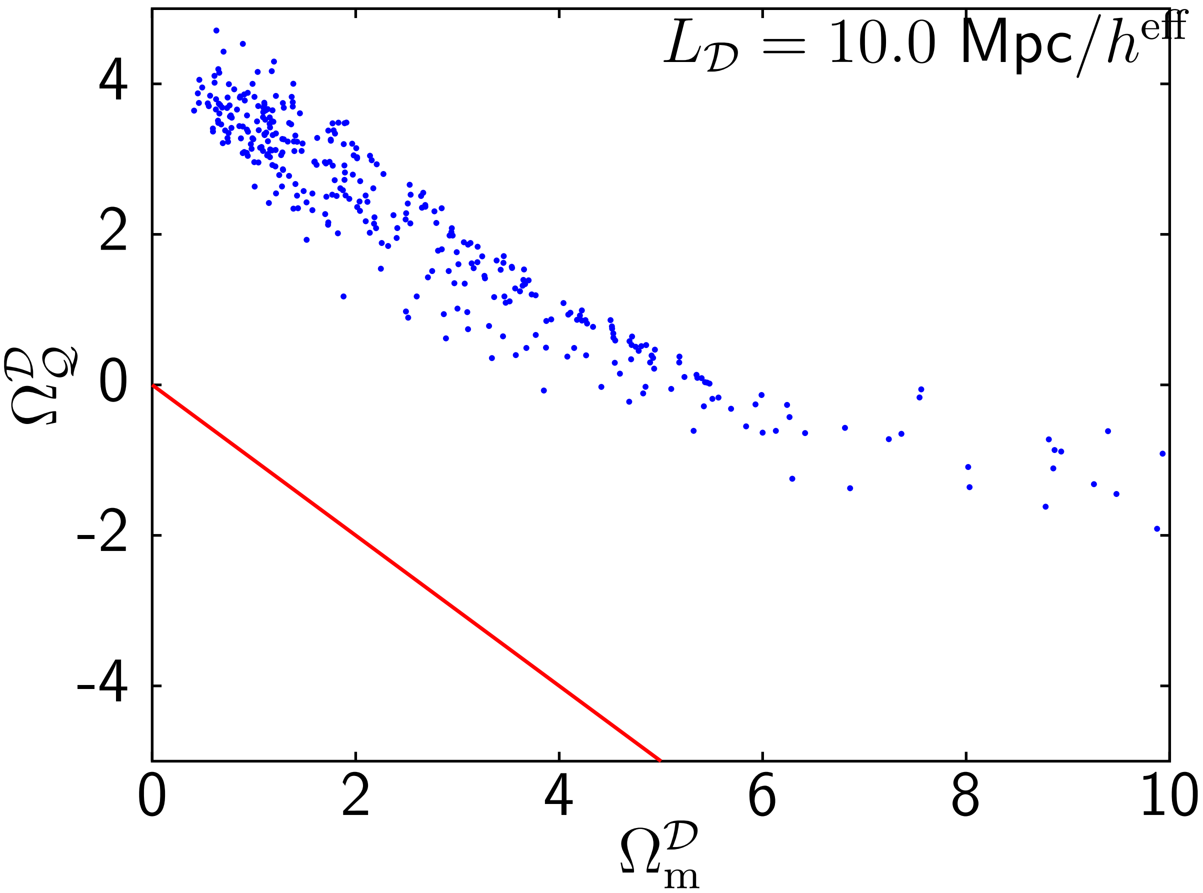

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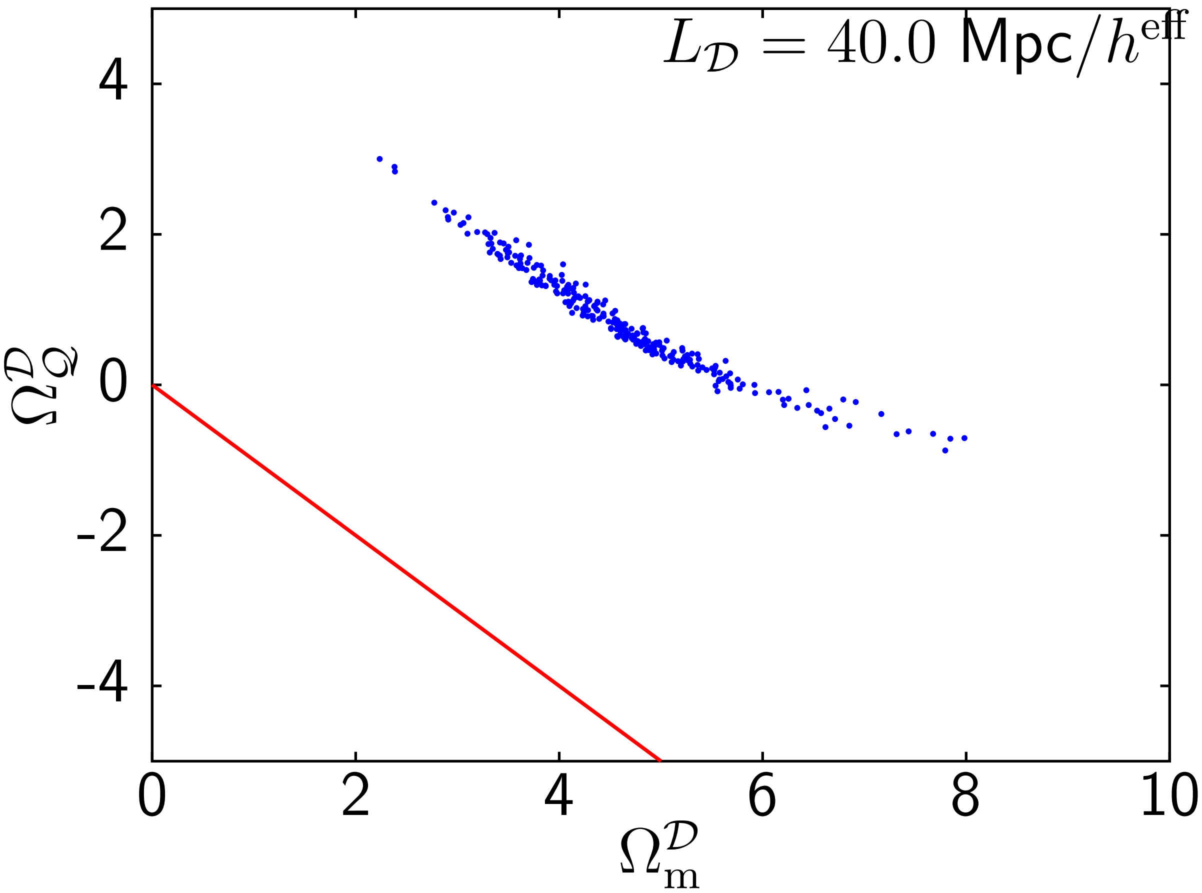

