Cosmic inhomogeneity and topology I

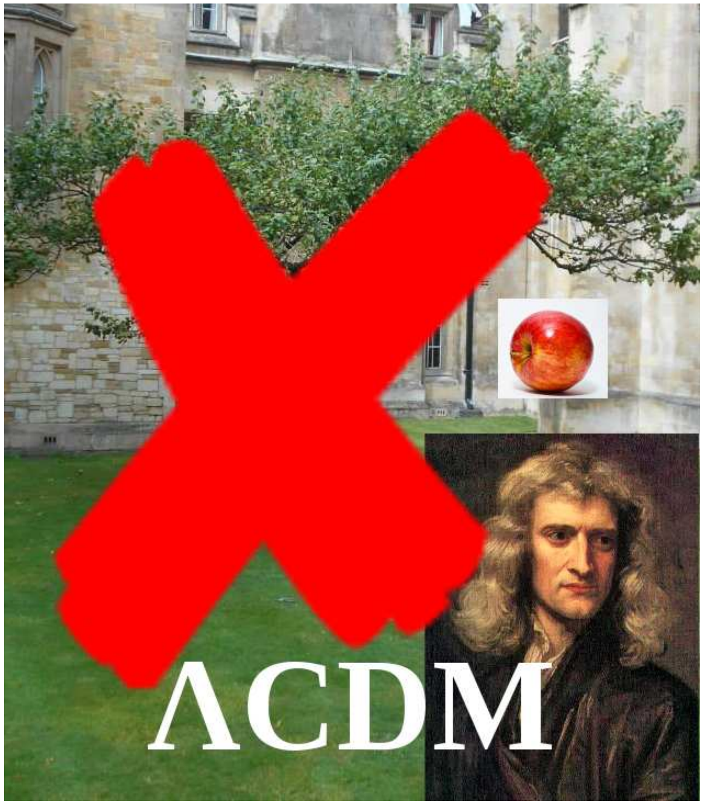
Boud Roukema
Institute of Astronomy
Nicolaus Copernicus University

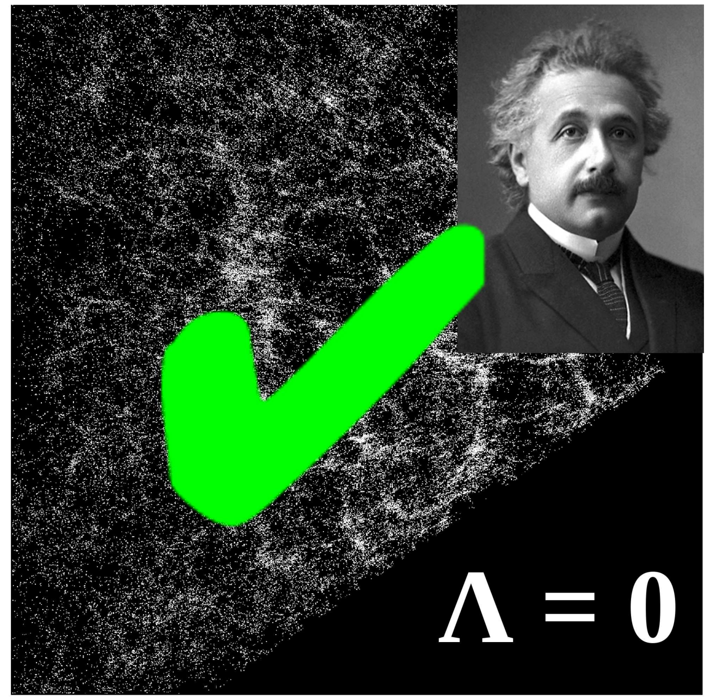
I: 2021-07-27 + 2021-07-29

https://cosmo.torun.pl/∼boud/Roukema20210727CIRM.pdf
https://cosmo.torun.pl/∼boud/Roukema20210729CIRM.pdf

Newton vs Einstein

space–time = Universe





?

