Towards a geometric & covariant interpretation of Milgrom's acceleration

Roberto A Sussman, Xavier Hernández, Juan Carlos Hidalgo
(UNAM, Mexico)

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Milgrom's acceleration

$$a_0 = 1.2 \times 10^{-8}\,\frac{\mathrm{cm}}{\mathrm{s}^2}$$

Milgrom's length scale: depends on the mass

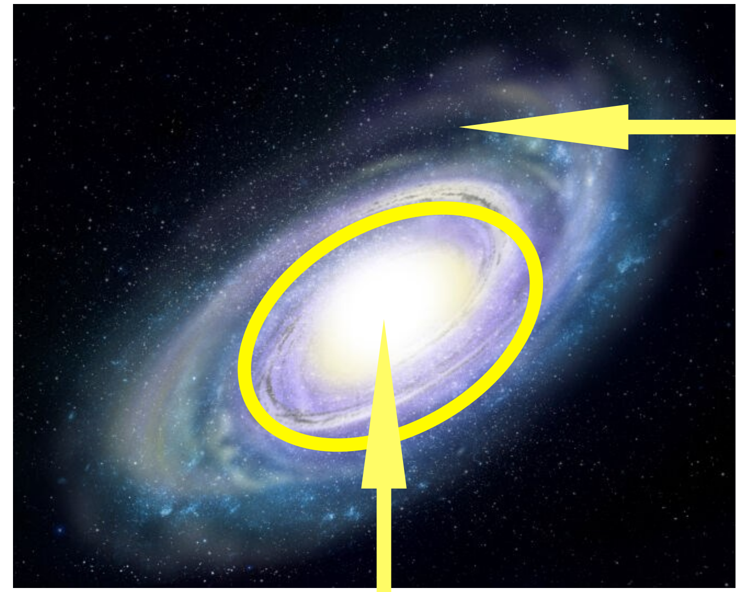
$$R_M = \left(\frac{GM}{a_0}\right)^{1/2}$$

- **These parameters were proposed by Milgrom in 1984 to describe galactic dynamics by modifying Newtonian gravity (MOND)**

- **Instead of assuming the existence of dark matter, Milgrom assumes that Newtonian gravity must be modified for accelerations** $a < a_0$ **that occur at scales** $r > R_m$ **for each** ***M***

SO FAR THERE IS NO COVARIANT INTERPRETATION

Dark Matter approach:
Only Newtonian gravity
Baryonic + Dark matter



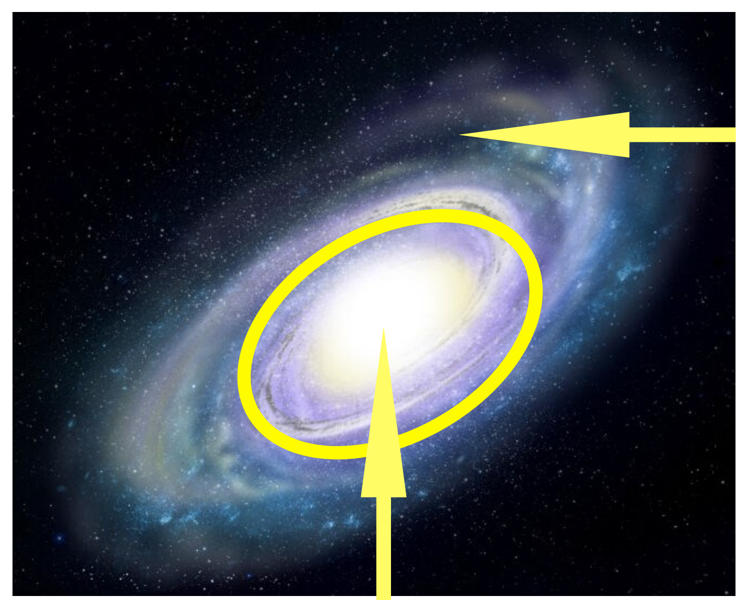
$r > R_M, \ a < a_0$

Dark matter dominates

$r < R_M, \ a > a_0$

Baryonic matter dominates

MOND approach:
Only Baryonic matter
Modified Newtonian gravity



$r > R_M, \ a < a_0$

MOND regime

$r < R_M, \ a > a_0$

Newtonian regime

Known empiric results:

Relation with cosmological expansion

$$a_0 \approx cH_0 \qquad\qquad a_0 = \frac{1}{5.83} \times cH_0$$

Tully-Fisher law

Galactic systems with baryonic mass M_b and observed terminal rotation velocity V_T satisfy

$$V_T = \left[G M_b\, a_0\right]^{1/4}$$

Known empiric results:

Systems with good agreement

- **Galaxies (rotation and dispersion velocities)**
- **Dwarf Galaxies and Globular Clusters (Hernandez+Jimenez, 1108.4021)**
- **Wide Binary systems. (Hernandez++,1105:1873)**

Systems with poor agreement[1]

- **CMB angular spectrum**
- **Matter Powerspectrum and BAO signal**

[1] **Cf. Angus, 0805.4014**

Relate a_0 to curvature scalars

- **Kretschmann scalar:** $\mathcal{K} = \mathcal{R}_{abcd}\mathcal{R}^{abcd}$

  The most general curvature scalar, contains the Ricci and Weyl part of Riemannian curvature. Must be nonzero for any non-trivial source in any metric theory

- **The ratio $\frac{a_0}{c^2}$ has units cm^{-1} but it is not useful (we tried it)**
- **Simplest combination of c and a_0 in units cm^{-2} is $\left[\frac{a_0}{c^2}\right]^2$**
- **Kretschmann scalar has units cm^{-4}, therefore to obtain a quantity with units cm^{-2} to equate to a_0^2/c^4, we multiply it by a surface area**

$$\kappa \equiv 4\pi\ell^2 \times \mathcal{K}(\ell) \propto \left(\frac{a_0}{c^2}\right)^2 = \mathrm{const.}$$

where ℓ is a suitable length scale and $\mathcal{K}(\ell)$ must be computed for a suitable metric

The natural length scale to compute Kretschmann scalar times area is the Milgrom radius [redacted].

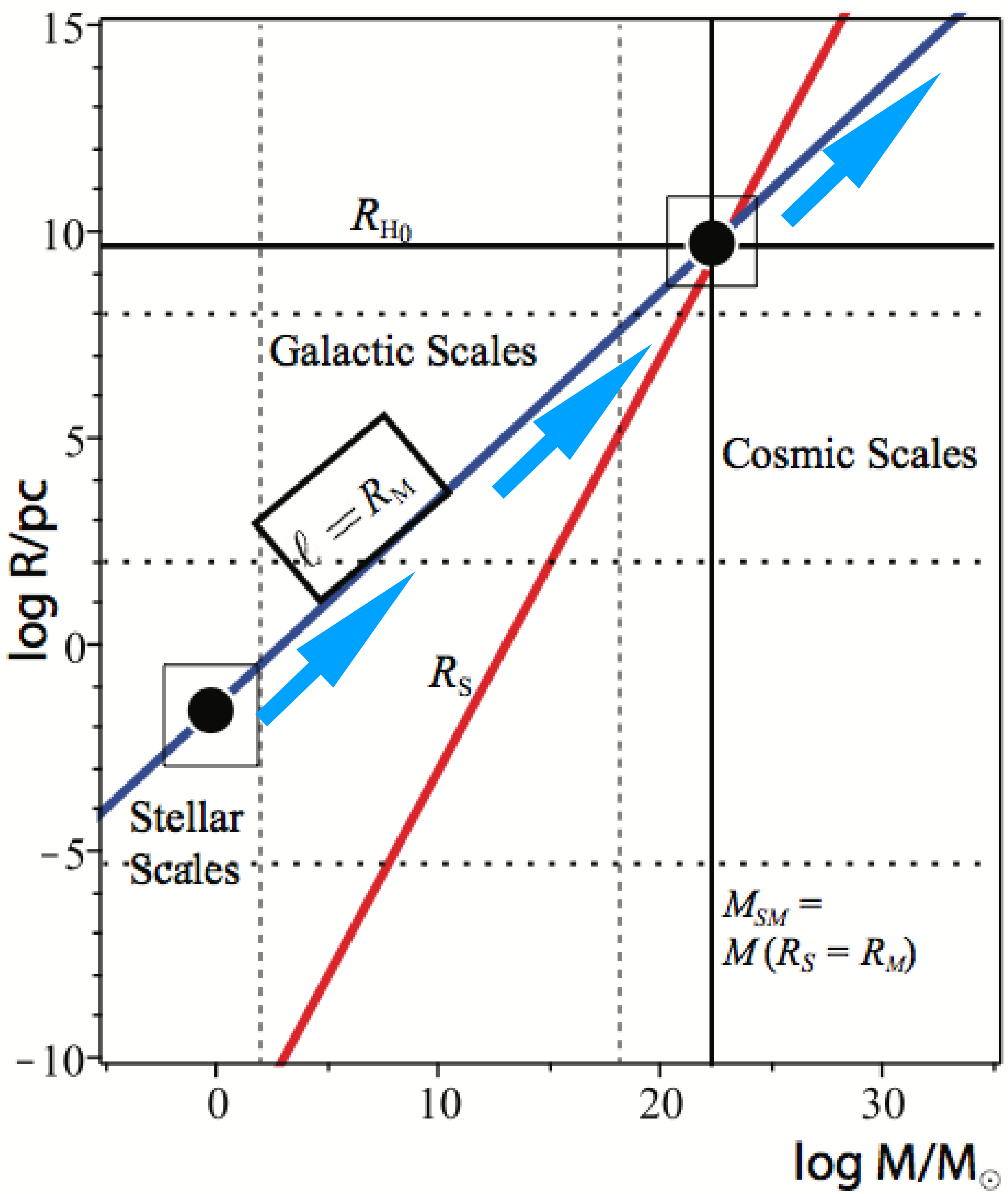
Look at fundamental length scales at our cosmic time