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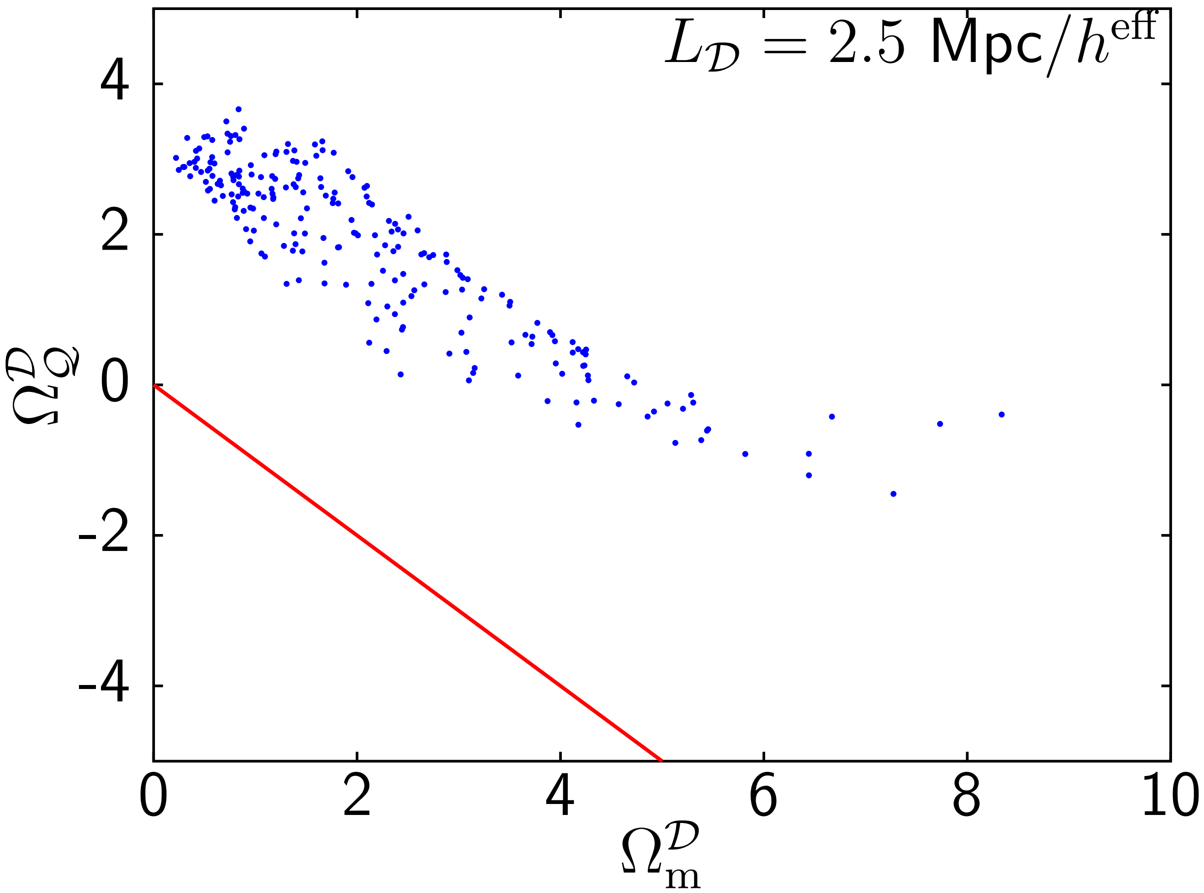