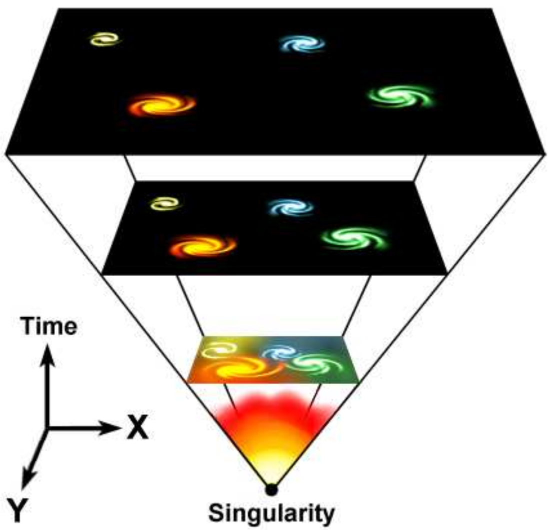
Gpc-scale galaxies/4MOST + SNe Ia/LSST ⇒ Einstein ?

local 100 Mpc/h^{eff} + "local" H_0 vs "global" H_0 ⇒ Einstein ?

Inhomogeneous cosmology

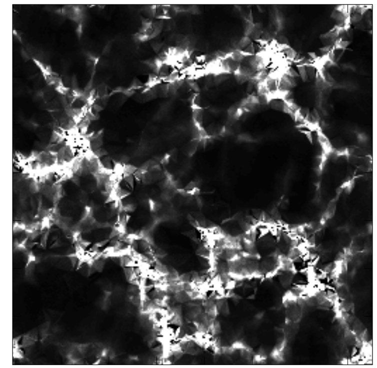
- theory
  - analytical
    - exact solutions — LTB, Szekeres, +... Sussman @CIRM
    - scalar averaging — Mourier @CIRM Delgado Gaspar @CIRM
  - numerical
    - scalar averaging; numerical relativity
    - reproducibility: Akhlaghi+2021, CiSE, 23, 82
      Peper & Roukema 2021, MNRAS, 505, 1223
- observations
  - methods Korzyński @CIRM
  - observational credibility estimates for dark energy as recently emerging negative average curvature

ΛCDM: the biverse model

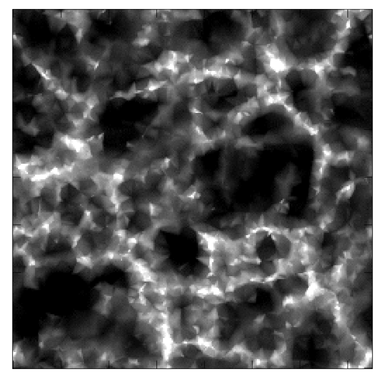


FLRW-universe

$\Rightarrow$



$\not\Leftarrow$



$||$ $\delta(t, k)$-universe

ΛCDM: the biverse model

- ΛCDM: exactly FLRW Universe — perfectly 3-Ricci flat;

  but

- ΛCDM: the Universe has structure: early epochs — curvature perturbations;
- Is curvature significant at late epochs?

ΛCDM: the biverse model

- ΛCDM: exactly FLRW Universe — perfectly 3-Ricci flat;

  but

- ΛCDM: the Universe has structure: early epochs — curvature perturbations;
- Is curvature significant at late epochs?
- Does gravitational collapse require positive curvature?
- Does the curvature–expansion rate relation imply that FLRW is inaccurate at late epochs?

turnaround epoch

- rumour: "everyone knows" that curvature is associated with structure formation

turnaround epoch

- rumour: "everyone knows" that curvature is associated with structure formation
- problem: is there a simple proof with very few assumptions?
- yes: clear, elegant argument using the turnaround epoch + RZA

Newtonian cosmology

- Newtonian cosmology — no such thing as 3-Ricci curvature

Newtonian cosmology

- Newtonian cosmology — no such thing as 3-Ricci curvature
- *Newtonian-gauge restriction* in GR cosmology, "Newtonian limit":

$$\mathrm{d}s^2 = a^2 \left\{ -(1+2\psi)\,\mathrm{d}\tau^2 + (1-2\varphi)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right] \right\}$$

