Special and General Relativity

Boud Roukema

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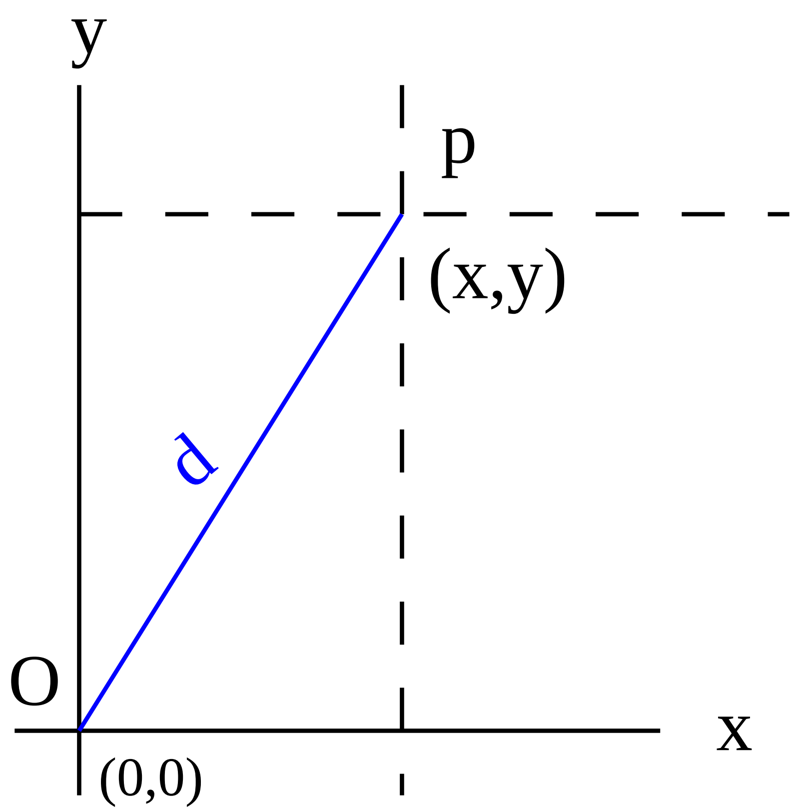
- GR: spacetime = a solution of the w:Einstein field equations

SR: Minkowski spacetime



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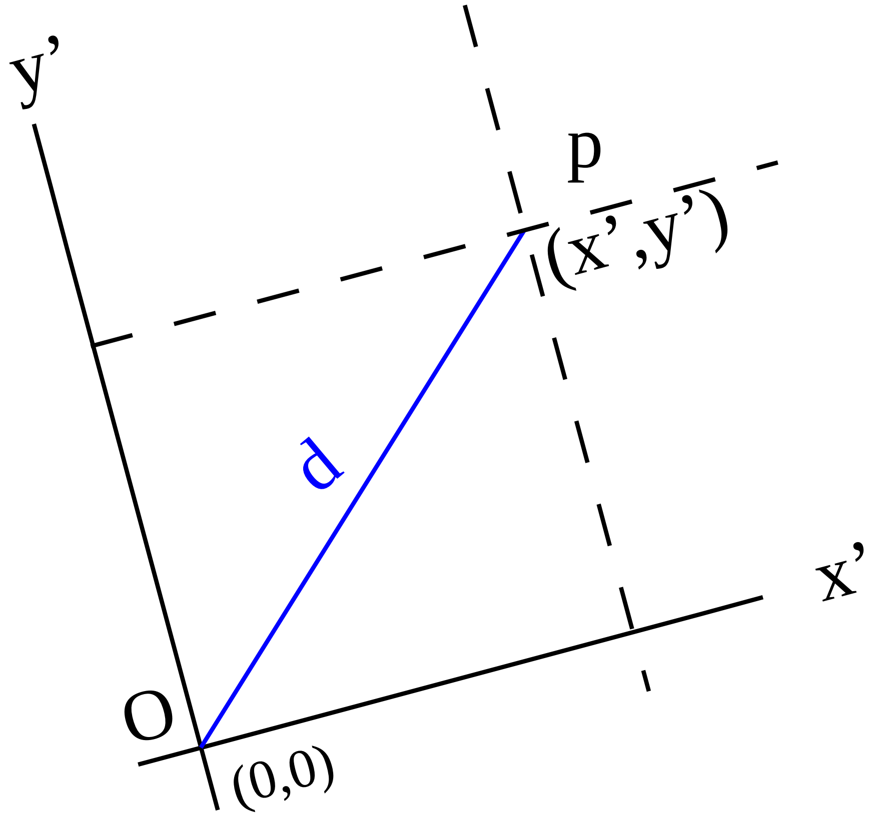

$$\begin{pmatrix} p_{x'} \\ p_{y'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

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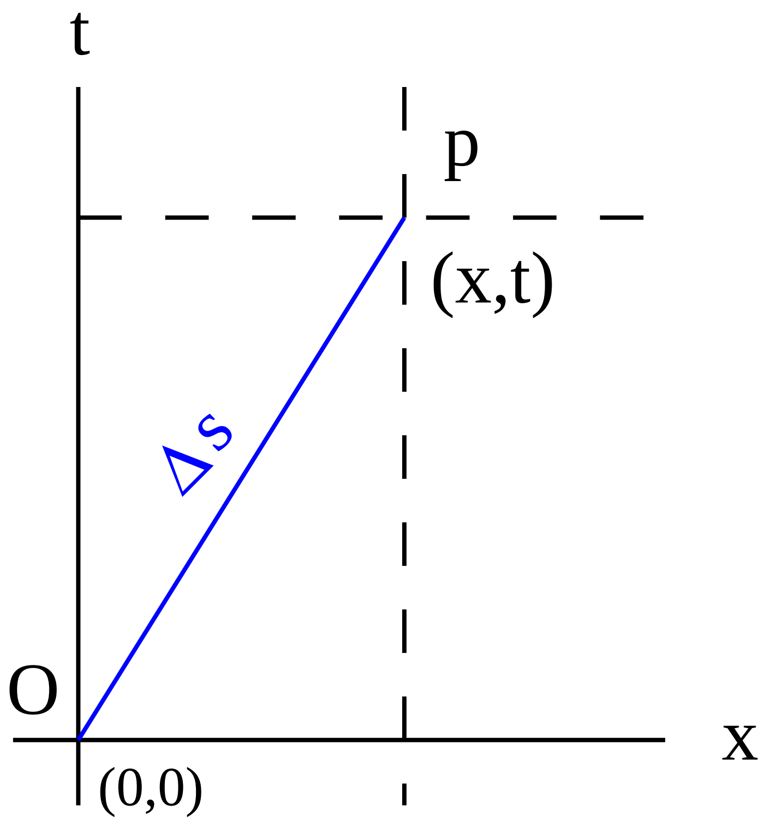

p at (x, t), w:invariant interval from observer at O is Δs
where $(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$

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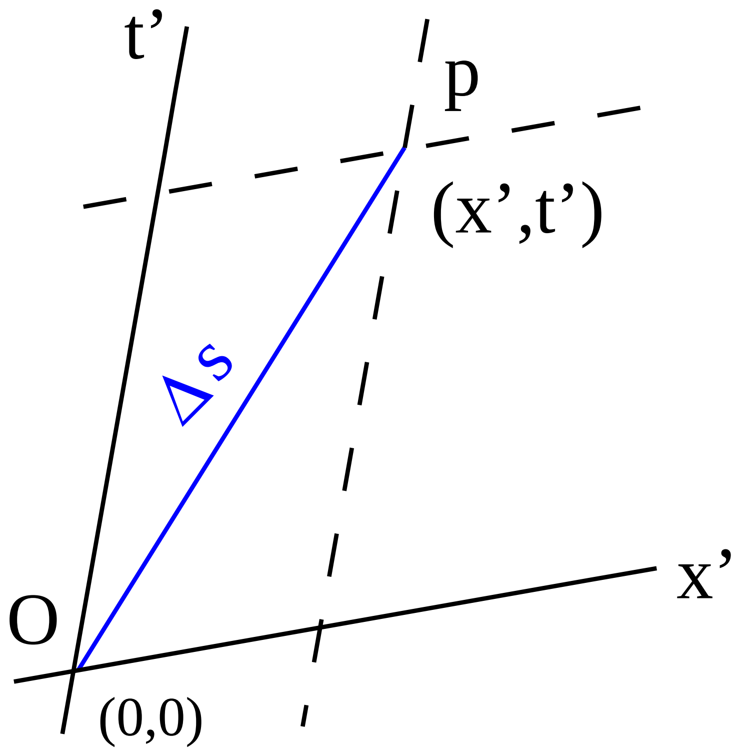

$$\begin{pmatrix} p_{\mathrm{x}'} \\ p_{\mathrm{t}'} \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} p_{\mathrm{x}} \\ p_{\mathrm{t}} \end{pmatrix}$$

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$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

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$= \frac{1 \text{ s}}{1 \text{ s}} = 1$ (dimensionless)

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in A's coordinate system, B's worldline is:

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$$\Rightarrow x = t' \sinh\phi = t \tanh\phi = \beta t$$

where velocity $\beta := v/c \equiv v = \tanh\phi$

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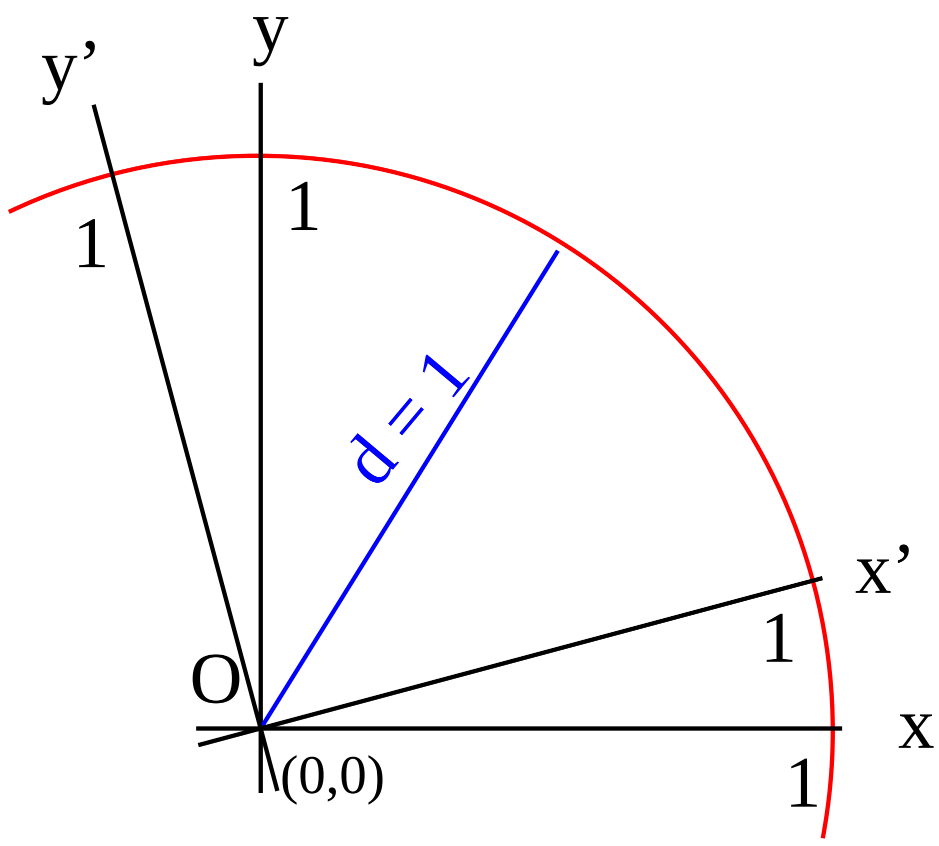

constant distance $d = 1$ from $(0, 0)$

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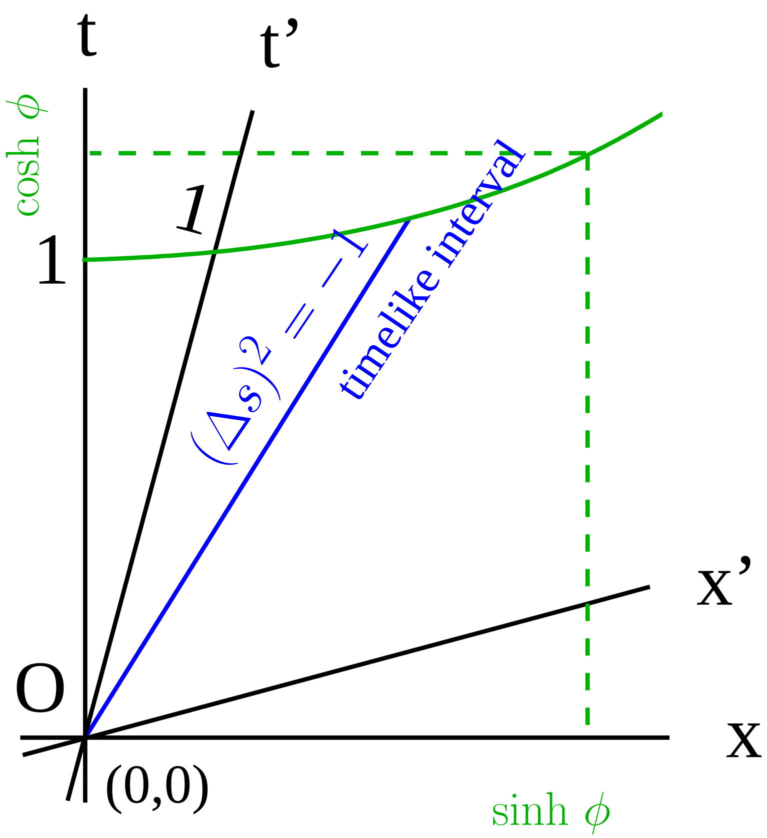

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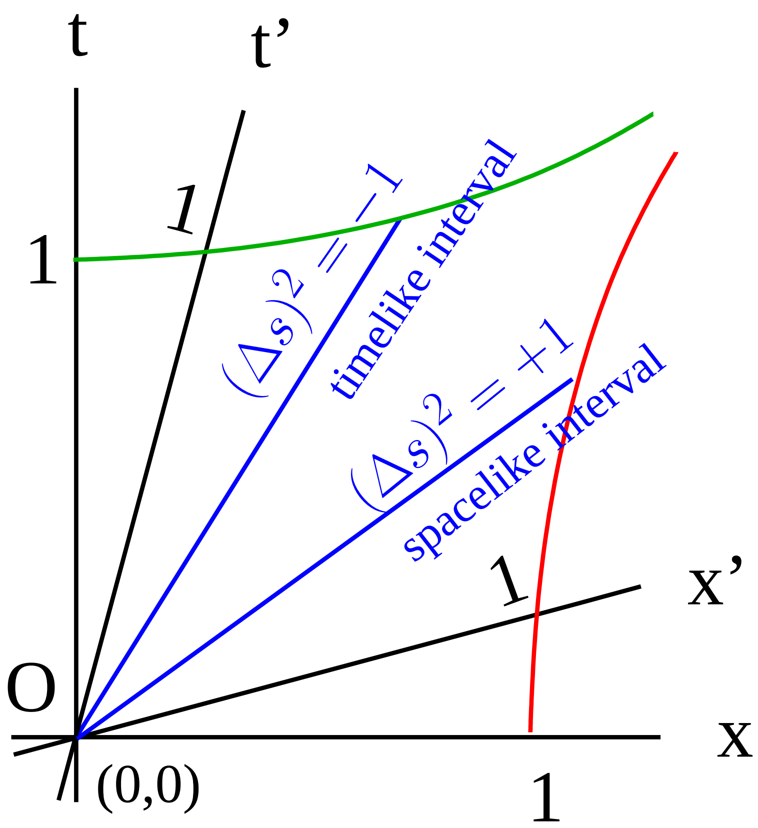

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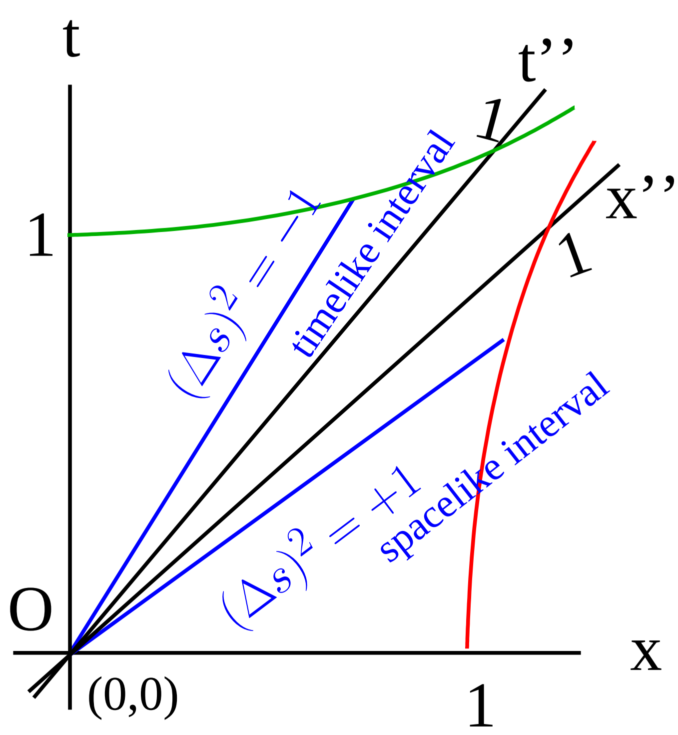

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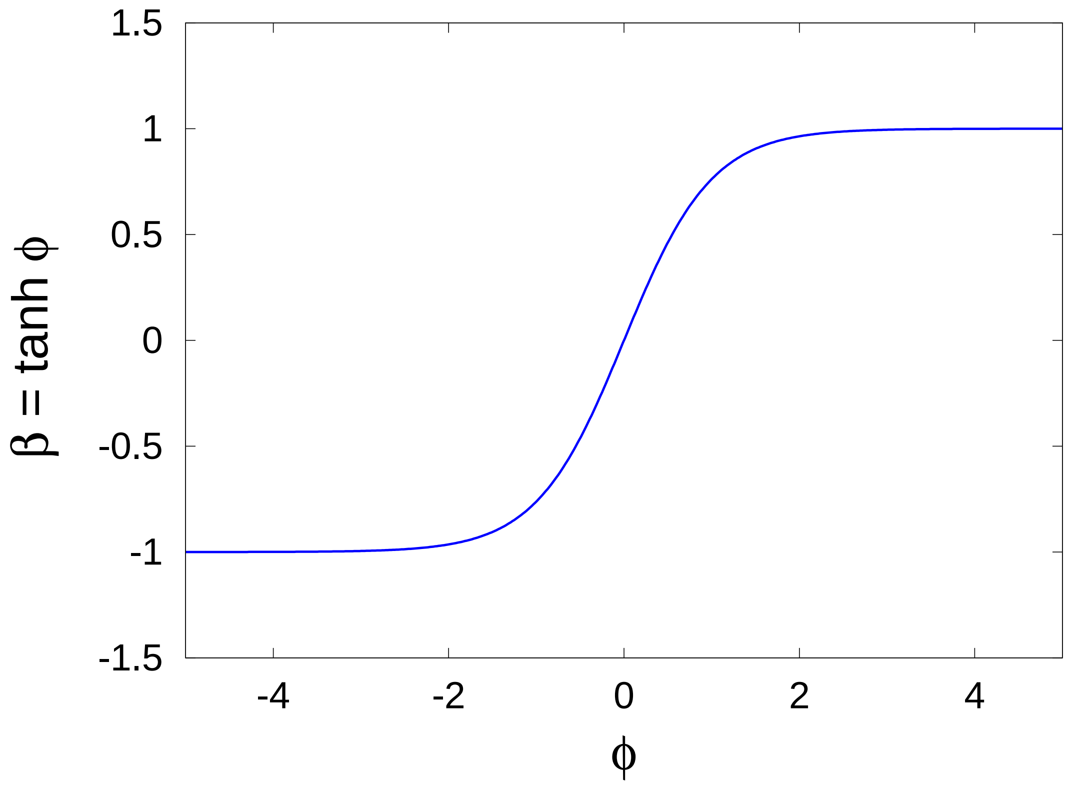

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- w:Michelson-Morley experiment (1887)

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interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?

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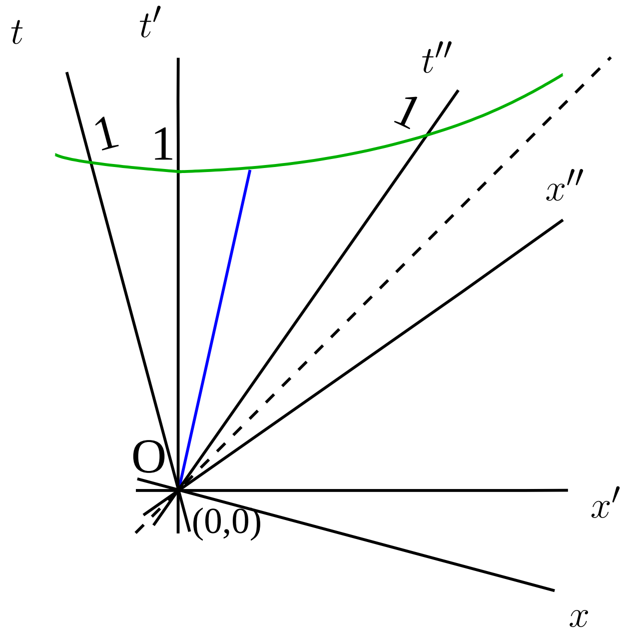

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where $\tanh\phi_1 = \beta_1 = 0.1$

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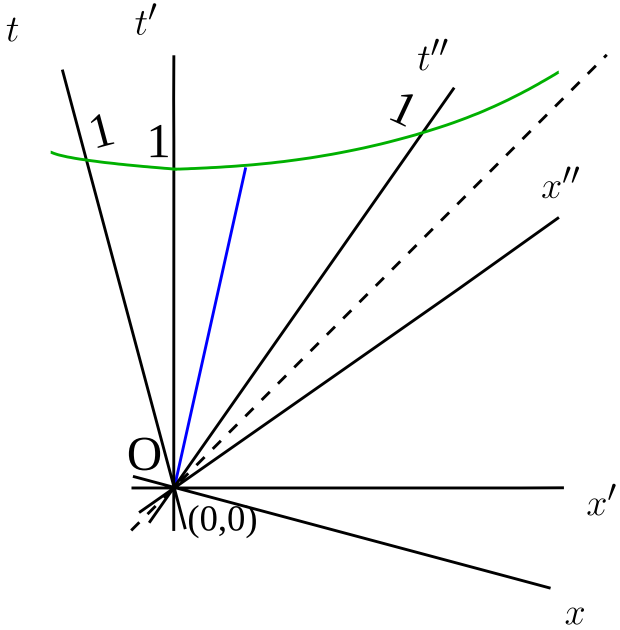

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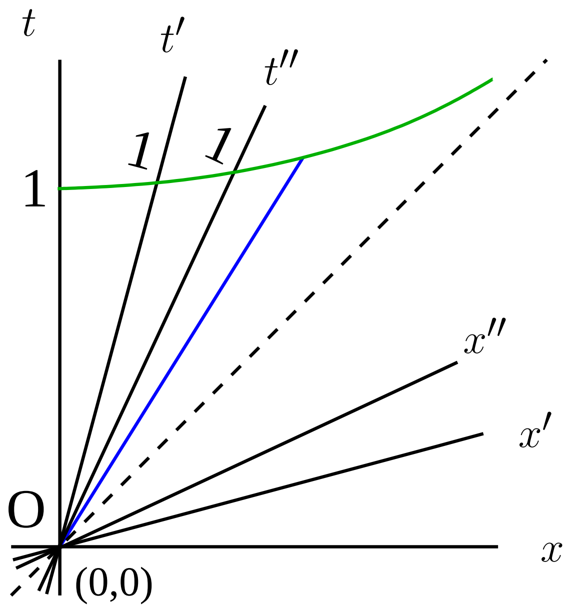

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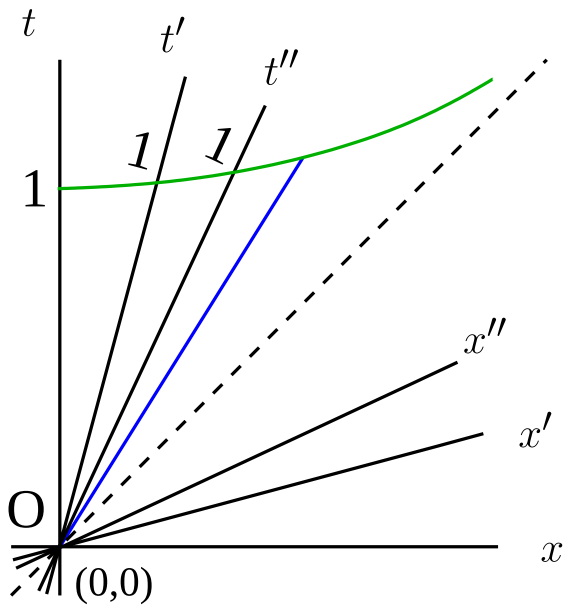

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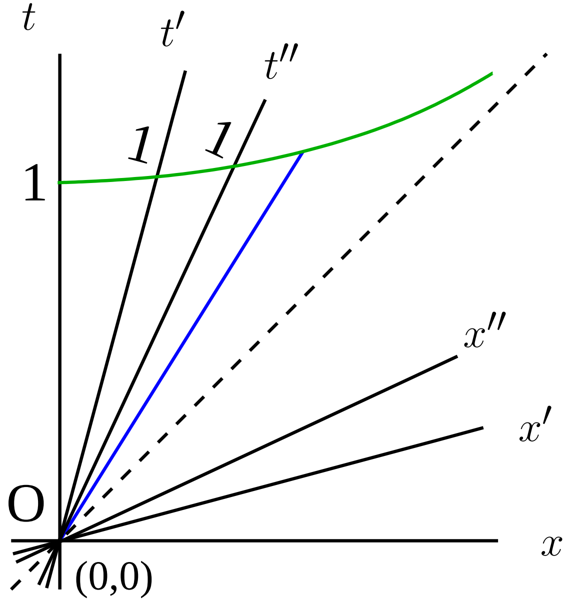

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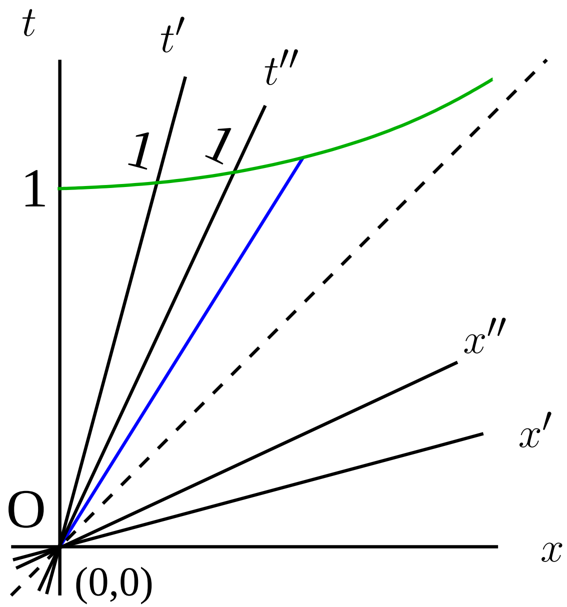

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cf. rotation θ_1 "plus" rotation θ_2 = rotation $\theta_1 + \theta_2$

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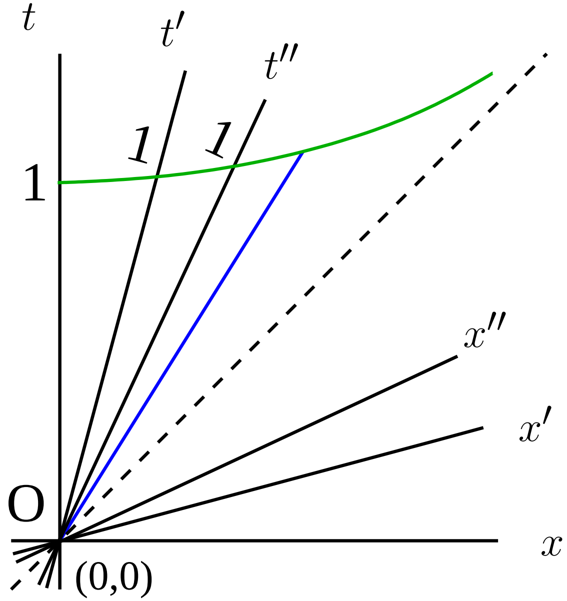

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so $\beta_3 = \tanh(\phi_1 + \phi_2)$

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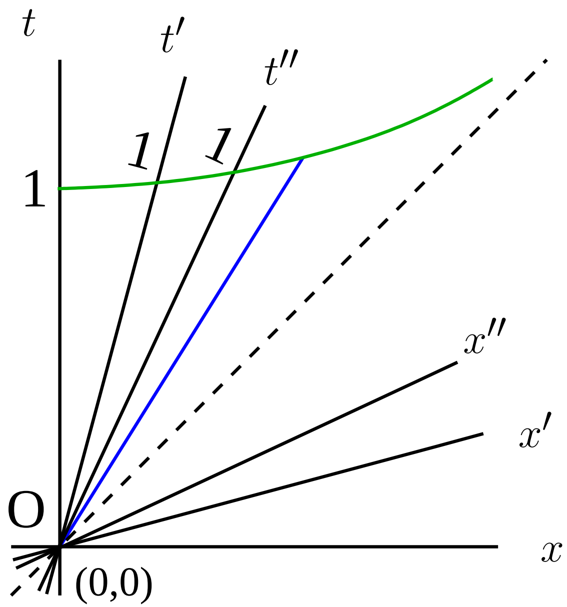

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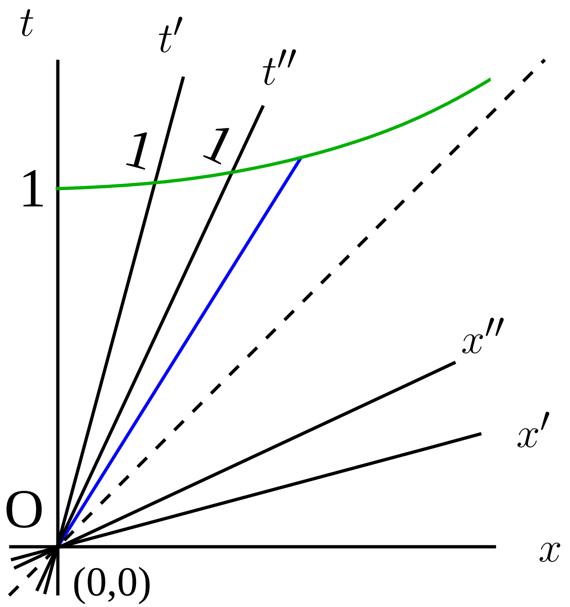
so $\beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\tanh\phi_1 + \tanh\phi_2}{1 + \tanh\phi_1 \tanh\phi_2}$

SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



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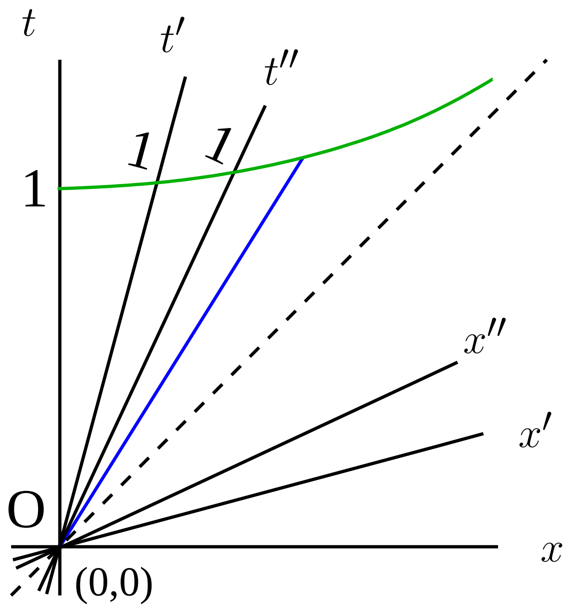