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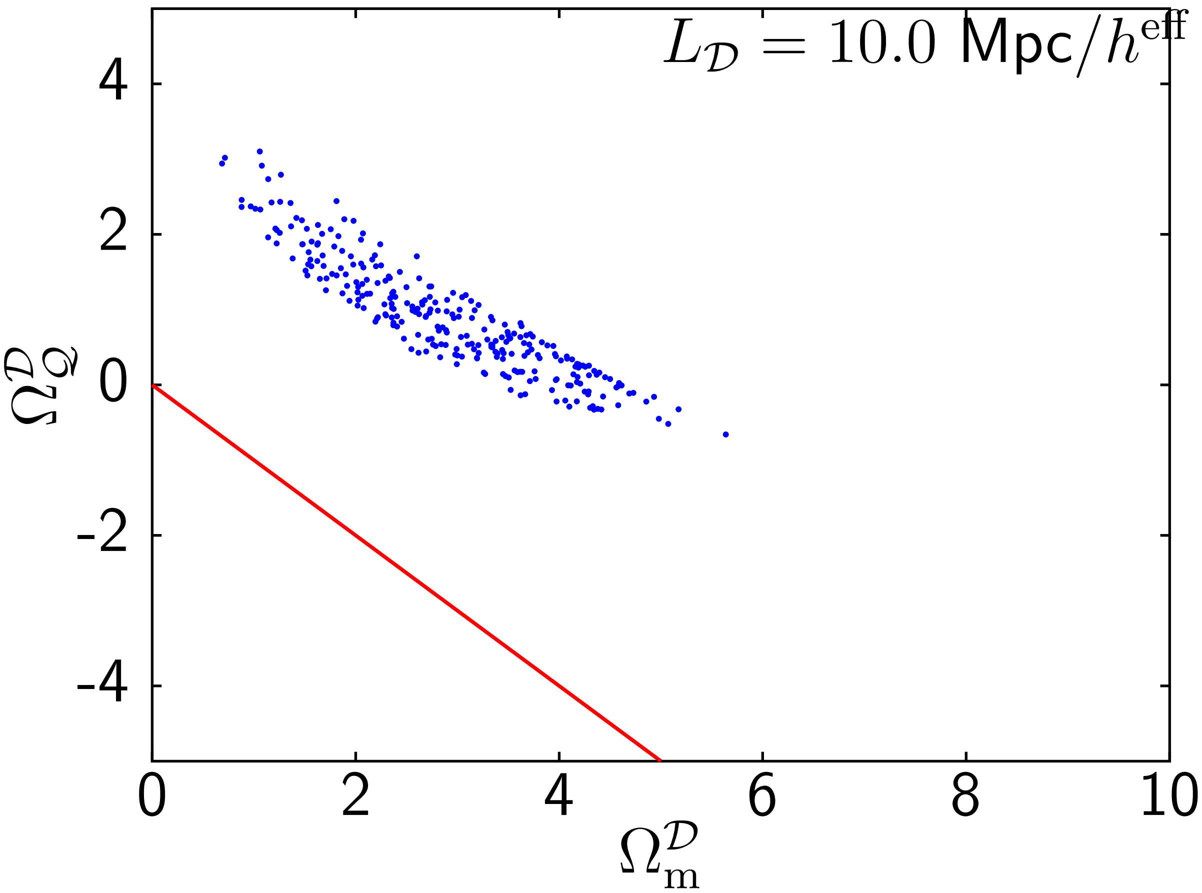