$\Rightarrow$

$$^3\mathcal{R} = \frac{8\,\varphi\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right) - 4\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right)}{a^2\,\left(2\,\varphi - 1\right)^3} + \frac{-6\,\left(\varphi_{,x}^2 + \varphi_{,y}^2 + \varphi_{,z}^2\right)}{a^2\,\left(2\,\varphi - 1\right)^3}$$

Newtonian cosmology

- Newtonian cosmology — no such thing as 3-Ricci curvature
- *Newtonian-gauge restriction* in GR cosmology, "Newtonian limit":
$$\mathrm{d}s^2 = a^2 \left\{-(1+2\psi)\,\mathrm{d}\tau^2 + (1-2\varphi)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]\right\}$$

$\Rightarrow$

$$^3\mathcal{R} = \frac{8\,\varphi\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right) - 4\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right)}{a^2\,\left(2\,\varphi - 1\right)^3} + \frac{-6\,\left(\varphi_{,x}^2 + \varphi_{,y}^2 + \varphi_{,z}^2\right)}{a^2\,\left(2\,\varphi - 1\right)^3}$$

if $|\varphi| \ll 1,\ |\varphi_{,x^i}| \ll 1,\ |\varphi_{,x^i x^i}| \ll 1\ \forall x^i \in \{x, y, z\}$, then

Newtonian cosmology

- Newtonian cosmology — no such thing as 3-Ricci curvature
- *Newtonian-gauge restriction* in GR cosmology, "Newtonian limit":

$$\mathrm{d}s^2 = a^2 \left\{-(1+2\psi)\,\mathrm{d}\tau^2 + (1-2\varphi)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]\right\}$$

if $|\varphi| \ll 1,\ |\varphi_{,x^i}| \ll 1,\ |\varphi_{,x^i x^i}| \ll 1\ \forall x^i \in \{x, y, z\}$, then

$$\begin{aligned} {}^3\mathcal{R} &\approx 4\,a^{-2}\,\left(\varphi_{,x\,x} + \varphi_{,y\,y} + \varphi_{,z\,z}\right) \\ &= 4\,a^{-2}\,\nabla^2_{\mathbb{E}^3}\varphi \\ &= 16\pi\,G\,\delta\rho \end{aligned}$$

- positive spatial curvature tends to associate with overdensities

Hamiltonian constraint

• assume: expanding dust universe with flow-orthogonal foliation, irrotational fluid

Hamiltonian constraint

• assume: expanding dust universe with flow-orthogonal foliation, irrotational fluid

Einstein Eqn time–time part gives (pointwise):

$$\frac{1}{3}\Theta^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

Buchert (2000a) — Hamiltonian constraint

Hamiltonian constraint

• assume: expanding dust universe with flow-orthogonal foliation, irrotational fluid
Einstein Eqn time–time part gives (pointwise):

$$\frac{1}{3}\Theta^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

Buchert (2000a) — Hamiltonian constraint

$$3\,H^2 = 3\,H^2\Omega_{\rm m} + 0 + 3\,H^2\Omega_{\rm k} + 3\,H^2\Omega_\Lambda \qquad \text{cf FLRW}$$

Hamiltonian constraint

• assume: expanding dust universe with flow-orthogonal foliation, irrotational fluid
Einstein Eqn time–time part gives (pointwise):

$$\frac{1}{3}\Theta^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

Buchert (2000a) — Hamiltonian constraint

$$3\,H^2 = 3\,H^2\Omega_{\rm m} + 0 + 3\,H^2\Omega_{\rm k} + 3\,H^2\Omega_\Lambda \qquad \text{cf FLRW}$$

- turnaround: $\Theta = 0 \Rightarrow \mathcal{R} > 0$
- "collapsing" domain must have $\mathcal{R} > 0$ to turn around

  arXiv:1902.09064 (RO19)

averaged case

What happens for an averaged domain?

