



Steps towards relativistic cosmology

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14/07/2017



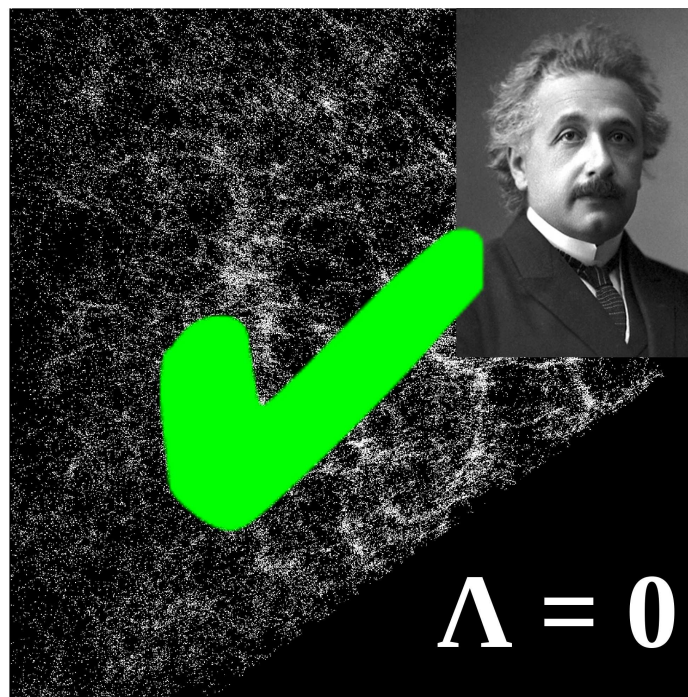
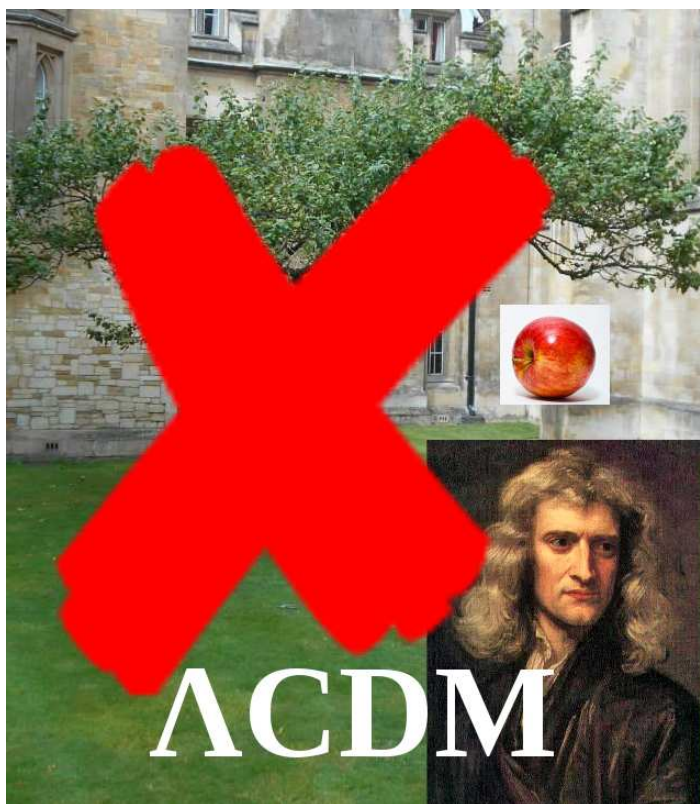


Newton vs Einstein



models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

Universe = space–time



?

FLRW ($\ni$ Λ CDM) models

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T} + \Lambda\mathbf{g}$

- stress–energy tensor:

“dust:” $p := 0$, $\mathbf{T} = \rho \mathbf{u} \otimes \mathbf{u}$

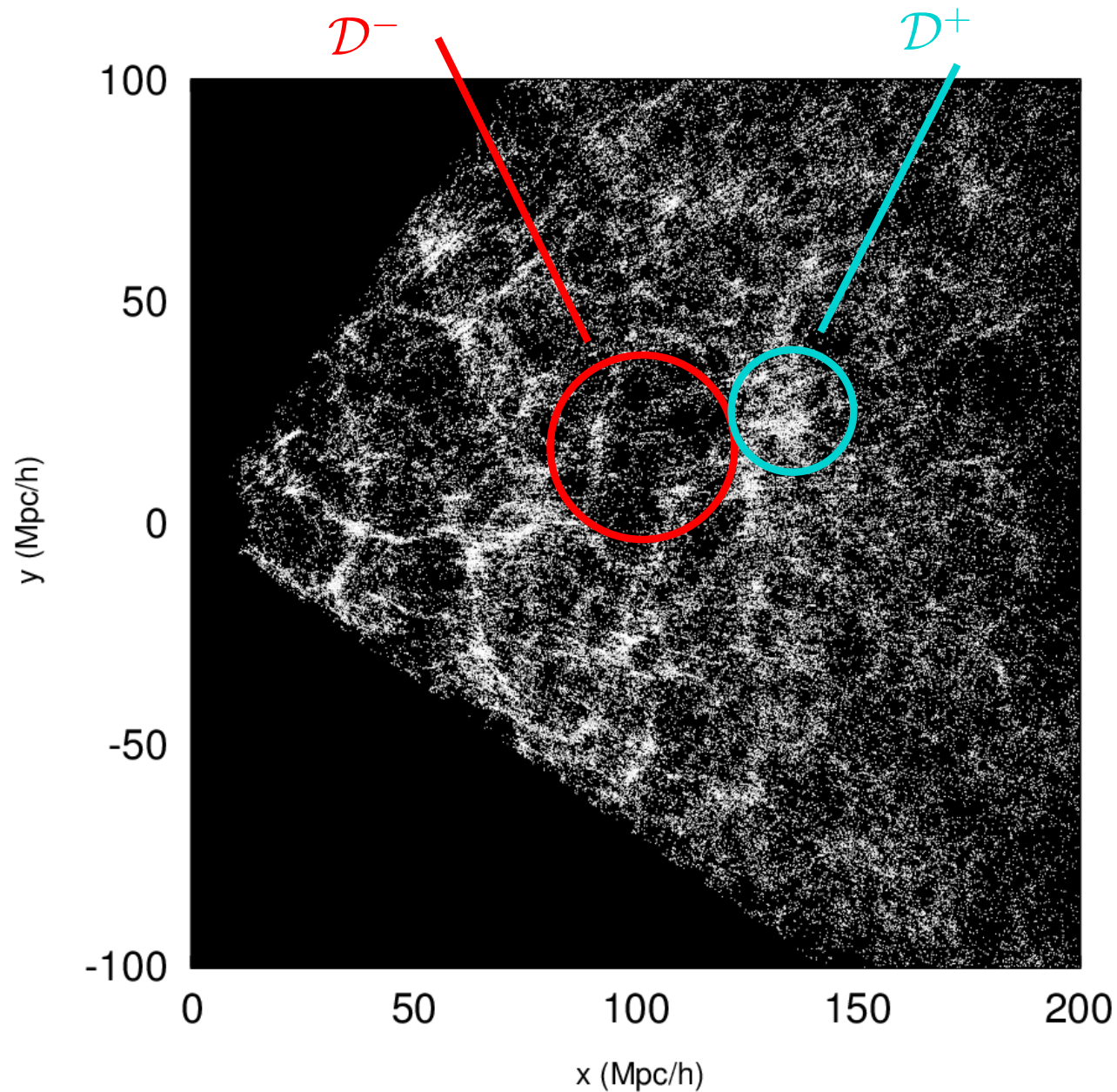
- homogeneity assumption of FLRW

$\Rightarrow$

- ◆ inhomogeneous curvature is ignored;
- ◆ evolution of average curvature is ignored;
- ◆ non-linear structure grav. collapse+virialisation process is relativistically ignored