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so $\beta_3 = \tanh(\phi_1 + \phi_2) = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2} = \frac{0.1+0.5}{1+0.1\times0.5} \approx 0.57$

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

$$\cosh\phi := \frac{e^{\phi}+e^{-\phi}}{2}$$

$$\sinh\phi := \frac{e^{\phi}-e^{-\phi}}{2}$$

w:hyperbolic function

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

$$\Lambda(\beta) := \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}$$

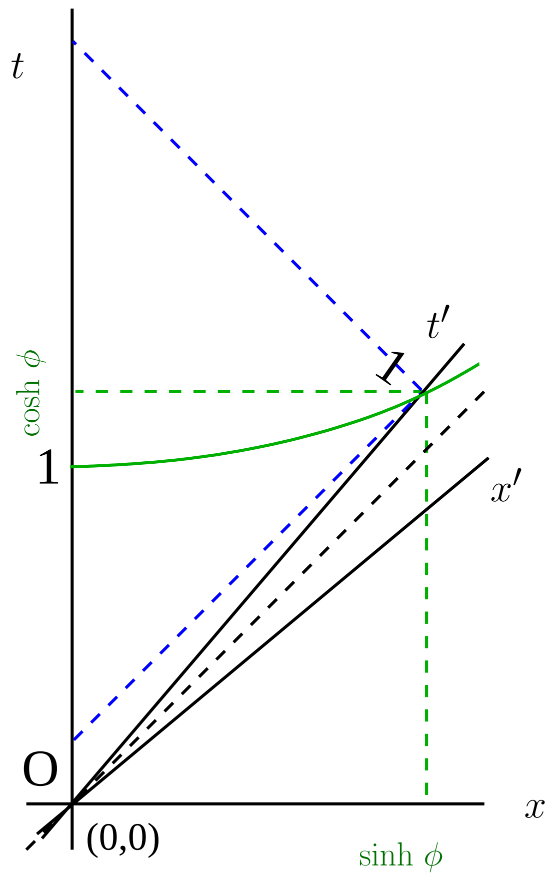
$\beta = \tanh\phi$

$\gamma := (1 - \beta^2)^{-1/2}$ = Lorentz factor

$\gamma = \cosh\phi$

$\beta\gamma = \sinh\phi$

SR: worldline time dilation



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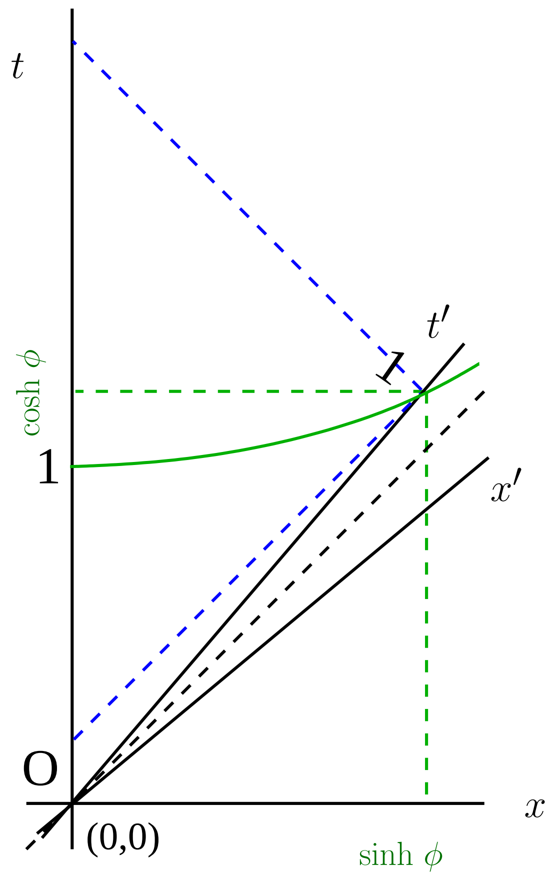
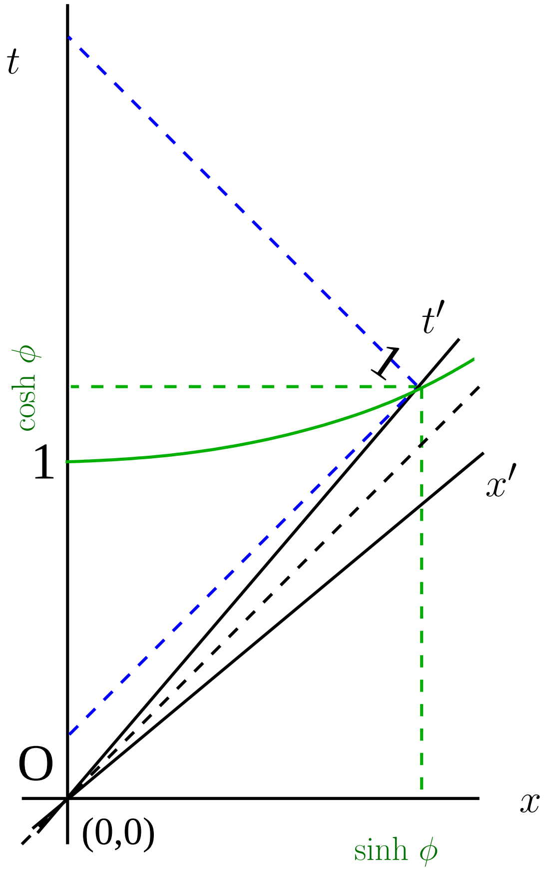


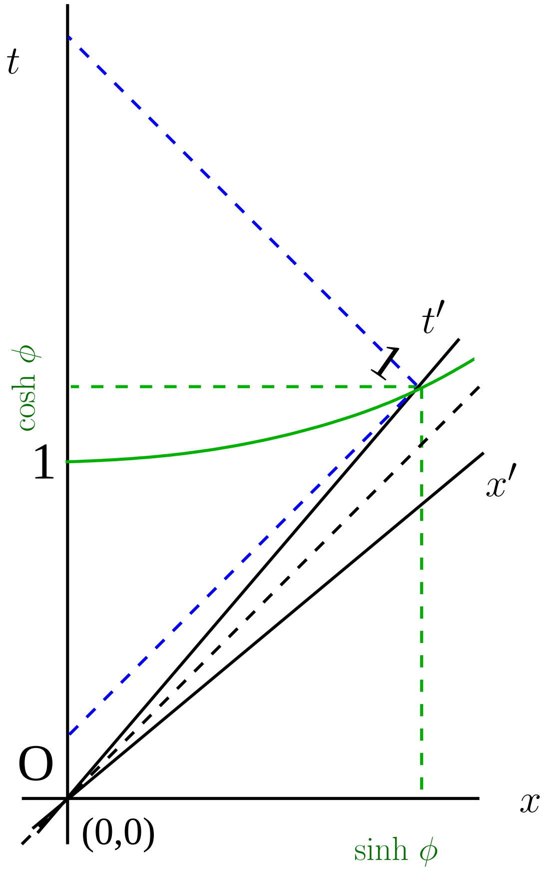
$$\cosh\phi \equiv \gamma$$

SR: worldline time dilation



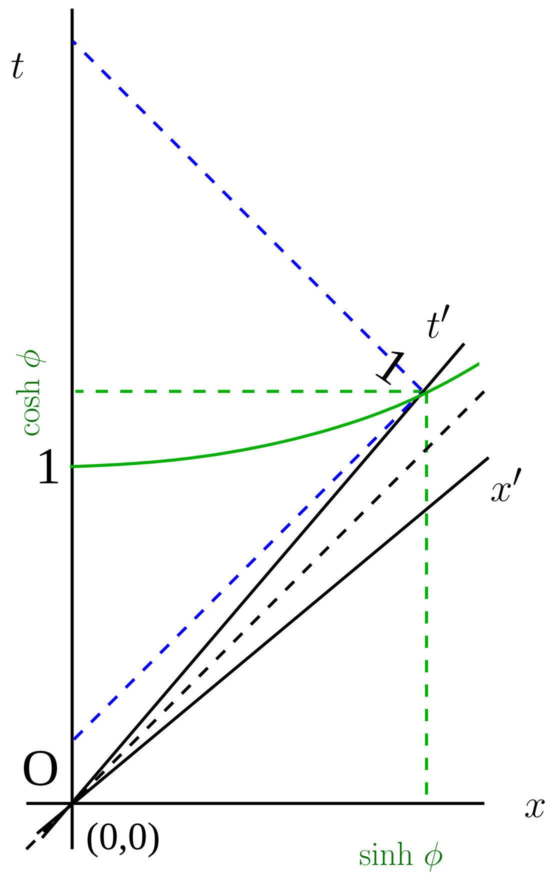
$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

SR: worldline time dilation



$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

SR: worldline time dilation



worldline "time dilation"

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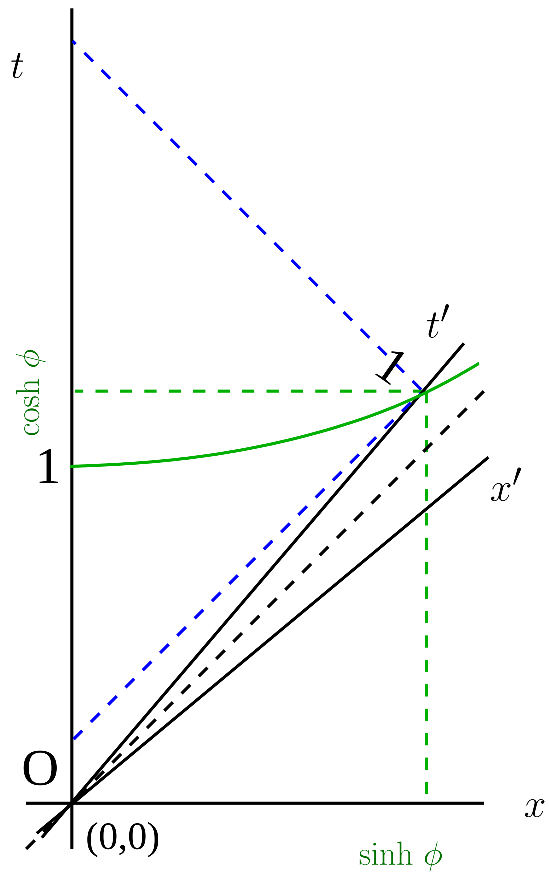

worldline "time dilation"

muons: mean lifetime 2197 ns $\ll 15$ km

$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

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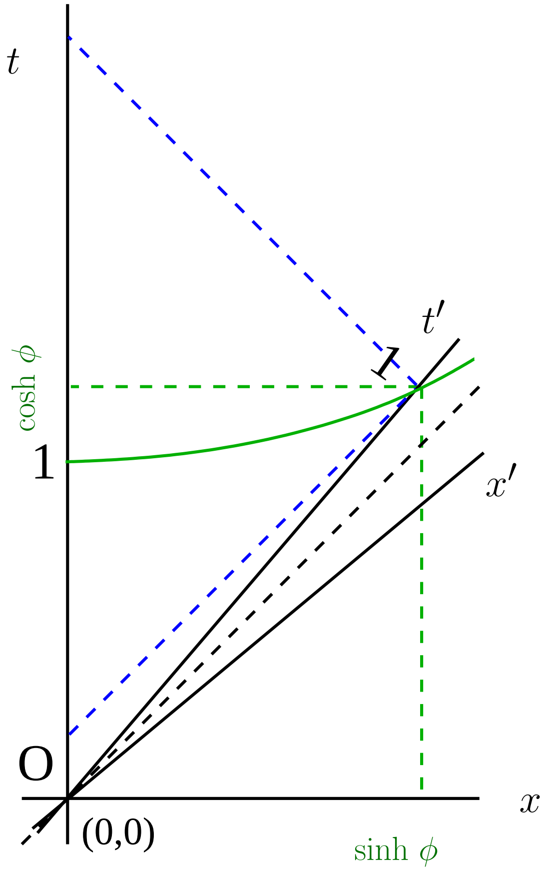


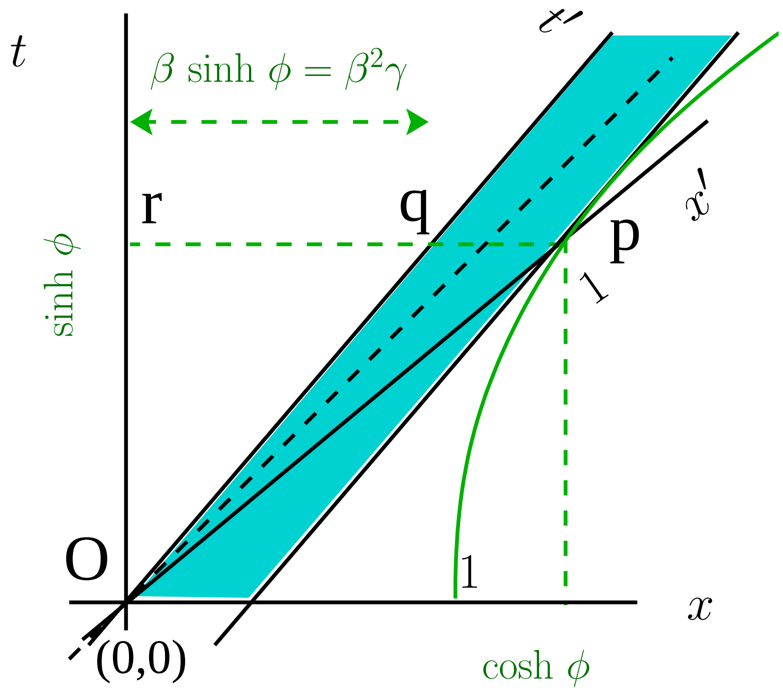
$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

worldline “time dilation”

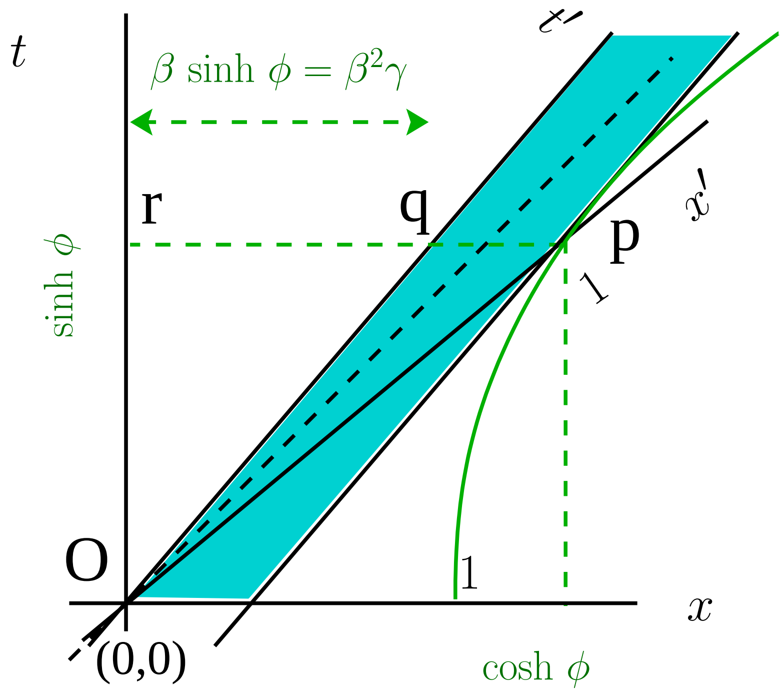
muons: mean lifetime 2197 ns $\ll 15$ km

time dilation $\Rightarrow$ muons can hit the ground

SR: worldsheet space contraction

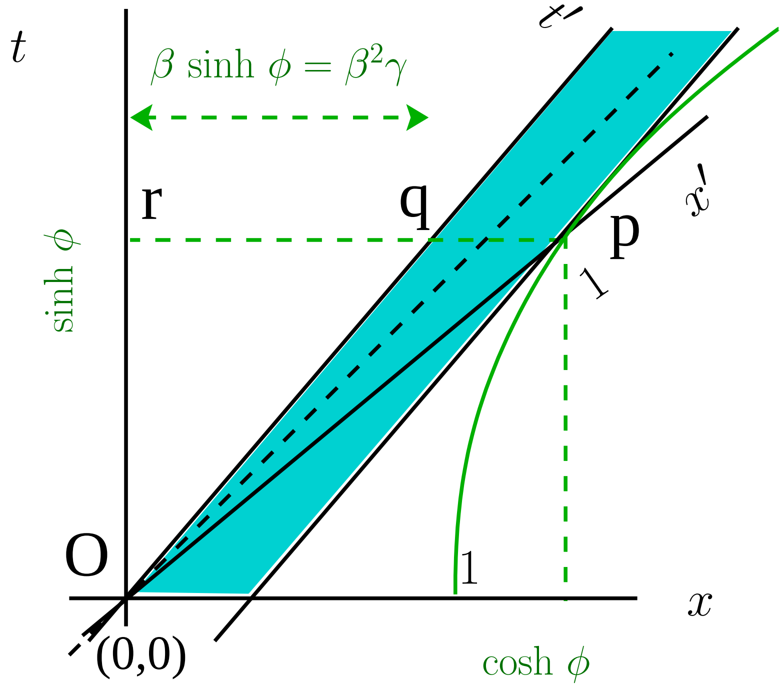


SR: worldsheet space contraction



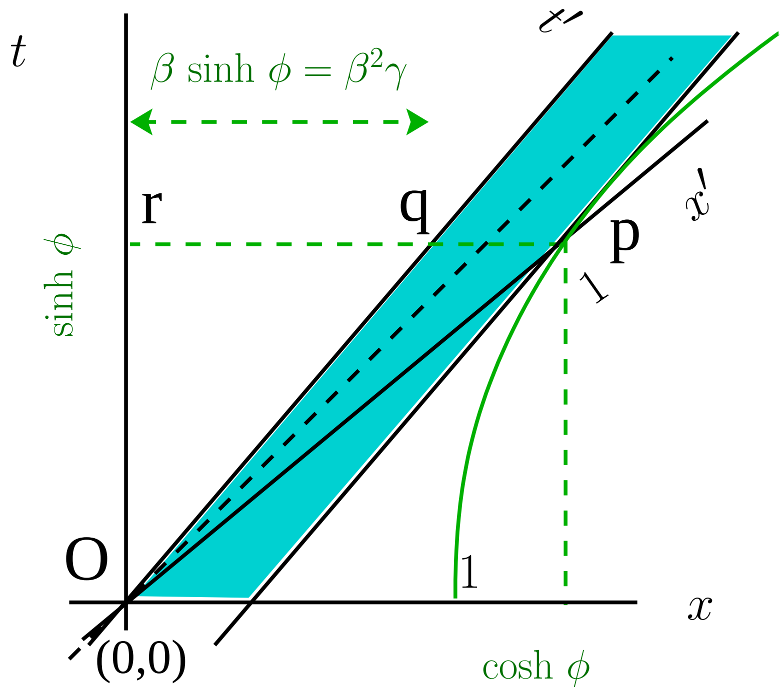
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} =$$

SR: worldsheet space contraction



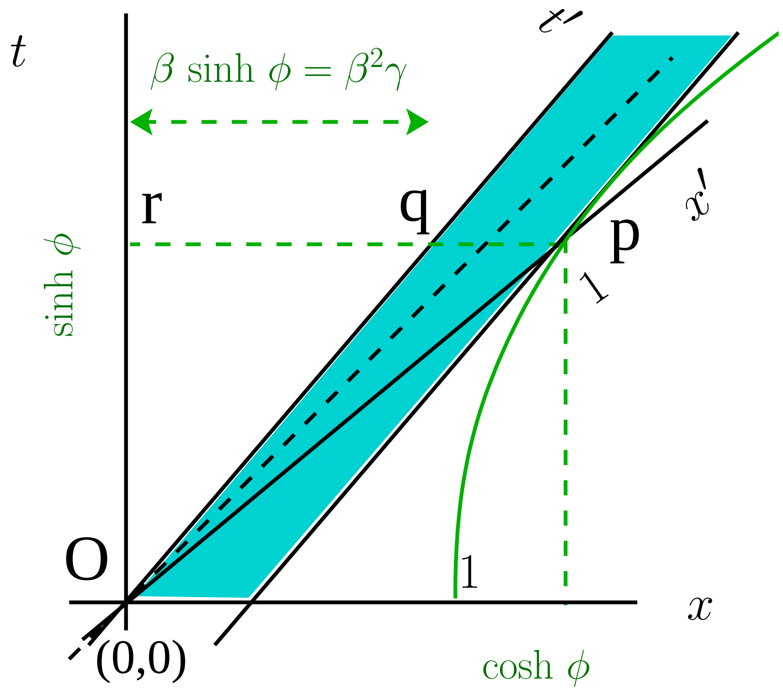
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \cosh\phi - \beta\sinh\phi$$

SR: worldsheet space contraction



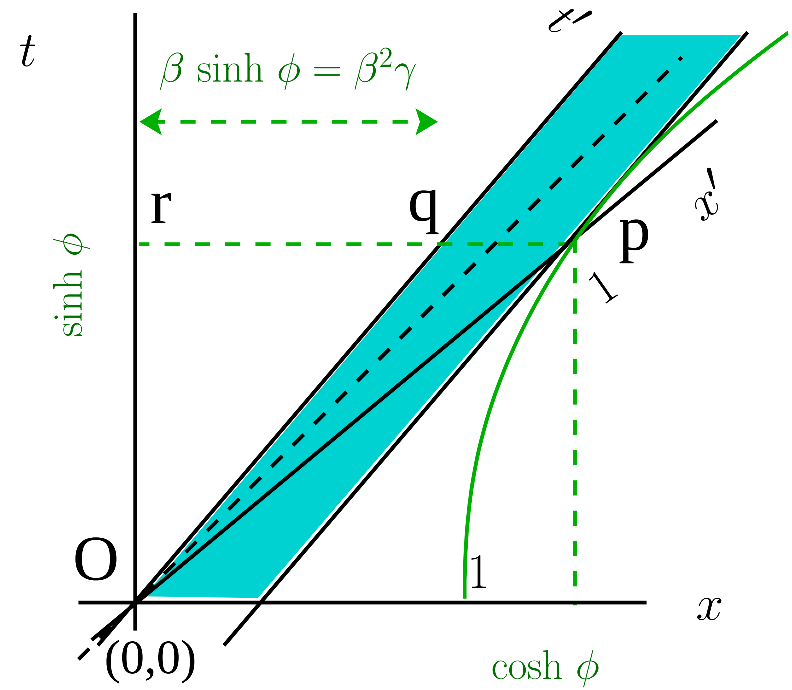
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma - \beta\beta\gamma$$

SR: worldsheet space contraction



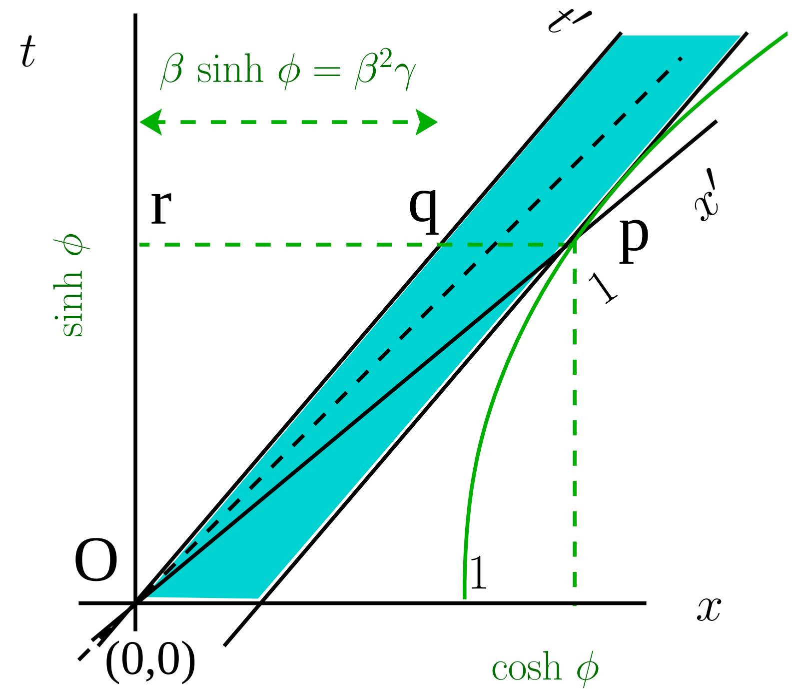
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)\gamma$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)^{1+(-1/2)}$$

SR: worldsheet space contraction



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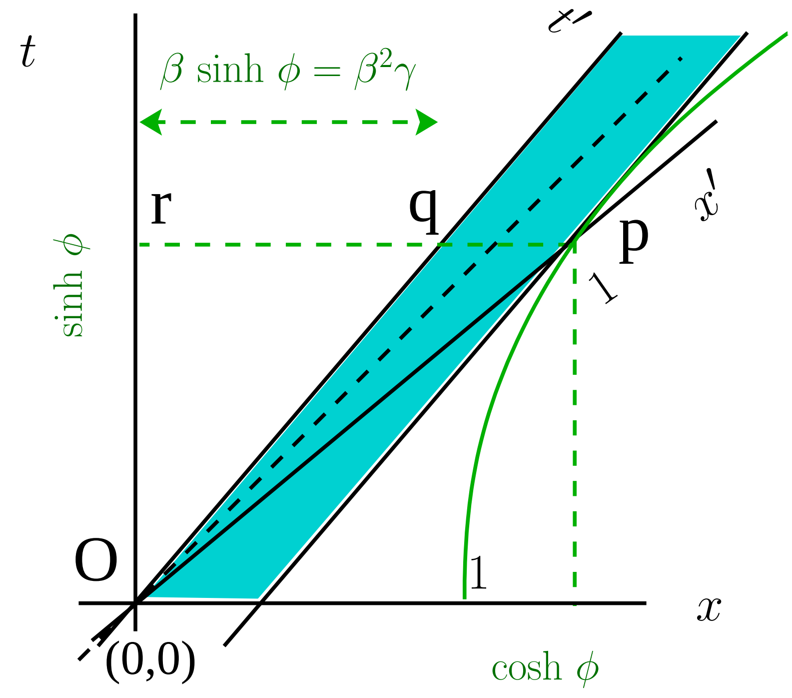
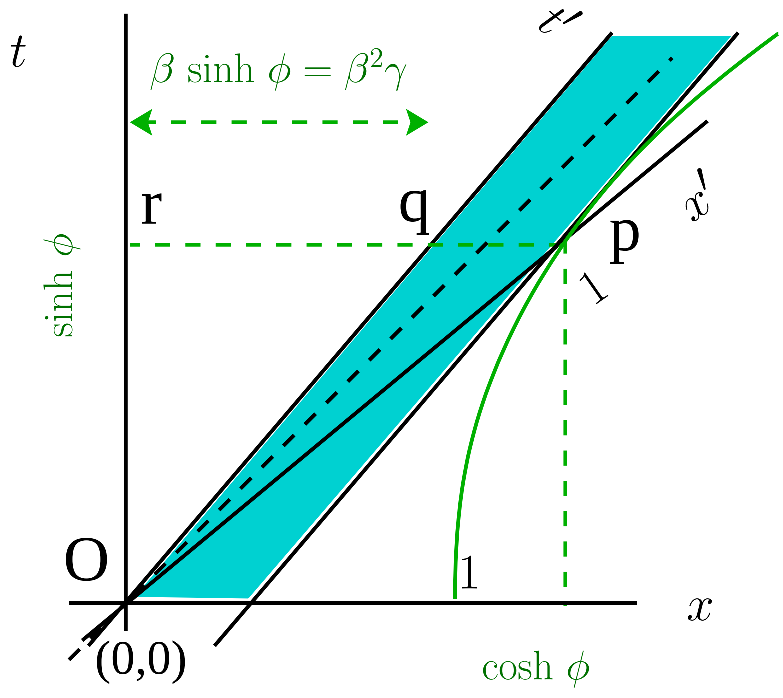


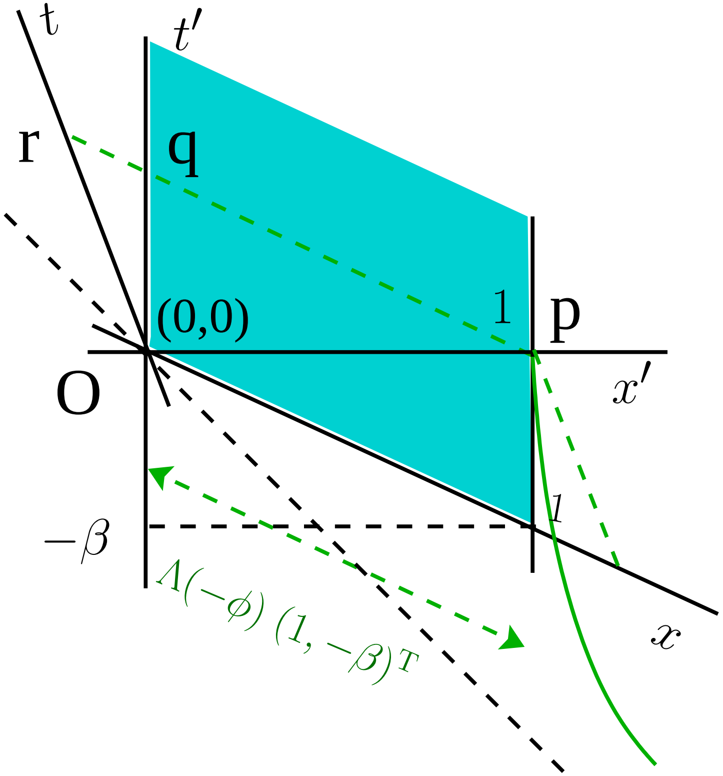
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$

SR: worldsheet space contraction

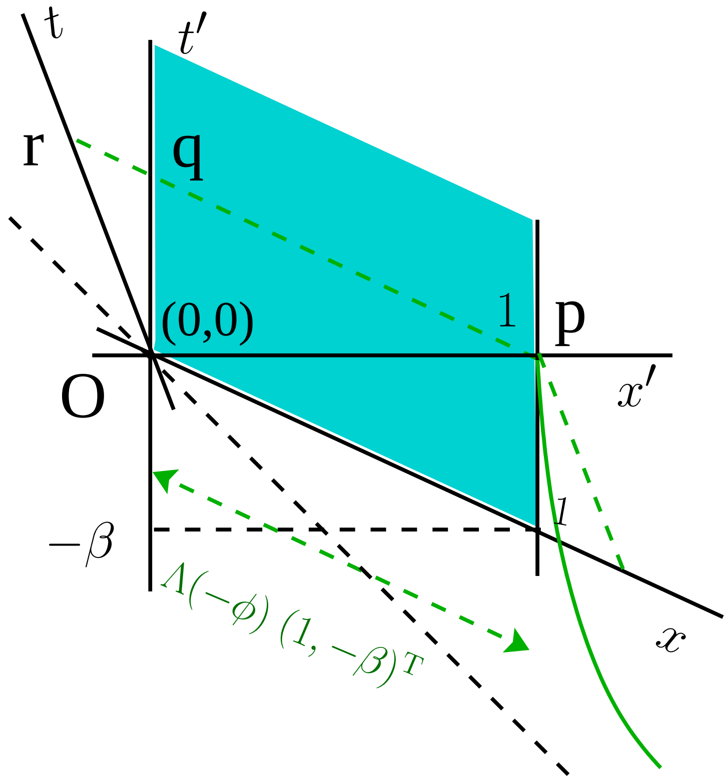


$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$ worldsheet "space contraction"