averaged Hamiltonian constraint

averaged Hamiltonian constraint

$$\frac{1}{3}\left\langle\Theta\right\rangle_{\mathcal{D}}^{2} = 8\pi G\left\langle\rho\right\rangle_{\mathcal{D}} + \left\langle\sigma^2\right\rangle_{\mathcal{D}} - \frac{1}{3}\left\langle\left(\Theta - \left\langle\Theta\right\rangle_{\mathcal{D}}\right)^2\right\rangle_{\mathcal{D}} - \frac{1}{2}\left\langle\mathcal{R}\right\rangle_{\mathcal{D}} + \Lambda$$

$$\mathcal{Q}_{\mathcal{D}} := \frac{2}{3}\left\langle\left(\Theta - \left\langle\Theta\right\rangle_{\mathcal{D}}\right)^2\right\rangle_{\mathcal{D}} - 2\left\langle\sigma^2\right\rangle_{\mathcal{D}}$$

RZA2

averaged Hamiltonian constraint

averaged Hamiltonian constraint

$$\Omega_{\mathrm{m}}^{\mathcal{D}} + \Omega_{\mathcal{Q}}^{\mathcal{D}} + \Omega_{\mathcal{R}}^{\mathcal{D}} + \Omega_{\Lambda}^{\mathcal{D}} = \frac{H_{\mathcal{D}}^2}{H_{\mathrm{eff}}^2} \,,$$

- turnaround: strong expansion variance term $\left\langle \left(\Theta - \langle\Theta\rangle_{\mathcal{D}}\right)^2 \right\rangle_{\mathcal{D}}$ might allow flat ($\mathcal{R} = 0$) turnaround ($\Theta = 0$)
- non-perturbative: $\mathcal{Q}_{\mathcal{D}}$ relativistic Zel'dovich approximation (QZA)

arXiv:1902.09064 (RO19)

Plane-symmetric case

general:

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \left[(1 + S(w,t))^2 \left(\mathrm{d}x^2 + \mathrm{d}y^2\right) + \big(1 + P(w,t)\big)^2 \,\mathrm{d}w^2 \right]$$

Plane-symmetric case

spatially flat subcase (RZA2 V.A):

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \left[\mathrm{d}x^2 + \mathrm{d}y^2 + \big(1 + P(w,t)\big)^2\,\mathrm{d}w^2\right],$$

Plane-symmetric case

spatially flat subcase (RZA2 V.A):

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a(t)^2 \left[\mathrm{d}x^2 + \mathrm{d}y^2 + \big(1 + P(w,t)\big)^2 \,\mathrm{d}w^2\right],$$

- either EdS or ΛCDM reference model:

$$\lim_{t\to\infty} \Theta = 3H := 3\frac{\dot{a}}{a}$$

- $\Theta > 0$ (and $H > 0$) initially $\Rightarrow$ no pancake collapse
- RZA2 V.A: “pancake collapse possible” — assumed growing mode allowed
- fundamental difference Newtonian vs GR:
  GR forbids growing mode in this case

Analytical calculation: special case

- assume $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$, $\langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$, $\Lambda = 0$
- define $\alpha := \frac{H}{H_{\mathrm{eff}}} \lesssim 1$ (normalise $\Omega^{\mathcal{D}}$'s by H^2_{eff})
- turnaround condition:

$$\langle \mathrm{I} \rangle_{\mathcal{D}} = -3 \frac{H}{H_{\mathbf{i}}}$$

Analytical calculation: special case

- assume $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0 \,, \quad \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0 \,, \Lambda = 0$
- define $\alpha := \frac{H}{H_{\mathrm{eff}}} \lesssim 1$ (normalise $\Omega^{\mathcal{D}}$'s by H_{eff}^2)
- turnaround condition:

$$\langle \mathrm{I} \rangle_{\mathcal{D}} = -3 \frac{H}{H_{\mathbf{i}}}$$

$\Rightarrow \quad \Omega_{\mathcal{R}}^{\mathcal{D}} = -5\alpha^2, \;\; \Omega_{\mathcal{Q}}^{\mathcal{D}} = \alpha^2, \;\; \Omega_{\mathrm{m}}^{\mathcal{D}} = 4\alpha^2$