ΛCDM

Curvature-induced sp.geod. devn

Define:

$$\varepsilon := \frac{1}{a_{\mathcal{D}}\, L_{\mathcal{D}}} \left(R_{\mathrm{C}}^{\mathrm{eff}} \sin \frac{a_{\mathcal{D}}\, L_{\mathcal{D}}}{R_{\mathrm{C}}^{\mathrm{eff}}} - a_{\mathcal{D}}\, L_{\mathcal{D}} \right)$$

$$= -\frac{1}{6} \left(a_{\mathcal{D}}\, H_{\mathrm{eff}}\, L_{\mathcal{D}}\right)^2 \Omega_{\mathcal{R}}^{\mathcal{D}} + \ldots \,,$$

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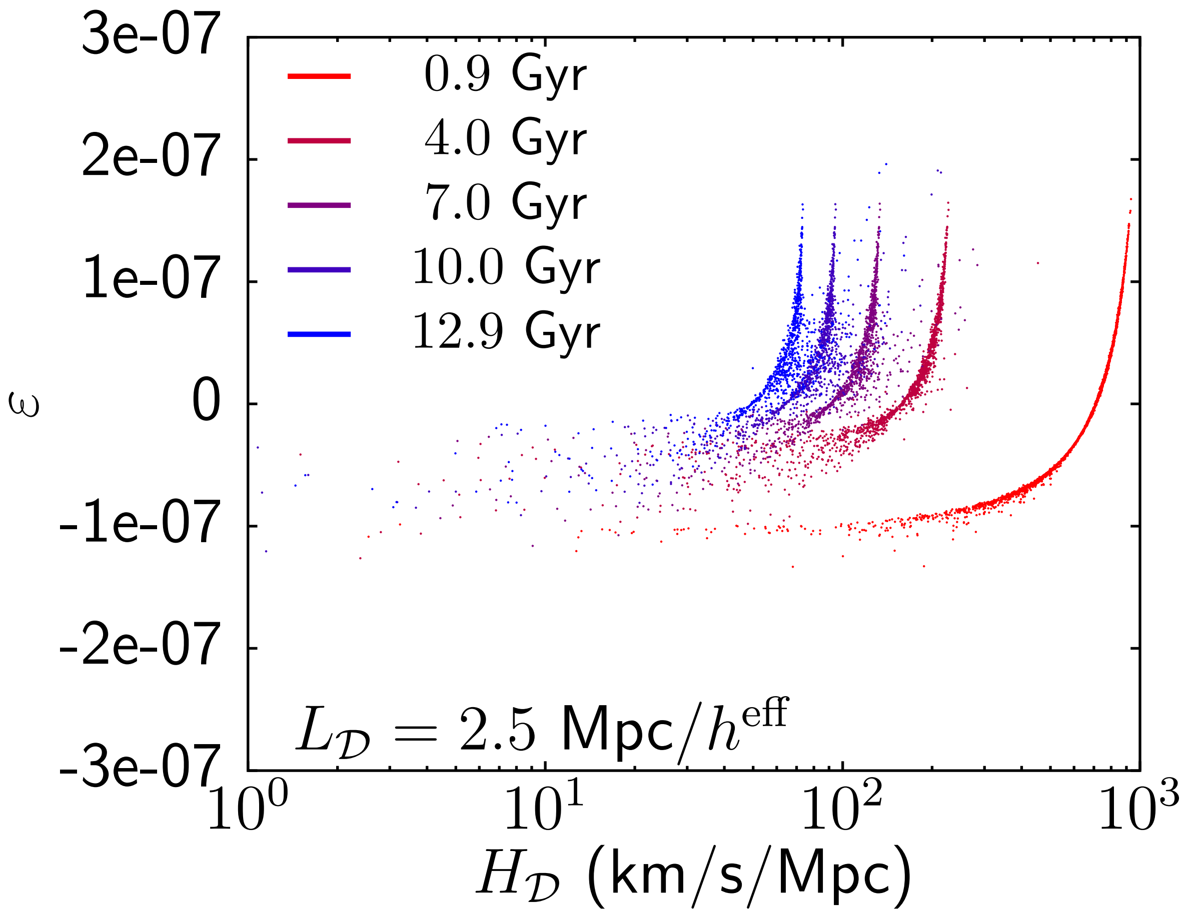