Average on a domain $\mathcal{D}$

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu



scalar averaging: Raychaudhuri eq

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

averaged Raychaudhuri eq [RZA2, PRD [arXiv:1303.6193](#), (9)]:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3} \frac{M_{\mathcal{D}_i} a_{\mathcal{D}_i}^3}{V_{\mathcal{D}_i} a_{\mathcal{D}}^3} + \frac{\mathcal{Q}_{\mathcal{D}}}{3} + \Lambda ,$$

remove a free parameter by setting $\Lambda := 0$

scalar averaging: Raychaudhuri eq

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

averaged Raychaudhuri eq [RZA2, PRD [arXiv:1303.6193](#), (9)]:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3} \frac{M_{\mathcal{D}_i} a_{\mathcal{D}_i}^3}{V_{\mathcal{D}_i} a_{\mathcal{D}}^3} + \frac{\mathcal{Q}_{\mathcal{D}}}{3} ,$$

where kinematical backreaction [Newtonian case: BKS, [arXiv:astro-ph/9912347](#), II.B., (5)]

$$\mathcal{Q}_{\mathcal{D}} := 2 \langle \text{II} \rangle_{\mathcal{D}} - \frac{2}{3} \langle \text{I} \rangle_{\mathcal{D}}^2 ,$$

scalar averaging: Raychaudhuri eq

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

with invariants of the peculiar expansion tensor [Newtonian case:
Buchert 94, MNRAS [arXiv:astro-ph/9309055](https://arxiv.org/abs/astro-ph/9309055)]:

$$\text{I}(v^i_{,j}) := \text{tr}(v^i_{,j}) = v^i_{,i} = \nabla \cdot \mathbf{v}$$

$$\begin{aligned} \text{II}(v^i_{,j}) &:= \frac{1}{2} \left\{ \left[\text{tr}(v^i_{,j}) \right]^2 - \text{tr} \left[(v^i_{,j})^2 \right] \right\} \\ &= \frac{1}{2} \left((v^i_{,i})^2 - v^i_{,j} v^j_{,i} \right) \\ &= \frac{1}{2} \nabla \cdot \left(\mathbf{v}(\nabla \cdot \mathbf{v}) - (\mathbf{v} \cdot \nabla) \mathbf{v} \right) \end{aligned}$$

$$\text{III}(v^i_{,j}) := \det(v^i_{,j}).$$

scalar averaging: QZA

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

$$\mathcal{Q}_{\mathcal{D}} = \frac{\dot{\xi}^2 (\gamma_1 + \xi \gamma_2 + \xi^2 \gamma_3)}{(1 + \xi \langle \text{I}_{\text{i}} \rangle_{\mathcal{I}} + \xi^2 \langle \text{II}_{\text{i}} \rangle_{\mathcal{I}} + \xi^3 \langle \text{III}_{\text{i}} \rangle_{\mathcal{I}})^2},$$

where

$$\begin{cases} \gamma_1 := 2 \langle \text{II}_{\text{i}} \rangle_{\mathcal{I}} - \frac{2}{3} \langle \text{I}_{\text{i}} \rangle_{\mathcal{I}}^2 \\ \gamma_2 := 6 \langle \text{III}_{\text{i}} \rangle_{\mathcal{I}} - \frac{2}{3} \langle \text{II}_{\text{i}} \rangle_{\mathcal{I}} \langle \text{I}_{\text{i}} \rangle_{\mathcal{I}} \\ \gamma_3 := 2 \langle \text{I}_{\text{i}} \rangle_{\mathcal{I}} \langle \text{III}_{\text{i}} \rangle_{\mathcal{I}} - \frac{2}{3} \langle \text{II}_{\text{i}} \rangle_{\mathcal{I}}^2. \end{cases}$$

QZA = $\mathcal{Q}_{\mathcal{D}}$ Zel'dovich approximation:

- algebraic structure same in Newtonian and GR cases
- initial invariants conceptually differ
- initial invariants numerically approximated for zero curvature

Volume averaging

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

instead of verbal averaging (FLRW $\ni$ Λ CDM): mathematical averaging

$$a_{\text{eff}}(t) := \left(\frac{\sum_{\mathcal{D}} a_{\mathcal{D}}^3(t)}{\sum_{\mathcal{D}} 1} \right)^{1/3} = \left(\frac{\sum_{\mathcal{D}} a_{\mathcal{D}}^3(t)}{n_{\mathcal{D}}} \right)^{1/3}$$

QZA model

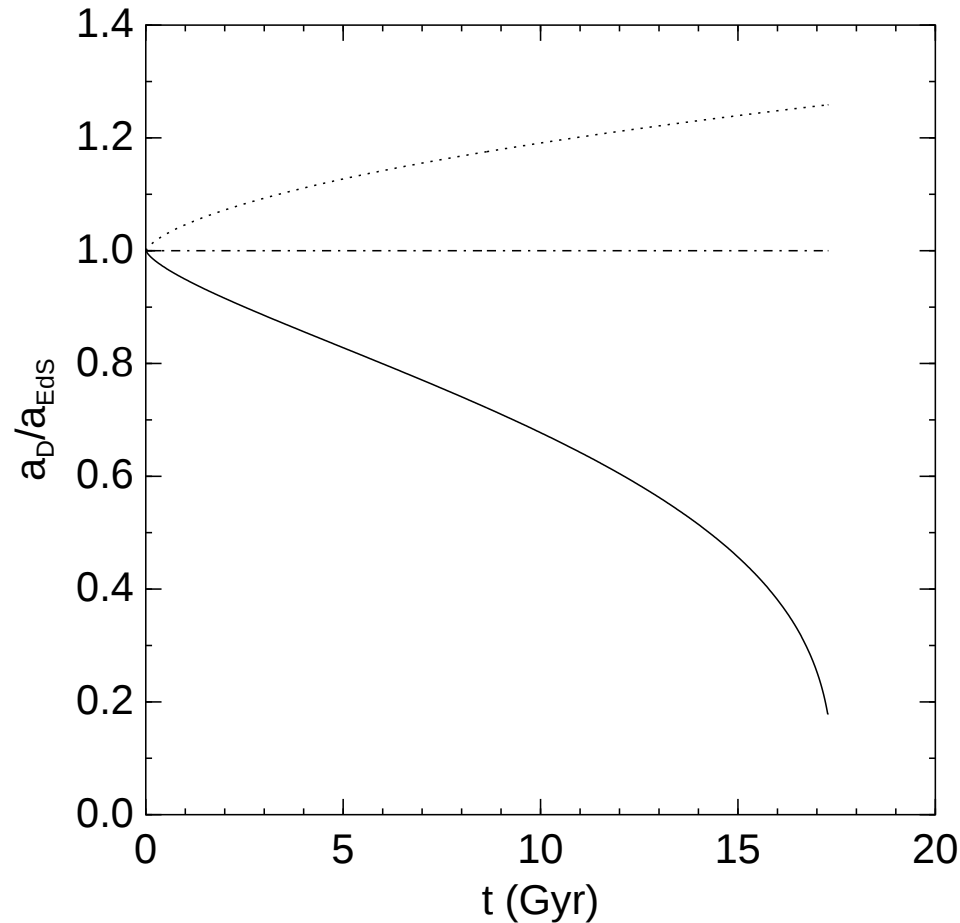
models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T}$ ($\Lambda := 0$)
- $a_{\mathcal{D}}$: domain-averaged scale factor and Raychaudhuri equation for $\ddot{a}_{\mathcal{D}}$ (“silent” universe) on a spatial domain $\mathcal{D}$
- non-linear scales but no shell-crossing (gravitational collapse)
- Raychaudhuri + $\mathcal{Q}_{\mathcal{D}}$ analytical approximation (QZA)
- calculate global average scale factor: $a_{\text{eff}}(t)$
- *expected result: generate $a_{\text{eff}}(t) \approx a_{\text{EdS}}(t)$ instead of assuming it*

Biscale example: QZA

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

$$\langle I \rangle_{\mathcal{D}_i^-} = 0.005$$



$a_{\text{EdSi}} = 0.005$, expanding domain $\mathcal{D}^-$:

$(\langle I \rangle_{\mathcal{D}_i^-}, \langle II \rangle_{\mathcal{D}_i^-}, \langle III \rangle_{\mathcal{D}_i^-}) = (0.005, 0, 0)$ (“planar” case)

non-linear $a_{\mathcal{D}^-}$, $a_{\mathcal{D}^+}$ cancel

V = virialisation model

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

Next step: how can we model virialisation?