- $\exists$ critical Ω parameters at turnaround

Analytical calculation: special case

- assume $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$, $\langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$, $\Lambda = 0$
- define $\alpha := \frac{H}{H_{\mathrm{eff}}} \lesssim 1$ (normalise $\Omega^{\mathcal{D}}$'s by H_{eff}^2)
- turnaround condition:

$$\langle \mathrm{I} \rangle_{\mathcal{D}} = -3 \frac{H}{H_{\mathbf{i}}}$$

$\Rightarrow$ $\Omega_{\mathcal{R}}^{\mathcal{D}} = -5\alpha^2, \;\; \Omega_{\mathcal{Q}}^{\mathcal{D}} = \alpha^2, \;\; \Omega_{\mathrm{m}}^{\mathcal{D}} = 4\alpha^2$

- $\exists$ critical Ω parameters at turnaround
- Ostrowski 2020: generic $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} \neq 0 \neq \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}}$

$\Rightarrow$ $\Omega_{\mathcal{R}}^{\mathcal{D}} + \Omega_{\mathcal{Q}}^{\mathcal{D}} = -\Omega_{\mathrm{m}}^{\mathcal{D}}, \;\; \Omega_{\mathrm{m}}^{\mathcal{D}} = 4\alpha^2$

scalar averaging: Raychaudhuri eq

with invariants of the peculiar expansion tensor [Newtonian case: Buchert 94, MNRAS arXiv:astro-ph/9309055]:

$$\begin{aligned}
\mathrm{I}(v^i_{\ ,j}) &:= \mathrm{tr}(v^i_{\ ,j}) = v^i_{\ ,i} = \nabla\cdot\mathbf{v} \\
\mathrm{II}(v^i_{\ ,j}) &:= \tfrac{1}{2}\left\{\left[\mathrm{tr}\left(v^i_{\ ,j}\right)\right]^2 - \mathrm{tr}\left[\left(v^i_{\ ,j}\right)^2\right]\right\} \\
&= \tfrac{1}{2}\left((v^i_{\ ,i})^2 - v^i_{\ ,j}\, v^j_{\ ,i}\right) \\
&= \tfrac{1}{2}\nabla\cdot\Big(\mathbf{v}(\nabla\cdot\mathbf{v}) - (\mathbf{v}\cdot\nabla)\mathbf{v}\Big) \\
\mathrm{III}(v^i_{\ ,j}) &:= \det(v^i_{\ ,j}).
\end{aligned}$$

scalar averaging: QZA

$$\mathcal{Q}_{\mathcal{D}} = \frac{\dot{\xi}^2 \left(\gamma_1 + \xi\gamma_2 + \xi^2\gamma_3\right)}{\left(1 + \xi \left\langle \mathrm{I}_{\mathbf{i}} \right\rangle_{\mathcal{I}} + \xi^2 \left\langle \mathrm{II}_{\mathbf{i}} \right\rangle_{\mathcal{I}} + \xi^3 \left\langle \mathrm{III}_{\mathbf{i}} \right\rangle_{\mathcal{I}}\right)^2}$$

$$\text{where} \begin{cases} \gamma_1 := 2 \left\langle \mathrm{II}_{\mathbf{i}} \right\rangle_{\mathcal{I}} - \frac{2}{3} \left\langle \mathrm{I}_{\mathbf{i}} \right\rangle_{\mathcal{I}}^2 \\ \gamma_2 := 6 \left\langle \mathrm{III}_{\mathbf{i}} \right\rangle_{\mathcal{I}} - \frac{2}{3} \left\langle \mathrm{II}_{\mathbf{i}} \right\rangle_{\mathcal{I}} \left\langle \mathrm{I}_{\mathbf{i}} \right\rangle_{\mathcal{I}} \\ \gamma_3 := 2 \left\langle \mathrm{I}_{\mathbf{i}} \right\rangle_{\mathcal{I}} \left\langle \mathrm{III}_{\mathbf{i}} \right\rangle_{\mathcal{I}} - \frac{2}{3} \left\langle \mathrm{II}_{\mathbf{i}} \right\rangle_{\mathcal{I}}^2 \end{cases}$$

QZA $= \mathcal{Q}_{\mathcal{D}}$ Zel'dovich approximation:

- algebraic structure same in Newtonian and GR cases
- initial invariants conceptually differ (Newt vs GR)
- initial invariants numerically approximated for zero curvature
- ξ is the reference model (EdS) linear growth rate