SR: worldsheet space contraction

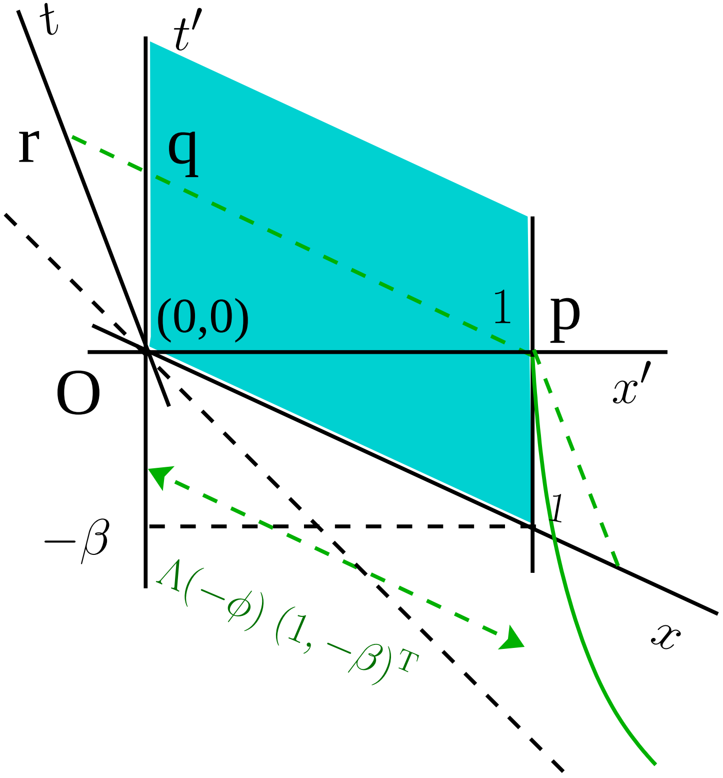


SR: worldsheet space contraction



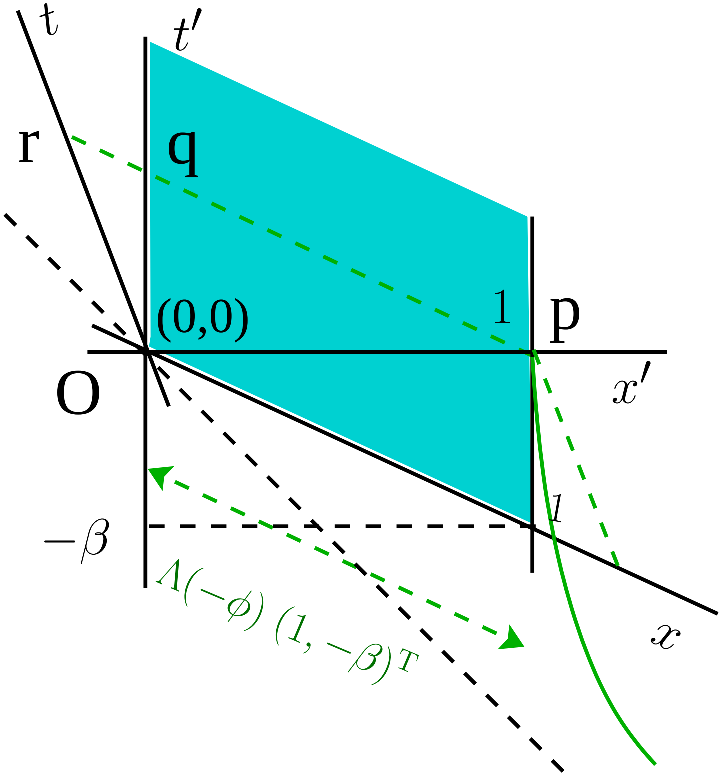
$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\cosh\phi-\beta\sinh\phi\\\sinh\phi-\beta\cosh\phi\end{pmatrix}$$

SR: worldsheet space contraction



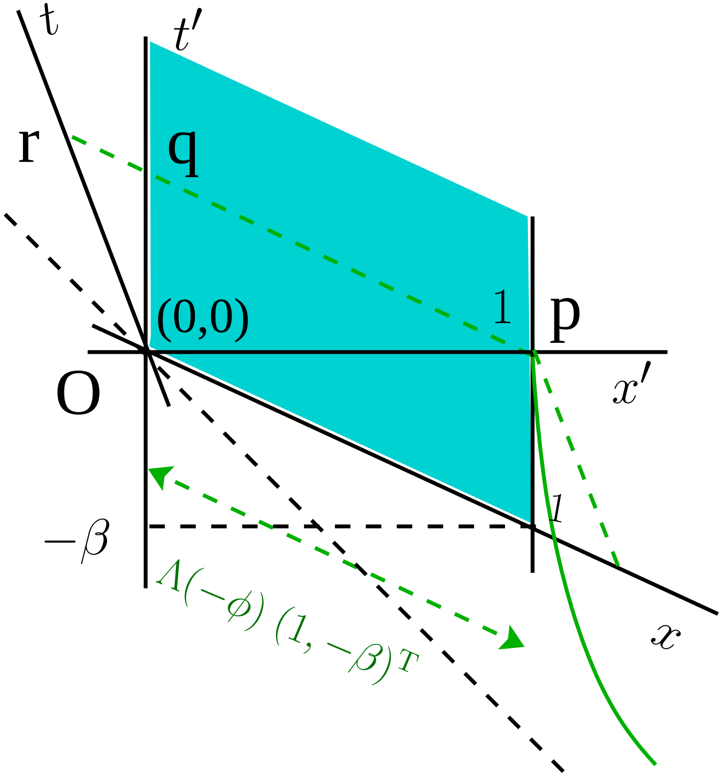
$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\cosh\phi(1-\beta^2)\\\cosh\phi(\tanh\phi-\tanh\phi)\end{pmatrix}$$

SR: worldsheet space contraction



$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix} = \begin{pmatrix}\gamma(1-\beta^2)\\0\end{pmatrix}$$

SR: worldsheet space contraction



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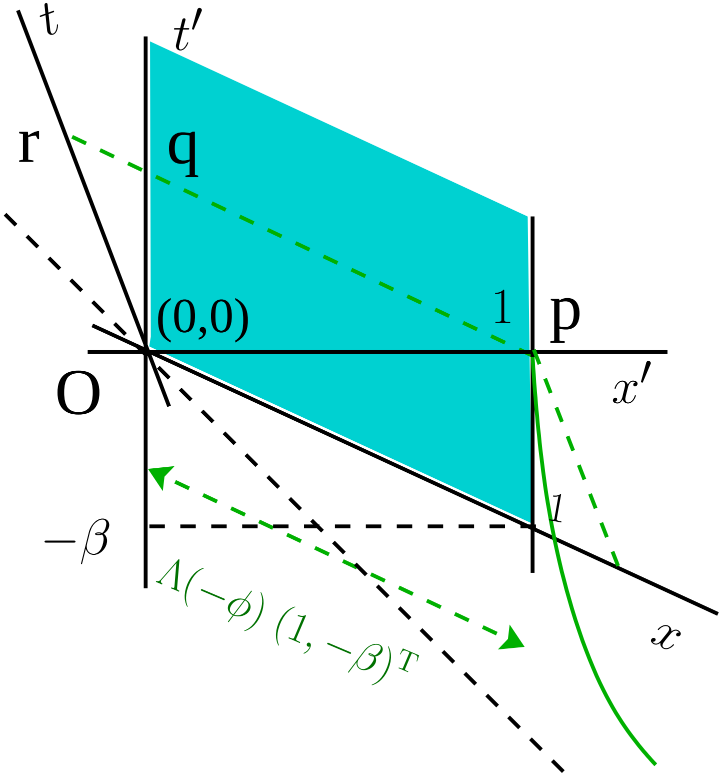

$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$

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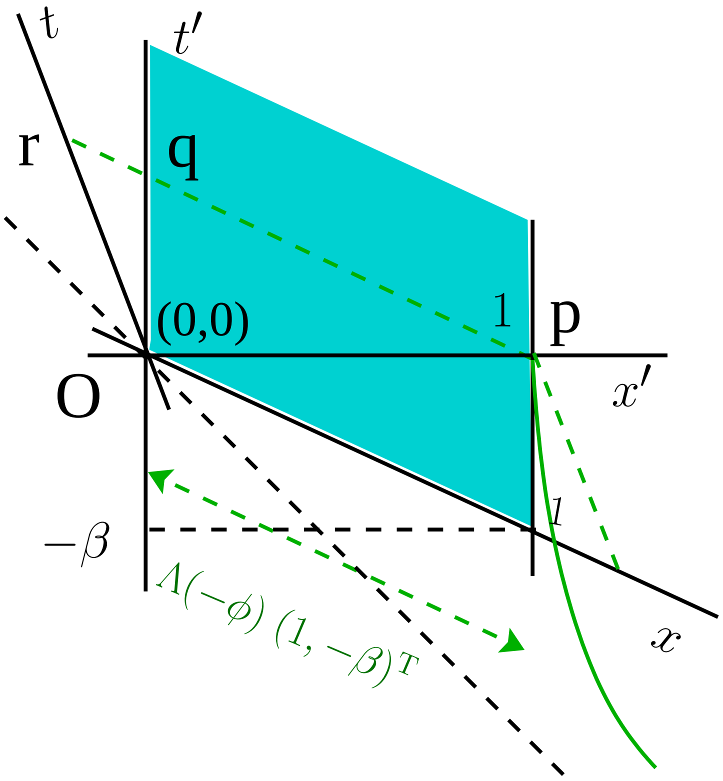
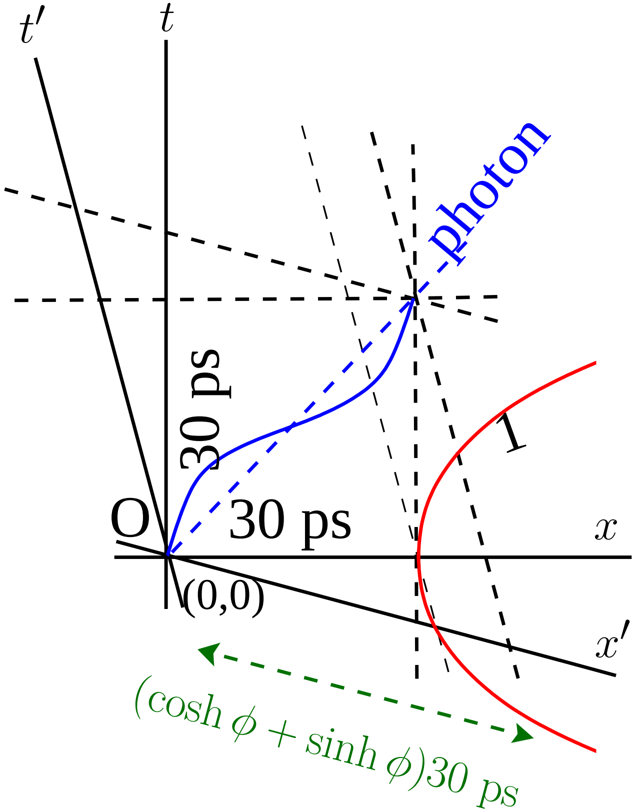
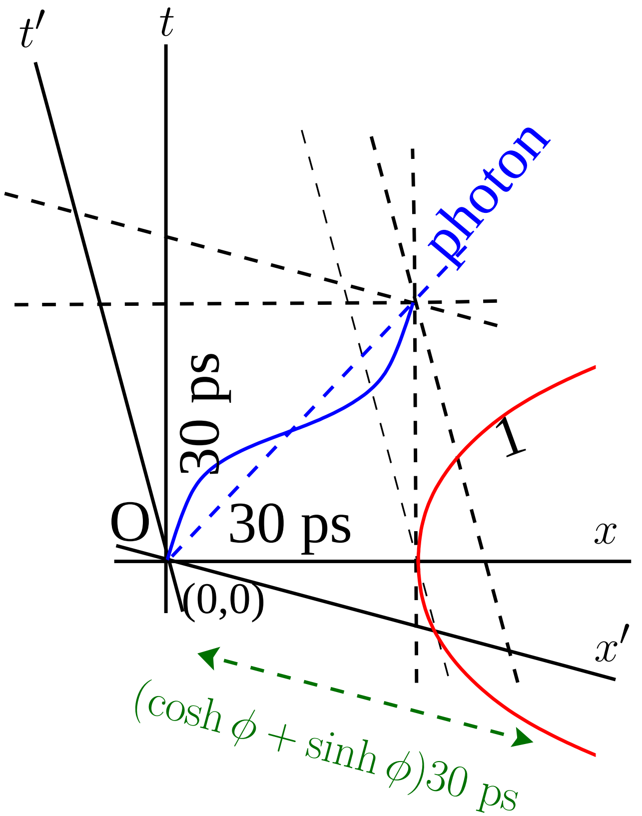


$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$ worldsheet “space contraction”

SR: Doppler shift

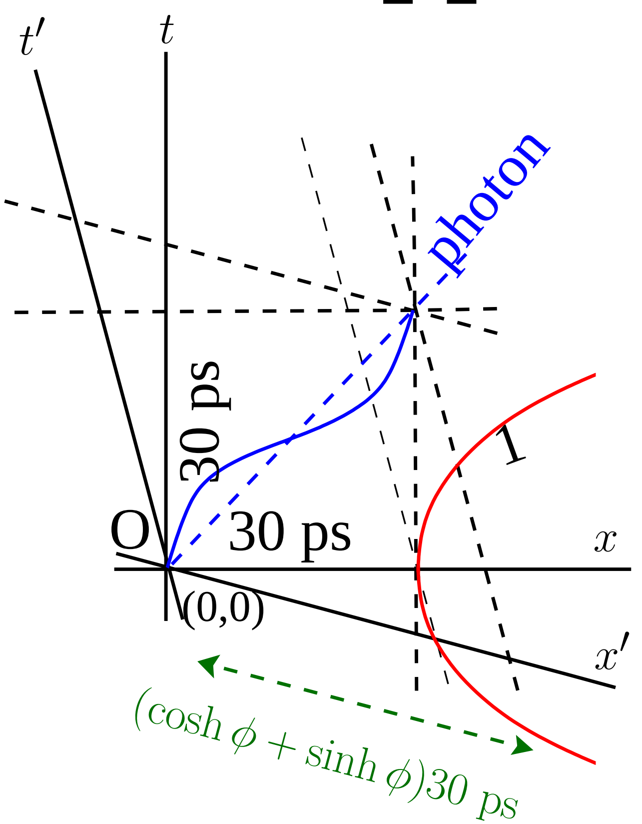


SR: Doppler shift



see photon worldline calculation

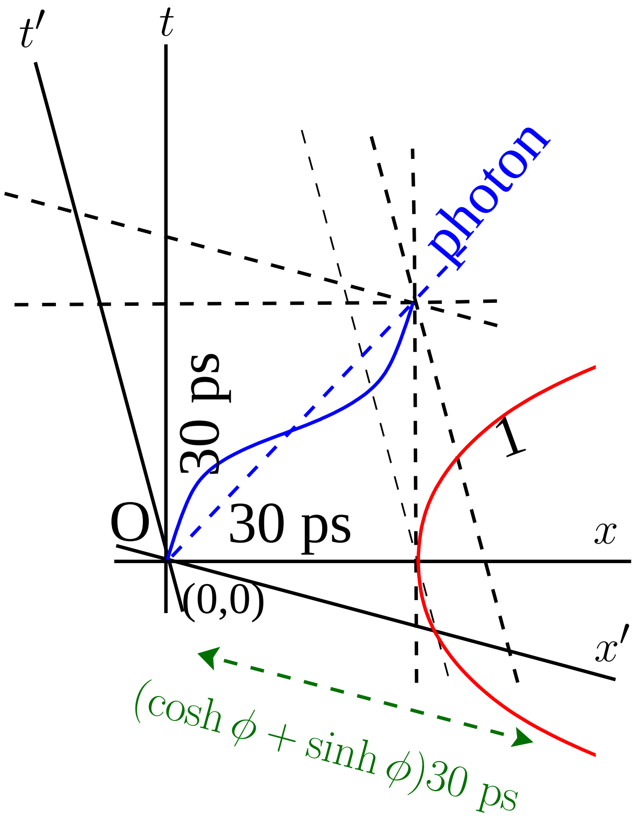
SR: Doppler shift



see photon worldline calculation

$$x' = (\cosh\phi + \sinh\phi)t$$

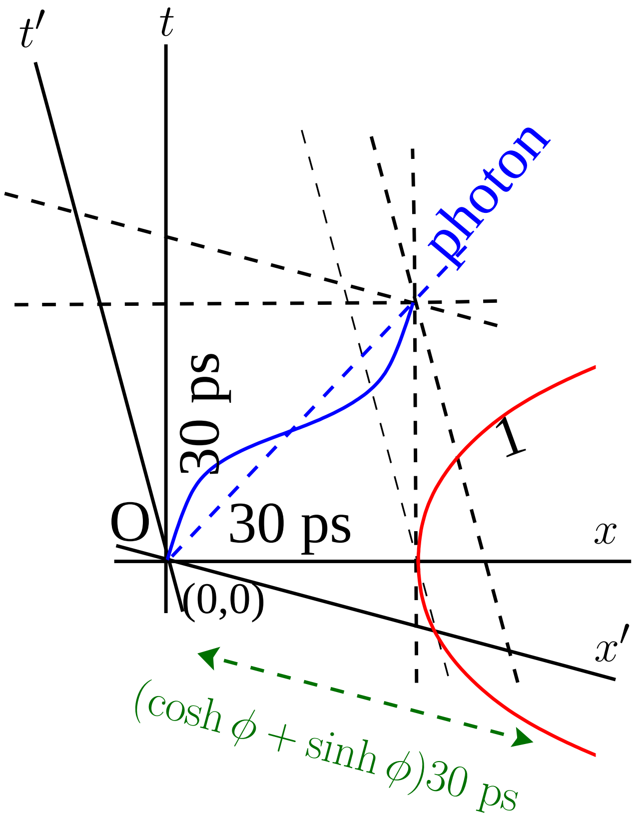
SR: Doppler shift



see photon worldline calculation

$$x' = (\cosh\phi + \sinh\phi)x$$

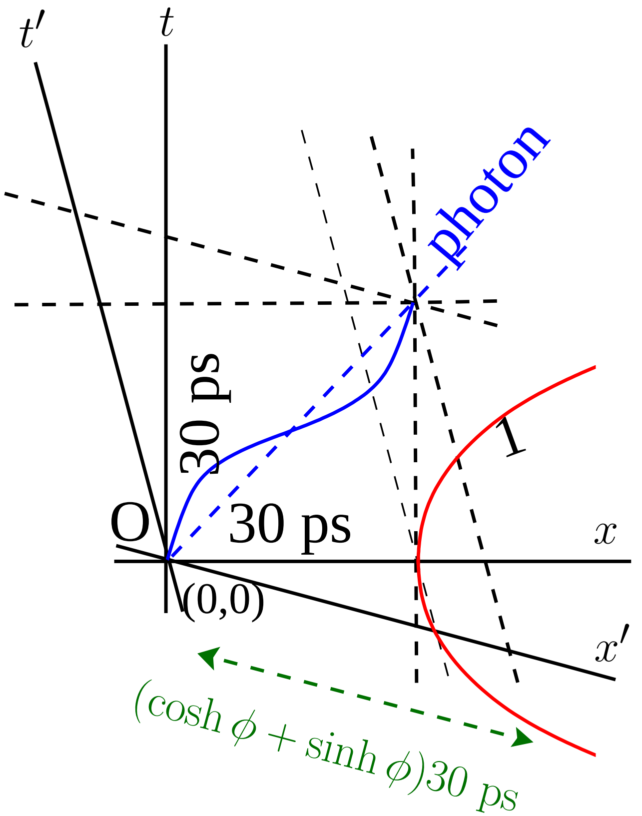
SR: Doppler shift



see photon worldline calculation

$$x'/x = \cosh\phi + \sinh\phi$$

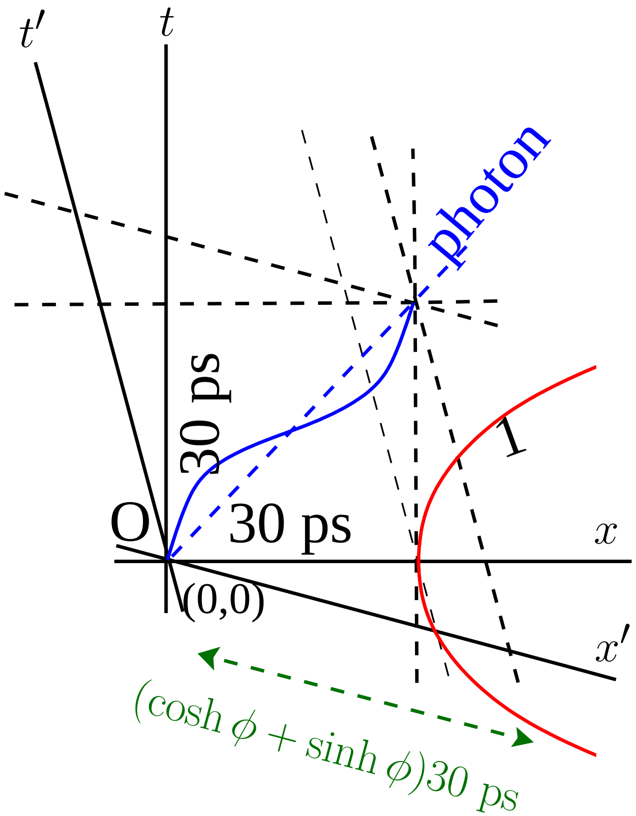
SR: Doppler shift



see photon worldline calculation

$x'/x = \gamma + \beta\gamma$

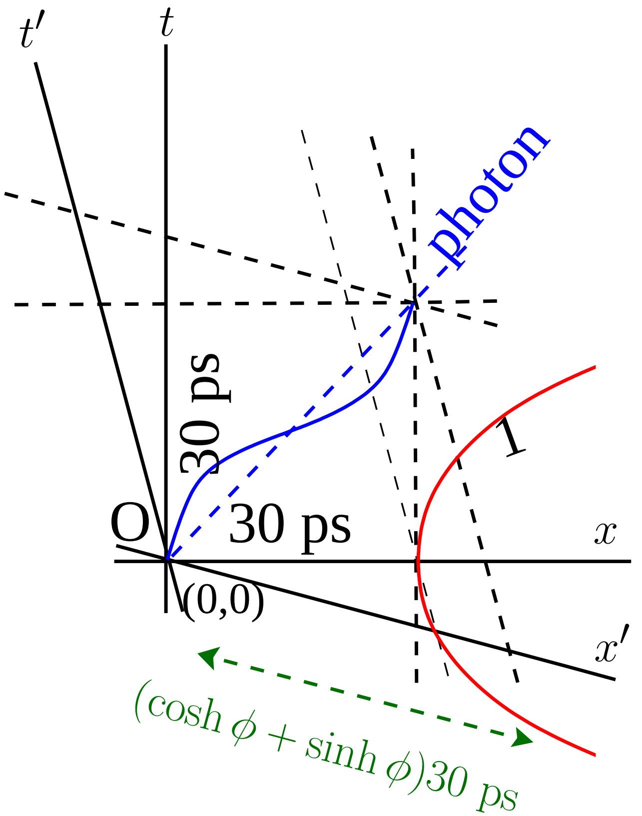
SR: Doppler shift



see photon worldline calculation

$x'/x = \gamma(1+\beta)$

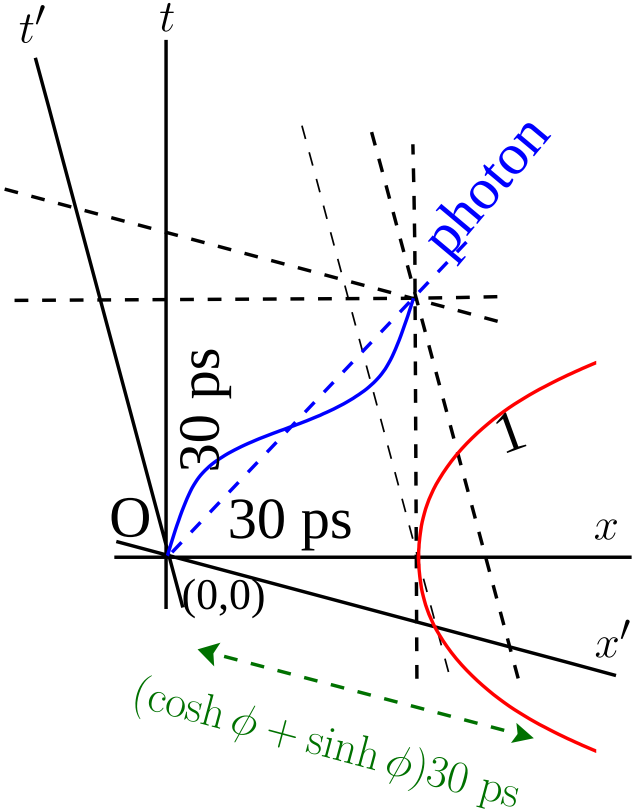
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \frac{1+\beta}{\sqrt{1-\beta^2}}$$

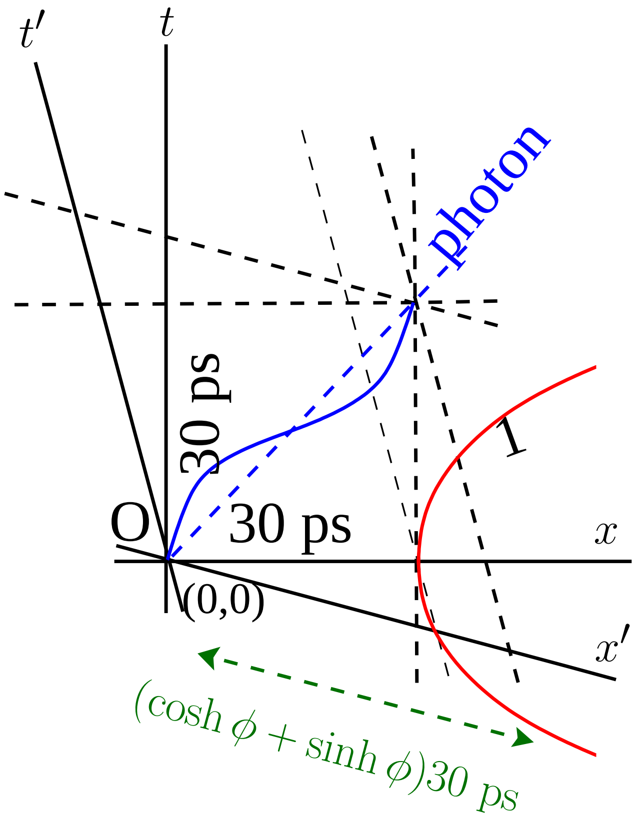
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \frac{\sqrt{(1+\beta)^2}}{\sqrt{(1-\beta)(1+\beta)}}$$

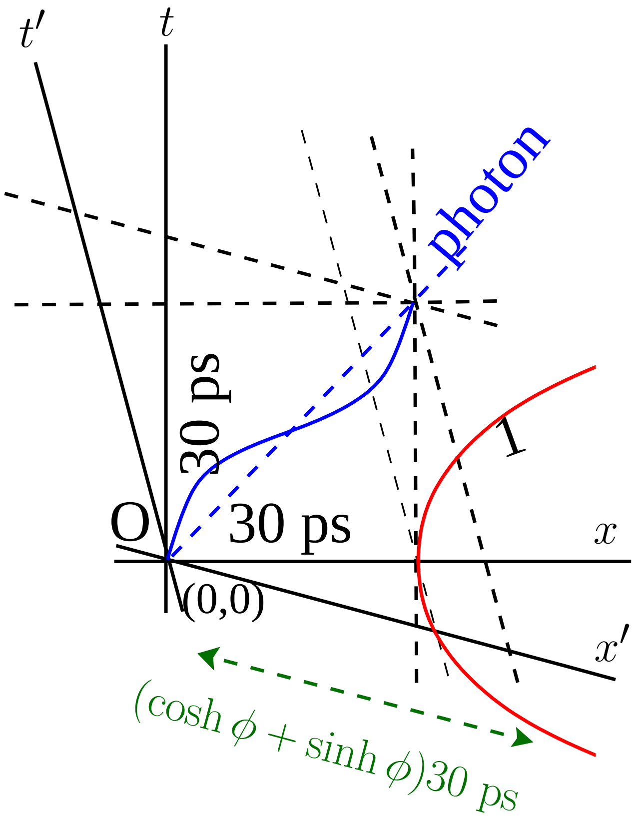
SR: Doppler shift



see photon worldline calculation

$$x'/x = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}}$$

SR: Doppler shift

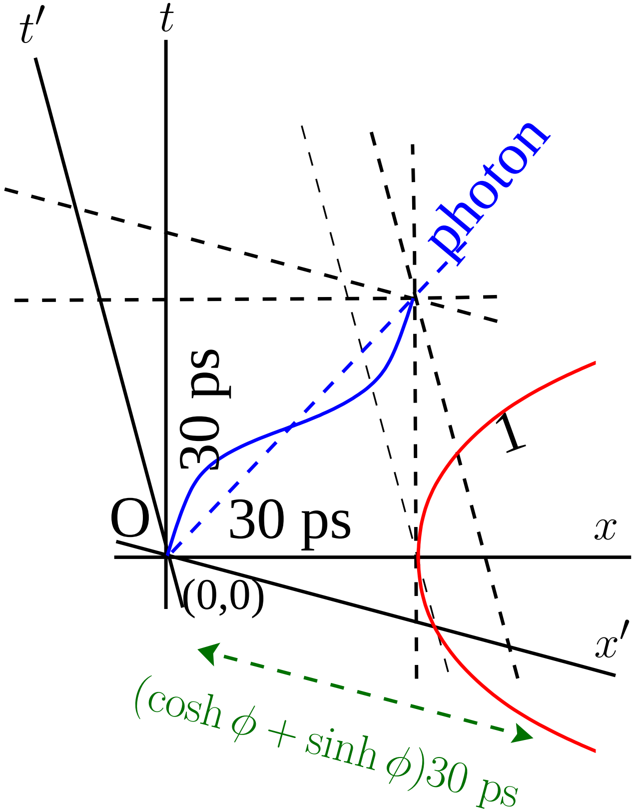


see photon worldline calculation

$$1 + z := \lambda'/\lambda = \gamma(1+\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}}$$

redshift

SR: Doppler shift

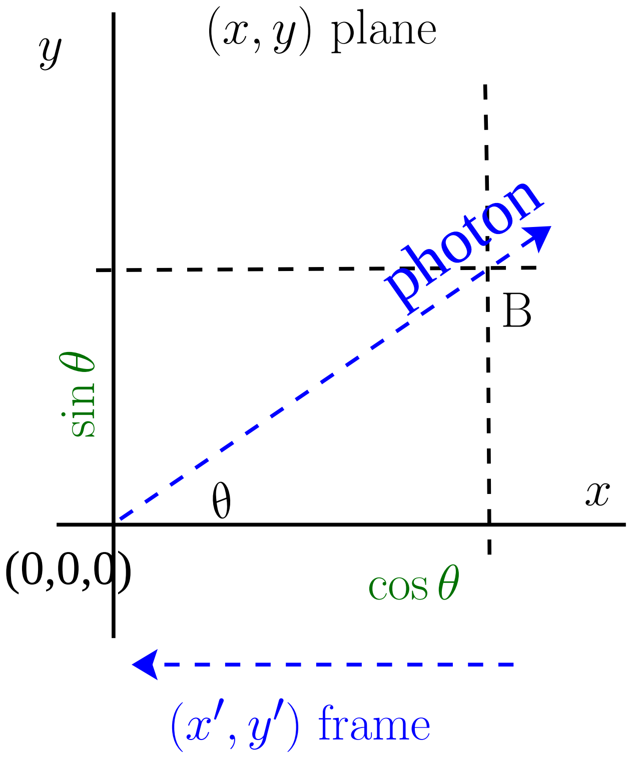


see photon worldline calculation

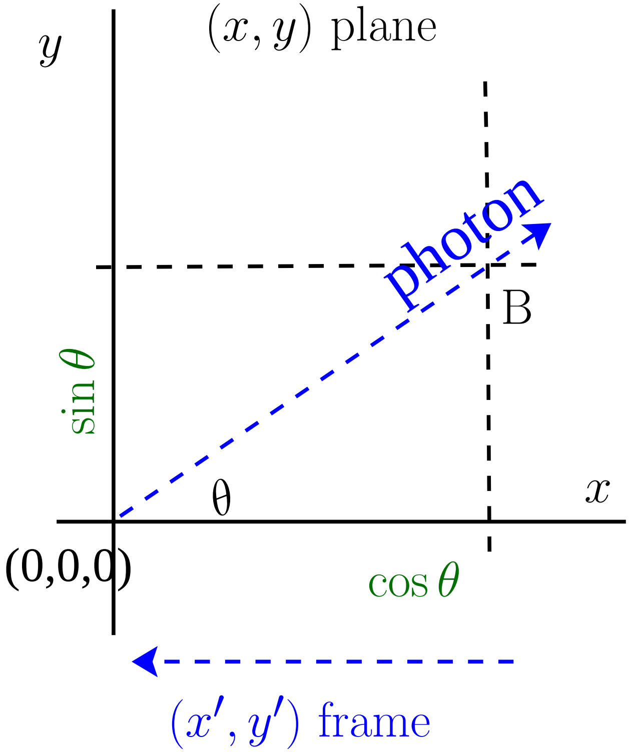
$$1 + z := \lambda'/\lambda = \gamma(1+\beta) \equiv \sqrt{\frac{1+\beta}{1-\beta}}$$ redshift