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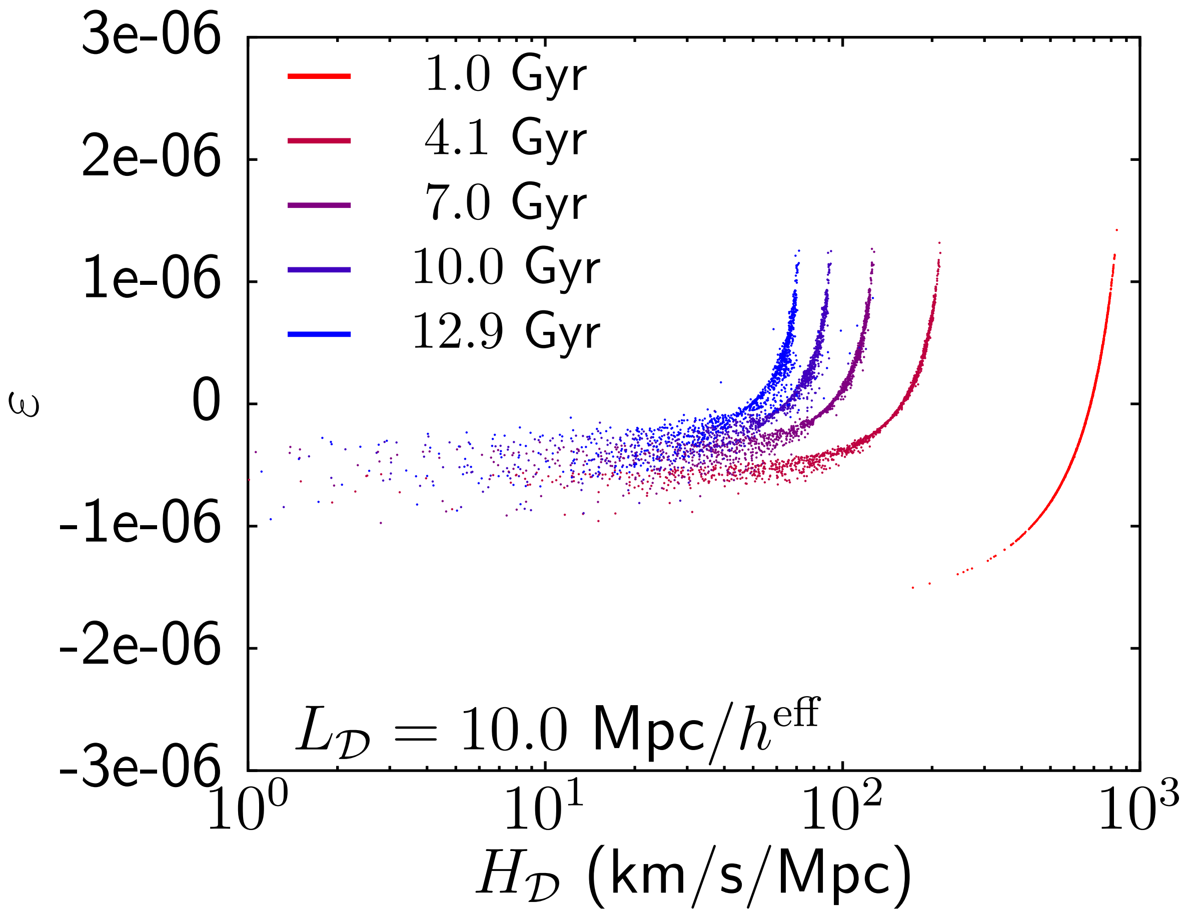