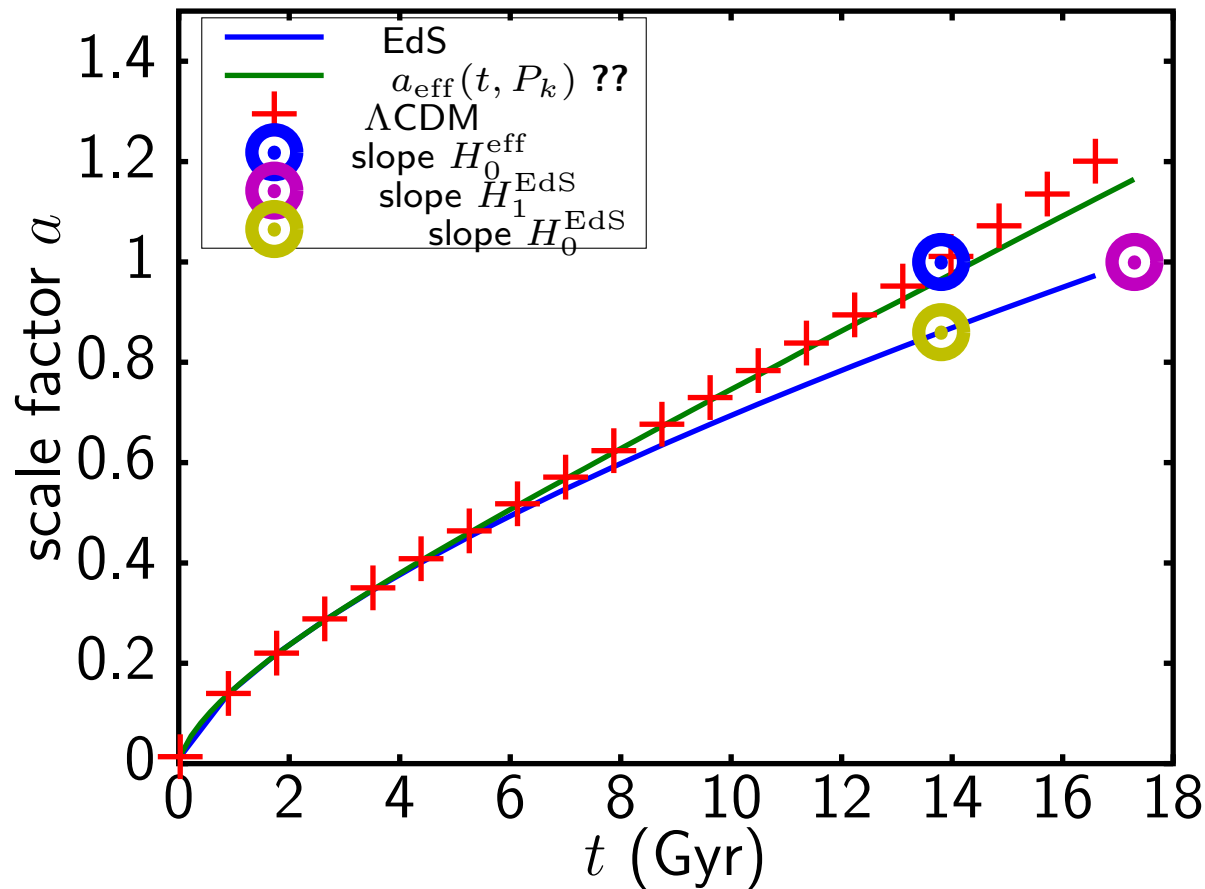
- **Newtonian:** virialisation backreaction term: $-(\Pi_{,k}^{ik}/\rho)_{,i}$?
[Al Roumi 2011; (2.20), (2.22), (2.23)]
- **GR:** $\mathcal{P}_{\mathcal{D}}^T$? [Mourier+2017]
i.e. add local negative effective pressure to **T**? **TODO!**
- **GR initial approximation:** assume standard EdS virialisation overdensity:

$$a_{\mathcal{D}}(t > t_{\text{coll}}) := a_{\text{EdS}}(t_{\text{coll}}) / (18\pi^2)^{1/3}$$

- *Is this GR?* It violates $\mathbf{T} = \rho \mathbf{u} \otimes \mathbf{u}$, but not $\mathbf{G} = 8\pi \mathbf{T}$.
- *Is this Newtonian?* $a_{\text{eff}}(t)$ will not be $a_{\text{EdS}}(t)$.

initial conds: Λ CDM proxy

obsvns $\Rightarrow H_0^{\text{eff}}$, H_1^{EdS} , $H_0^{\text{EdS}} = 67.74, 37.7, 47.24$ km/s/Mpc
 ([arXiv:1608.06004](#) Roukema+2016)



EdS +
 RZA(P_k) $\Rightarrow$
 $\sim \Lambda$ CDM ?

RZA = relativistic Zel'dovich approximation (PRD [arXiv:1303.6193](#))

TCfA+CRAL sims N -body + RZA : *work in progress...*

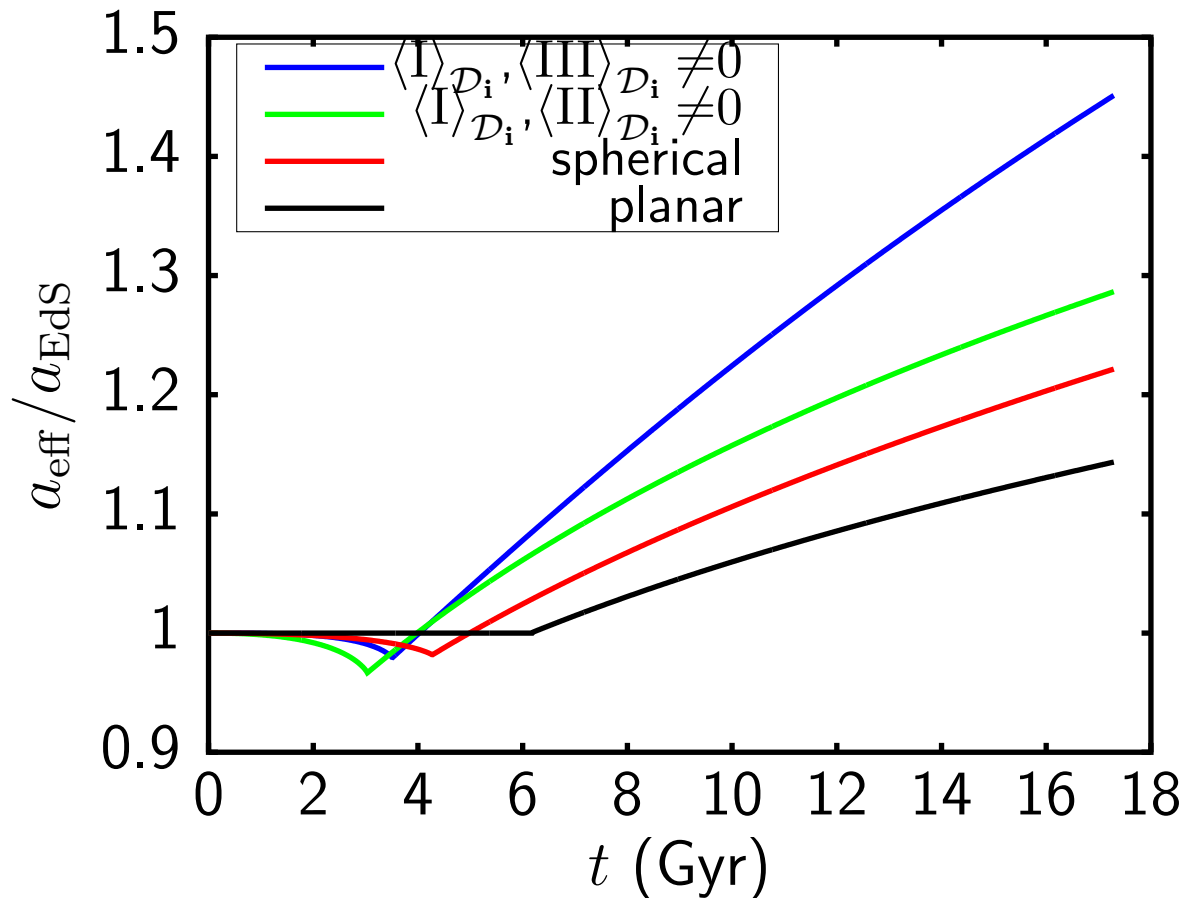
VQZA model

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T}$ ($\Lambda := 0$)
- $a_{\mathcal{D}}$: domain-averaged scale factor and Raychaudhuri equation for $\ddot{a}_{\mathcal{D}}$ (“silent” universe) on a spatial domain $\mathcal{D}$
- non-collapsed domains: Raychaudhuri + $\mathcal{Q}_{\mathcal{D}}$ analytical approximation (QZA)
- grav. collapsed/virialised domains:
- assume stable clustering (V)
[implicitly: introduce local negative effective pressure $p_{\mathcal{D}}^{\text{vir}}$ into $\mathbf{T}$]
virialisation statistically halts gravitational collapse
- V+QZA implicitly represents including $p_{\mathcal{D}}^{\text{vir}}$ in Einstein eq RHS
- calculate global average scale factor: $a_{\text{eff}}(t)$