$\Rightarrow$ when $\beta \ll 1$, $z \approx \beta$

SR: relativistic aberration



SR: relativistic aberration



event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

SR: relativistic aberration

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$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ 0 & 1 & 0 \\ \beta\gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma\cos\theta + \beta\gamma \\ \sin\theta \\ \beta\gamma\cos\theta + \gamma \end{pmatrix}$$

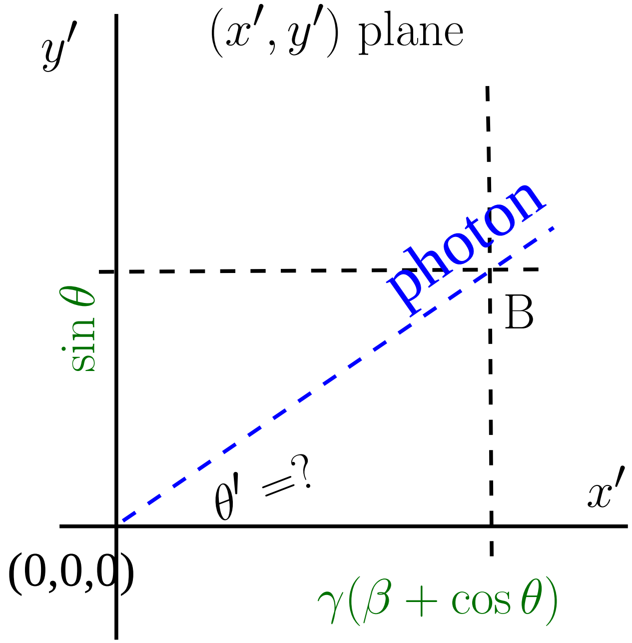
SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma(\cos\theta + \beta) \\ \sin\theta \\ \gamma(1 + \beta\cos\theta) \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$



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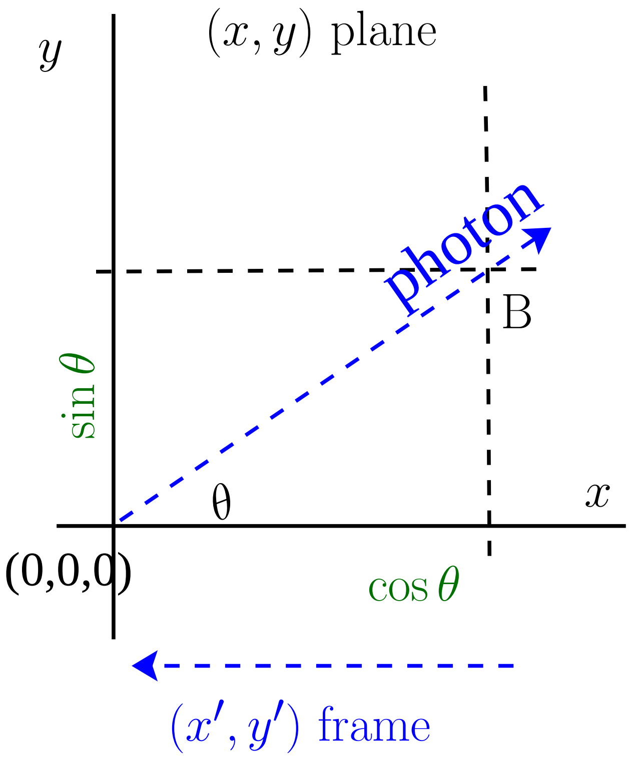

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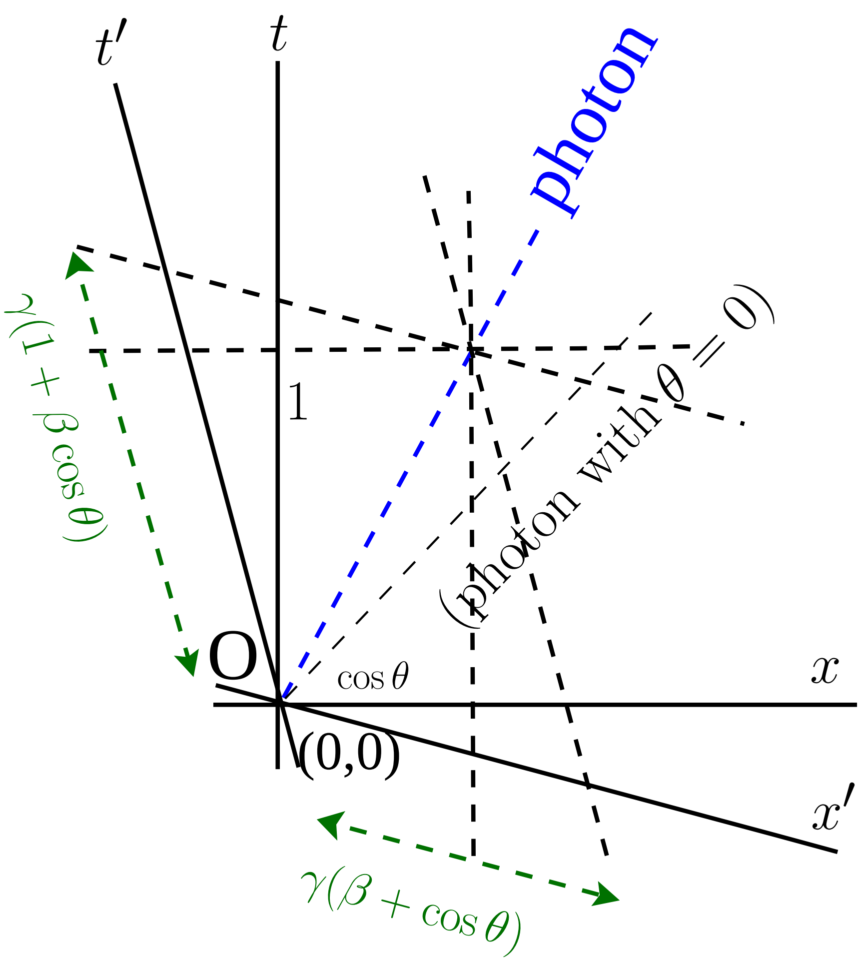

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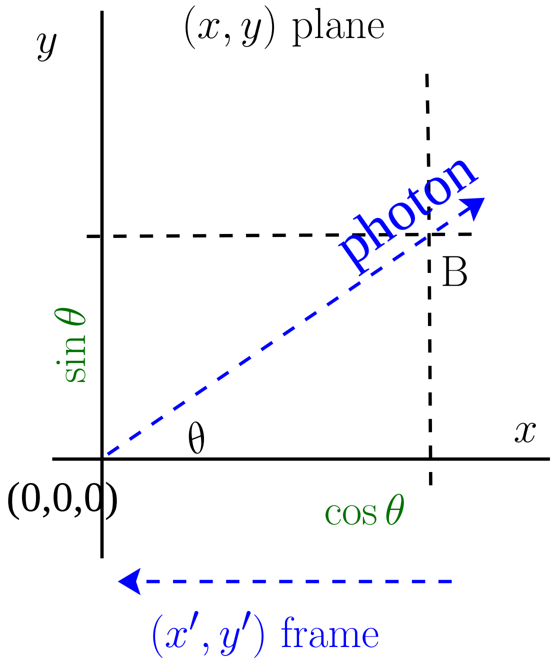
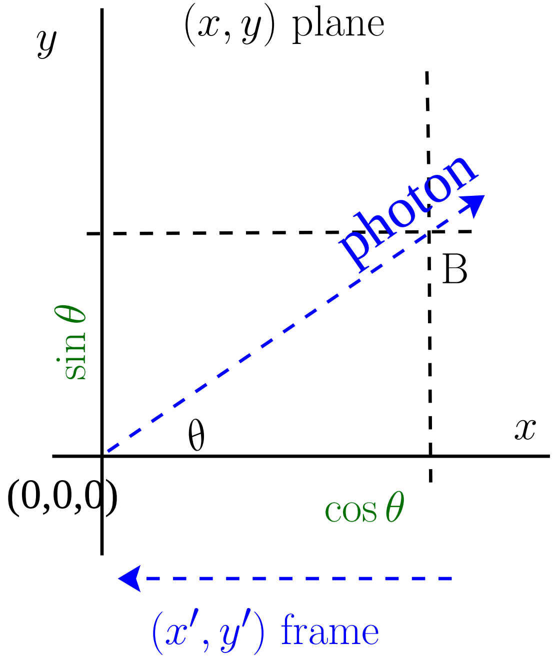


$$\tan\theta' = \frac{\sin\theta}{\gamma(\beta + \cos\theta)}$$

SR: relativistic aberration

event B: $(x,y,t) = (\cos\theta, \sin\theta, 1)$

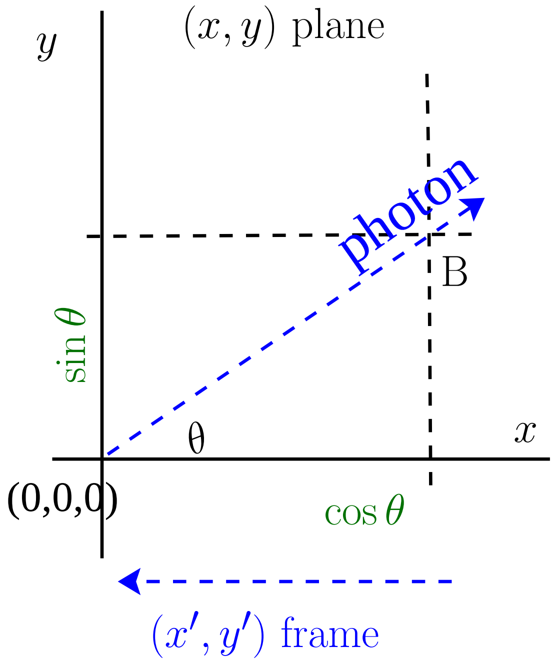


$\tan\theta' = \frac{\sin\theta}{\gamma(\beta+\cos\theta)}$ $< \tan\theta$ if $0<\beta<1$

w:Relativistic aberration (2011-02-22: quality=weak)

SR: relativistic aberration

event B: $(x,y,t) = (\cos\theta, \sin\theta, 1)$

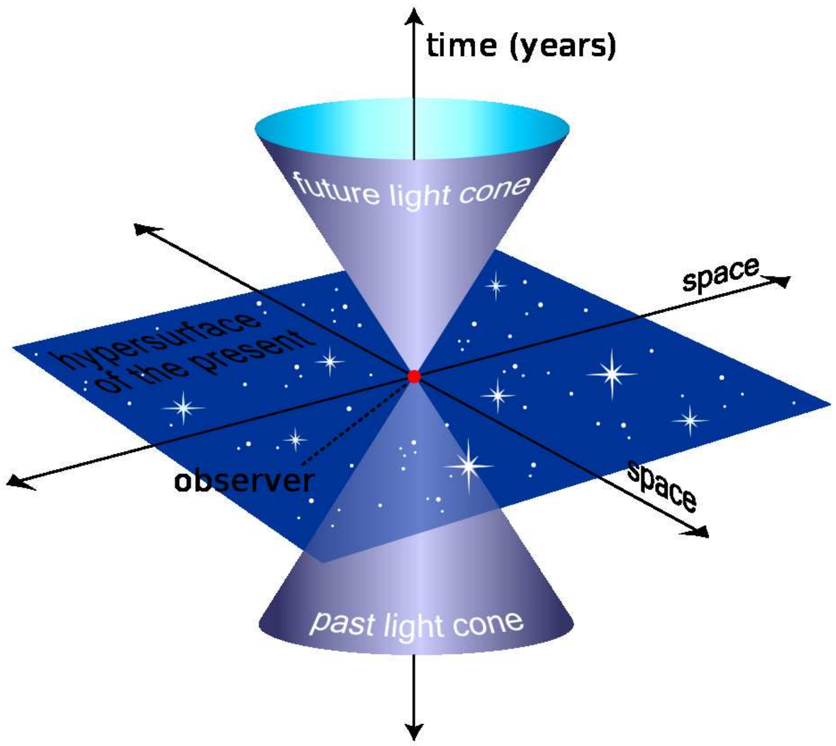


$$\tan\theta' = \frac{\sin\theta}{\gamma(\beta+\cos\theta)} < \tan\theta \text{ if } 0 < \beta < 1$$

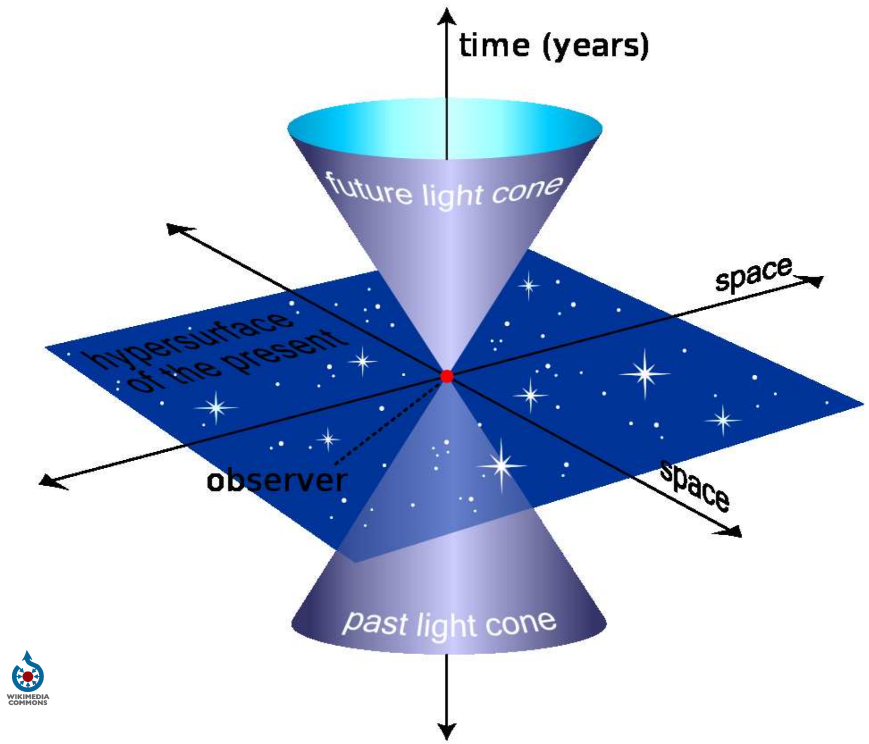
w:Relativistic aberration (2011-02-22: quality=weak)

$\Rightarrow$ relativistic beaming, e.g. AGN jets

SR: world line



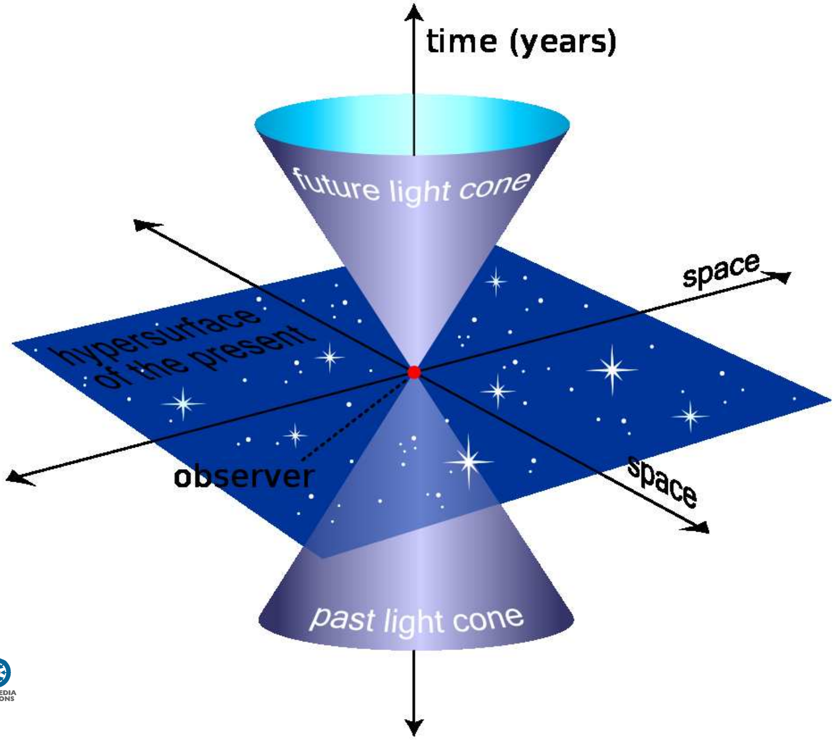
SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime =

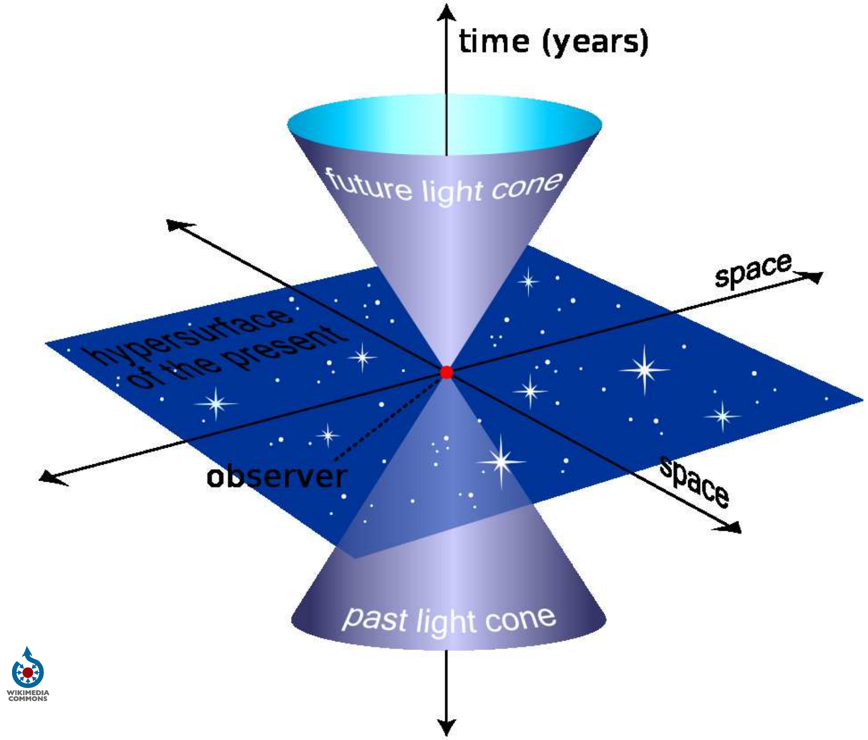
SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone

SR: world line

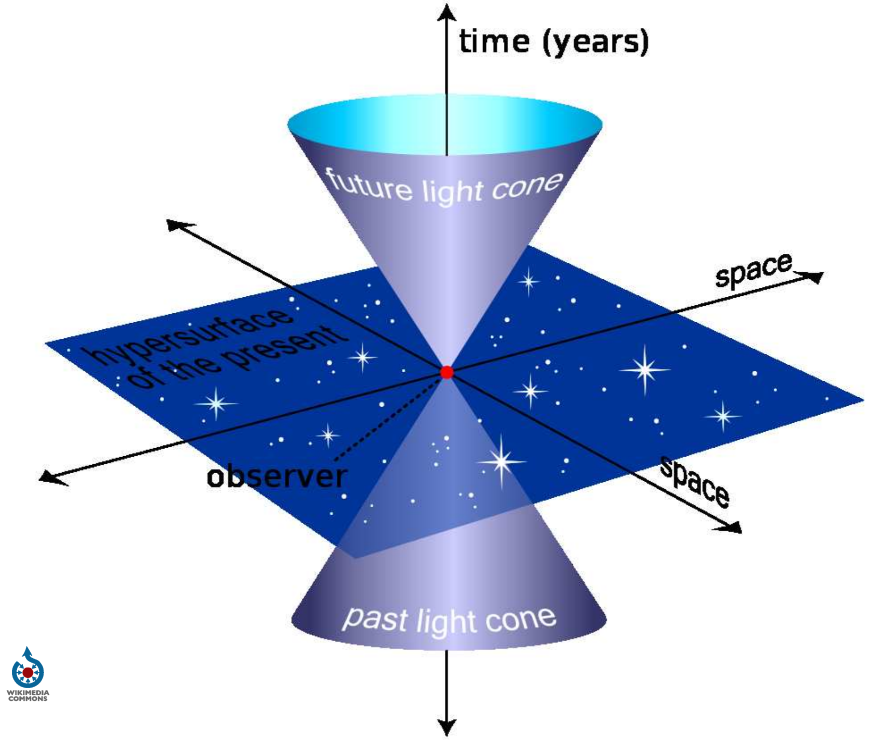


lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone

+ on future light cone + inside future light cone

SR: world line



lightlike interval = null interval: $(\Delta s)^2 = 0$

spacetime = on past w:light cone + inside past light cone

+ on future light cone + inside future light cone

+ elsewhere

SR: world line

Lorentz transform of world line

SR: world line

Lorentz transform of world line

SR: world line

Lorentz transform of world line

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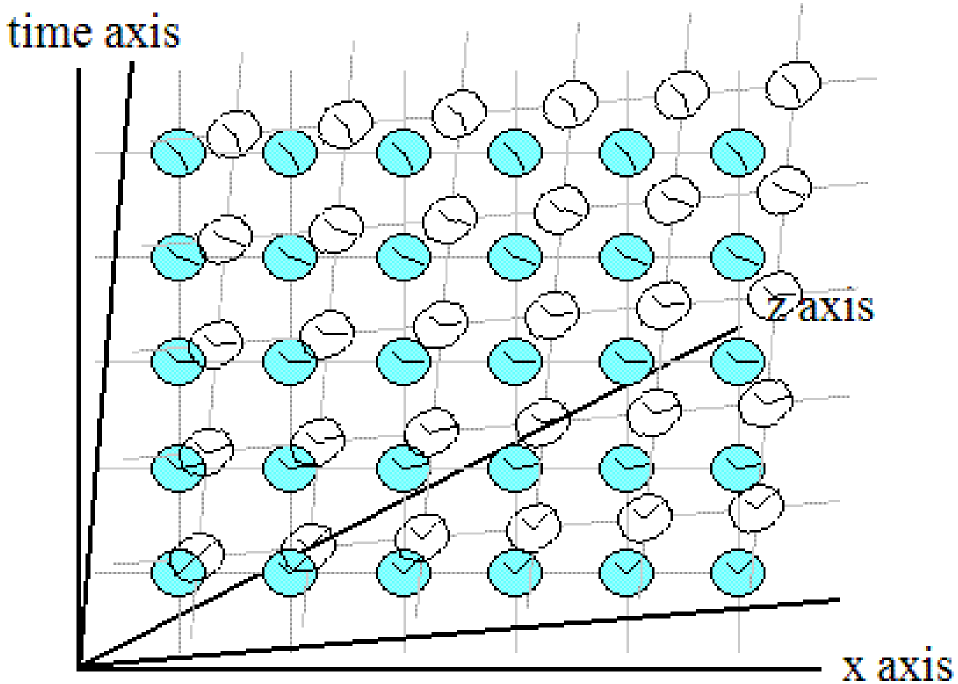
- often $\mathrm{d}\tau$ is useful for integrating

SR: Rietdijk–Putnam–Penrose p.



Relativity shows that the inertial frames of reference of relatively moving objects do not overlie each other.

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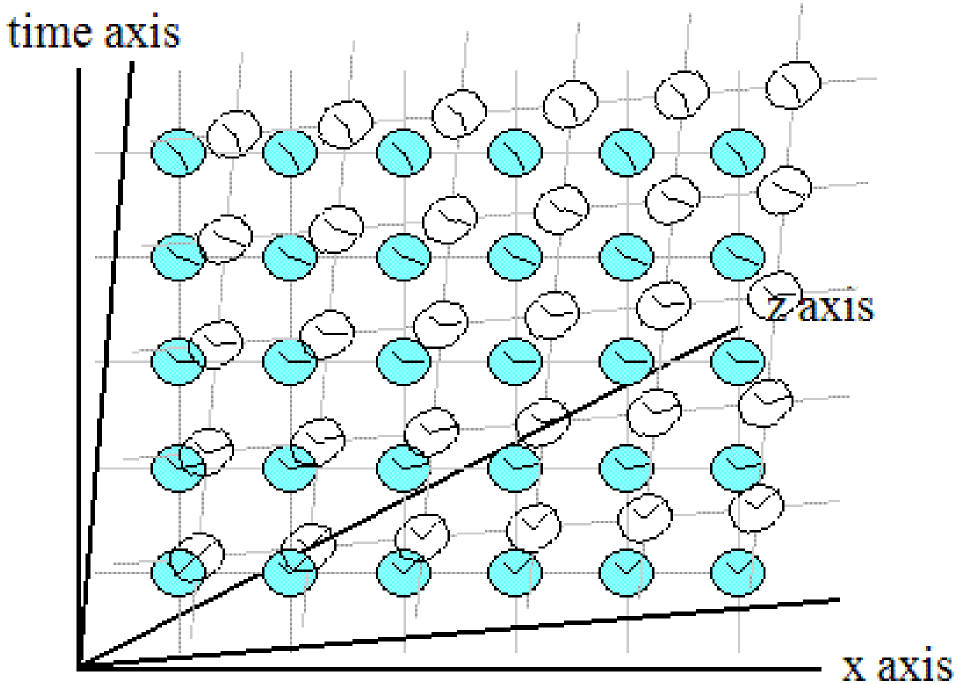

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b:Inertialoverlay.GIF

- each observer can synchronise clocks + rods

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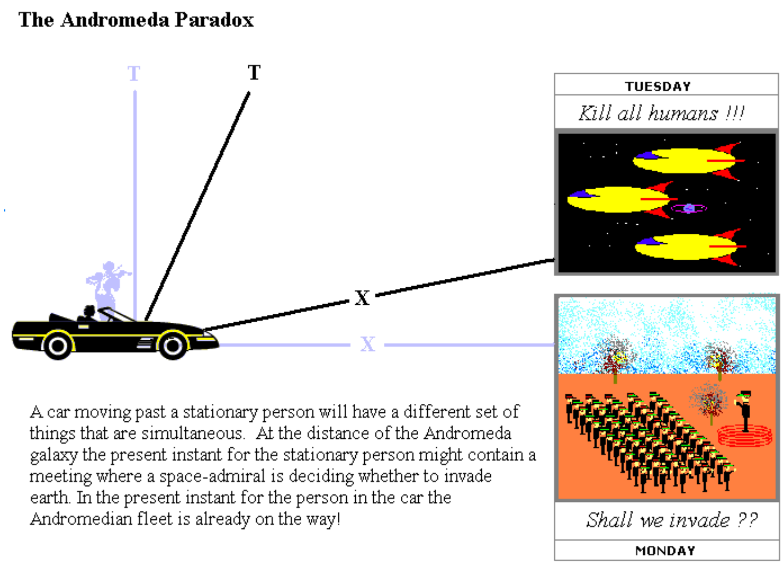
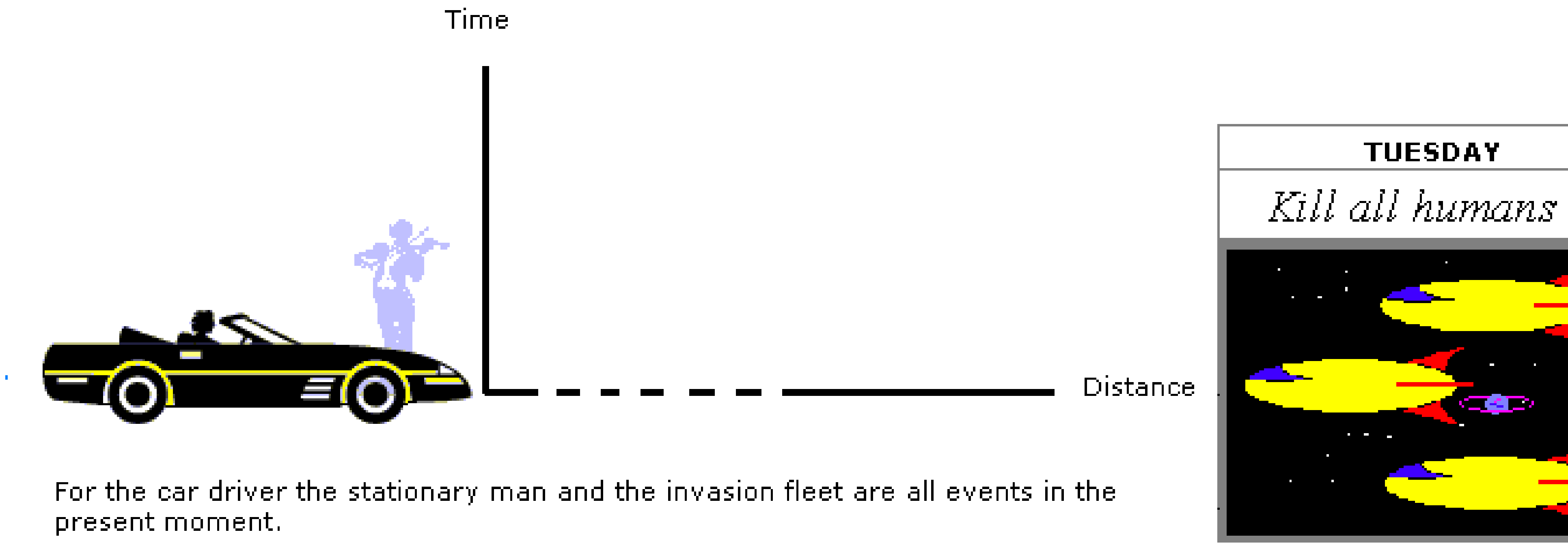


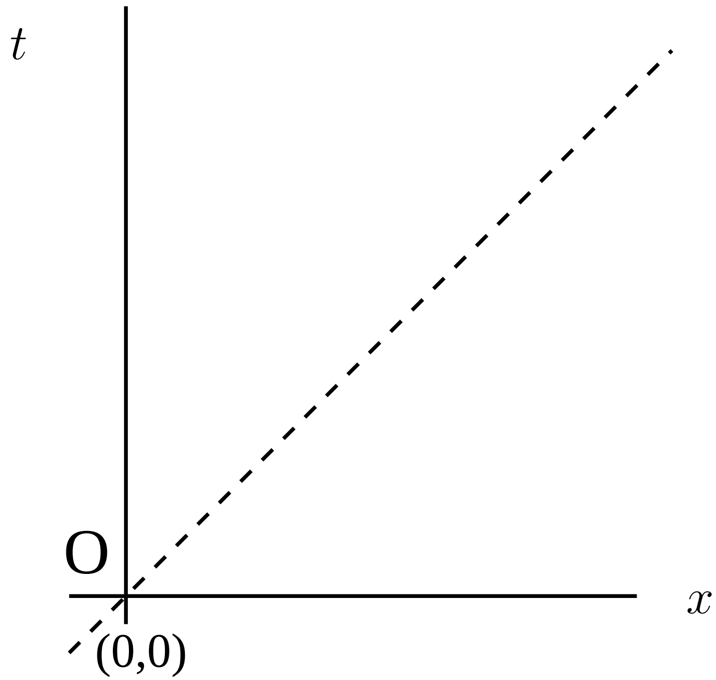
w:Rietdijk-Putnam argument b:Rel2.gif

SR: Rietdijk–Putnam–Penrose p.