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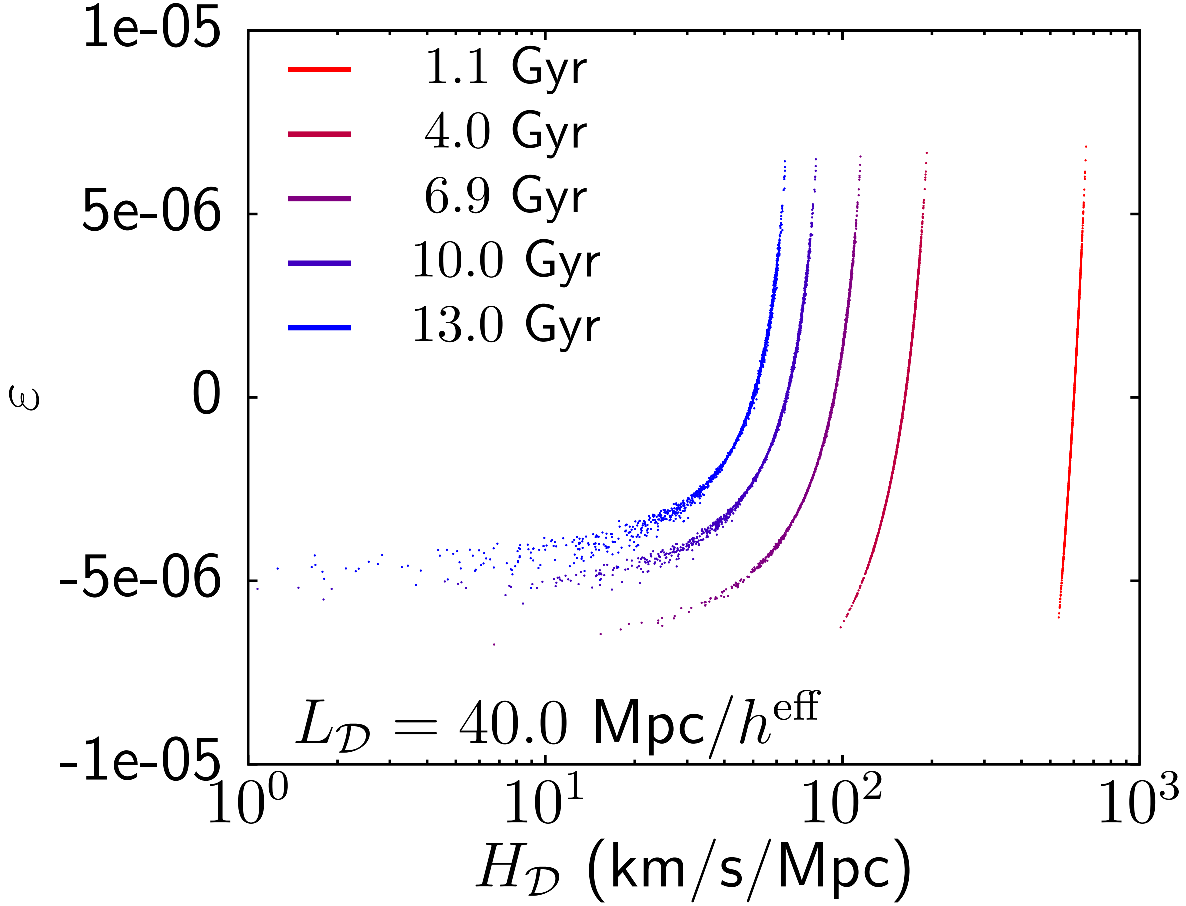