Biscale example: global evolution

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

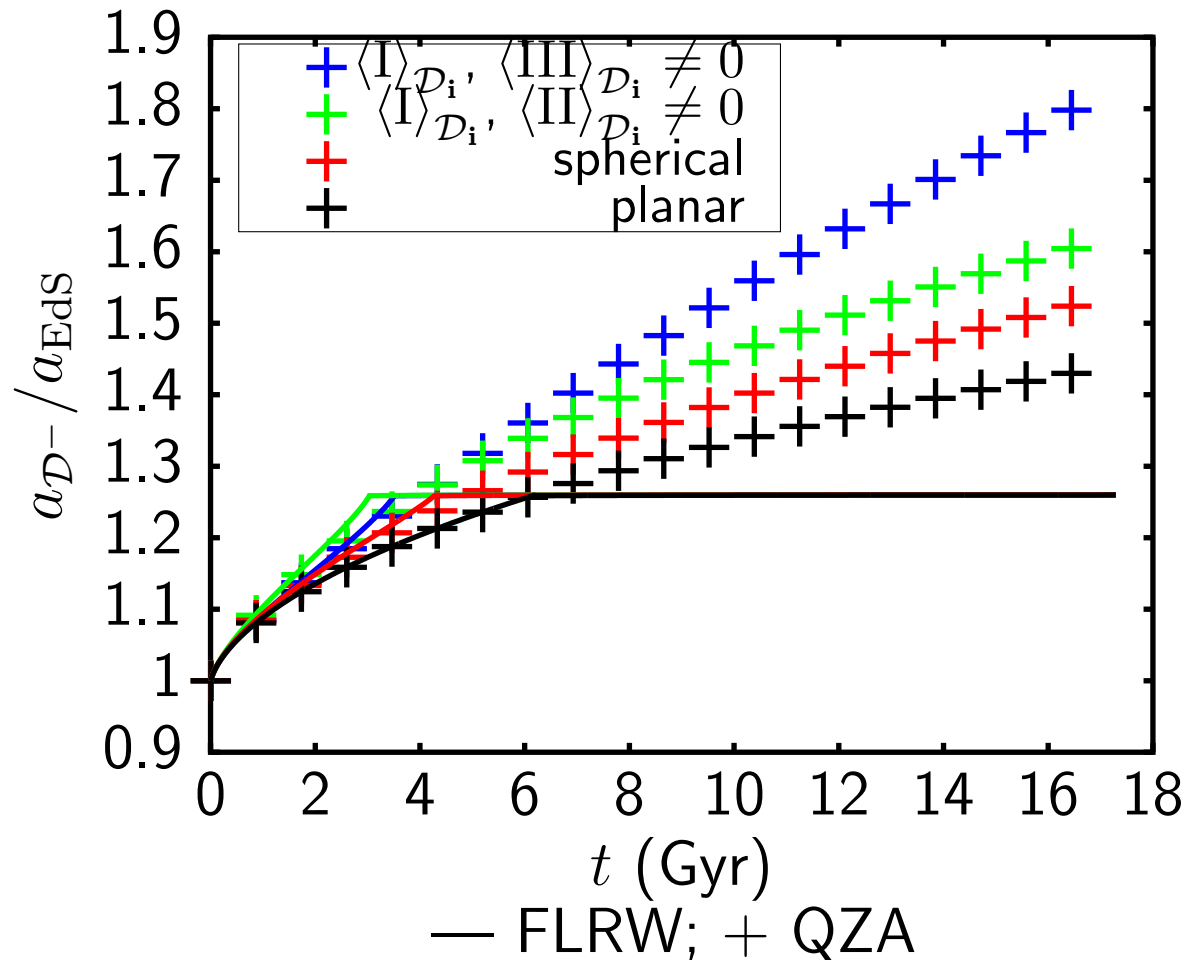


$a_{\text{EdS}i} = 0.005$, expanding domain D^- :

$(\langle I \rangle_{\mathcal{D}_i^-}, \langle II \rangle_{\mathcal{D}_i^-}, \langle III \rangle_{\mathcal{D}_i^-}) = (0.01, 0, 0)$ (“planar” case);
 $(0.01, 10^{-4}/3, 10^{-6}/27)$ (“spherical” $\mathcal{D}^-$ case); $(0.01, 10^{-4}, 0)$;
 $(0.01, 0, 10^{-6})$;

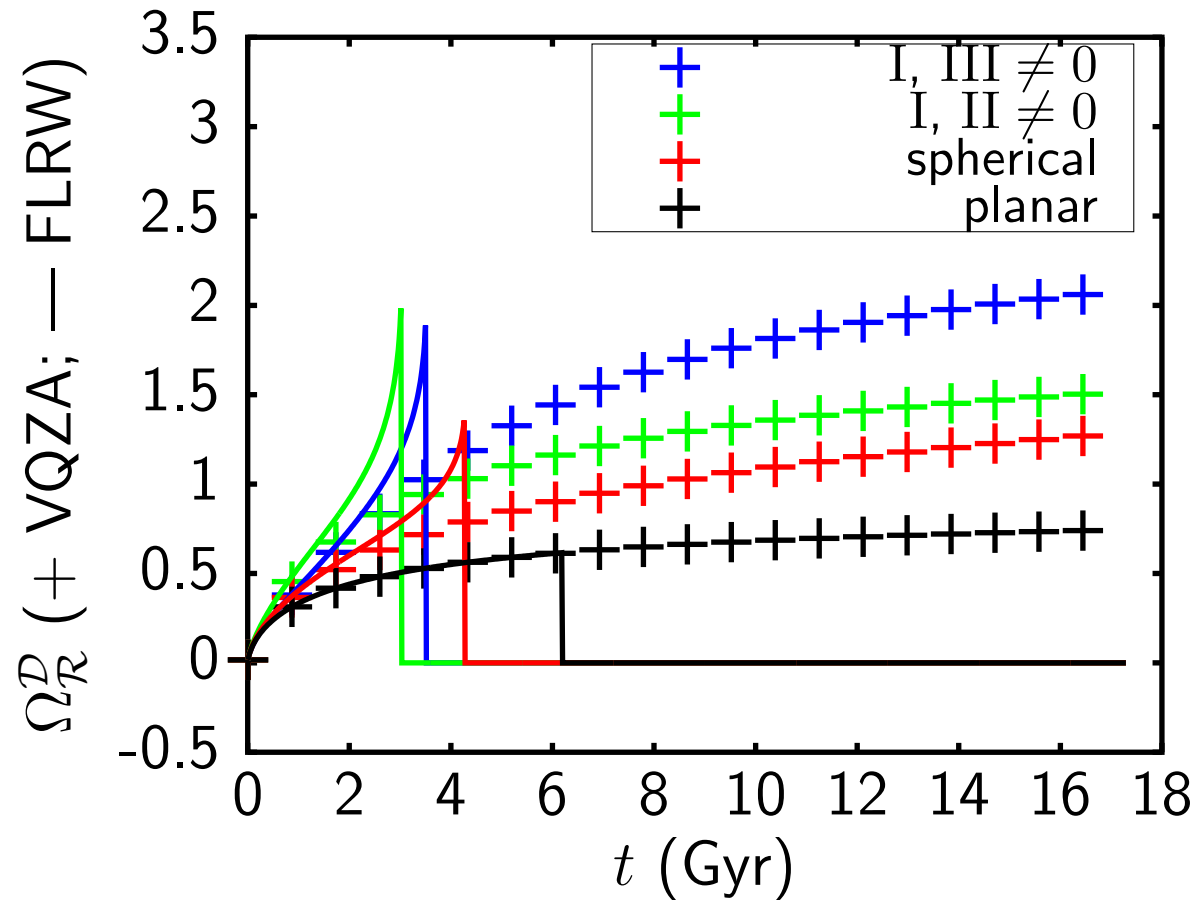
Biscale example: $a_{\mathcal{D}-}$

models – QZA – a_{eff} – Vir – $\mathcal{D}^{\pm}$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu



Biscale example: $\mathcal{D}^-$ curvature

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu



Hamiltonian constraint interpretation

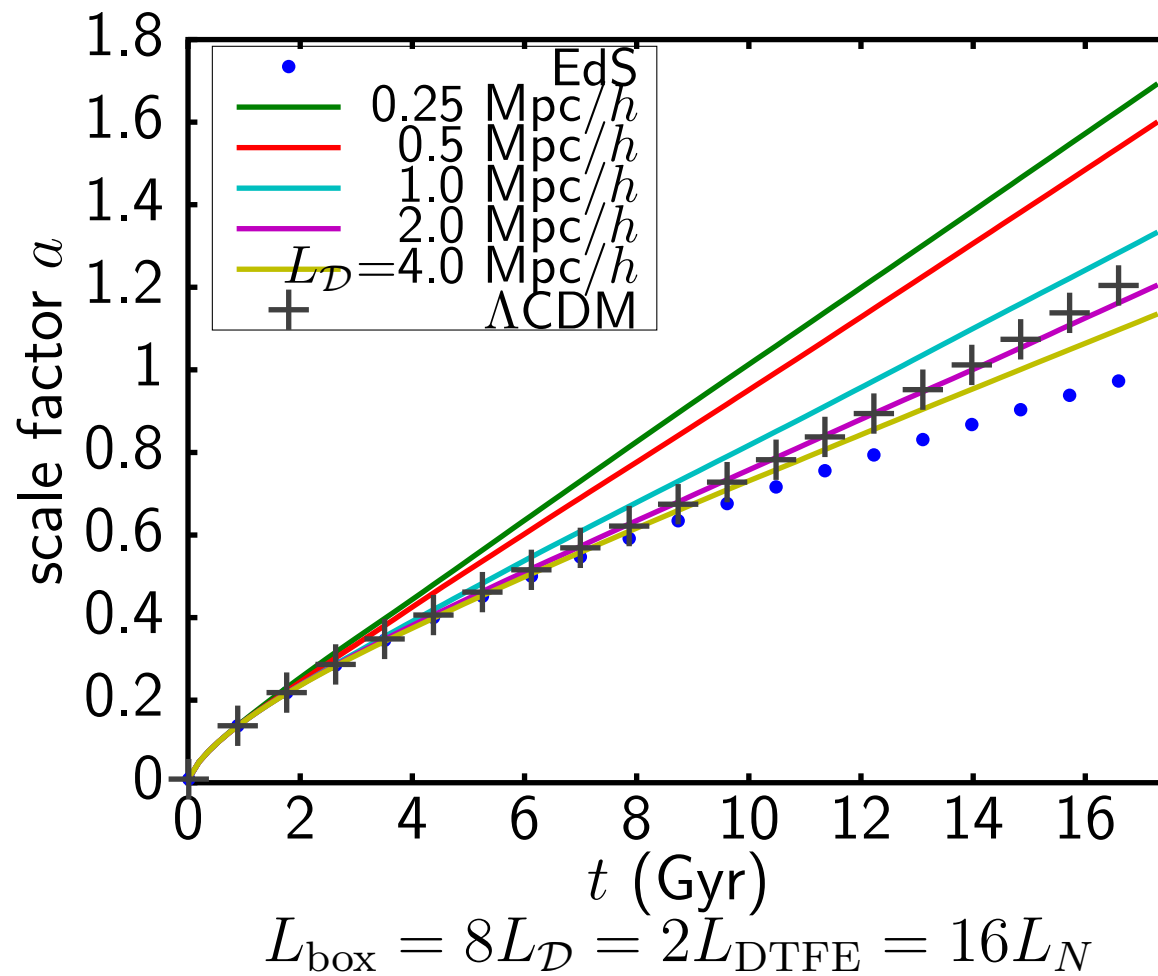
software

models – QZA – a_{eff} – Vir – $\mathcal{D}^{\pm}$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