Milgrom radius $$R_M = \left(\frac{GM}{a_0}\right)^{1/2}$$

Schwarzschild radius $$R_S = \frac{2GM}{c^2}$$

Hubble radius (h = 0.7) $$R_{H_0} = \frac{c}{H_0} = 4285\,\mathrm{Mpc}$$

The Milgrom radius over a range of scales



Evaluate κ along the blue arrows that mark $\ell = R_M$

- **Cosmic scales**

Mass for which $R_S = R_M$

$$M_{SM} = 1.26 \times 10^{23}\,\mathrm{M}_\odot$$

Hubble Mass $M_{H_0} = \frac{4}{3}\pi\,\rho_{\mathrm{crit}}\,R_{H_0}^3$

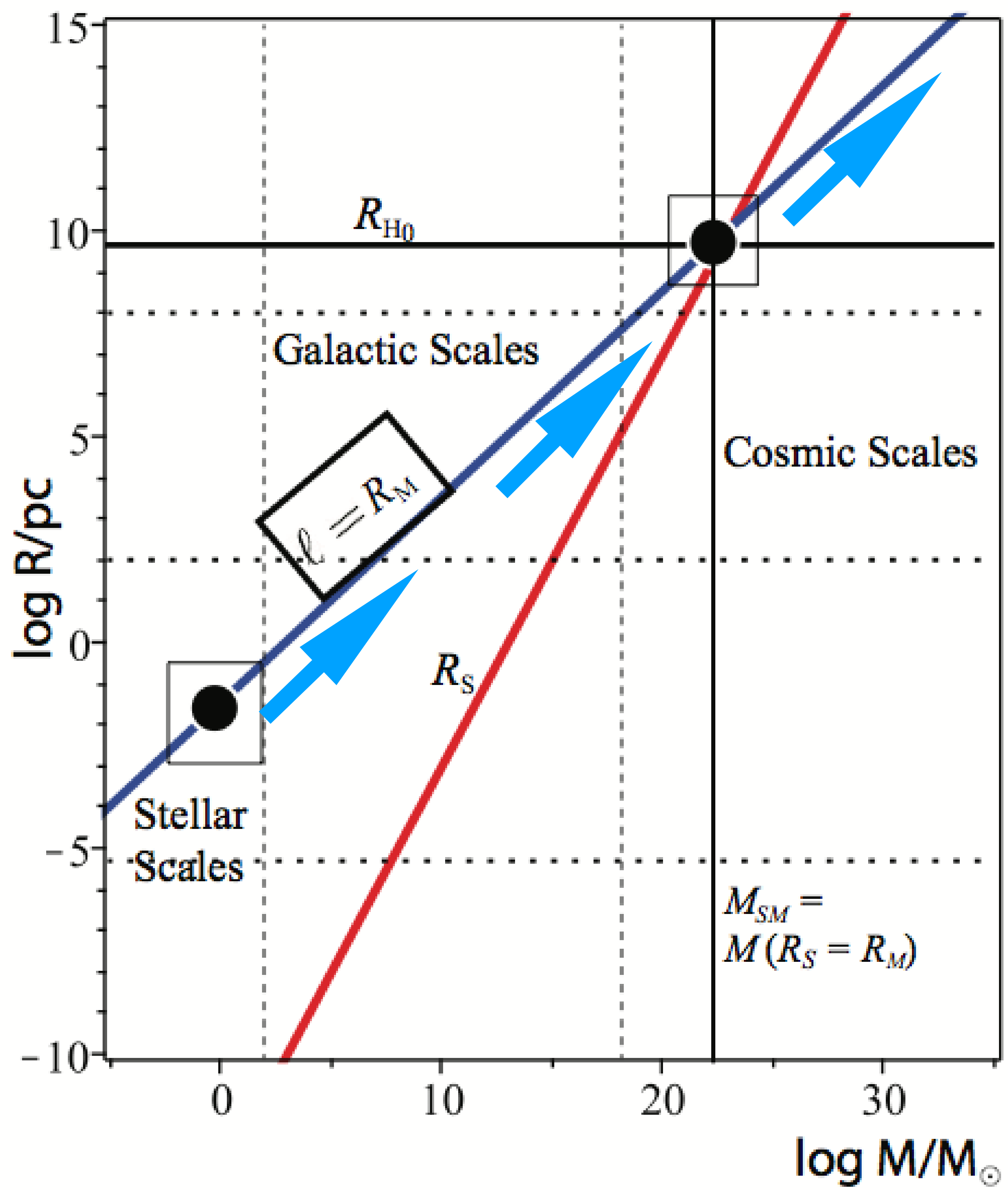
$$M_{H_0} = 4.33 \times 10^{22}\,\mathrm{M}_\odot$$

At cosmic scales $M_{SM} \approx M_{H_0}$

- **THEREFORE:**

$$\ell = R_M \approx R_{H_0}$$

The Milgrom radius over a range of scales



- **Stellar scales**

  Single stars & compact binaries or multiple systems

  $$M \approx M_{\mathbf{stars}}$$
  $$0.01\,\mathrm{M}_\odot < M < 100\,\mathrm{M}_\odot$$