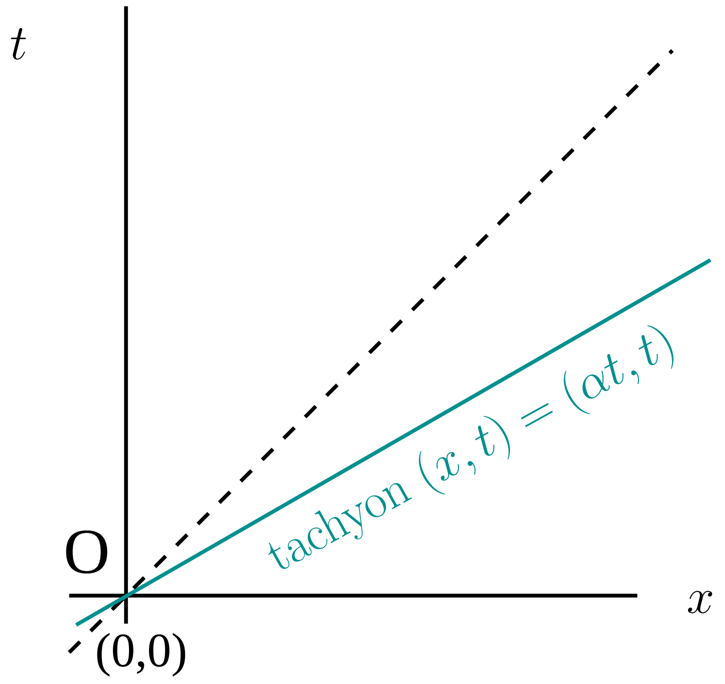
w:Rietdijk-Putnam argument b:Rel3.gif

SR: tachyons and causality



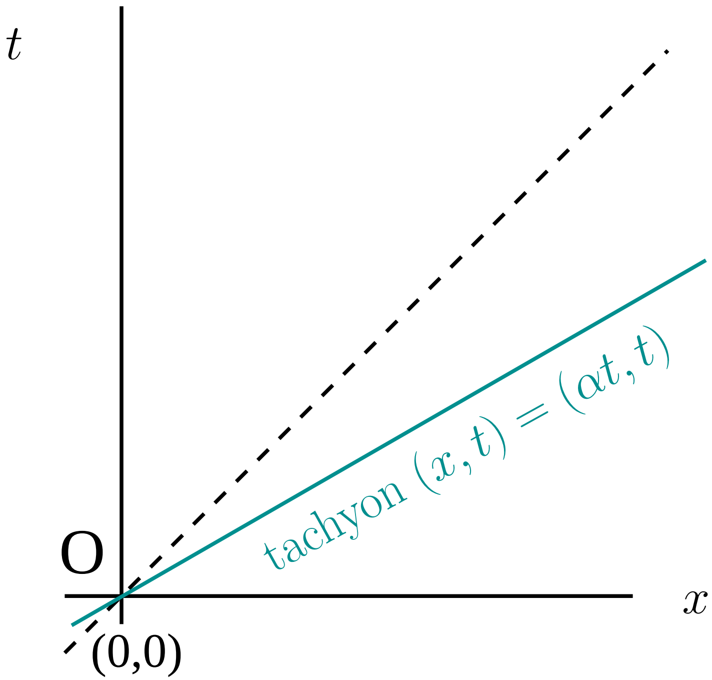
observer “at rest”

SR: tachyons and causality



add a tachyon with speed $\alpha > 1$

SR: tachyons and causality



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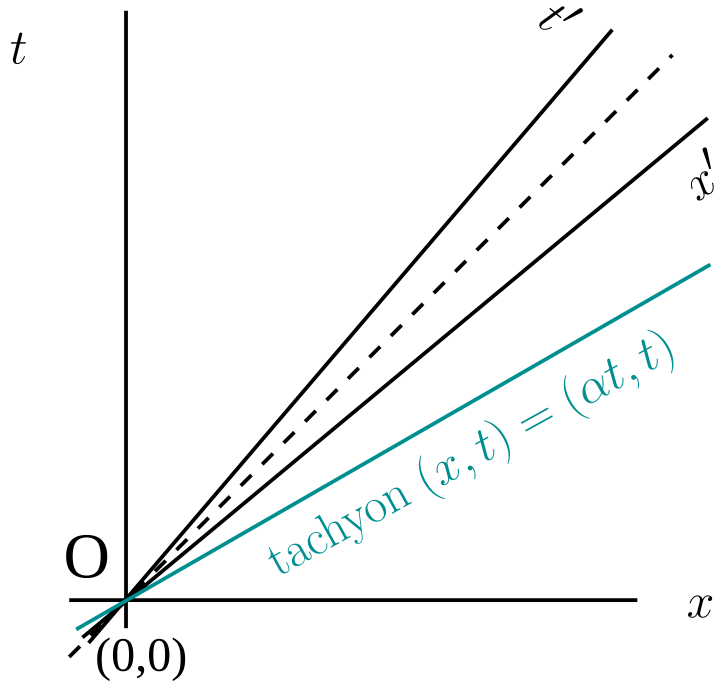

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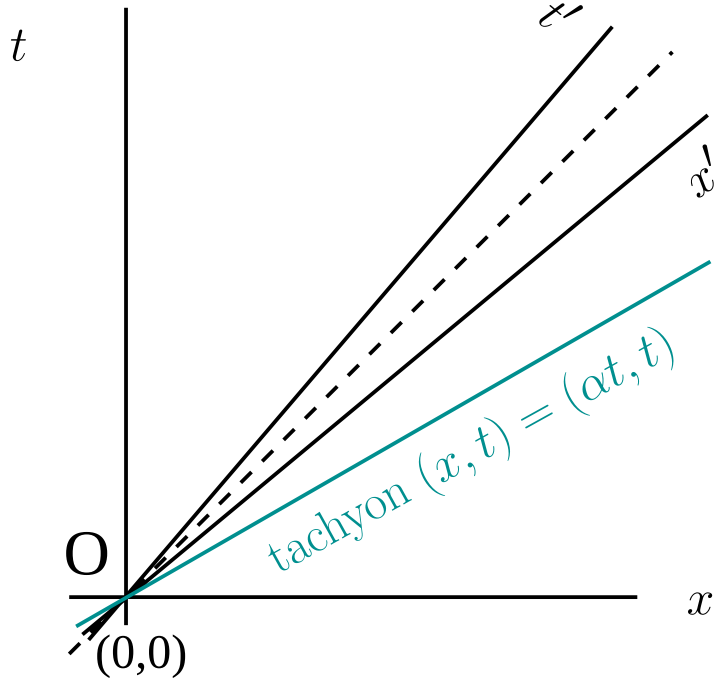

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rocket frame: $(\alpha t, t)$ becomes $\Lambda\ (\alpha t, t)^{\mathrm{T}}$

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$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \begin{pmatrix} \gamma\alpha t - \beta\gamma t \\ -\alpha\beta\gamma t + \gamma t \end{pmatrix}$$

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$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \gamma t \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

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same sequence of spacetime events = tachyon worldline:

t increases for observer “at rest”,

t' decreases for rocket observer (with $\beta > 1/\alpha$)

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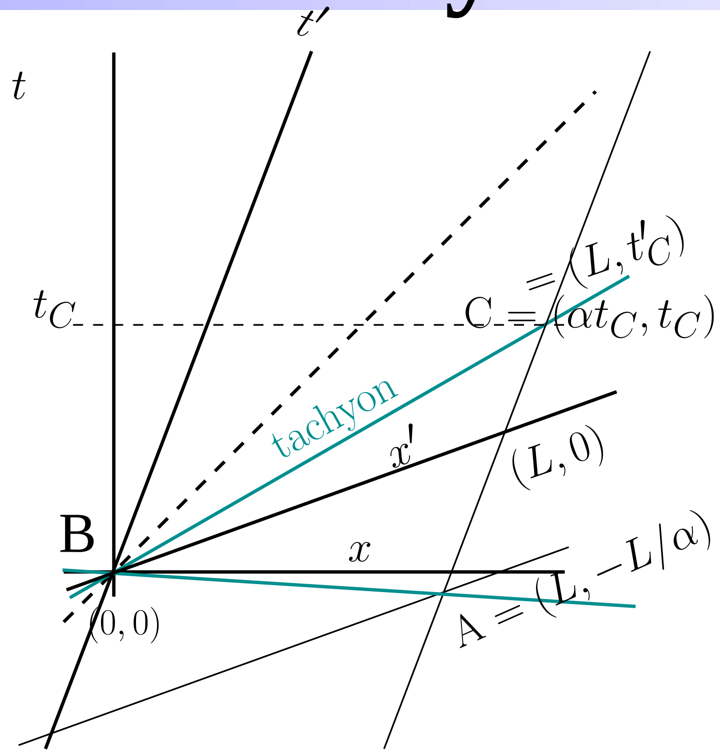
same sequence of spacetime events = tachyon worldline:

t increases for observer "at rest",

t' decreases for rocket observer (with $\beta > 1/\alpha$)

- observer at rest: tachyon emitted at origin
- rocket: tachyon absorbed at origin

SR: tachyonic antitelephone



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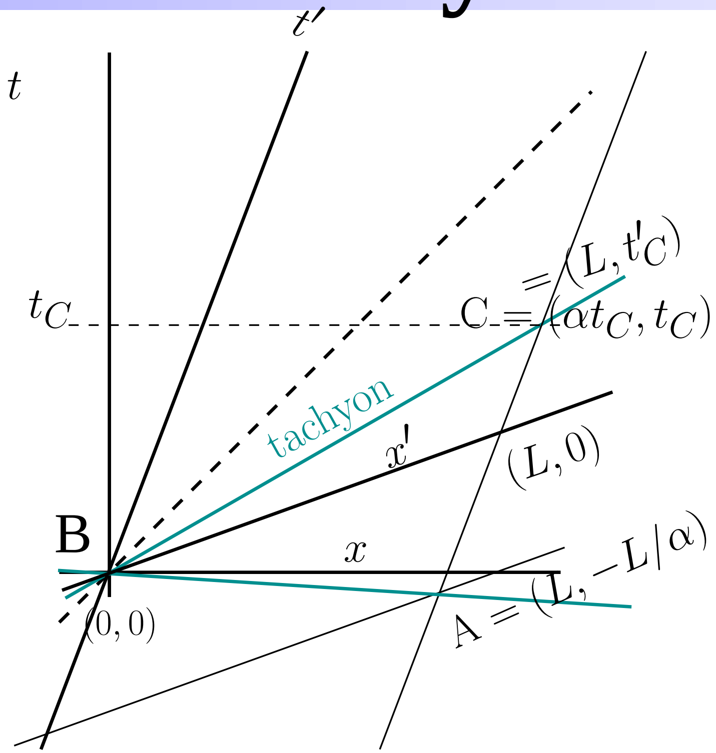
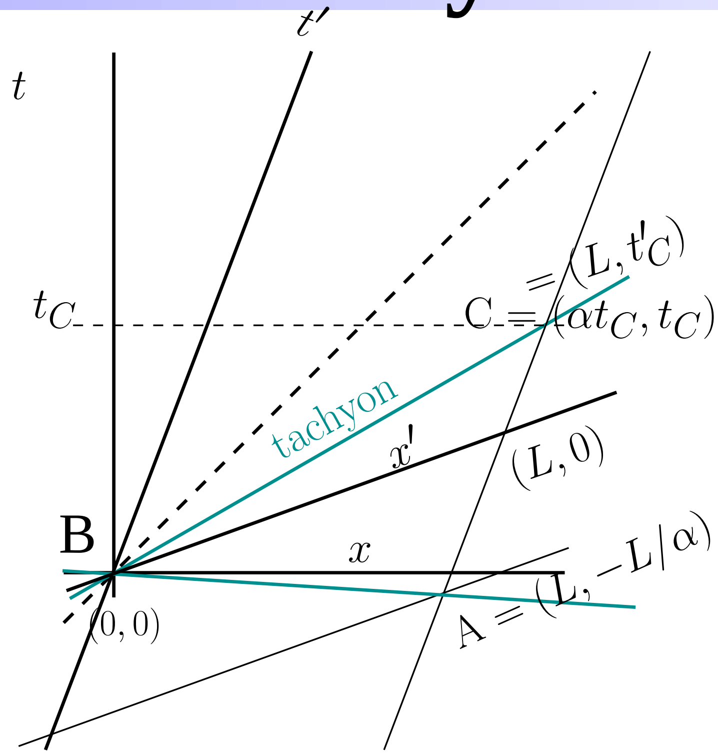


B stationary: frame (x, t)

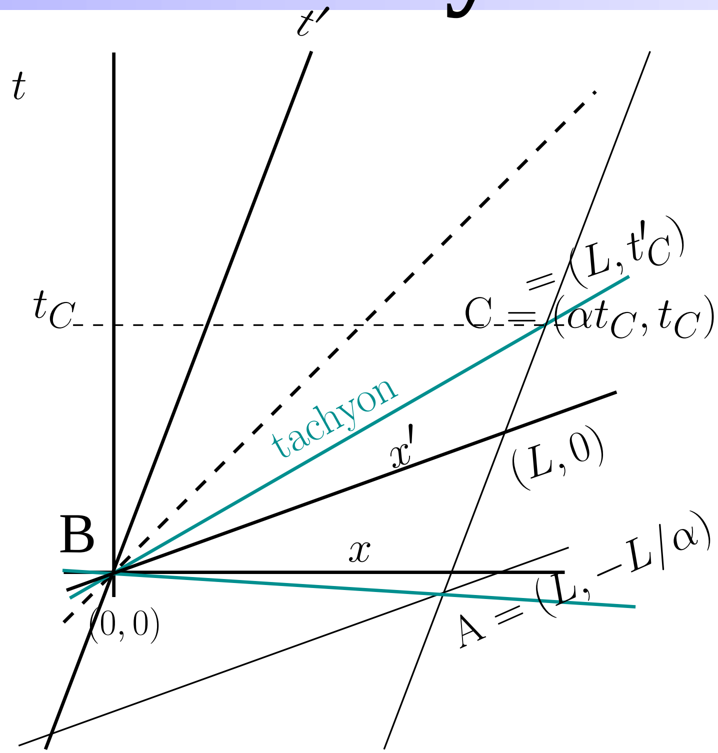
SR: tachyonic antitelephone



B stationary: (x, t) frame

A moving at speed β: (x', t') frame

SR: tachyonic antitelephone



A: tachyon at $\alpha > 1$ to B

B stationary: (x, t) frame

A moving at speed β: (x', t') frame

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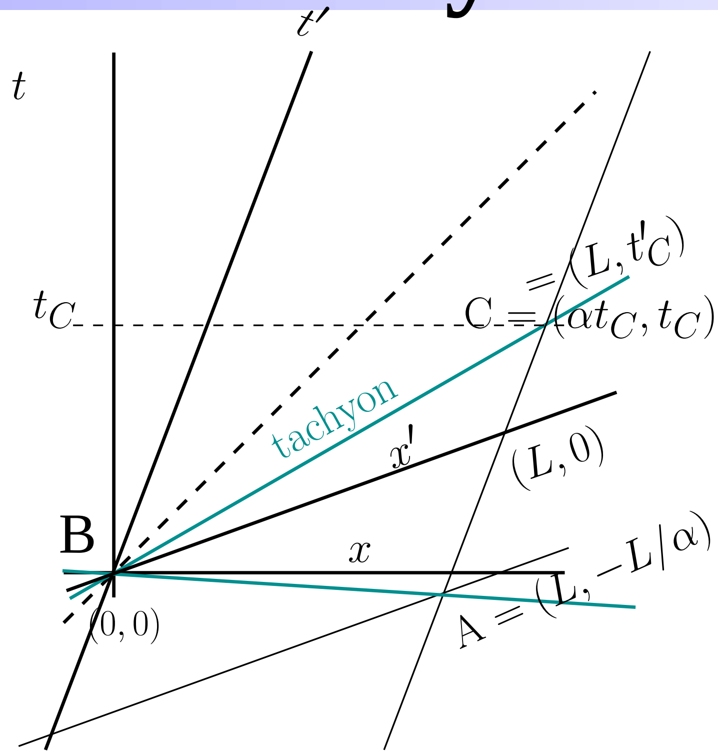

B: tachyon at $\alpha > 1$ to C

B stationary: (x, t) frame

A moving at speed β: (x', t') frame

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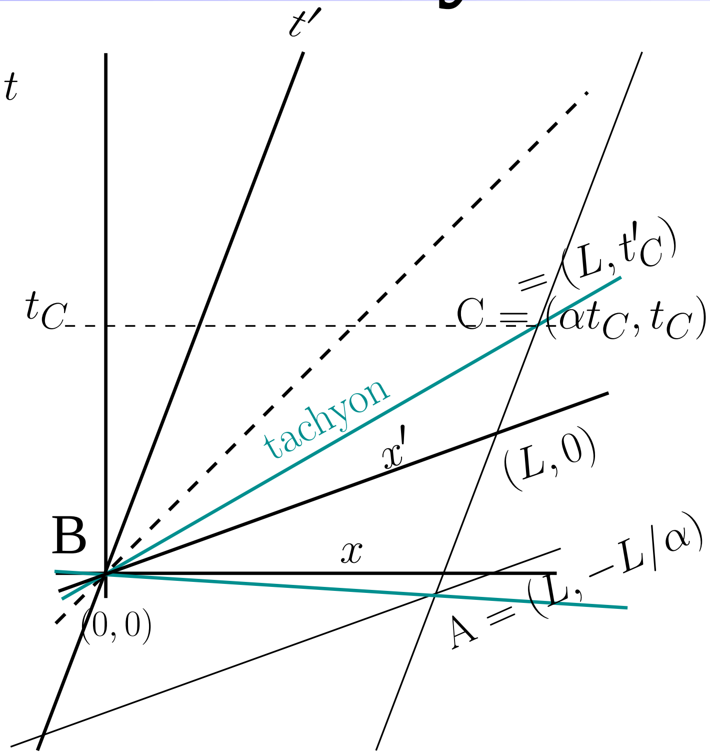
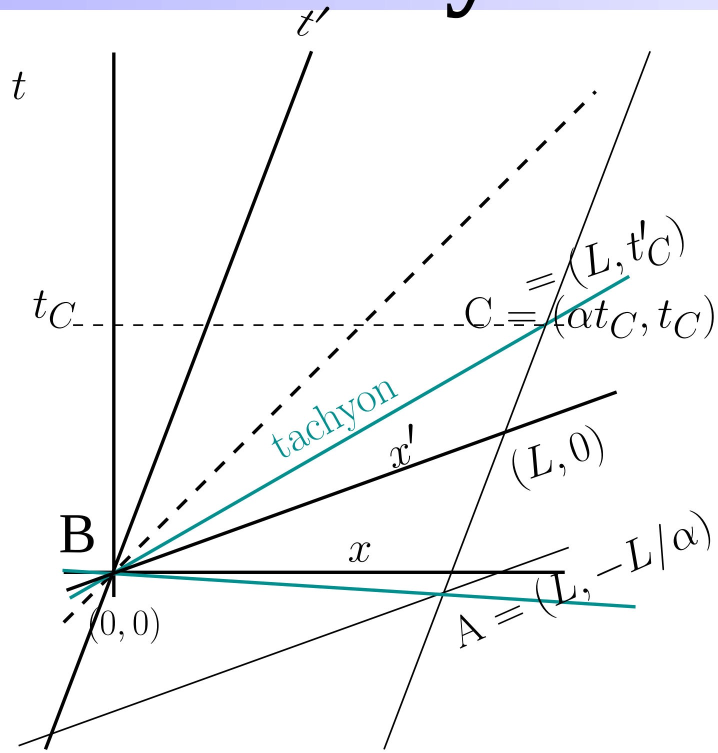


C: $$\begin{pmatrix} L \\ t'_{\mathrm{C}} \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t_{\mathrm{C}} \\ t_{\mathrm{C}} \end{pmatrix}$$

B stationary: (x,t) frame
A moving at speed β: (x',t') frame

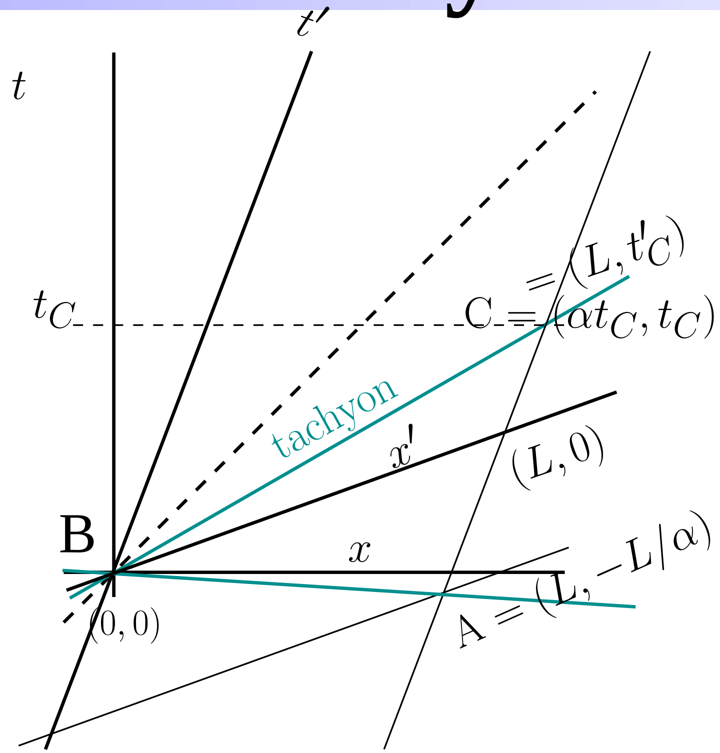
SR: tachyonic antitelephone



$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

B stationary: (x, t) frame
A moving at speed β: (x', t') frame

SR: tachyonic antitelephone



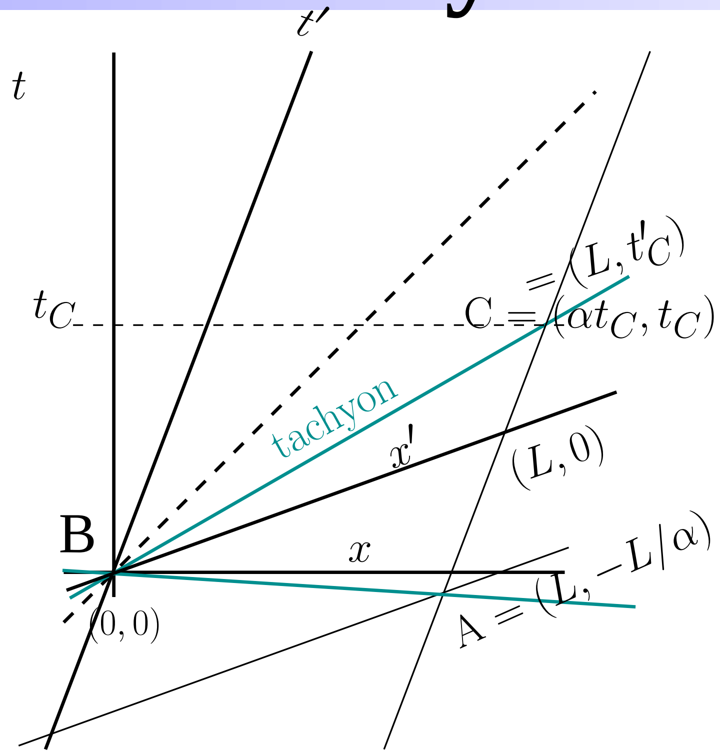
$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$t'_C = \gamma t_C (1 - \alpha\beta)$$

B stationary: (x, t) frame

A moving at speed β: (x', t') frame

SR: tachyonic antitelephone



$$\begin{pmatrix} L \\ t'_C \end{pmatrix} = \gamma t_C \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$$

$$t'_C = \gamma \frac{L}{\gamma(\alpha-\beta)}(1 - \alpha\beta)$$

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A moving at speed β: (x', t') frame

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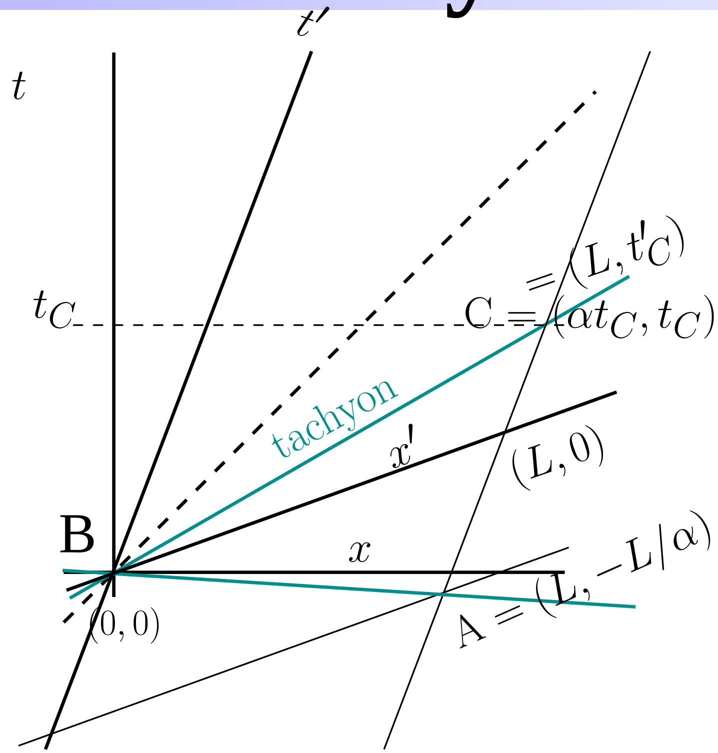

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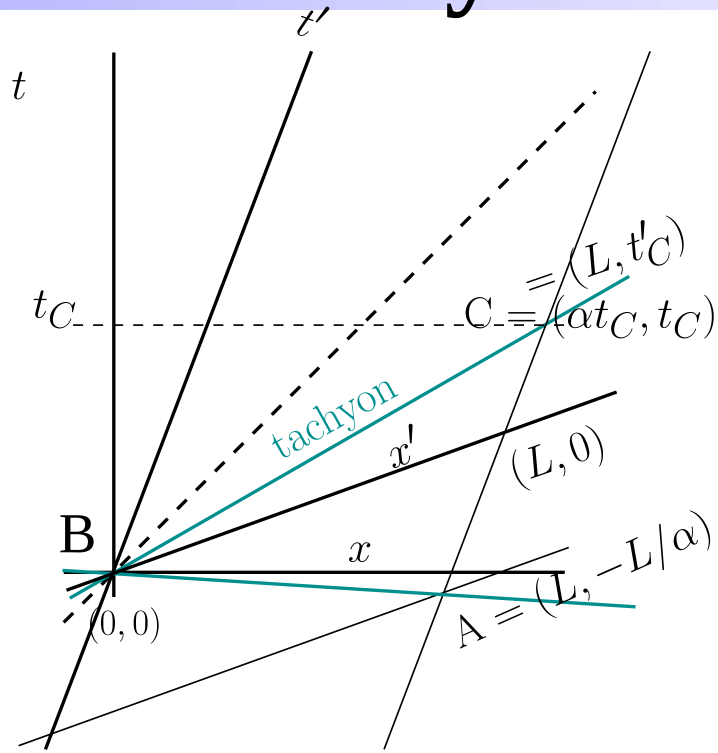

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$$t'_{\mathrm{C}} = L\frac{1-\alpha\beta}{\alpha-\beta}$$

$$t'_{\mathrm{C}} - t'_{\mathrm{A}} = L\left(\frac{1-\alpha\beta}{\alpha-\beta} + \frac{1}{\alpha}\right)$$

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A moving at speed β: (x', t') frame

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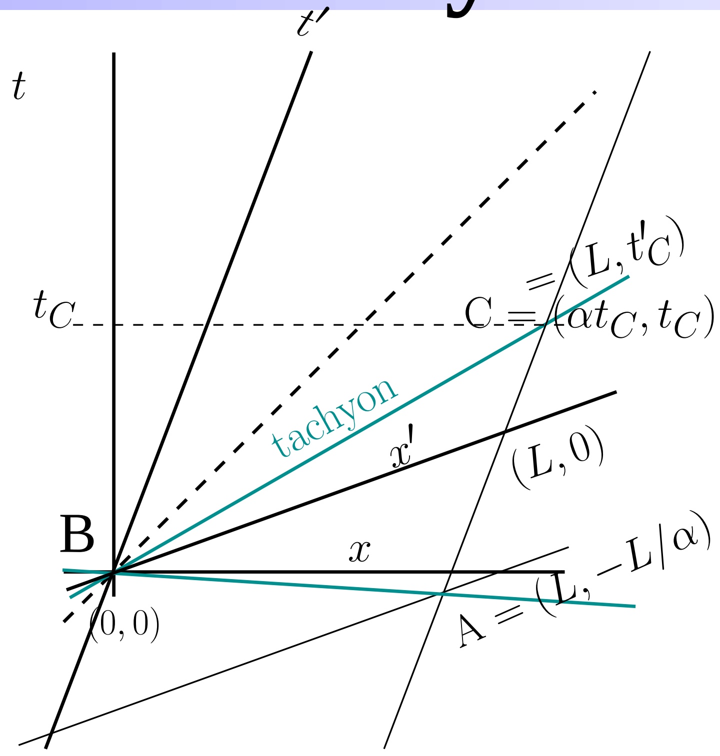

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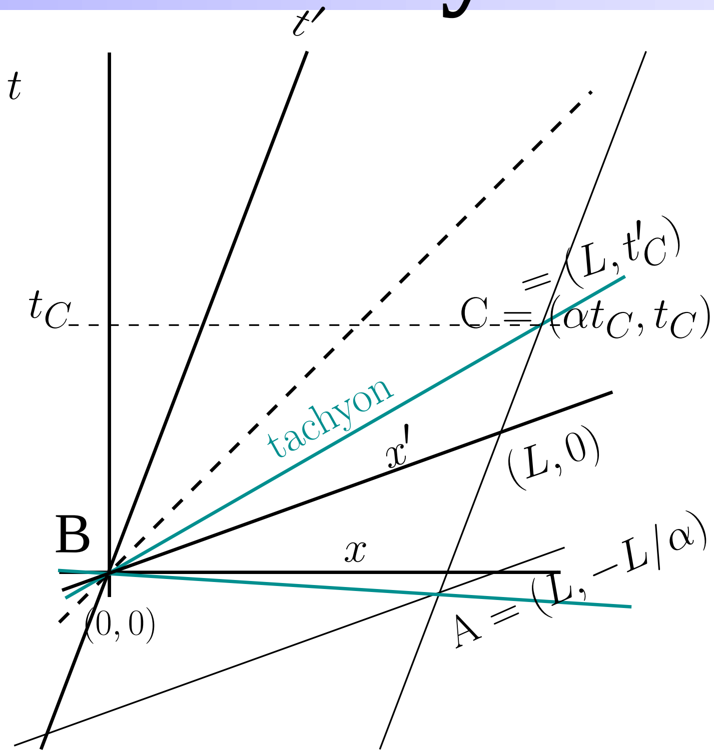

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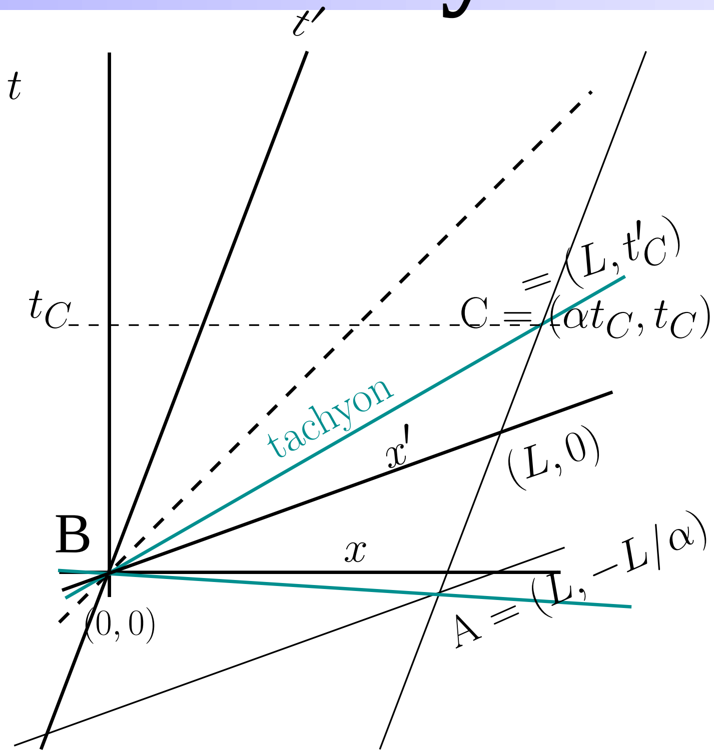