- MPGRAFIC — initial conditions (GPL, f90)
- DTFE — measure I, II, III (GPL, C++)
- INHOMOG — evolve QZA; stabilise virialised domains (GPL, C)
- RAMSES-SCALAV — extension of RAMSES as front end to the above (Cecill, f90)
- start at <http://bitbucket.org/broukema/ramses-scalav>

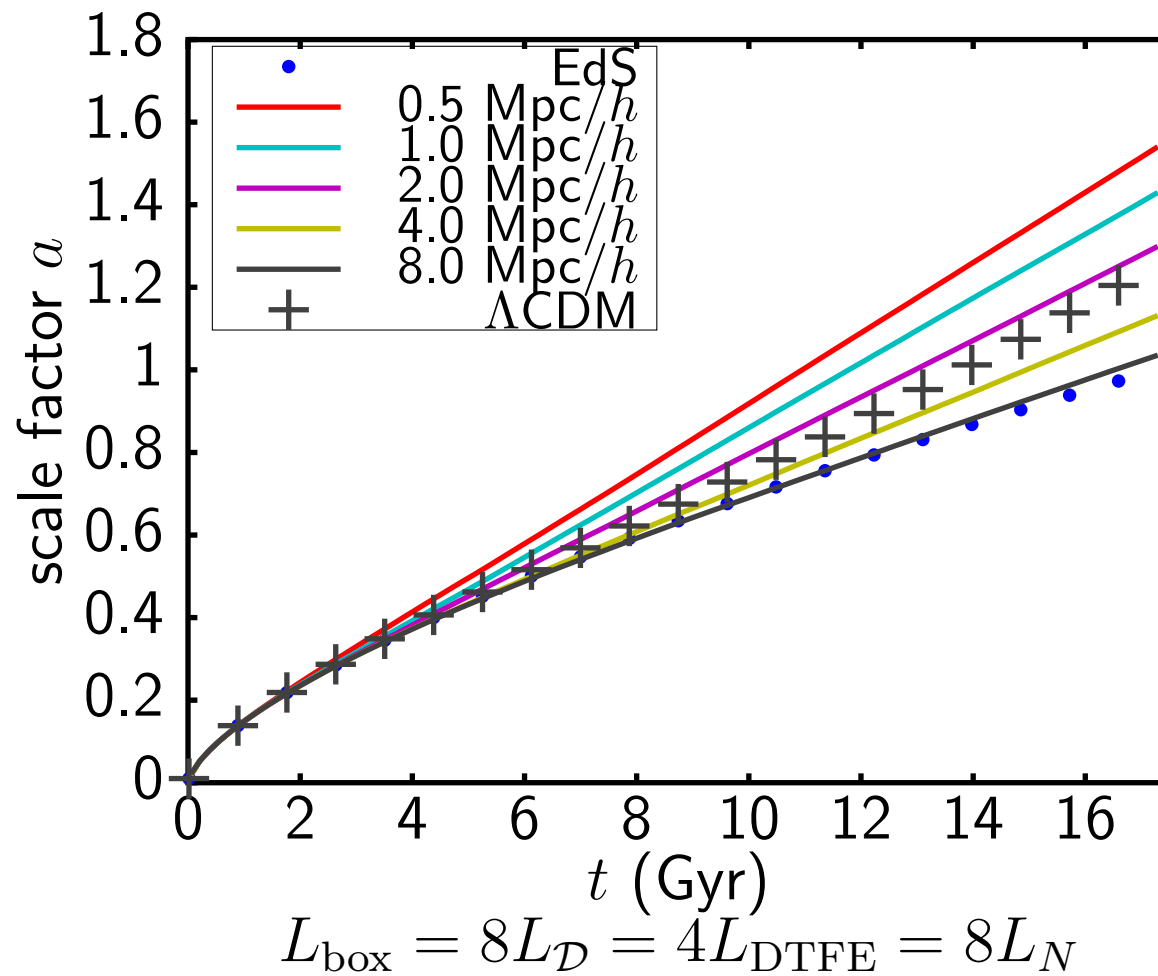
VQZA $a_{\text{eff}}(t)$

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_\mathcal{D}$ – Conclu



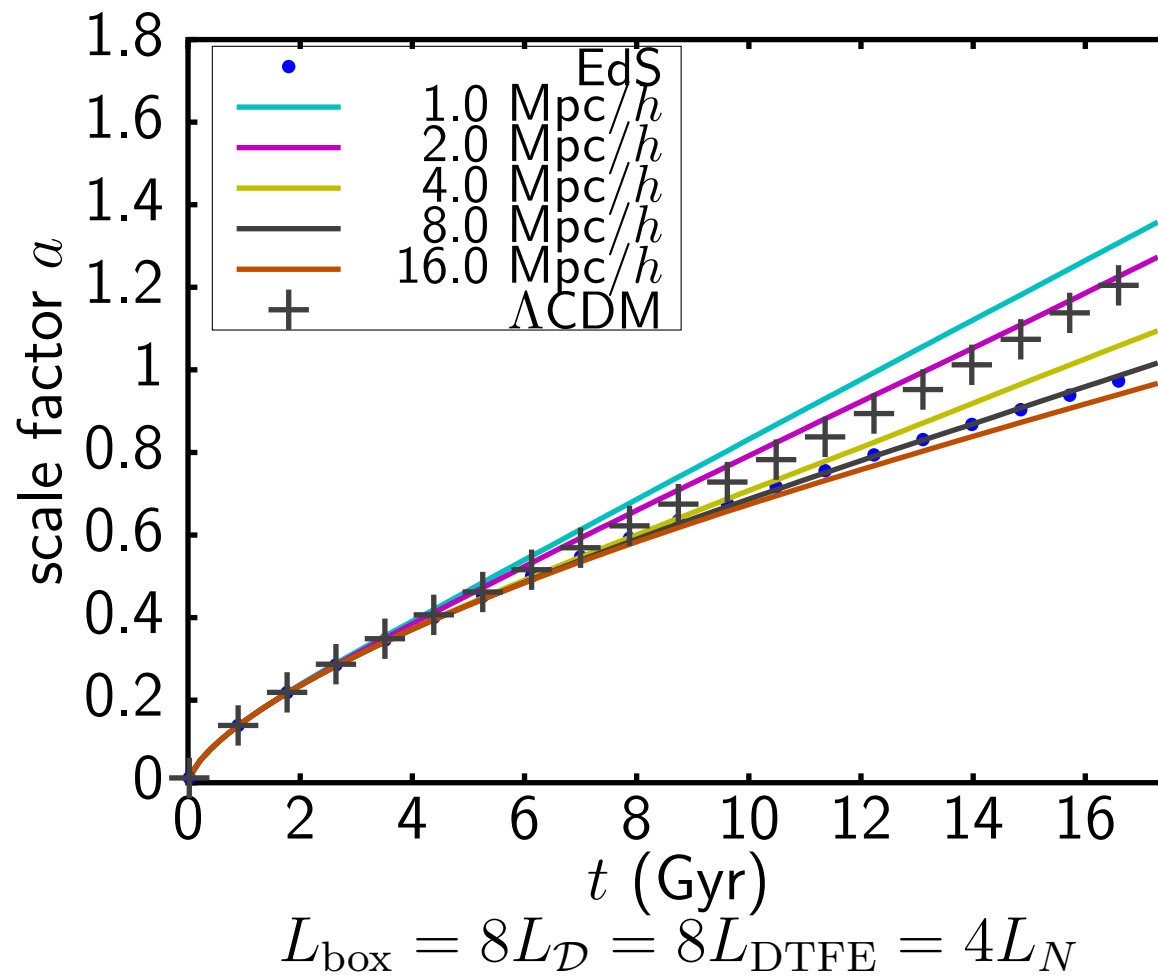
VQZA $a_{\text{eff}}(t)$

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_\mathcal{D}$ – Conclu