  Milgrom's radius

  $$0.0035\,\mathrm{pc} < R_M < 0.35\,\mathrm{pc}$$

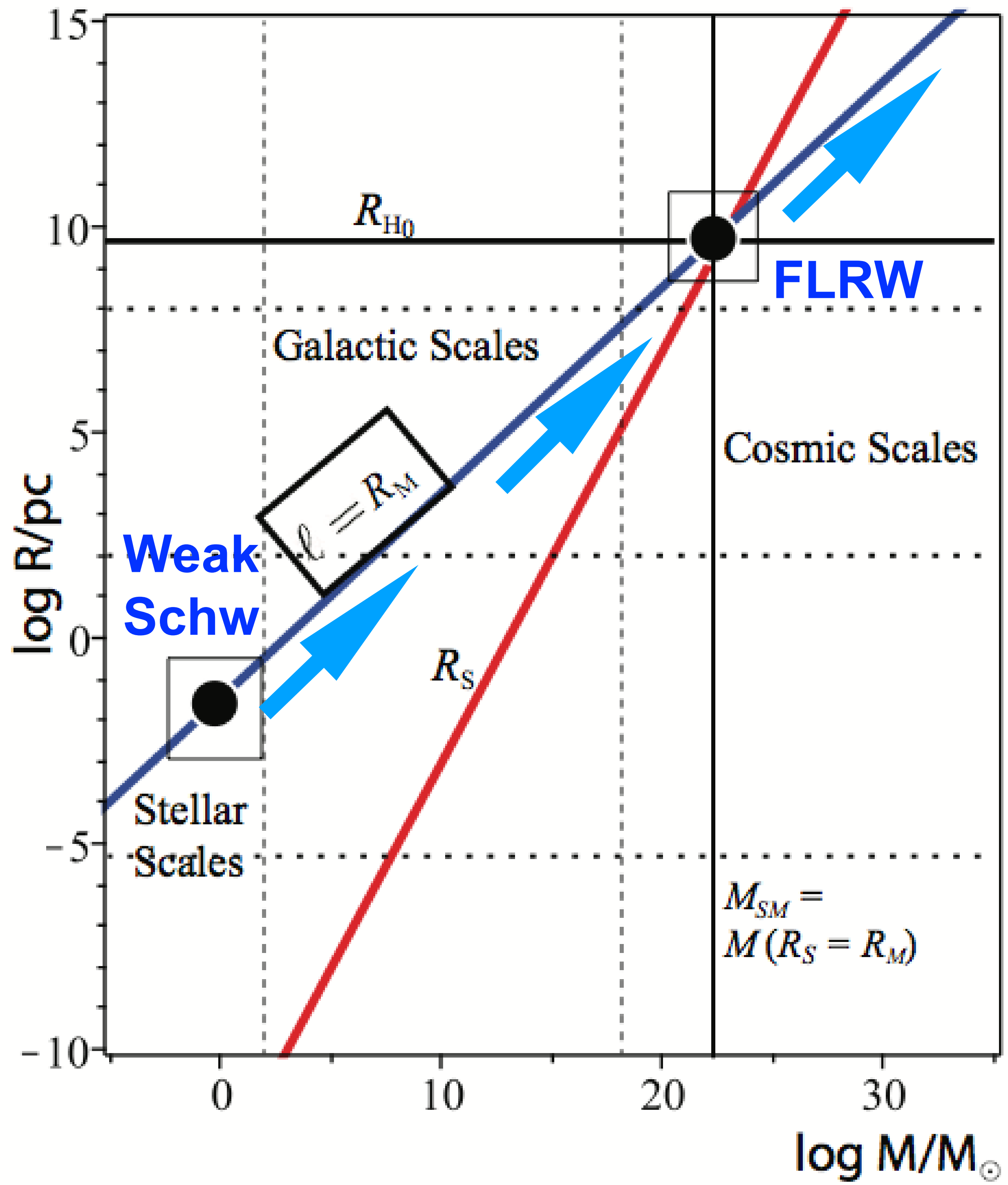
  Characteristic length

  $$0.1\,\mathrm{AU} < R_{\mathbf{char}} < 100\,\mathrm{AU}$$
  $$5 \times 10^{-7}\,\mathrm{pc} < R_{\mathbf{char}} < 5 \times 10^{-4}\,\mathrm{pc}$$

- **THEREFORE**

  $$R_M \gg R_{\mathbf{char}} \gg R_S$$

Metrics at stellar & cosmic scales to evaluate κ at $\ell = R_M$



- **Stellar scales (static)**

$$\ell = R_M \gg R_{\text{char}}$$

At $\ell = R_M$ stellar systems can be described by weak field **Schwarzschild metric**