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< 0 if $\beta > \frac{2\alpha}{\alpha^2+1}$

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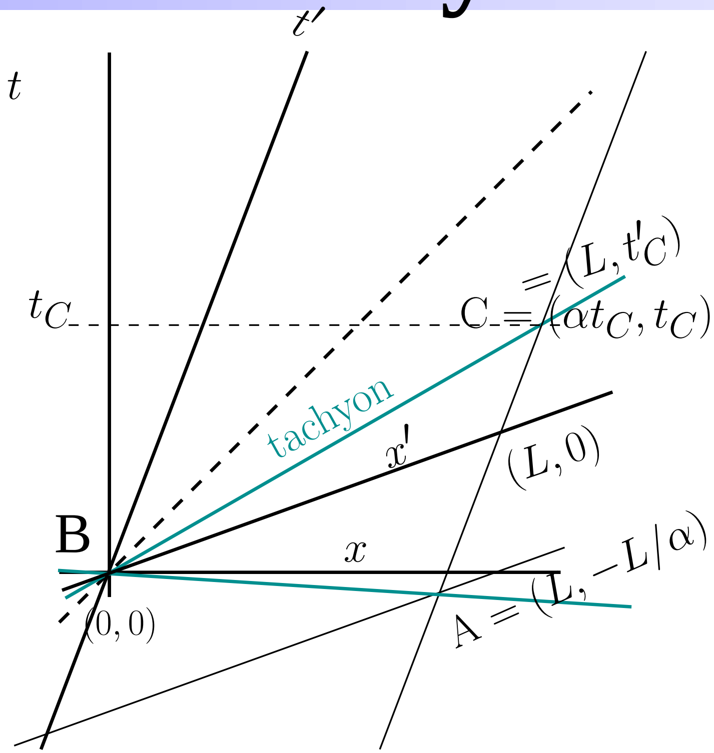

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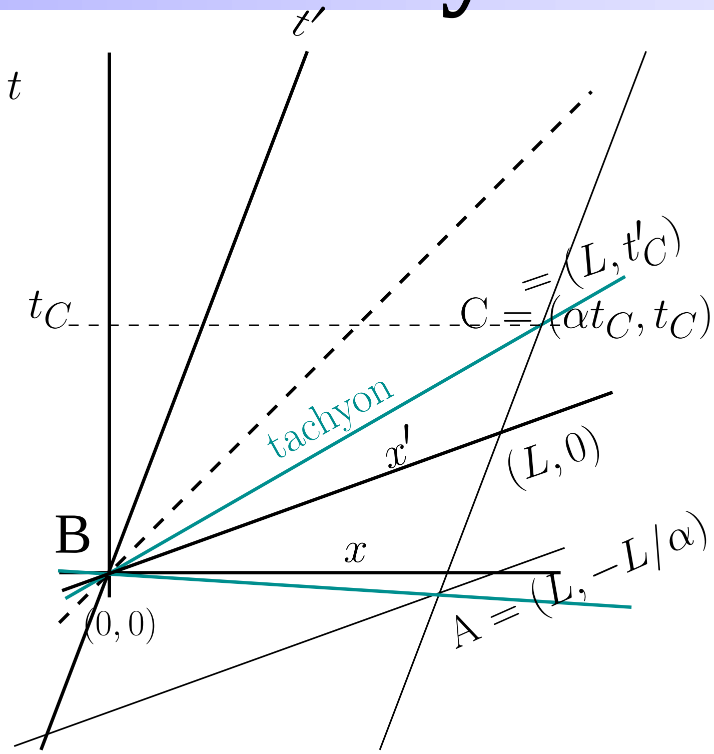
A receives tachyonic response at C before sending it

SR: tachyonic antitelephone



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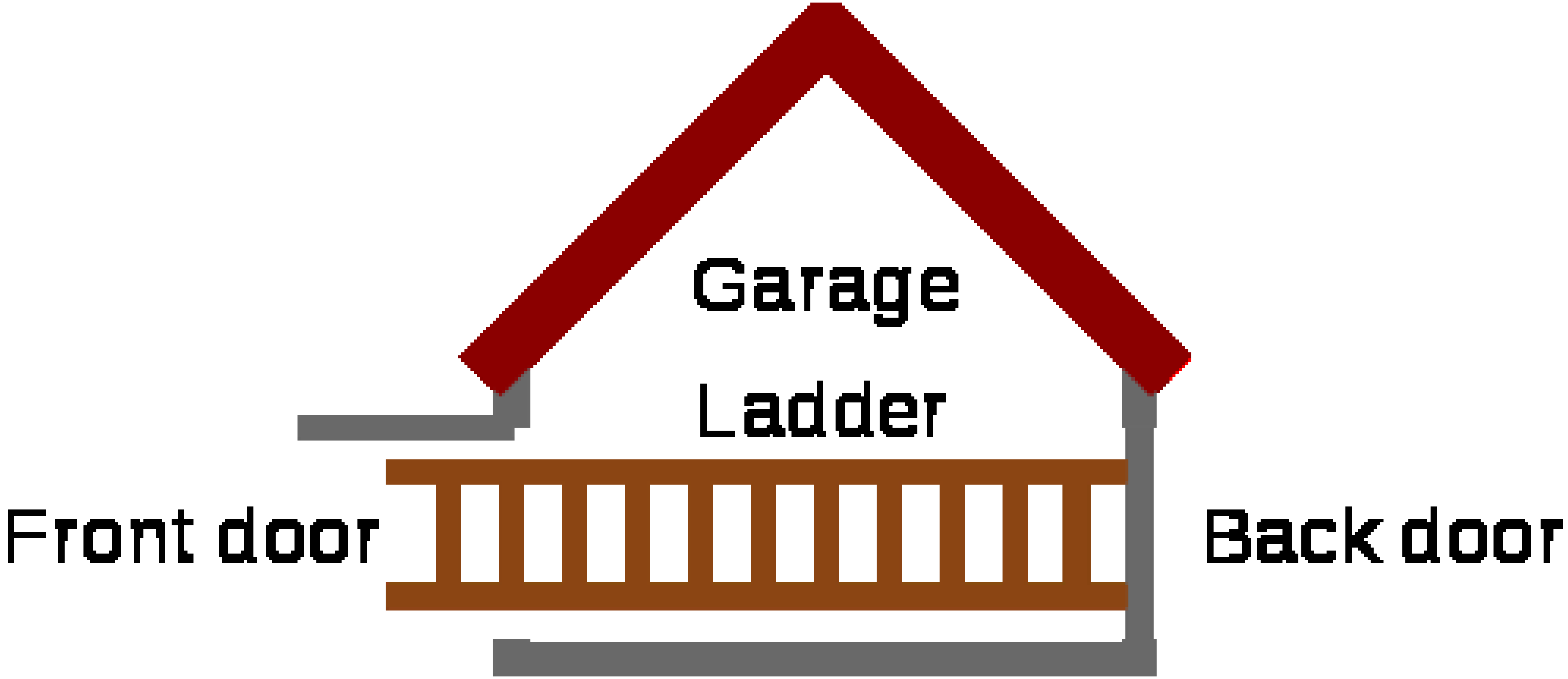
w:tachyonic antitelephone

SR: pole-barn/ladder paradox



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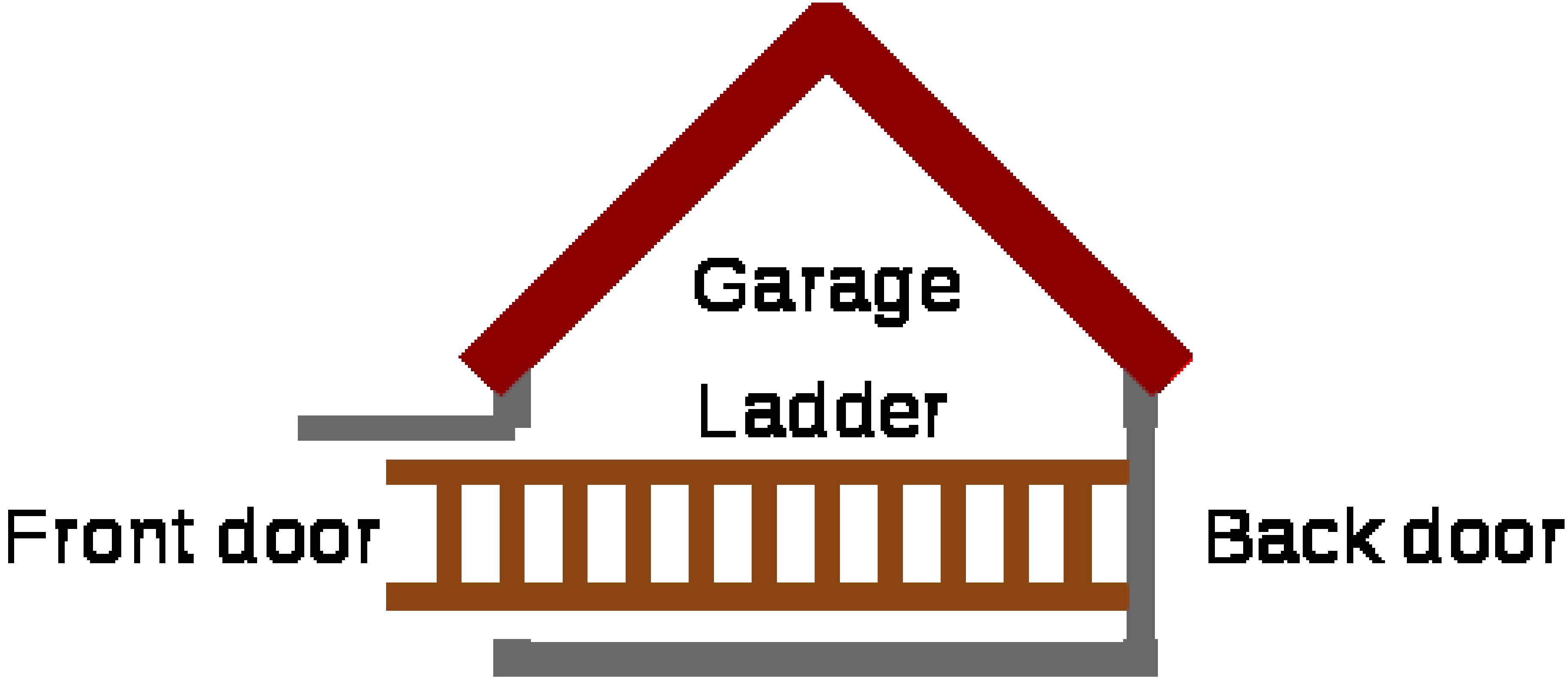
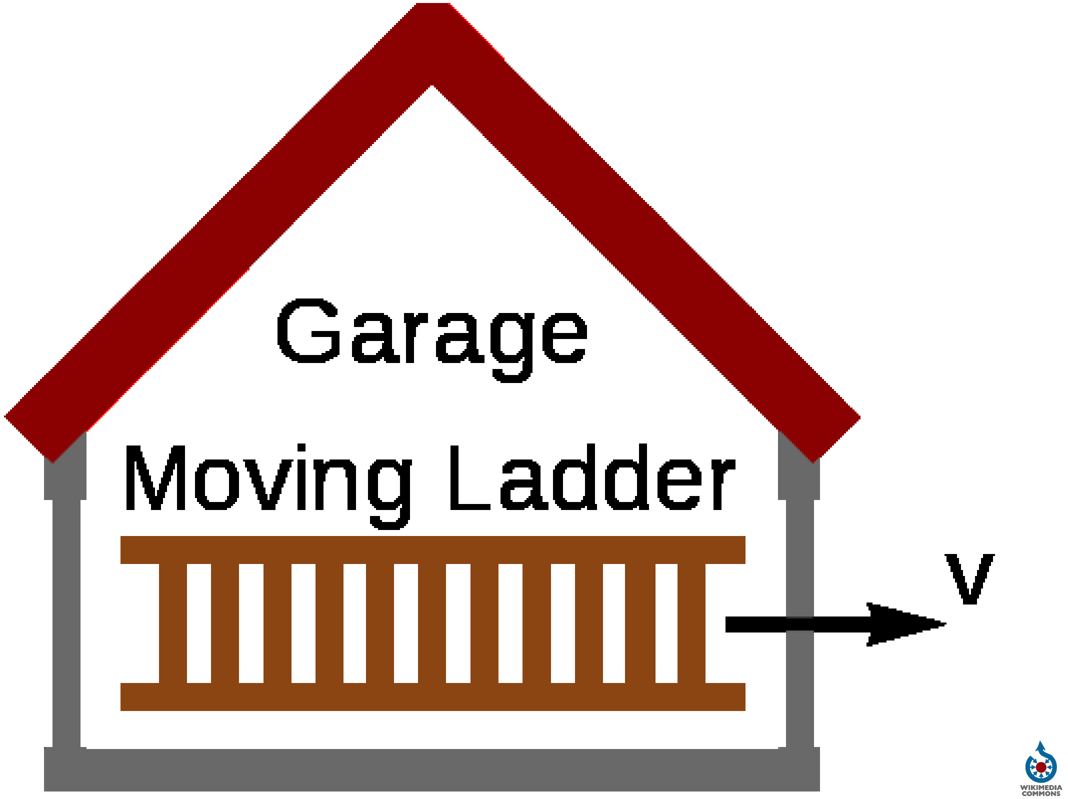


- ladder of length 29.9γ ns, garage length 30 ns

SR: pole-barn/ladder paradox



- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors

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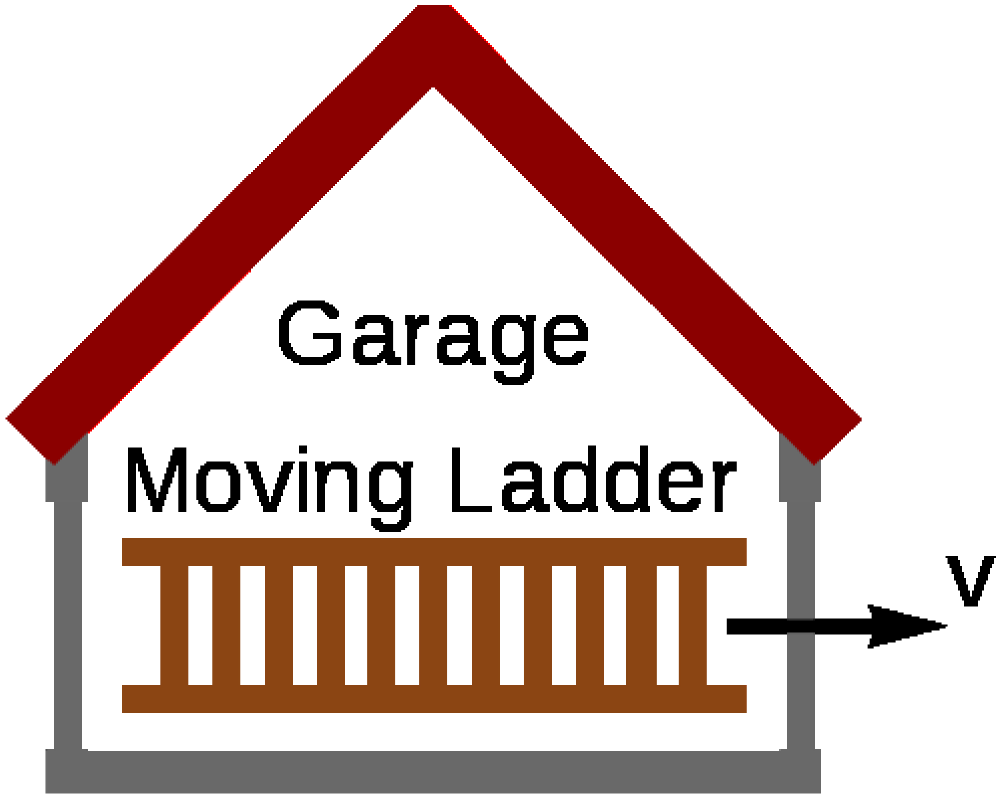

- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- 29.9γ ns $/\gamma < 30$ ns $\Rightarrow$ OK

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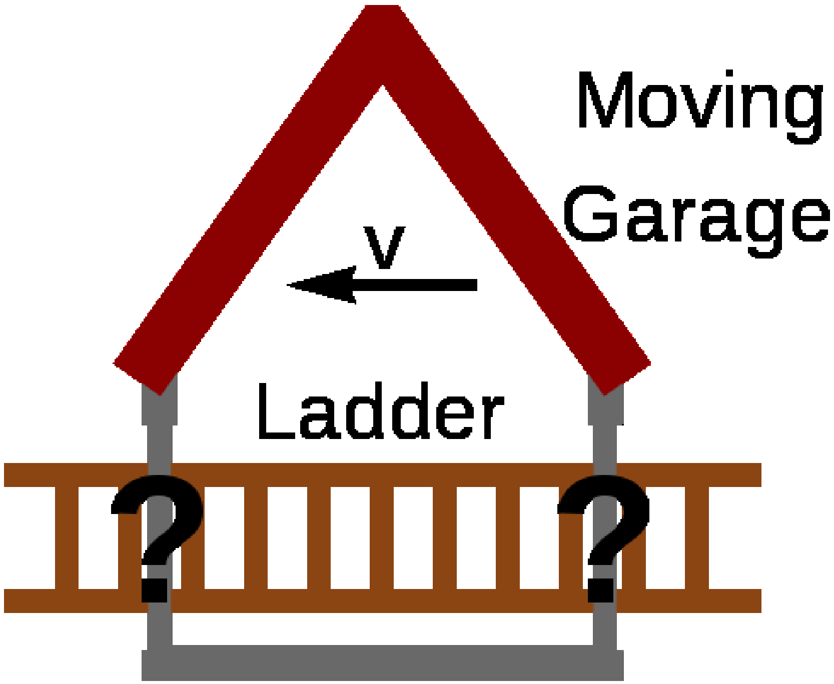
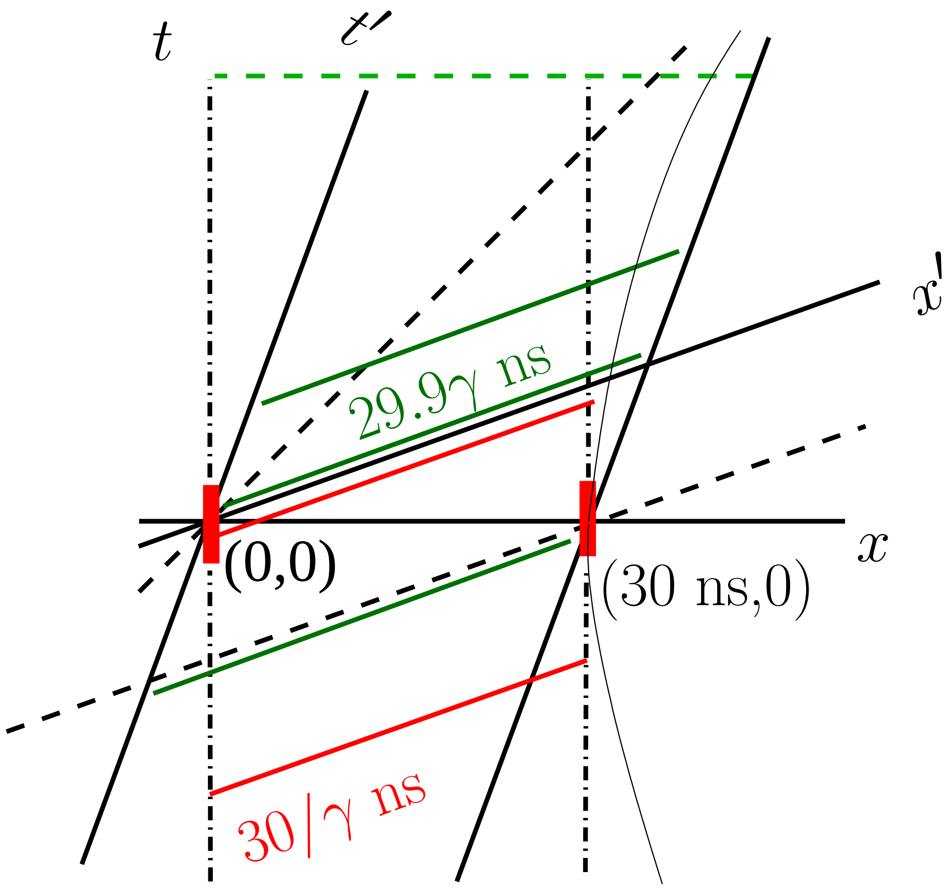


- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- ladder frame: garage $30/\gamma$ ns long $\ll 29.9\gamma$ ns!!

Is this possible or not? Make a spacetime diagram.

SR: pole-barn/ladder paradox



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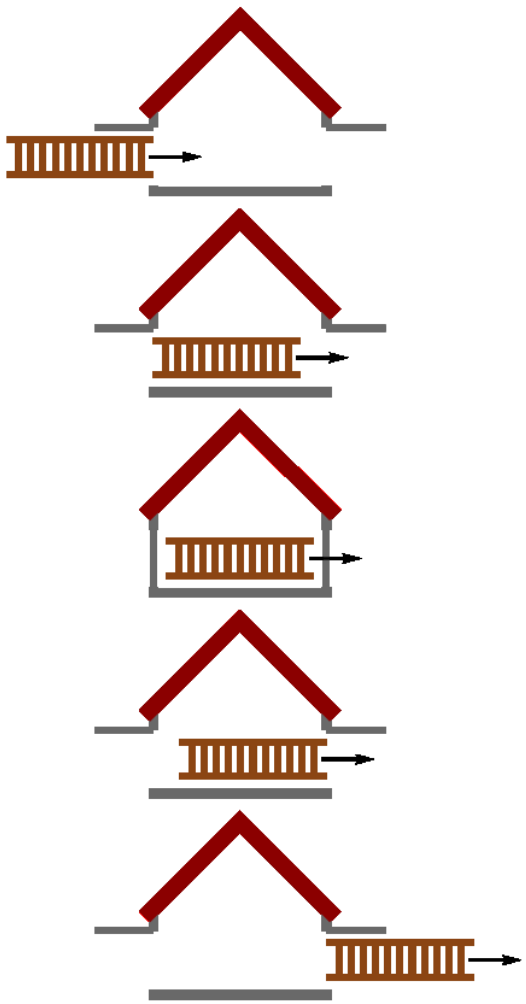

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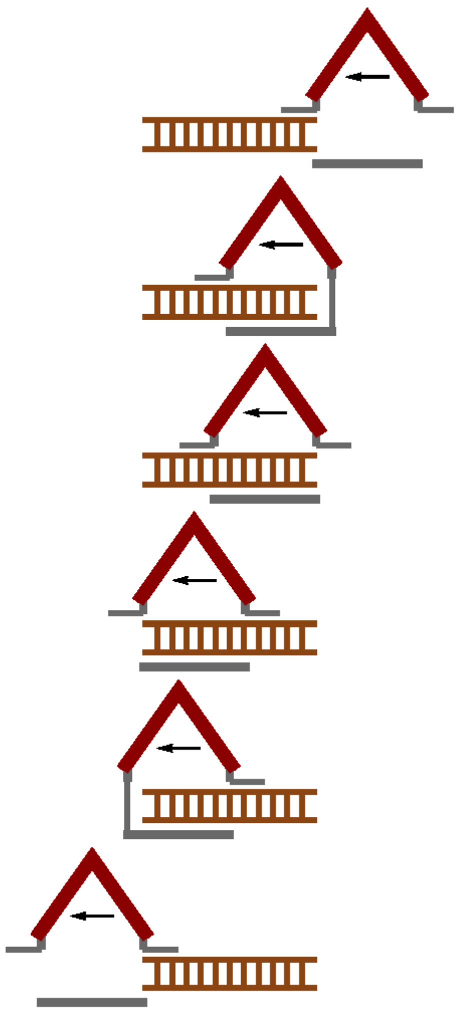

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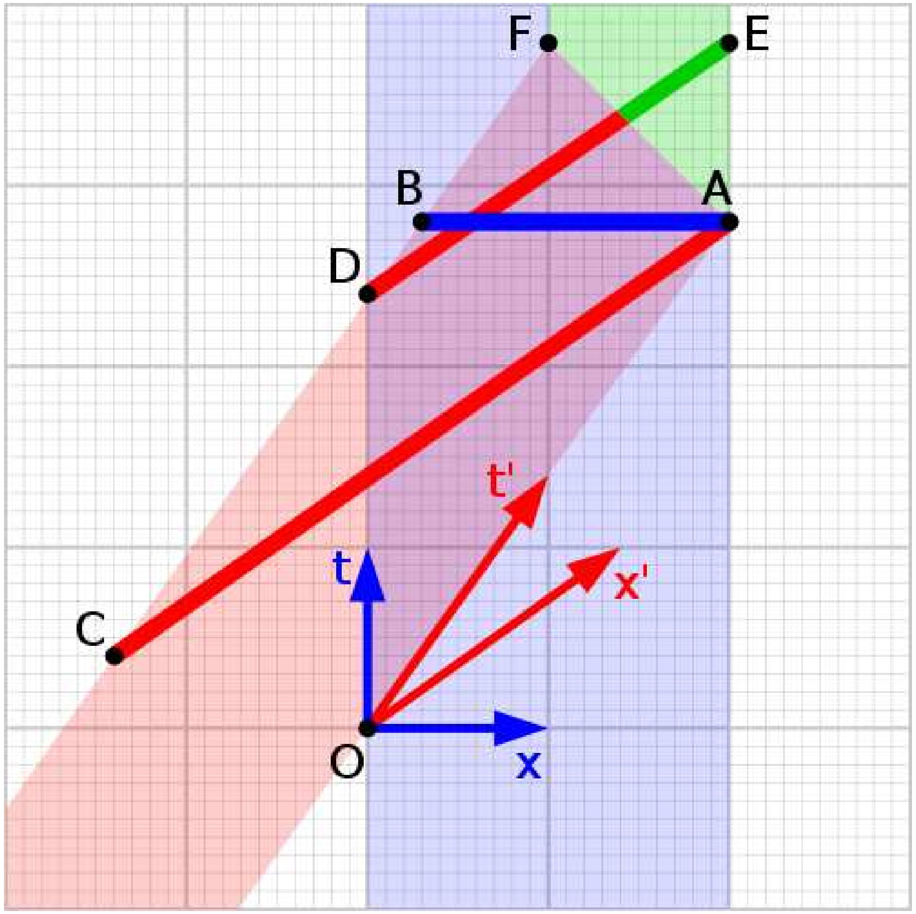
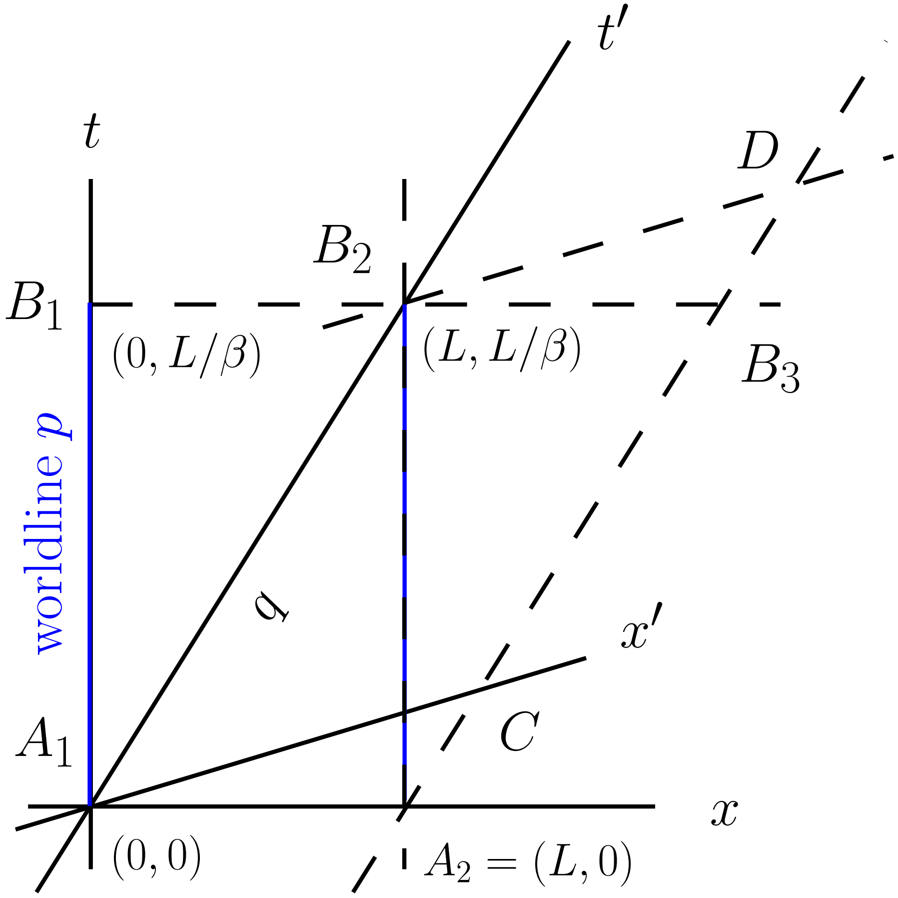