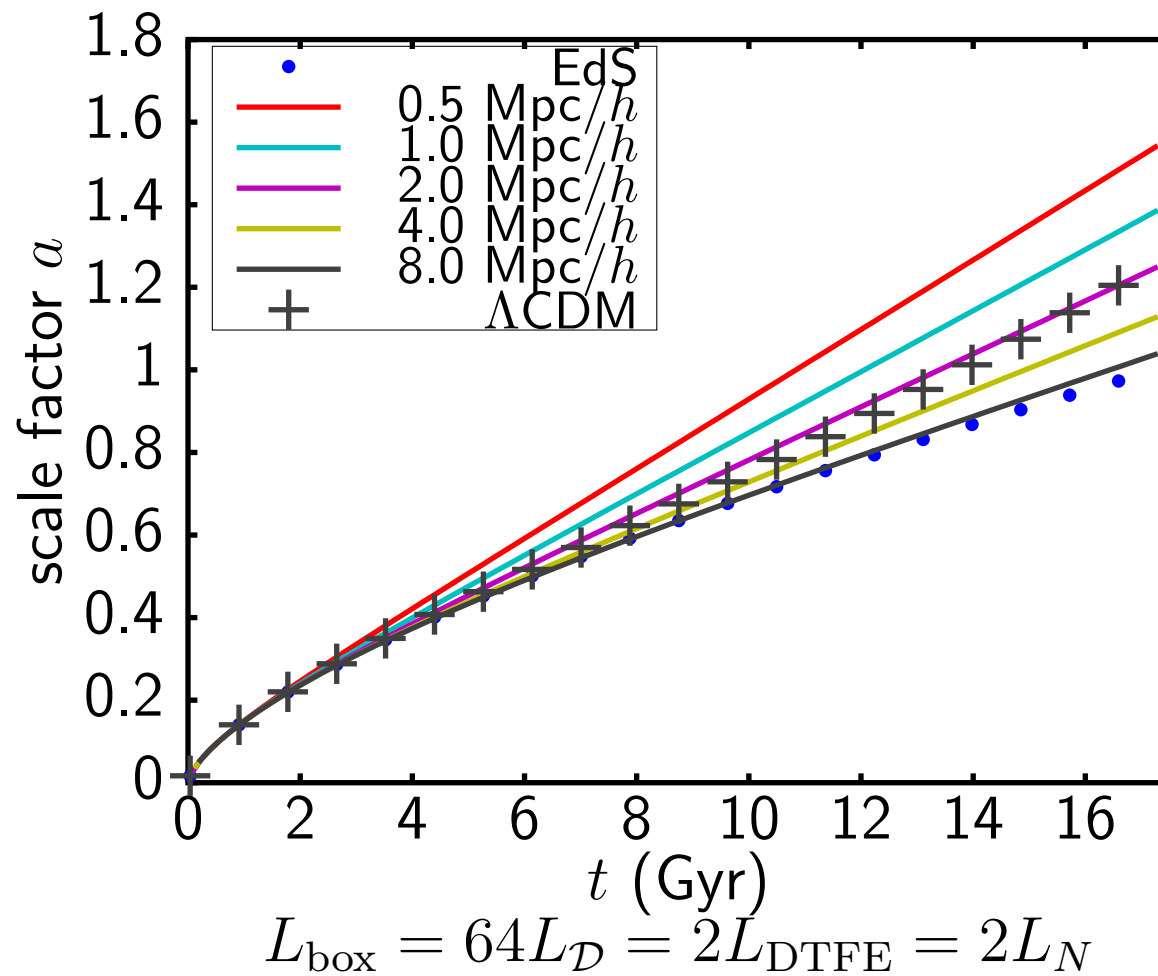
VQZA $a_{\text{eff}}(t)$

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_\mathcal{D}$ – Conclu



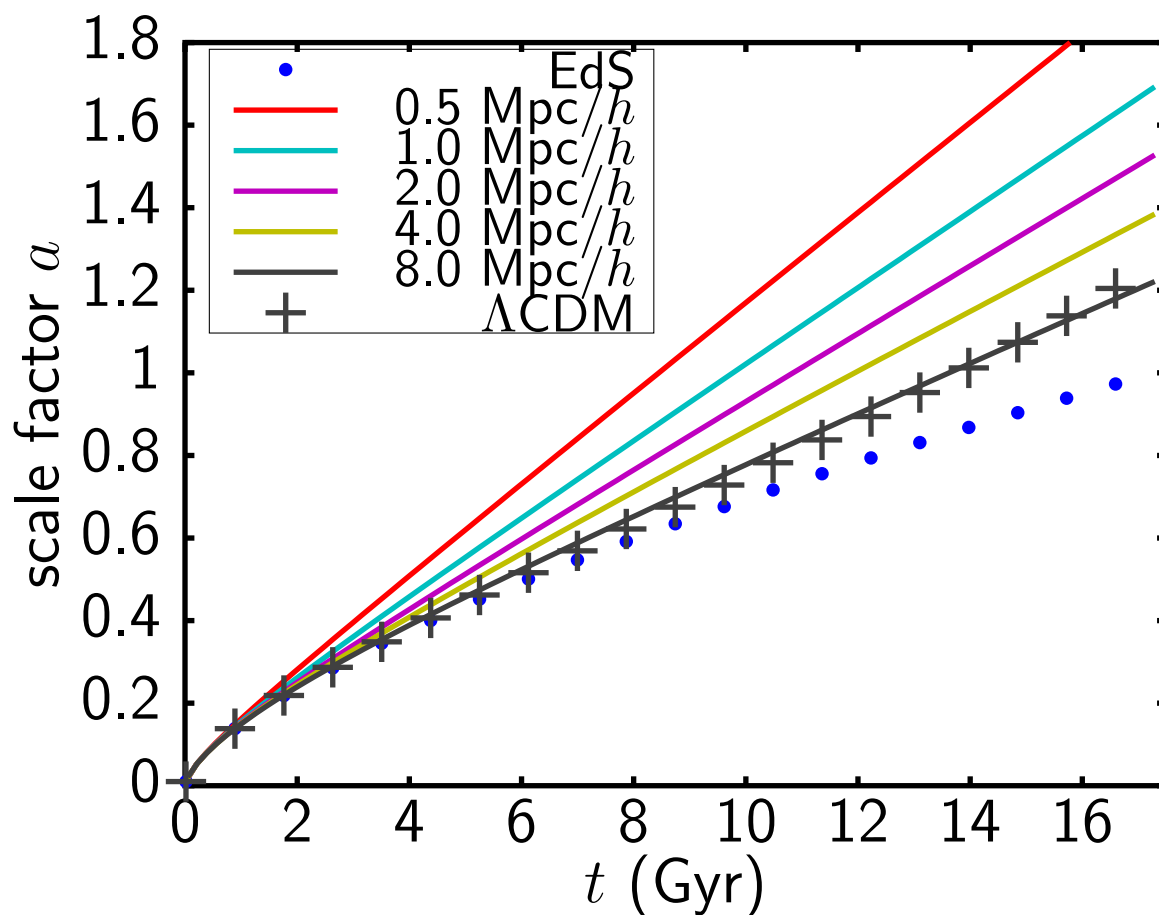
VQZA $a_{\text{eff}}(t)$

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_\mathcal{D}$ – Conclu



$$Q_{\mathcal{D}} := 0$$

models – QZA – a_{eff} – Vir – $\mathcal{D}^{\pm}$ – soft – $a_{\text{eff}}(t)$ – $Q_{\mathcal{D}}$ – Conclu



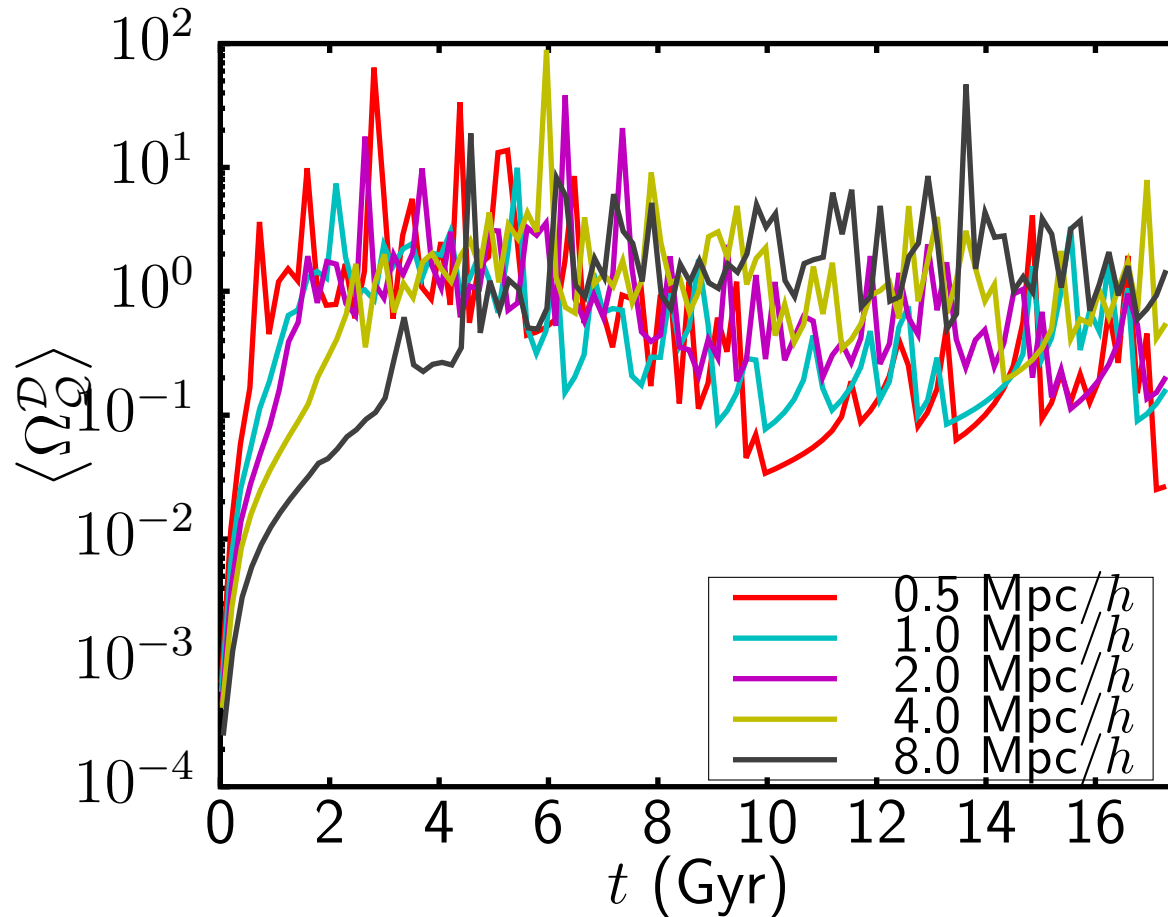
if ignoring kinematical backreaction: $Q_{\mathcal{D}} := 0$