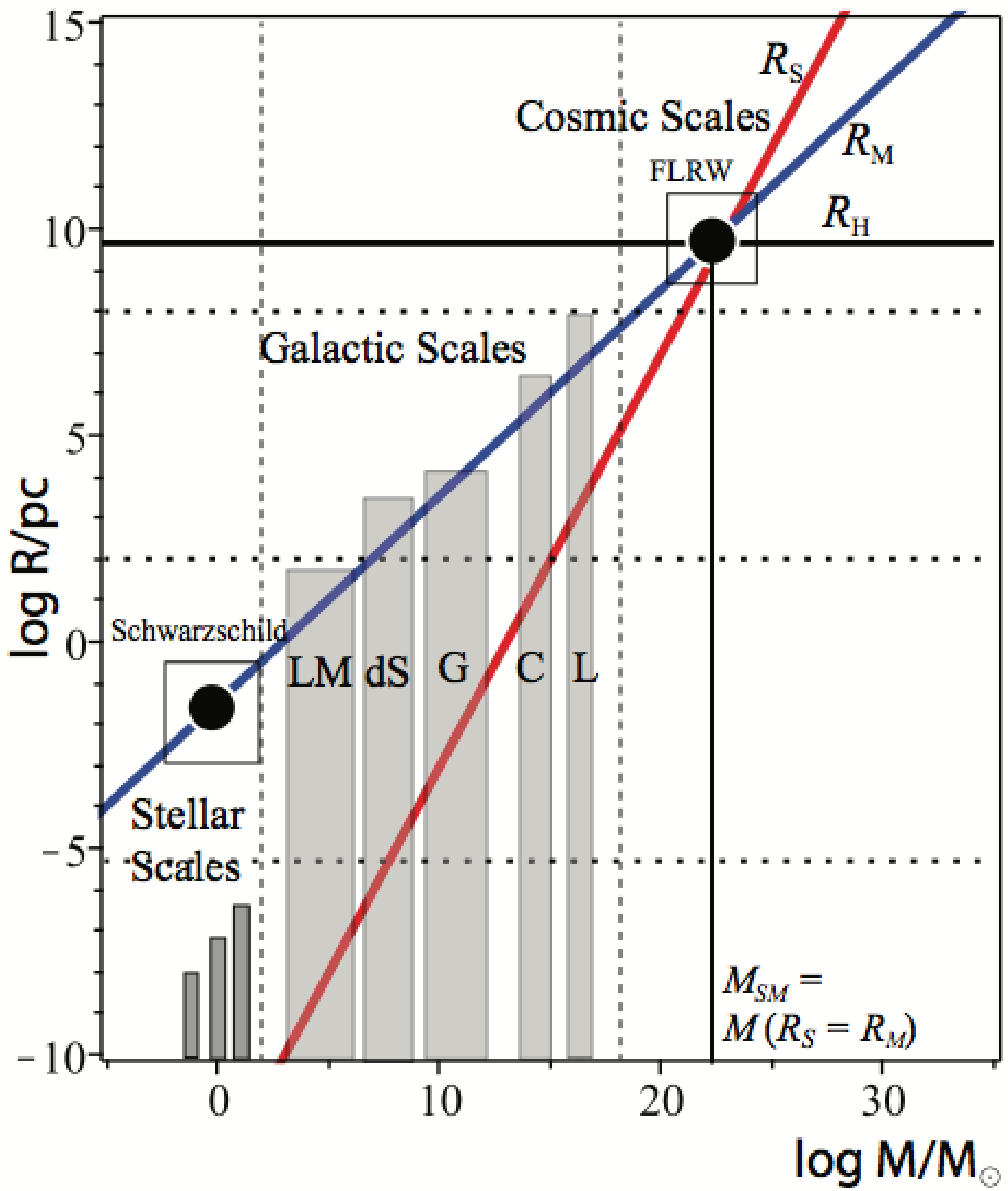
- **Cosmic scales (expanding)**

$$\ell_0 = [R_M]_0 \approx R_{H_0}$$

$$\ell(t) = [R_M](t) \approx R_H = \frac{c}{H(t)}$$

At $\ell = R_M$ cosmic scales can be described by the **FLRW metric**

Galactic scale: uncertainty



Milgrom radius inside or very near the end of visible matter distribution

$$\ell = R_M \approx R_{\mathrm{char}} \gg R_S$$

R_{char} is the characteristic size of baryonic matter structures

- **LM** = Low Mass systems: globular clusters, dwarf galaxies, etc
- **dS** = Dwarf spheroidal galaxies
- **G** = Spiral and elliptic galaxies
- **C** = Clusters of galaxies
- **L** = Superclusters (Laneakea)

Metric to compute κ and evaluate at $\ell = R_M$???