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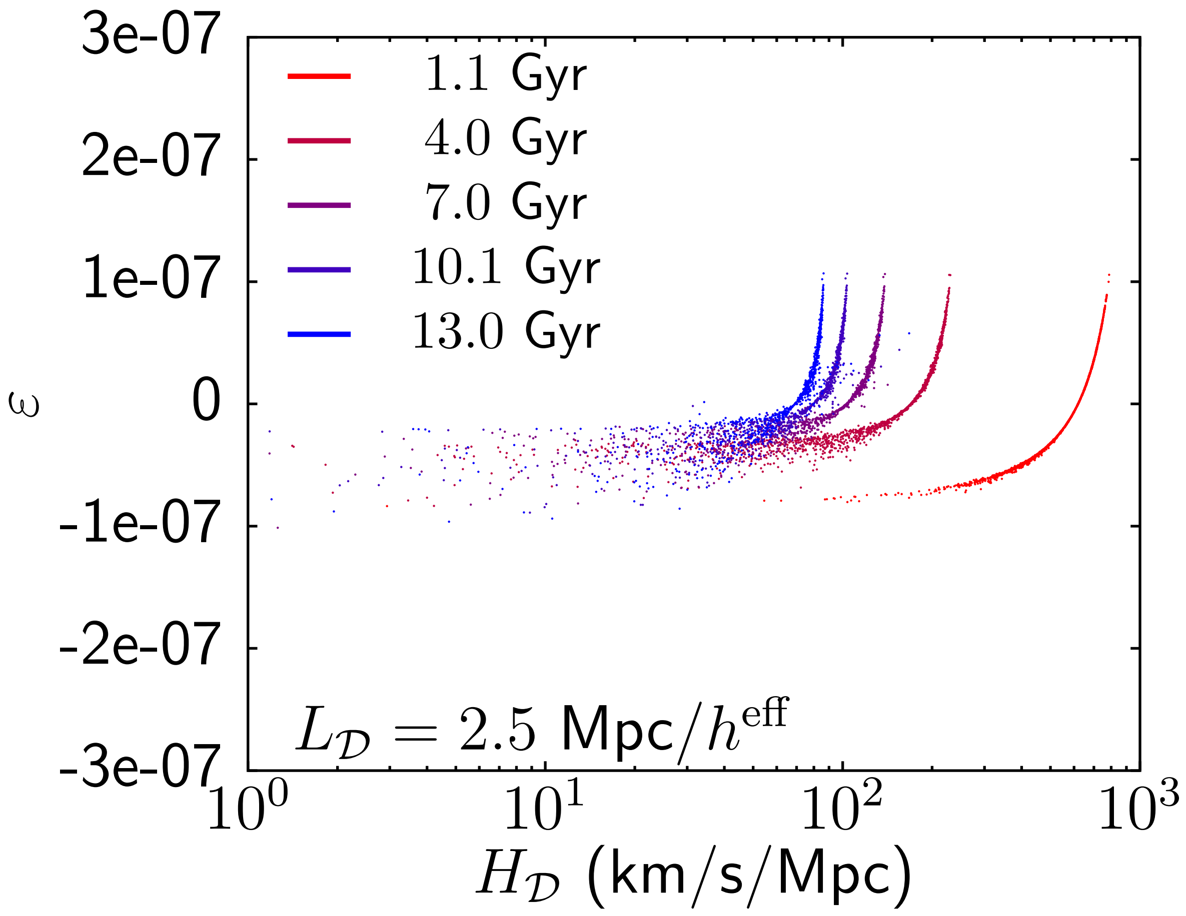