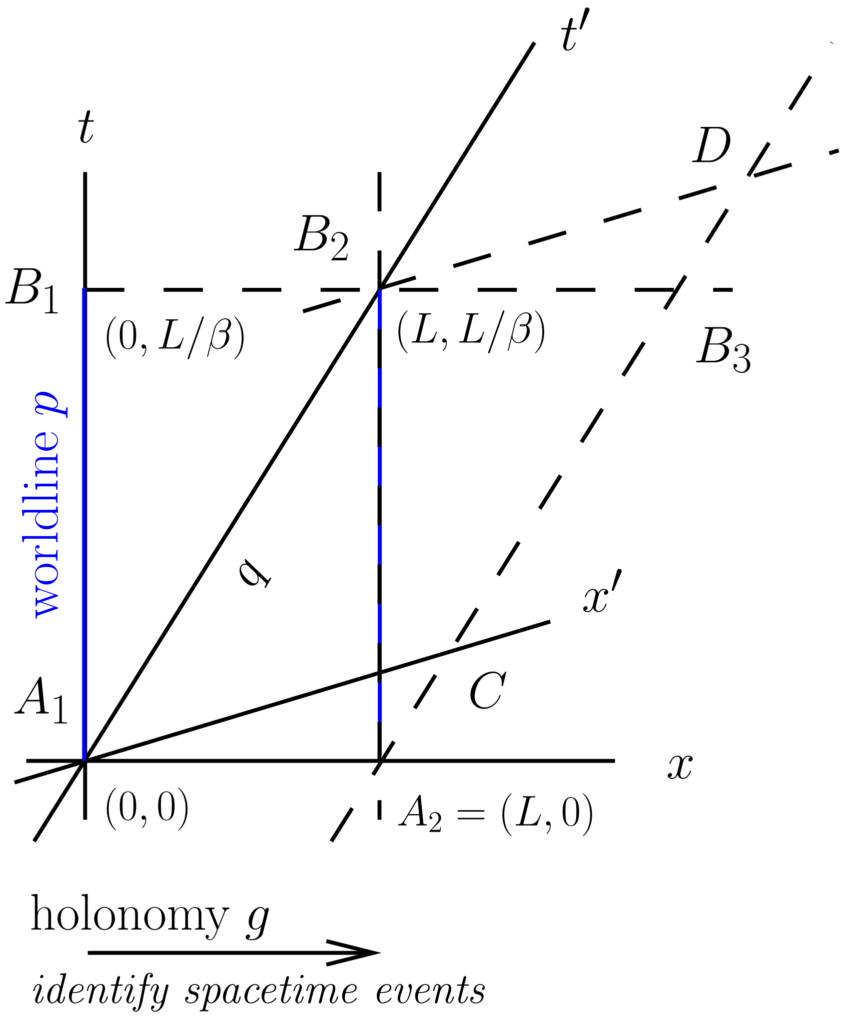
w:Ladder paradox

SR: twins paradox

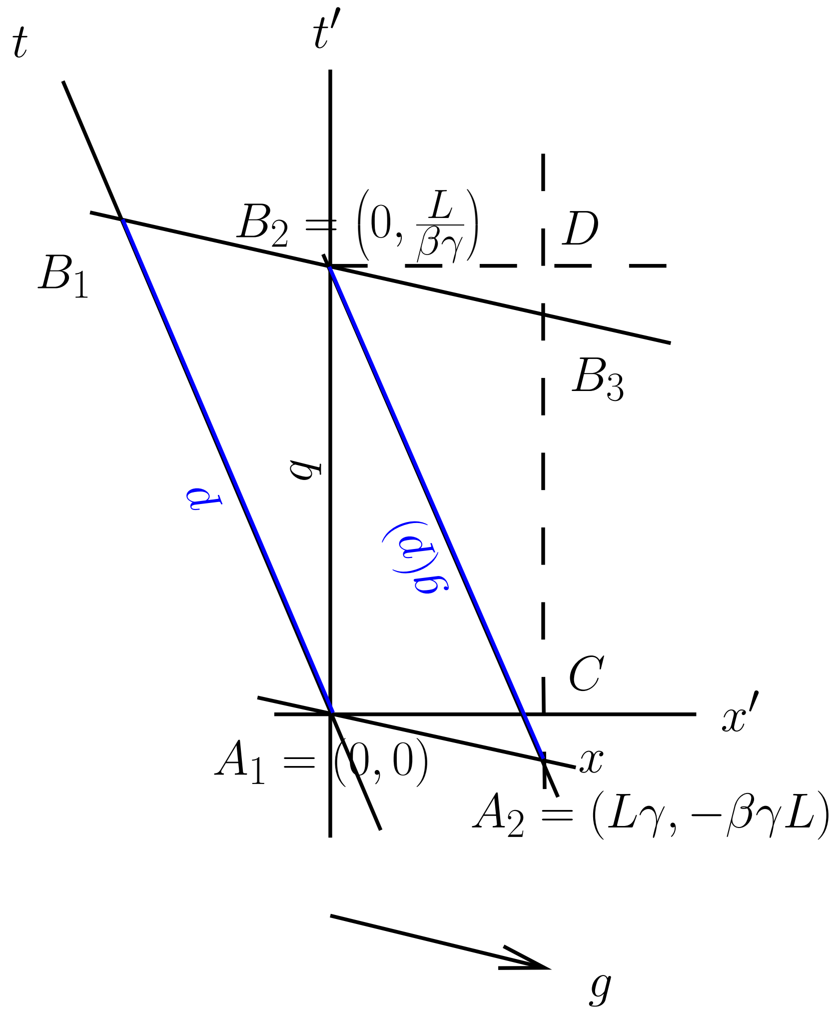


simply connected Minkowski

SR: twins paradox



SR: twins paradox



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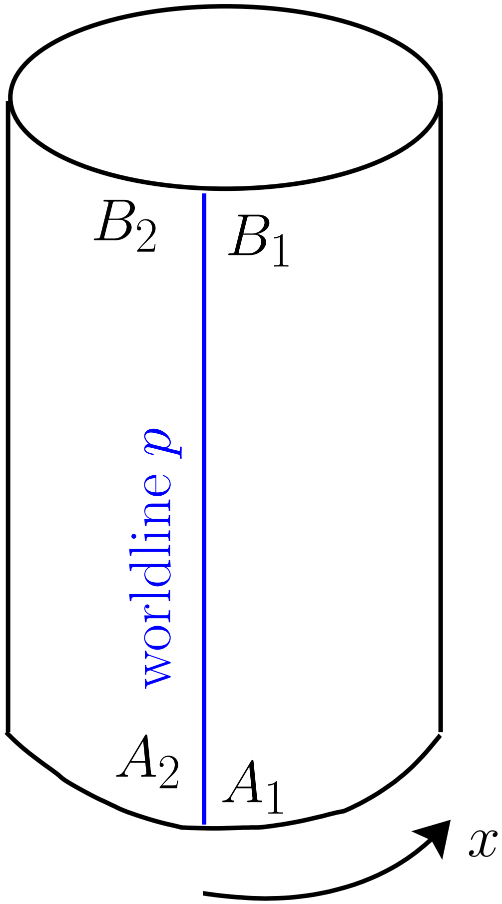
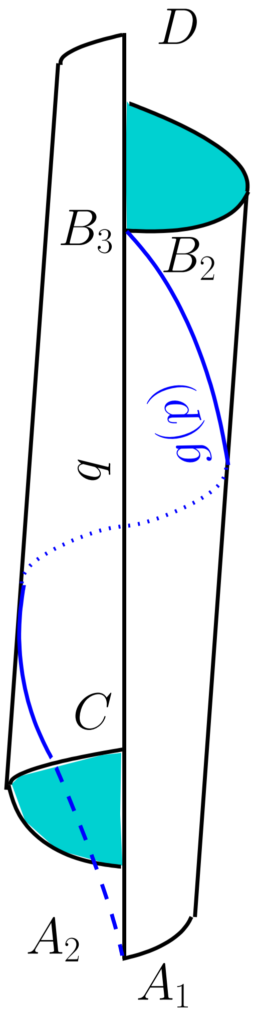
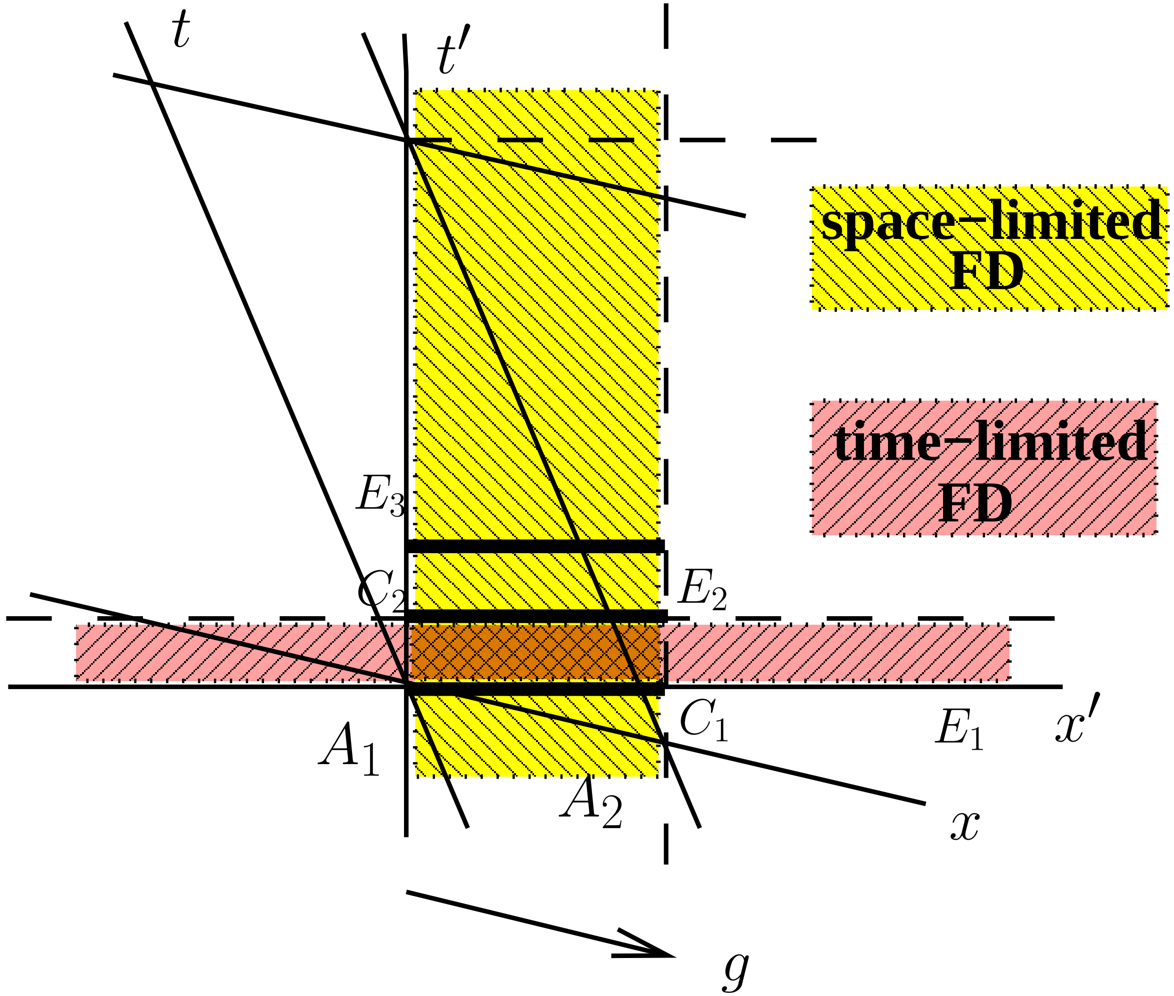


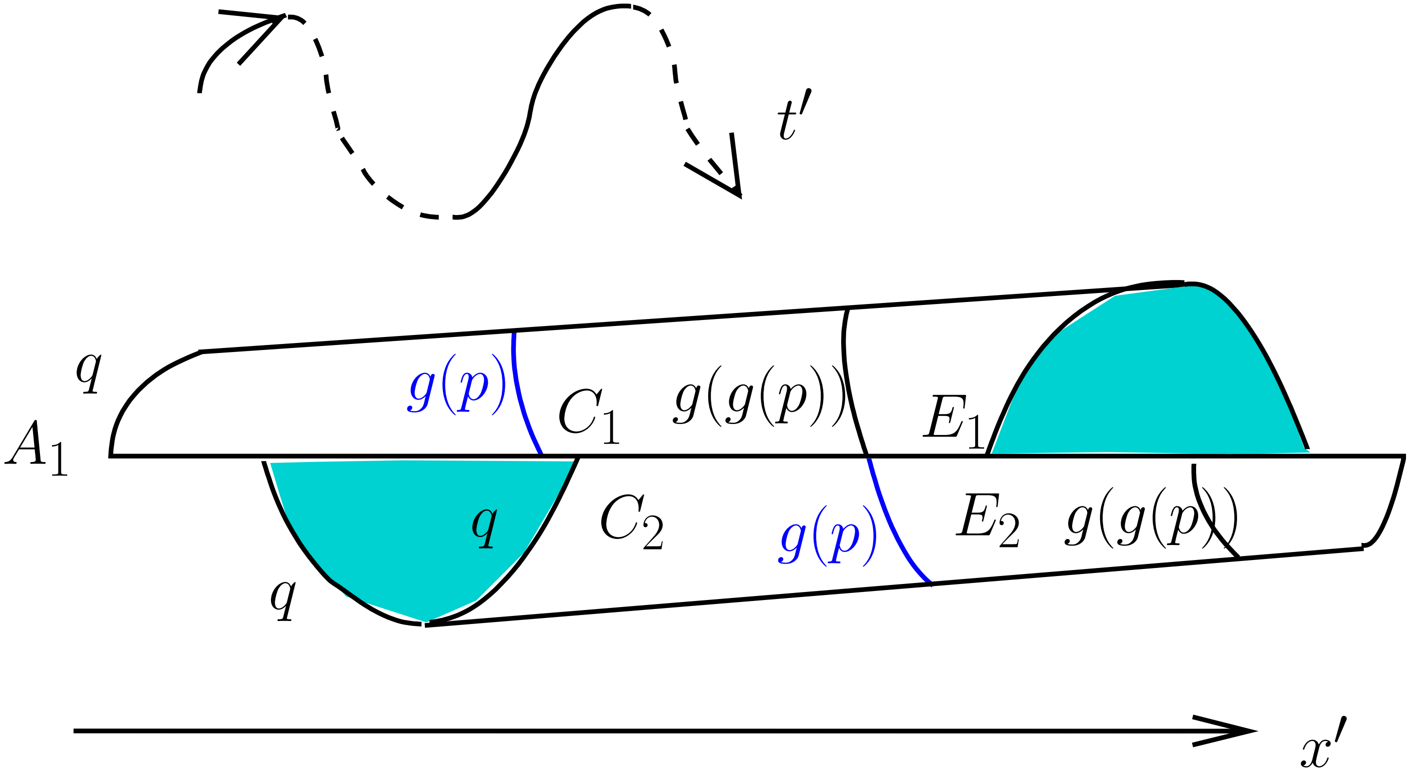
SR: twins paradox



SR: twins paradox



SR: twins paradox



Roukema & Bajtlik 2008, MNRAS, 390, 655

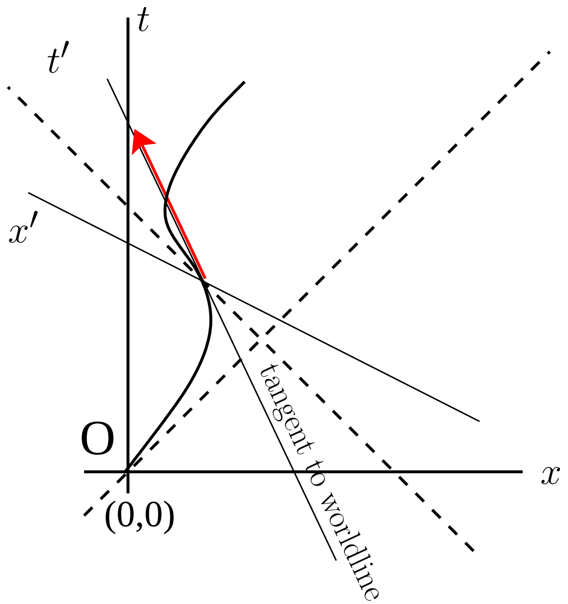
arXiv:astro-ph/0612155

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\mathrm{Galilean}} = (\beta, 0, 0)^{\mathrm{T}}$ for observer with $(t, x, y, z)^{\mathrm{T}}$ coord system

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\mathrm{Galilean}} = (\beta, 0, 0)^{\mathrm{T}}$ for observer with (t, x, y, z) coord system



- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector = tangent to worldline

SR: four-velocity, four-momentum

choose x axis so that 3-velocity $u_{\text{Galilean}} = (\beta, 0, 0)^{\text{T}}$ for observer with (t, x, y, z) coord system

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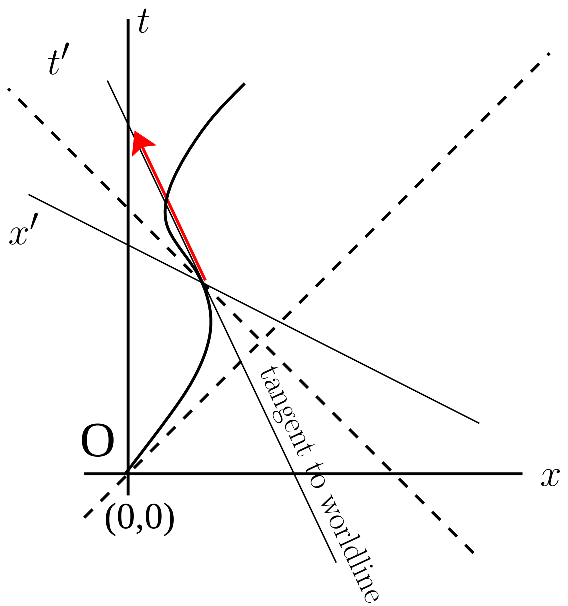
w:four-velocity



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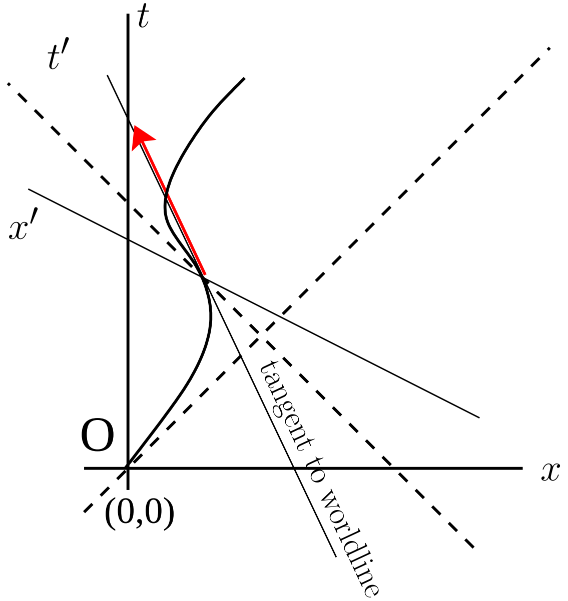

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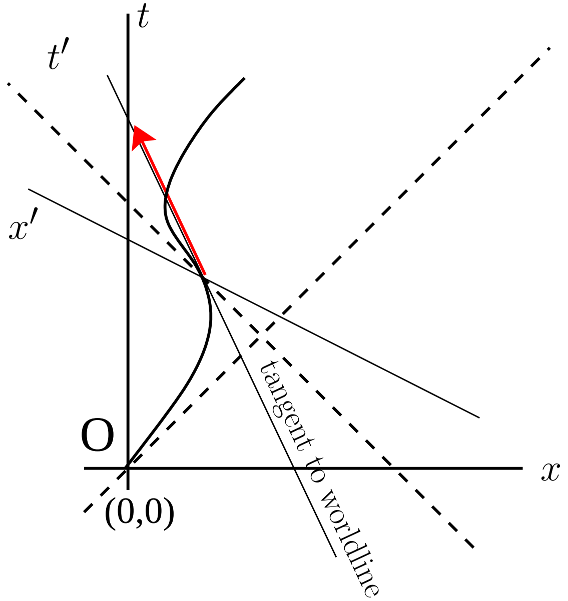

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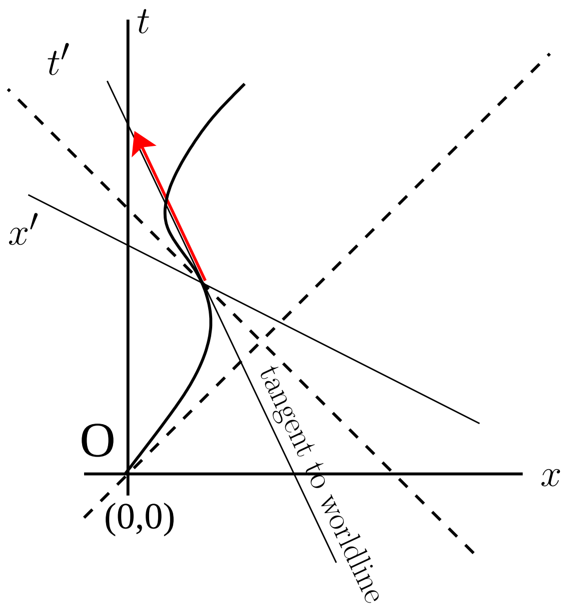

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want $\vec{u}$ Lorentz invariant $\Rightarrow$ $(u^t, u^x)^{\text{T}} = \Lambda^{-1}(1, 0)^{\text{T}}$

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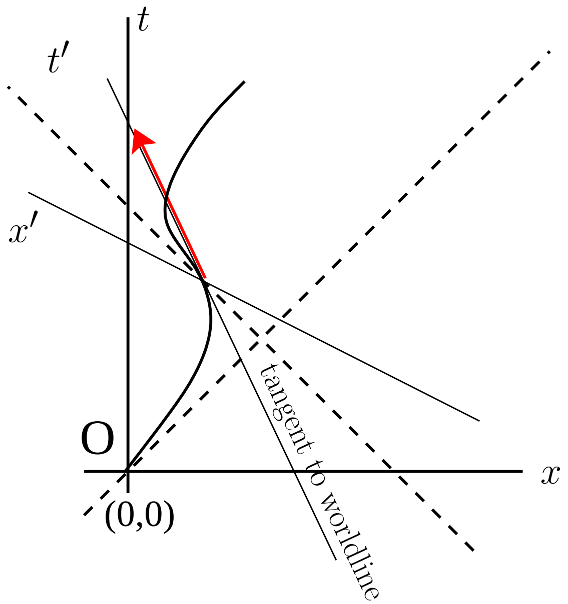

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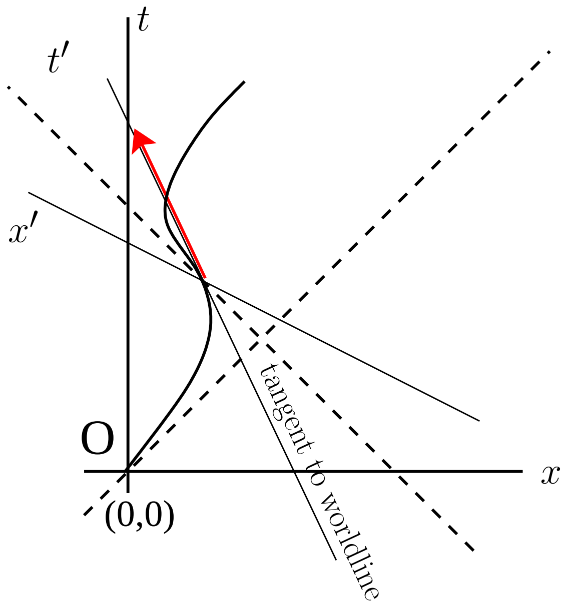

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- in (t, x) spacetime 2-plane, extend from scalar speed β to spacetime vector = tangent to worldline

4D: $\vec{u} = \gamma(1, \beta^x, \beta^y, \beta^z)^{\mathrm{T}}$

notation in this pdf:

$\vec{u}$ = 4-vector, ${}^{(3)}\vec{u}$ = spatial component

SR: four-velocity, four-momentum

Is the 3-component (spatial component) of $\vec{u}$ the same as the non-relativistic velocity?

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$$\neq \frac{\mathrm{d}}{\mathrm{d}t}(x, y, z)^{\mathrm{T}}$$ except if $\beta = 0 \Leftrightarrow \gamma = 1$

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momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^{\mathrm{T}}$, where m = constant w:invariant mass

${}^x \ldots$ = tensor-style component notation, not powers

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What does the time component of momentum = $p^0 = m\gamma$ mean physically?

- first look at spatial component in a given ref. frame

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let us define 4-acceleration, 4-force

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$= \frac{\mathrm{d}}{\mathrm{d}\tau}\vec{p}$

SR: invariance of ${}^{(4)}u, {}^{(4)}a, {}^{(4)}f$

Euclidean norm: $\|\vec{x}\|^2 = \sum_\mu (x^\mu)^2$

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Minkowski pseudo-norm: $\|\vec{x}\|^2 = \sum_{\mu,\nu} \eta_{\mu\nu} x^\mu x^\nu$

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w:Einstein summation sum is implicit

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Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0 x^0 + \delta_{ij} x^i x^j$

$\delta_{ij} = 1$ if $i = j$, otherwise $= 0$; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2$ = same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common

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similarly: $\|\vec{a}\|^2$, $\|\vec{f}\|^2$ invariant

SR: energy: varies with ref frame

Newtonian $K = (1/2)m\beta^2$ = 0 in rest frame

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$$= \int_0^{\beta_2} \frac{\mathrm{d}}{\mathrm{d}t}(m\beta\gamma)\mathrm{d}x$$

(assume ${}^{(3)}\vec{f}/\gamma \parallel \vec{x}$)

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$$= m \int_{\gamma=1}^{\gamma=\gamma_2} [\beta^2 + (1 - \beta^2)]\mathrm{d}\gamma$$

SR: energy: varies with ref frame

in (x,t) frame,

$K =$ work done

$$
\begin{aligned}
K &= \int_0^{\beta_2} \frac{{}^{(3)}\vec{f}}{\gamma} \cdot \mathrm{d}\vec{x} \\
&= \int_0^{\beta_2} \frac{\mathrm{d}}{\mathrm{d}t}(m\beta\gamma)\mathrm{d}x = m \int_0^{\beta_2} \mathrm{d}(\beta\gamma)\beta \\
&= m \int_0^{\beta_2} \beta(\beta\mathrm{d}\gamma + \gamma\mathrm{d}\beta) \\
&= m \int_{\gamma=1}^{\gamma=\gamma_2} (\beta^2 + \gamma^{-2})\mathrm{d}\gamma \ \Leftarrow \mathrm{d}\gamma = \beta\gamma^3\mathrm{d}\beta \\
&= m \int_{\gamma=1}^{\gamma=\gamma_2} \mathrm{d}\gamma
\end{aligned}
$$

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$$= m \int_{\gamma=1}^{\gamma=\gamma_2} \mathrm{d}\gamma = m\gamma_2 - m$$

SR: energy: varies with ref frame

in (x, t) frame, K = work done

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$\Rightarrow K + m = m\gamma$ *drop* "$_2$"

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$\Rightarrow K + m = m\gamma = p^0$
so $p^0 =$ kinetic energy + rest mass

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Does small β limit agree with Newtonian K?

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Does small β limit agree with Newtonian K?

momentum time component:

$p^0 = m\gamma = m(1-\beta^2)^{-1/2}$

$= m[1-(1/2)(-\beta^2) + \mathcal{O}(\beta^4)]$ if $\beta \ll 1$

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momentum time component:

$p^0 = m\gamma = m(1-\beta^2)^{-1/2}$

$\approx m + (1/2)m\beta^2$ if $\beta \ll 1$

Yes.

SR: $\vec{p}$ . . .: invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^{\mathrm{T}}$

$p^0 = m + K = m\gamma$

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m = w:invariant mass $\equiv$ rest mass: invariant

AND conserved (in interactions): $\|\vec{p} + \vec{q}\|^2 = \|\vec{r}\|^2$

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AND conserved (in interactions): $\|\vec{p} + \vec{q}\|^2 = \|\vec{r}\|^2$

where interaction is (4-momenta):

$\vec{p} + \vec{q} \rightarrow \vec{r}$

SR: $\vec{p}\ldots$: invariant or not?

momentum: $\vec{p} = m\gamma(1, \beta^x, \beta^y, \beta^z)^{\mathrm{T}}$

$p^0 = m + K = m\gamma$

non-rest frame: $\|\vec{p}\|^2 = -m^2\gamma^2 + m^2\gamma^2\beta^2 = -m^2$

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WARNING: assume that 4-momentum vectors at different space-time positions can be parallel-transported; not the case in curved spacetime

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vector space $\Rightarrow p^i + q^i = r^i$ ($i = 1, 2, 3$)

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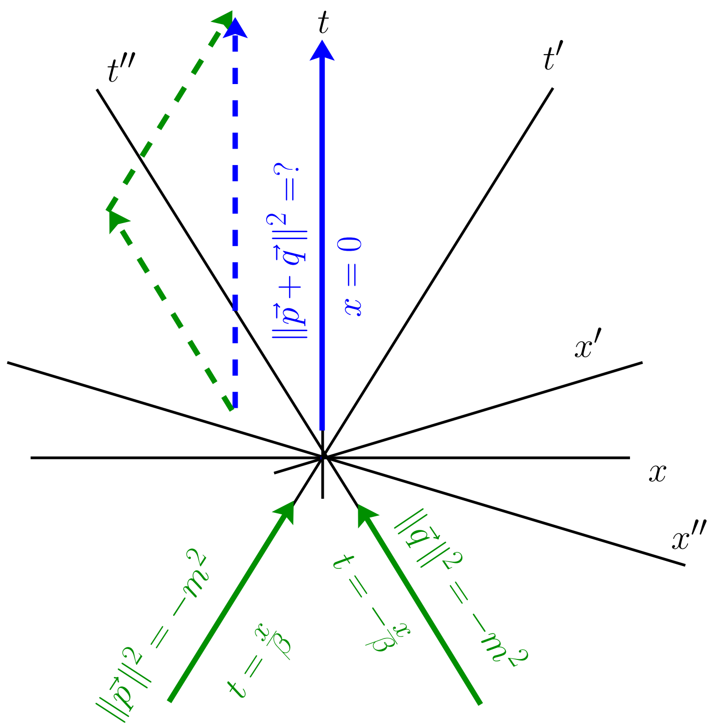
= conservation of (relativistic) “total energy” = $m + K$ (Newtonian: m conserved, K not conserved, $K+$ potential energy conserved)

but NOT invariant (Newtonian: m invariant, K not invariant)

SR: $\vec{p}\ldots$: invariant or not?