Proposal

Probe

$$\kappa \equiv 4\pi\ell^2\mathcal{K}(\ell) \propto \left(\frac{a_0}{c^2}\right)^2, \qquad \mathcal{K} = \mathcal{R}_{abcd}\mathcal{R}^{abcd}$$

- **Stellar scales (static weak field Schwarzschild)**

$$ds^2 = -\left(1 - \frac{R_S}{r}\right)c^2dt^2 + \left(1 + \frac{R_S}{r}\right)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\frac{R_S}{r} = \frac{2GM}{c^2 r} \ll 1 \qquad \ell = \alpha R_M = \alpha\left(\frac{GM}{a_0}\right)^{1/2} \qquad \alpha \sim O(1)$$

constant

- **Cosmic scales (expanding FLRW cosmology)**

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1 - k_0 r^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$\ell = R_M = \frac{c}{H}, \qquad H \equiv \frac{a_{,t}}{a}$$

- **Demand consistency**

Results

Stellar scales: weak Schwarzschild field

$$\kappa = \left[4\pi\ell^2\mathcal{K}(\ell)\right]_\ell = \left[\frac{192\pi G^2 M^2}{c^4 r^4}\right]_{r=\ell} = \frac{192\pi}{\alpha^2}\left(\frac{a_0}{c^2}\right)^2$$

Cosmic scales: expanding spatially flat FLRW metric

$$\kappa = \left[4\pi\ell^2\mathcal{K}(\ell)\right]_\ell = \frac{48\pi H^2\left(1+q^2\right)}{c^2}, \qquad q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -\left(1+\frac{\dot{H}}{H^2}\right)$$

Results: consistency stellar vs cosmic scales

$$\kappa = \frac{48\pi H_0^2 \left(1+q_0^2\right)}{c^2} = \frac{192\pi}{\alpha^4}\left(\frac{a_0}{c^2}\right)^2$$

Calibration: Planck 2015 values

$$\Omega_0^m = 0.315, \quad \Omega_0^\Lambda = 0.685, \quad H_0 = 68\frac{\mathrm{km}}{\mathrm{s\,Mpc}} \quad \Rightarrow \quad q_0 = -0.5275$$

$$\Rightarrow \quad a_0 = \frac{\alpha^2\sqrt{1+q_0^2}}{4\sqrt{3}} \times c\,H_0 \approx \frac{c\,H_0}{5.83} \quad \Rightarrow \quad \alpha = 1.0511 \approx 1$$

Within observational error

Covariant geometric forms for Milgrom's parameters

$$a_0 = \frac{\sqrt{1+q_0^2}}{4\sqrt{3}}\,cH_0, \qquad R_M = \frac{2\times 3^{1/4}}{\left(1+q_0^2\right)^{1/4}}\left(\frac{GM}{cH_0}\right)^{1/2}$$

New result: Theoretical covariant interpretation of the empiric proportionality between two fundamental accelerations a_0 and cH_0

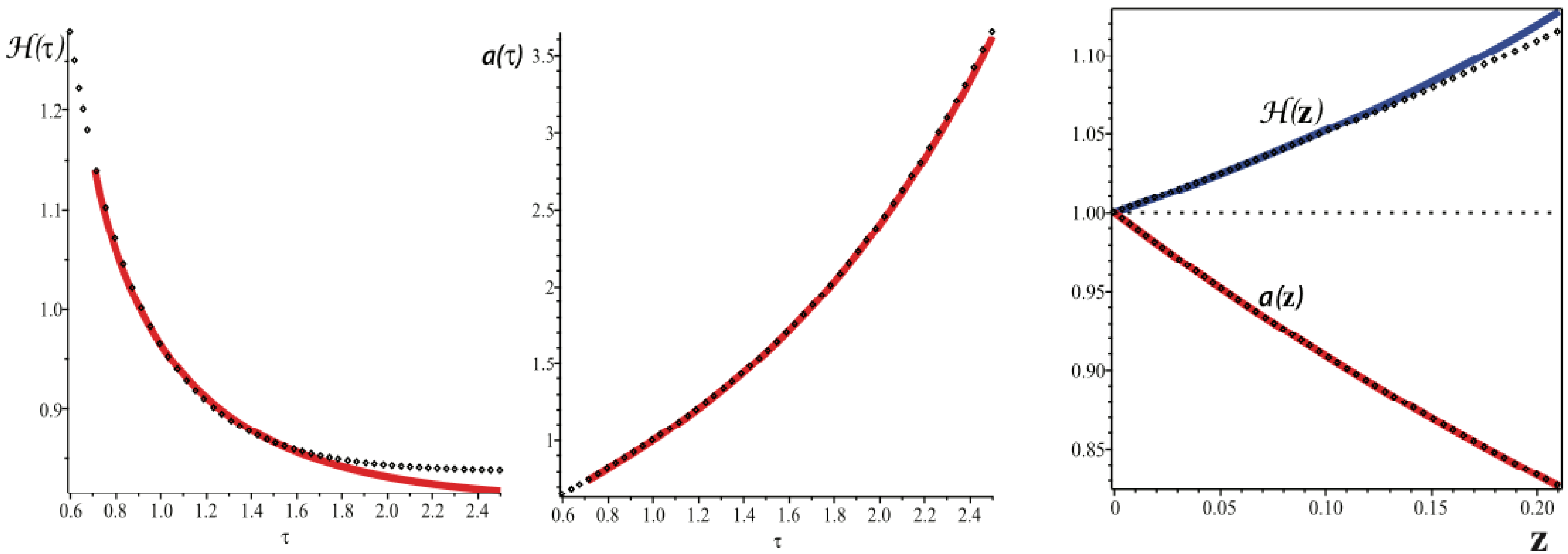
Results: predictions of the conservation of κ