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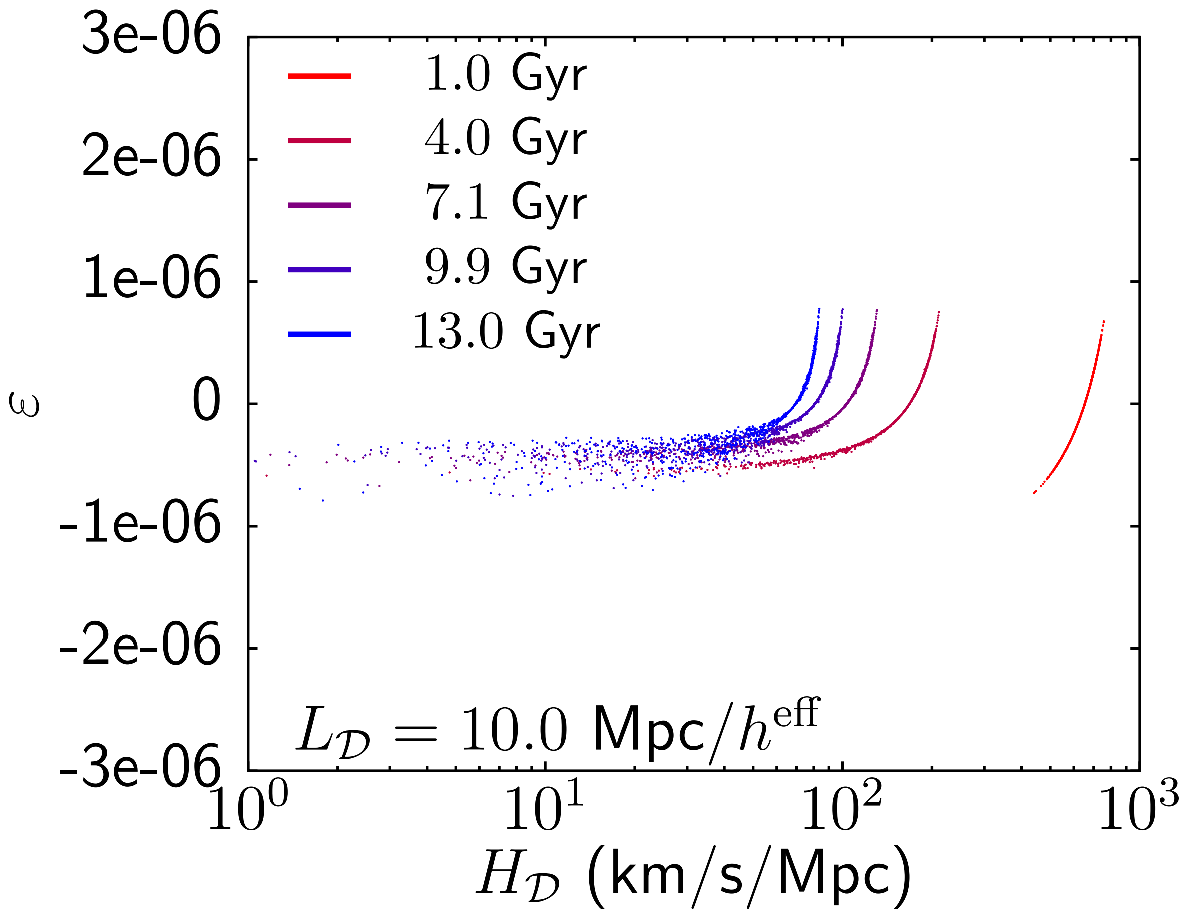

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  $\Omega^{\mathcal{D}}_{\mathcal{R}} = -5\alpha^2,\ \ \Omega^{\mathcal{D}}_{\mathcal{Q}} = \alpha^2,\ \ \Omega^{\mathcal{D}}_{\mathrm{m}} = 4\alpha^2$

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- arXiv:0902.09064 (RO19) — special thanks Mourier + Vigneron
- *Scientific reproducibility — check all figures and tables yourself!*