$\sim$ cf Rácz+2017

$$L_{\text{box}} = 8L_{\mathcal{D}} = 4L_{\text{DTFE}} = 8L_N$$

$\mathcal{Q}_D$ evolution

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_D$ – Conclu

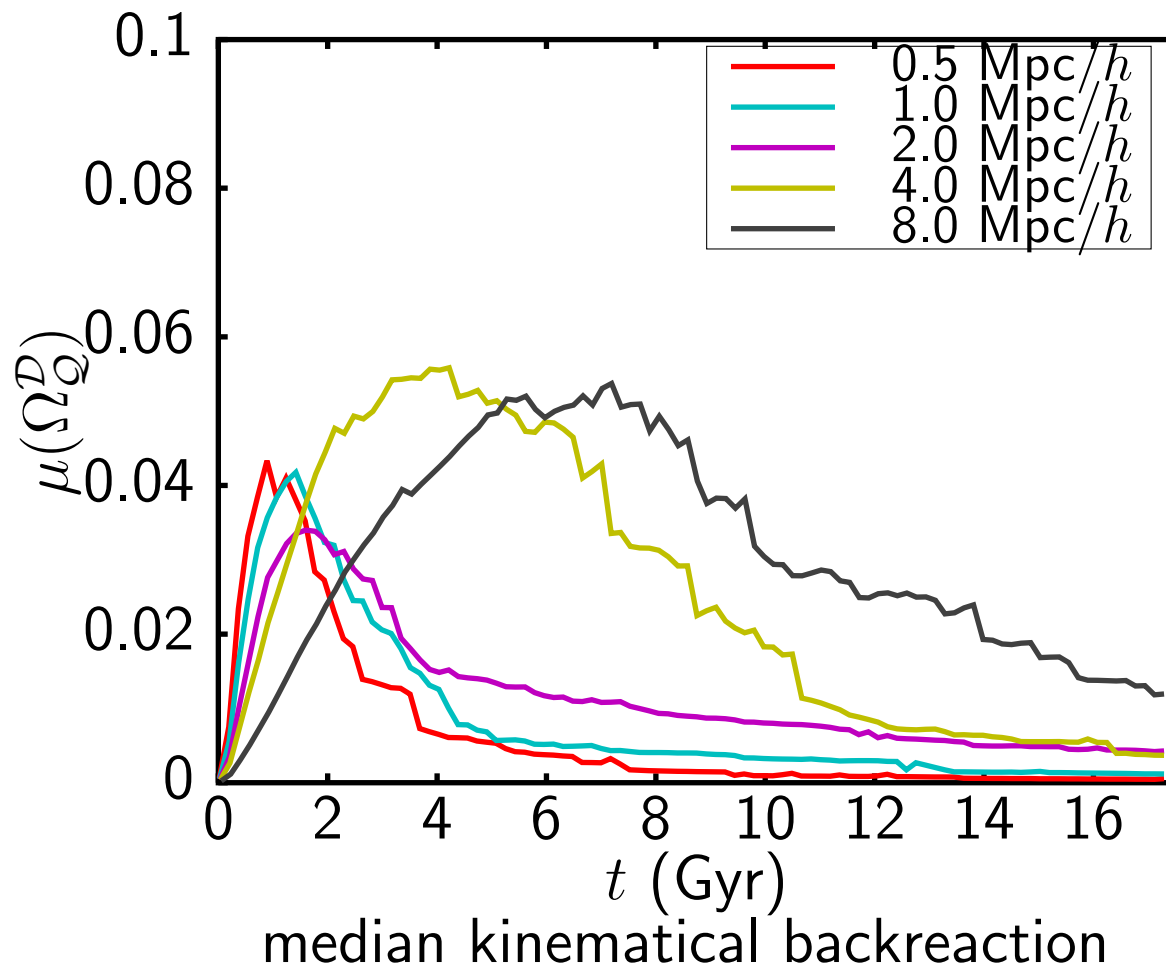


mean kinematical backreaction

$$\text{VQZA } L_{\text{box}} = 8L_D = 4L_{\text{DTFE}} = 8L_N$$

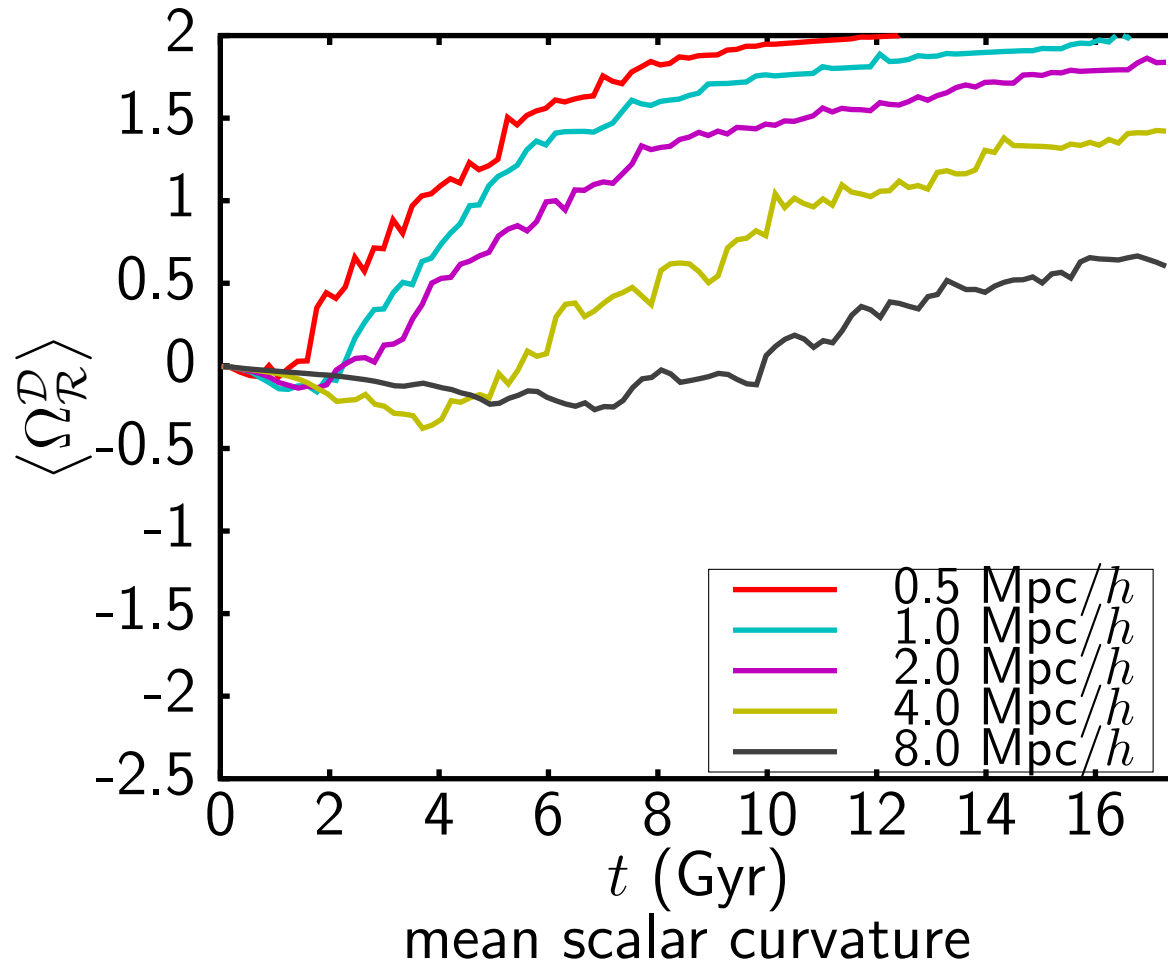
Q_D evolution

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – Q_D – Conclu



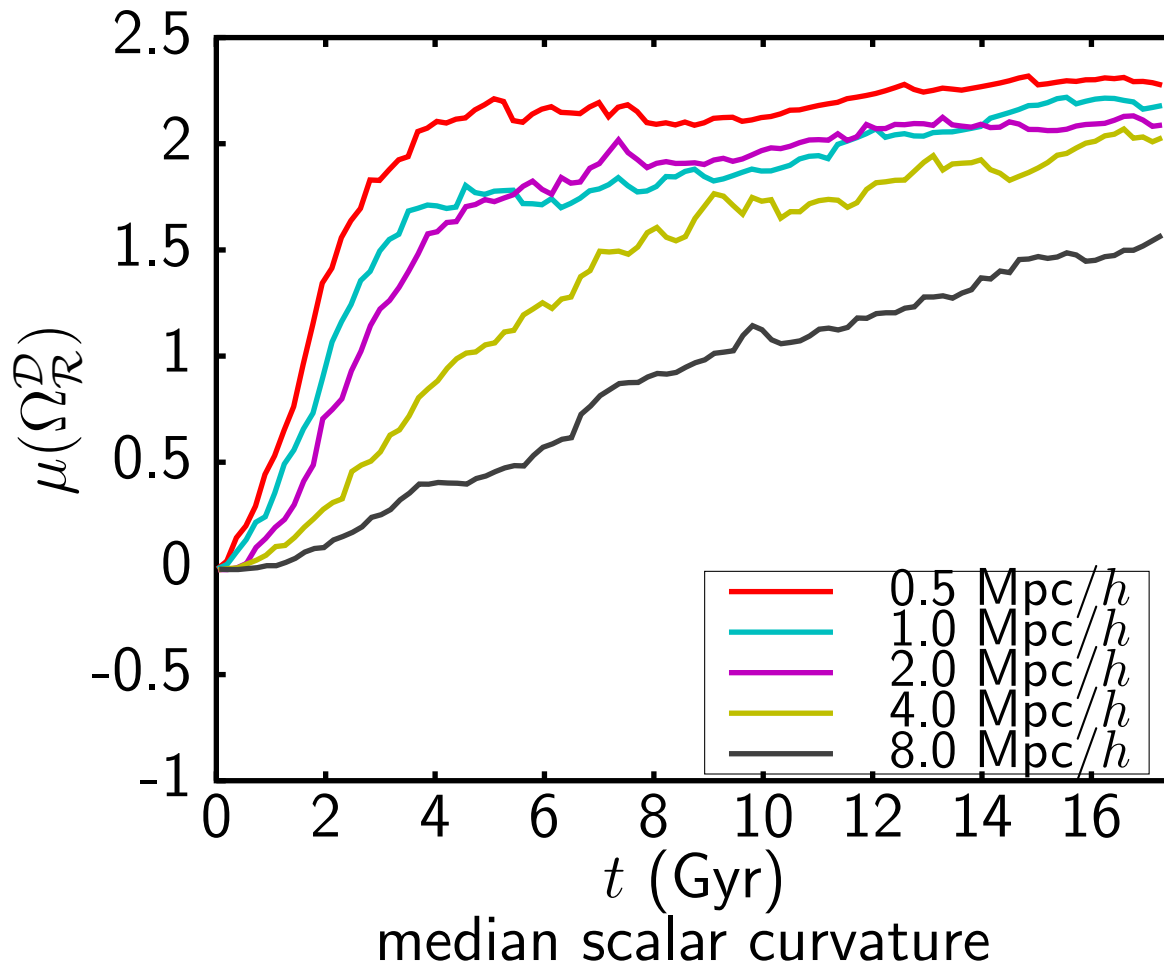
mean curvature evolution

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu



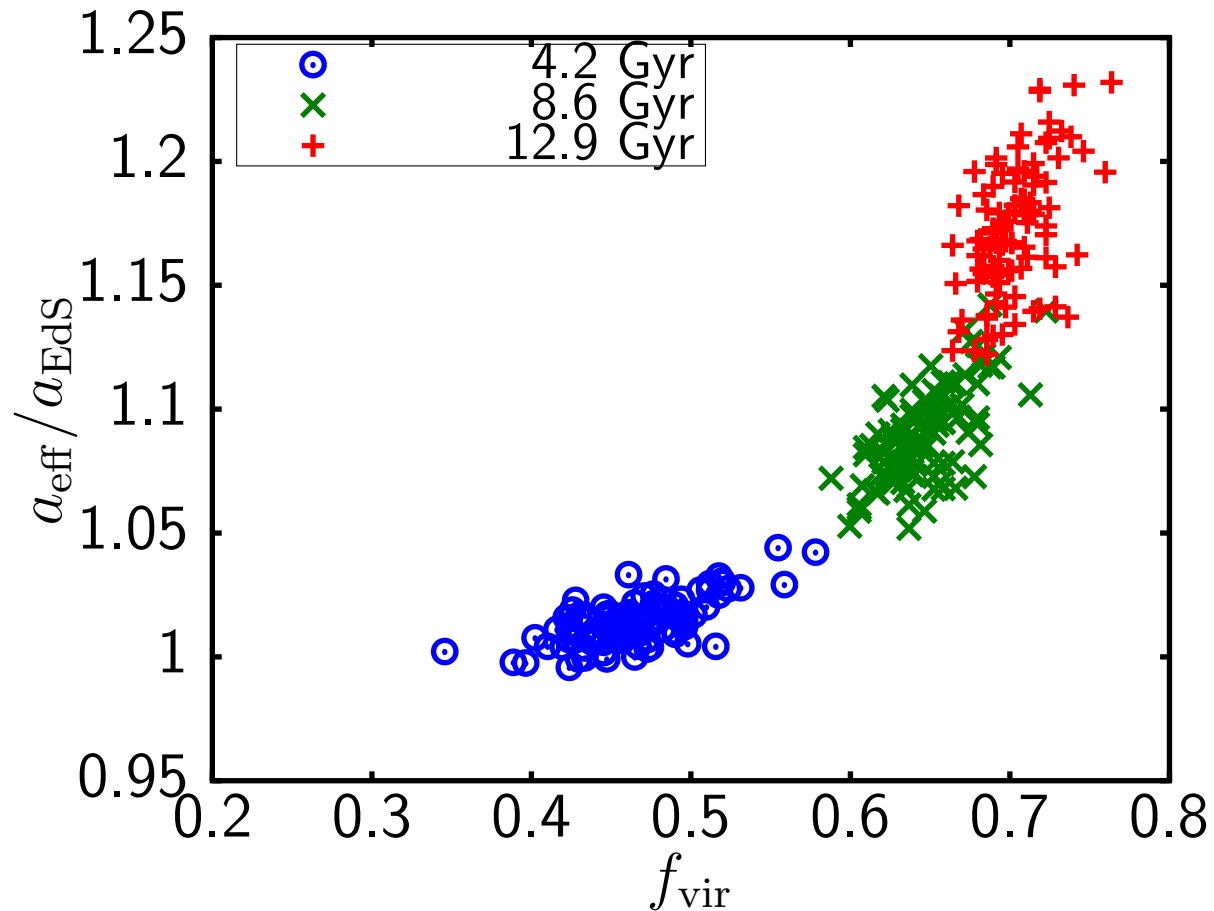
mean curvature evolution

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu



super-EdS growth vs f_{vir}

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu



100 VQZA simulations with $L_{\mathcal{D}} = 2 \text{ Mpc}/h^{\text{eff}}$ and
 $L_{\text{box}} = 8L_{\mathcal{D}} = 4L_{\text{DTFE}} = 8L_N$

Conclusion

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