Conservation law: evolution of *H*

$$\dot{\kappa} = 0 \quad \Rightarrow \quad H^2(1+q^2) = H_0^2(1+q_0^2)$$

$$\Rightarrow \quad \frac{d\mathcal{H}}{d\tau} = \mathcal{H}\left[-\mathcal{H} + \sqrt{1+q_0^2 - \mathcal{H}^2}\right], \qquad \mathcal{H} = \frac{H}{H_0}, \;\; \tau = H_0 t$$

Numerical solutions: very close fit to ΛCDM model (Planck 2015) near present time



Red solid curve = conservation of κ Black dots = ΛCDM (Planck 2015)

Error < 1% in $H(z)$ up to $z = 0.2$. Error < 1% in $a(z)$ up to $z = 2$

Results: "equation of state" w for this evolution law

What is the equation of state w that we can associate with a GR solution satisfying the evolution of *H* associated with the conservation of $\mathcal{K}$?

Compare the FLRW spatially flat GR Raychaudhuri equation with the evolution of *H* predicted by the conservation of $\mathcal{K}$

$$\frac{d\mathcal{H}}{d\tau} = -\mathcal{H}^2 + \mathcal{H}\sqrt{1 + q_0^2 - \mathcal{H}^2}, \quad \text{VS} \quad \frac{d\mathcal{H}}{d\tau} = -\mathcal{H}^2 \left[1 + \frac{1 + 3w(1 - \Omega^m)}{2}\right],$$

$$\Rightarrow \quad w(\tau) = -\frac{1 + 2\mathcal{H}\sqrt{1 + q_0^2 - \mathcal{H}^2}}{3(1 - \Omega^m)}$$

Evaluate at $t = t_0 \ (\mathcal{H}_0 = 1)$ using Planck 2015 values for $\Omega_0^m,\ q_0$

$$\Rightarrow \quad w_0 = -\frac{1 + 2|q_0|}{3(1 - \Omega_0^m)} = -1.0260$$

Very close to the ΛCDM value ***w = -1***

Conclusions & future work

- Conservation of $\kappa = 4\pi\ell^2\mathcal{K}(\ell) \propto a_0^2/c^4$ for ℓ given by a characteristic length scale provides a geometric covariant interpretation of Milgrom's acceleration.
- For stellar systems $\ell = \alpha R_M$ described by weak field Schwarzschild metric
- For cosmic scales $\ell = R_H = c/H$ described by spatially flat FLRW metric
- Consistency implies $a_0 = \frac{\sqrt{1+q_0^2}}{4\sqrt{3}}\, cH_0, \qquad R_M = \frac{2\times 3^{1/4}}{(1+q_0^2)^{1/4}}\left(\frac{GM}{cH_0}\right)^{1/2}$

  Covariant forms
- Conservation of κ yields an evolution law for ***H(z), a(z)*** that mimics that of a LCDM with < 1% error near present cosmic times
- Work to do: apply to GALACTIC SYSTEMS

Coincidence or something deeper?