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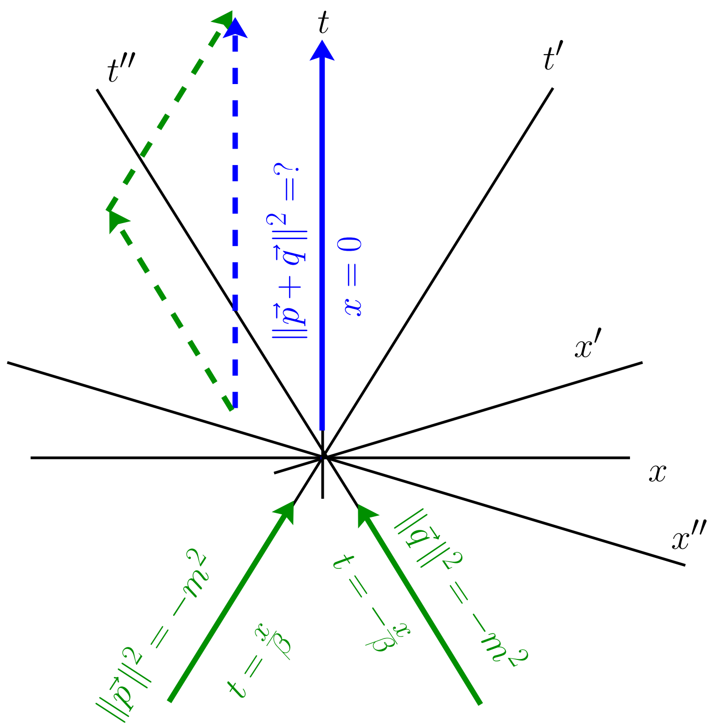

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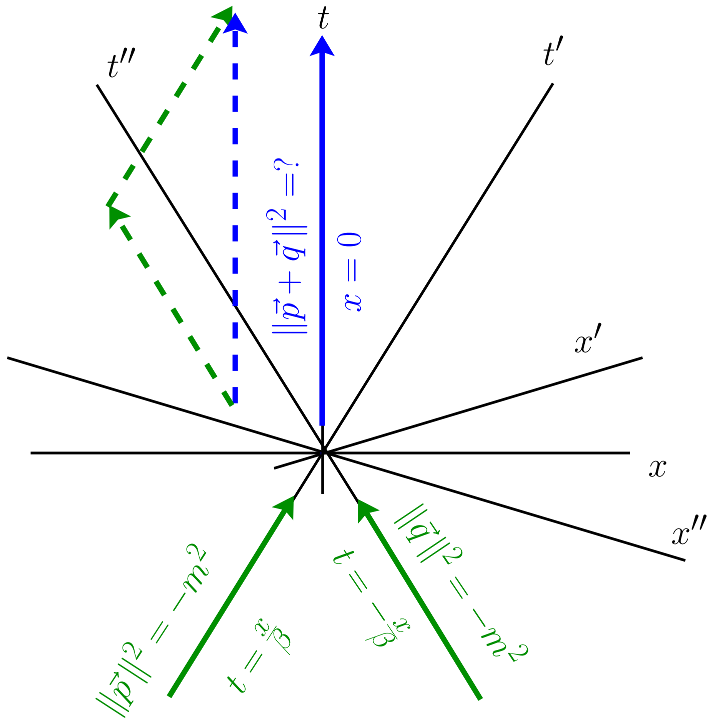

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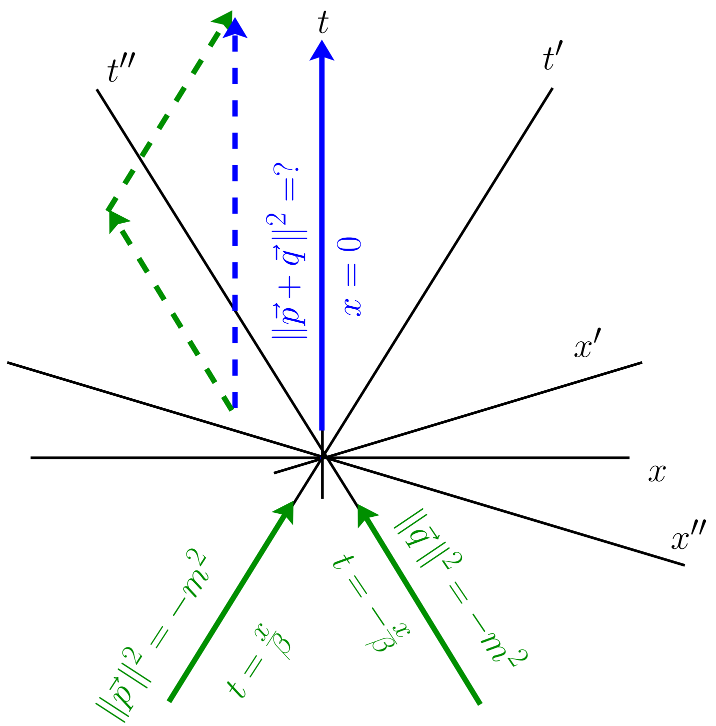

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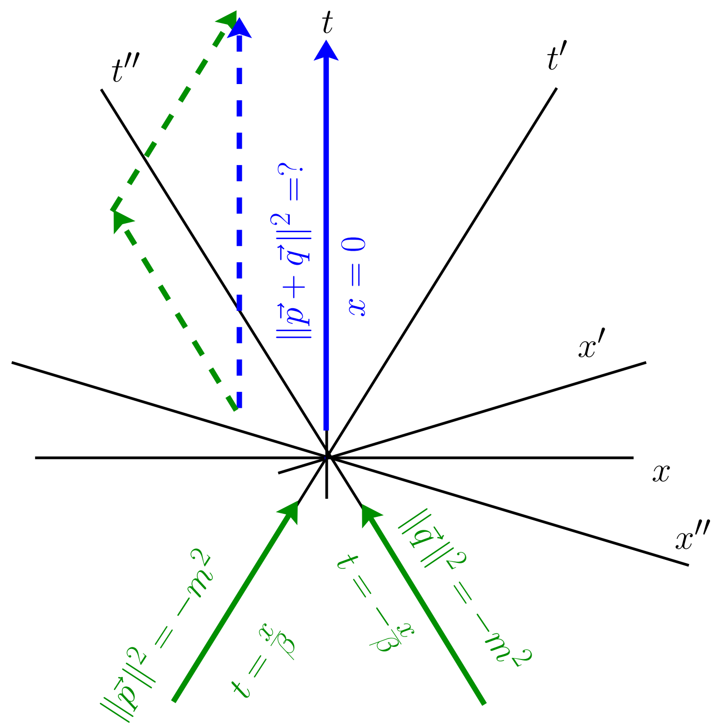

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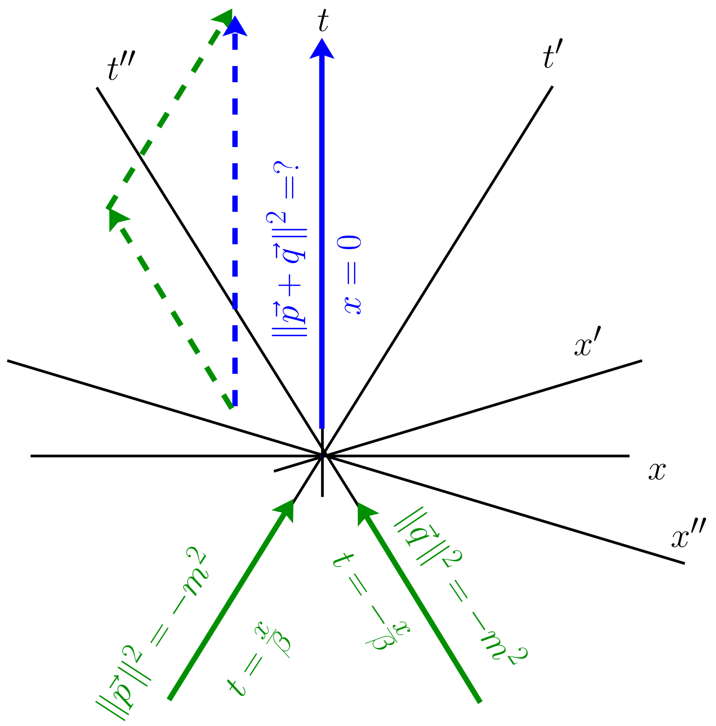

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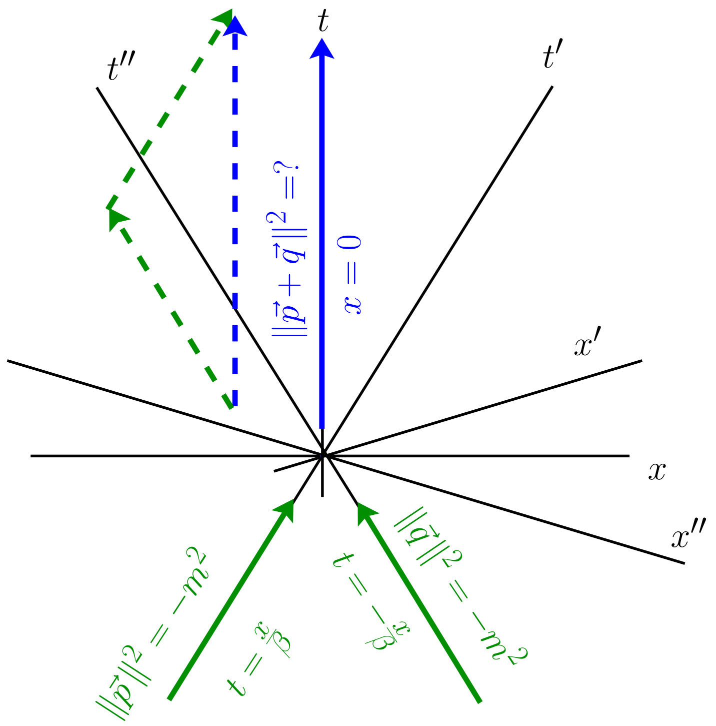

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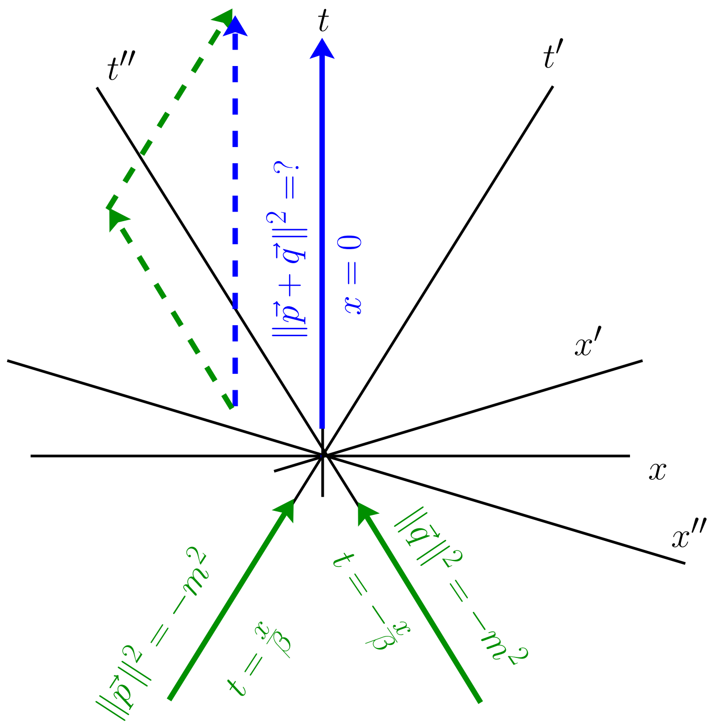

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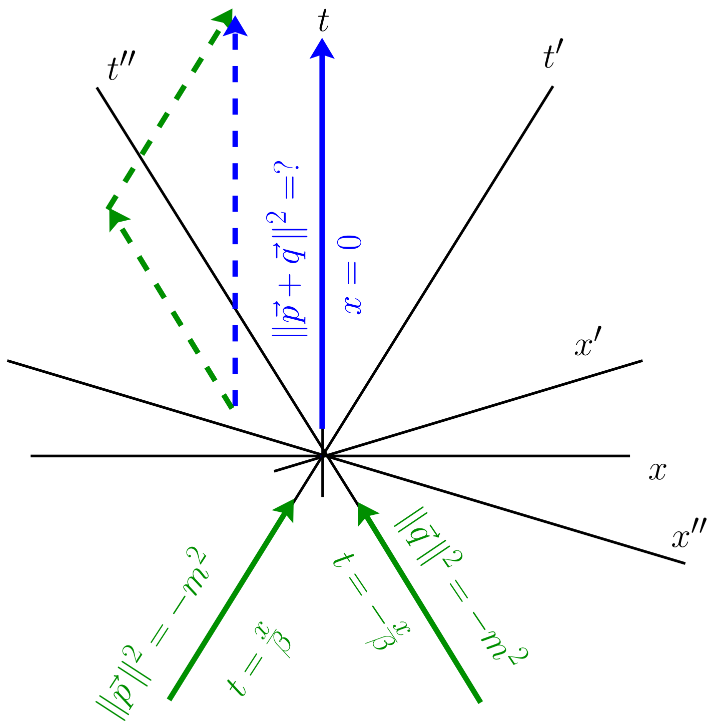

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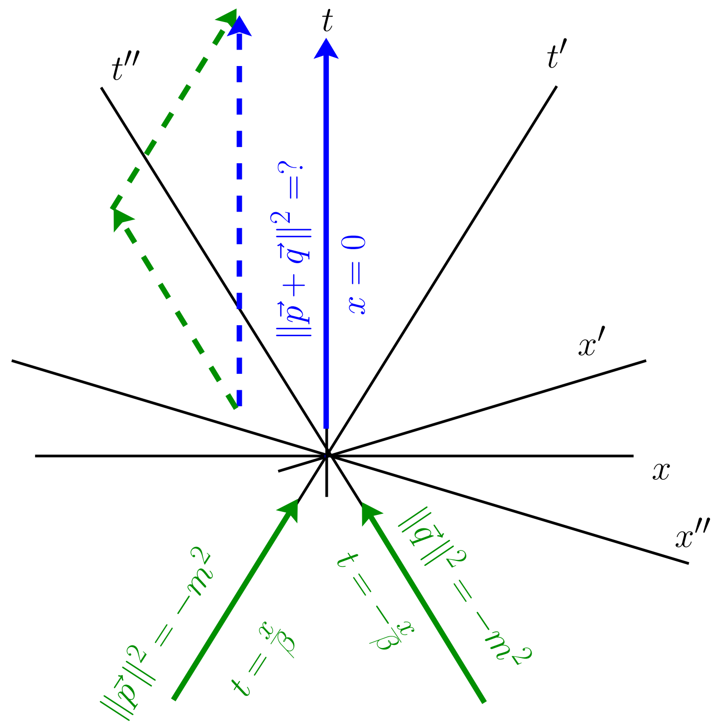

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$\sqrt{-\|\vec{p}\|^2} + \sqrt{-\|\vec{q}\|^2} \neq \sqrt{-\|\vec{r}\|^2}$
system mass is invariant, but can be divided into p^0 and $p^i, i \in \{1,2,3\}$ components in many different ways

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so $E = p$ for any massless particle

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Minkowski spacetime: draw a correct diagram

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refuse the assumption of absolute simultaneity (time)

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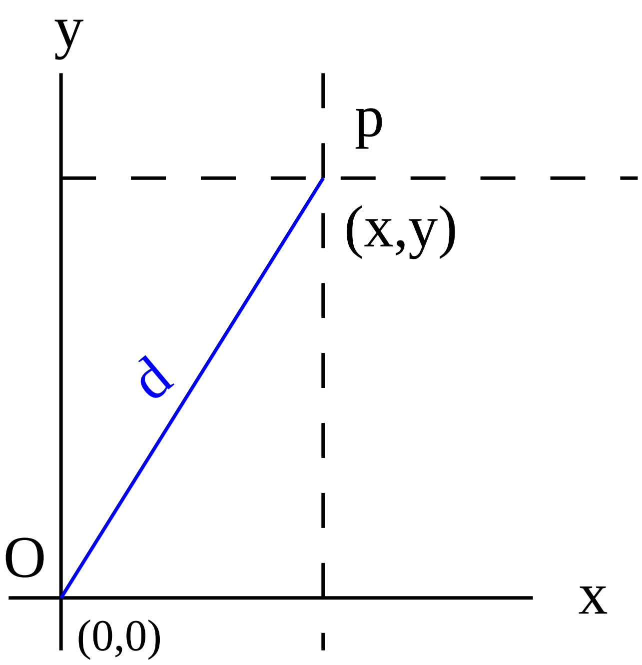
5. metric $\Leftarrow$ Einstein field equations

GR: coordinate transformations



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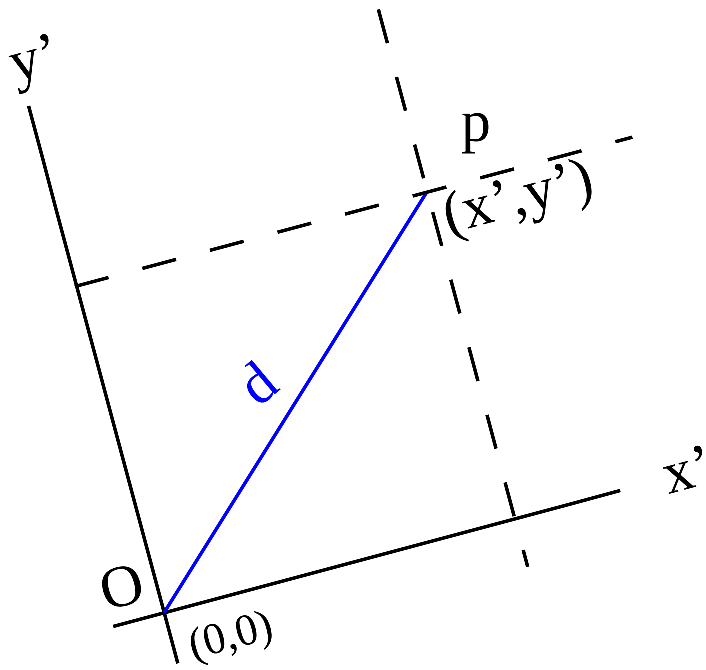

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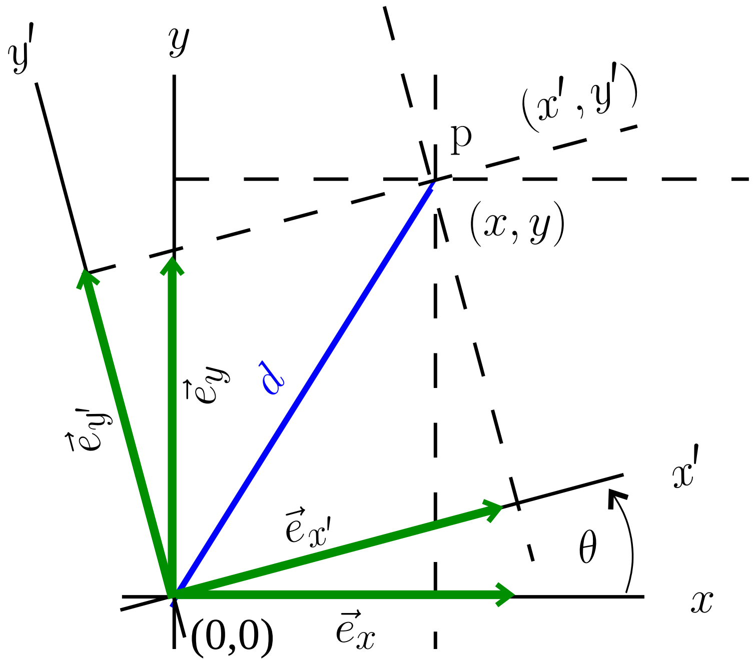

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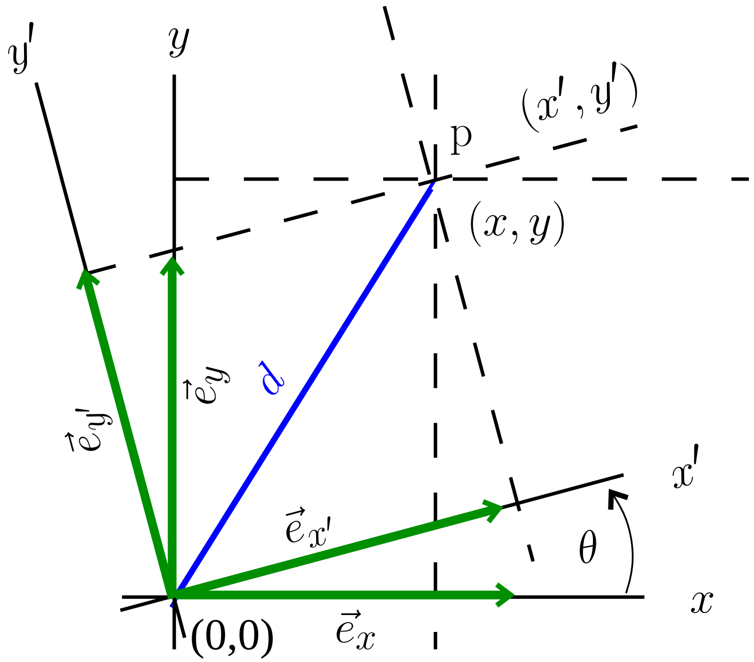

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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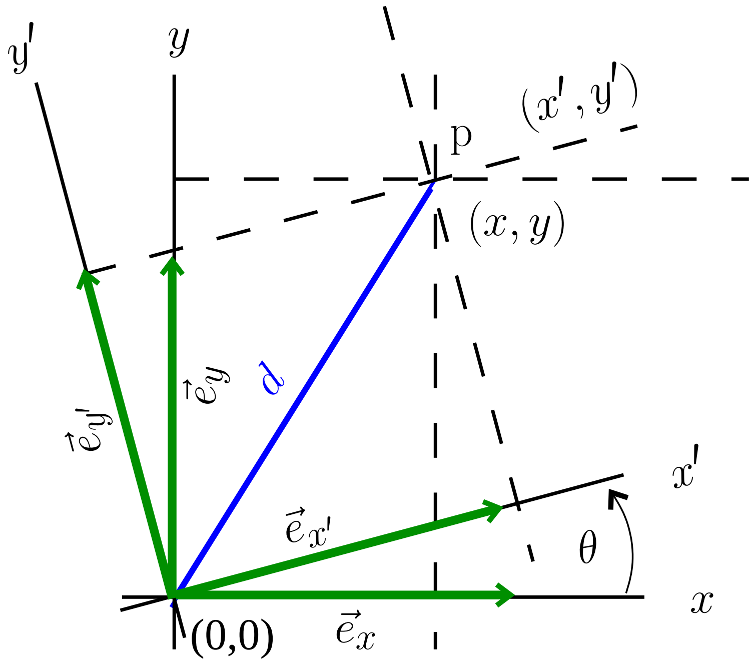

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \Lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

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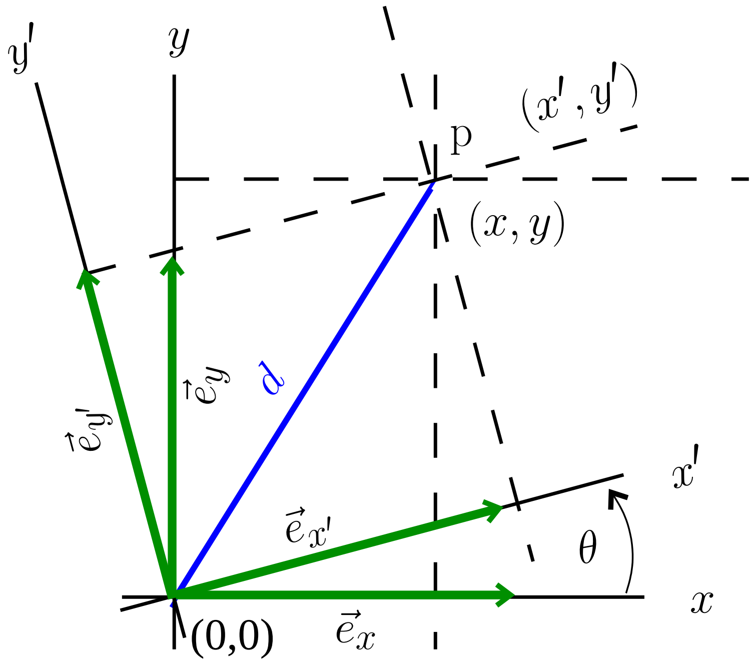

but: $$\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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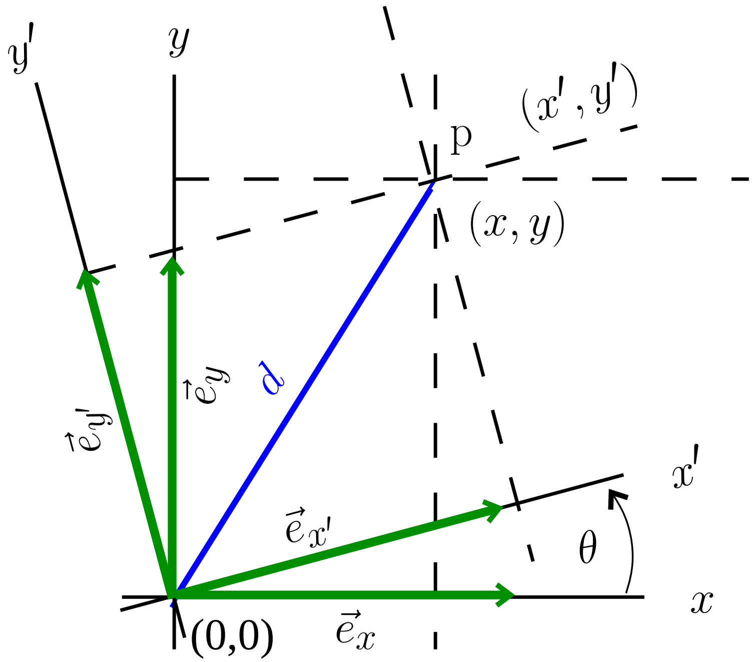

$$\vec{e}_{x'} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \vec{e}_x$$

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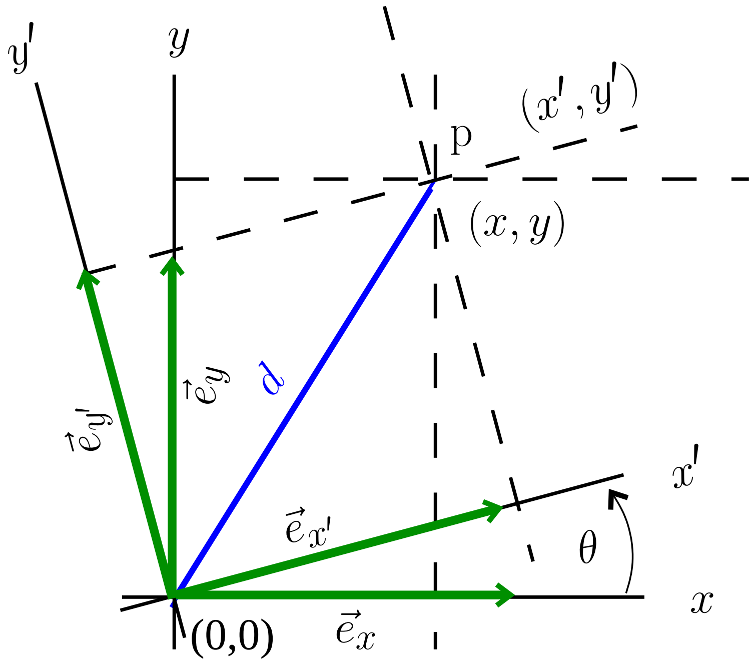

$$\vec{e}_{x'} = \Lambda^{-1}\,\vec{e}_x$$

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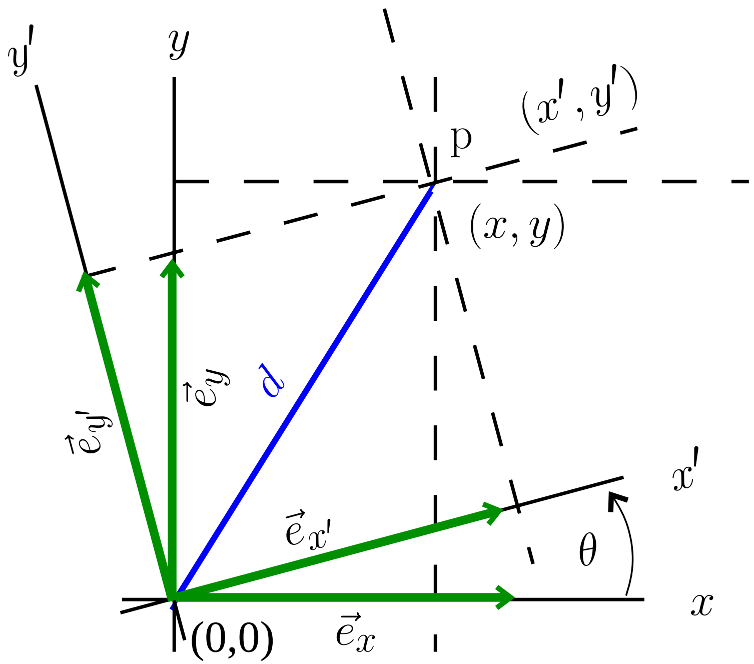

also:

$$\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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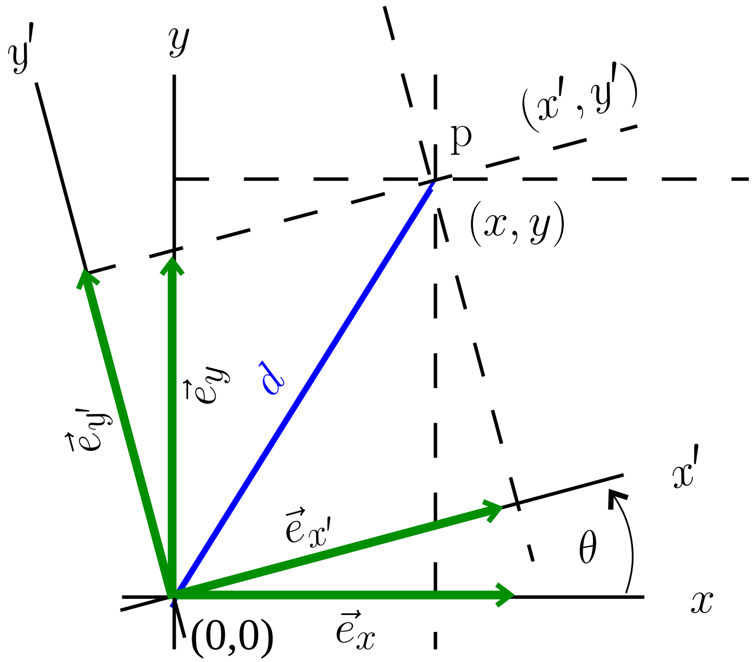

$$\vec{e}_{y'} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \vec{e}_y$$

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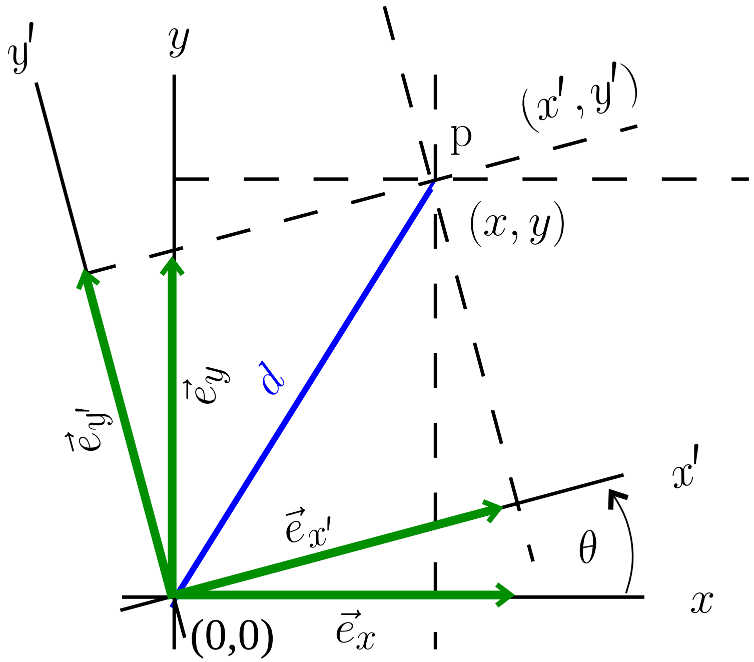

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$$\vec{p} = \sum_i p^i \vec{e}_i$$

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w:Covariance and contravariance of vectors

- $\vec{p}$ is invariant: no dependence on coords
- $\vec{p}$ is contravariant: p^i change inversely to $\vec{e}_i$

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ϕ = scalar field = $\phi(x, y) \equiv \phi(x', y')$

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What is the relation between

$$\begin{pmatrix} \phi_{,x'} \\ \phi_{,y'} \end{pmatrix}$$

and

$$\begin{pmatrix} \phi_{,x} \\ \phi_{,y} \end{pmatrix}?$$

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$$\Rightarrow \frac{\partial \phi}{\partial x'} = \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial \phi}{\partial y'}\frac{\partial y'}{\partial x'}$$

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$\Rightarrow \phi_{,x'} = \phi_{,x}\, x_{,x'} + \phi_{,y}\, y_{,x'}$

GR: coord. transf.: 1-forms

ϕ = scalar field = $\phi(x, y) \equiv \phi(x', y')$

write $\phi_{,x} := \frac{\partial \phi}{\partial x}$

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$$\begin{pmatrix} \phi_{,x'} \\ \phi_{,y'} \end{pmatrix} = \begin{pmatrix} \phi_{,x}\, x_{,x'} + \phi_{,y}\, y_{,x'} \\ \phi_{,x}\, x_{,y'} + \phi_{,y}\, y_{,y'} \end{pmatrix}$$

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$ (example: rotation)

$x_{,x'} = \frac{\partial x}{\partial x'} = \cos\theta$

$x_{,y'} = \frac{\partial x}{\partial y'} = -\sin\theta$

$\ldots$

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_{,x'} & x_{,y'} \\ y_{,x'} & y_{,y'} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$ (general)

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write $\phi_{,x} := \frac{\partial \phi}{\partial x}$

ϕ depends either on x and y, or on x' and y'

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$$\begin{pmatrix} \phi_{,x'} \\ \phi_{,y'} \end{pmatrix} = \begin{pmatrix} x_{,x'} & y_{,x'} \\ x_{,y'} & y_{,y'} \end{pmatrix} \begin{pmatrix} \phi_{,x} \\ \phi_{,y} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix} \quad \text{(general)}$$

$$\Rightarrow \begin{pmatrix} \phi_{,x'} \\ \phi_{,y'} \end{pmatrix} = (\Lambda^{-1})^{\mathrm{T}} \begin{pmatrix} \phi_{,x} \\ \phi_{,y} \end{pmatrix}$$ should be: Λ^{-1}

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$$\begin{pmatrix} x \\ y \end{pmatrix} = \Lambda^{-1} \begin{pmatrix} x' \\ y' \end{pmatrix}$$ (general)

$\tilde{\mathrm{d}}\phi \rightarrow_{\mathcal{O}} (\phi_{,x}, \phi_{,y})$ changes like Λ^{-1}, not like Λ

= 1-form = covariant

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+ w:Riemannian geometry (and pseudo-Riemannian geometry)

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GR: an approximation method: ADM

+ w:ADM formalism

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GR: a numerical method: cactus

+ Cactus - http://cactuscode.org

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