- assume EdS initial conditions, matched to Λ CDM obs proxy
- evolve Lagrangian domains (QZA) [silent universe approximation]
- biscale no-virialisation (QZA): $|a_{\text{eff}}(t) - a_{\text{EdS}}(t)| \ll 1$

Conclusion

models – QZA – a_{eff} – Vir – $\mathcal{D}^\pm$ – soft – $a_{\text{eff}}(t)$ – $\mathcal{Q}_{\mathcal{D}}$ – Conclu

- assume EdS initial conditions, matched to Λ CDM obs proxy
- evolve Lagrangian domains (QZA) [silent universe approximation]
- biscale no-virialisation (QZA): $|a_{\text{eff}}(t) - a_{\text{EdS}}(t)| \ll 1$
- assume stable clustering [implicitly: local statistical acceleration] (V)
- 2000 VQZA simulations at resolution 256^3 :
 $L_{13.8} = 2.5_{-0.4}^{+0.1} \text{ Mpc}/h^{\text{eff}} \Rightarrow a_{\text{eff}}(13.8 \text{ Gyr}) \approx 1$ (16% above EdS)
- “DE” provided at order unity level from typical non-linear structure scale with this virialisation model
- Can better GR virialisation models avoid this?
- Roukema [arXiv:1706.06179](https://arxiv.org/abs/1706.06179), submitted A&A