Volume averaging

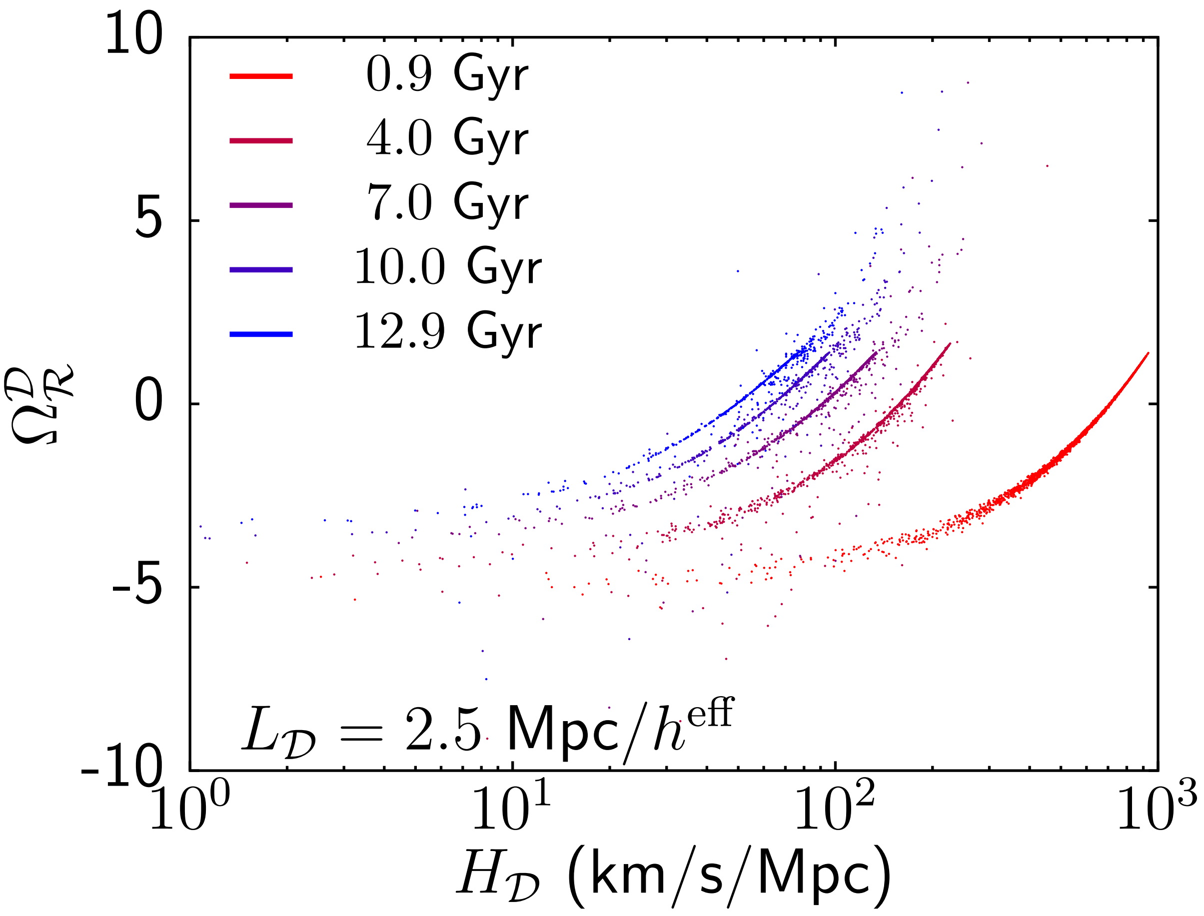
instead of verbal averaging (FLRW $\ni$ ΛCDM): mathematical averaging

$$a_{\text{eff}}(t) := \left(\frac{\sum_{\mathcal{D}} a_{\mathcal{D}}^3(t)}{\sum_{\mathcal{D}} 1} \right)^{1/3} = \left(\frac{\sum_{\mathcal{D}} a_{\mathcal{D}}^3(t)}{n_{\mathcal{D}}} \right)^{1/3}$$

QZA model

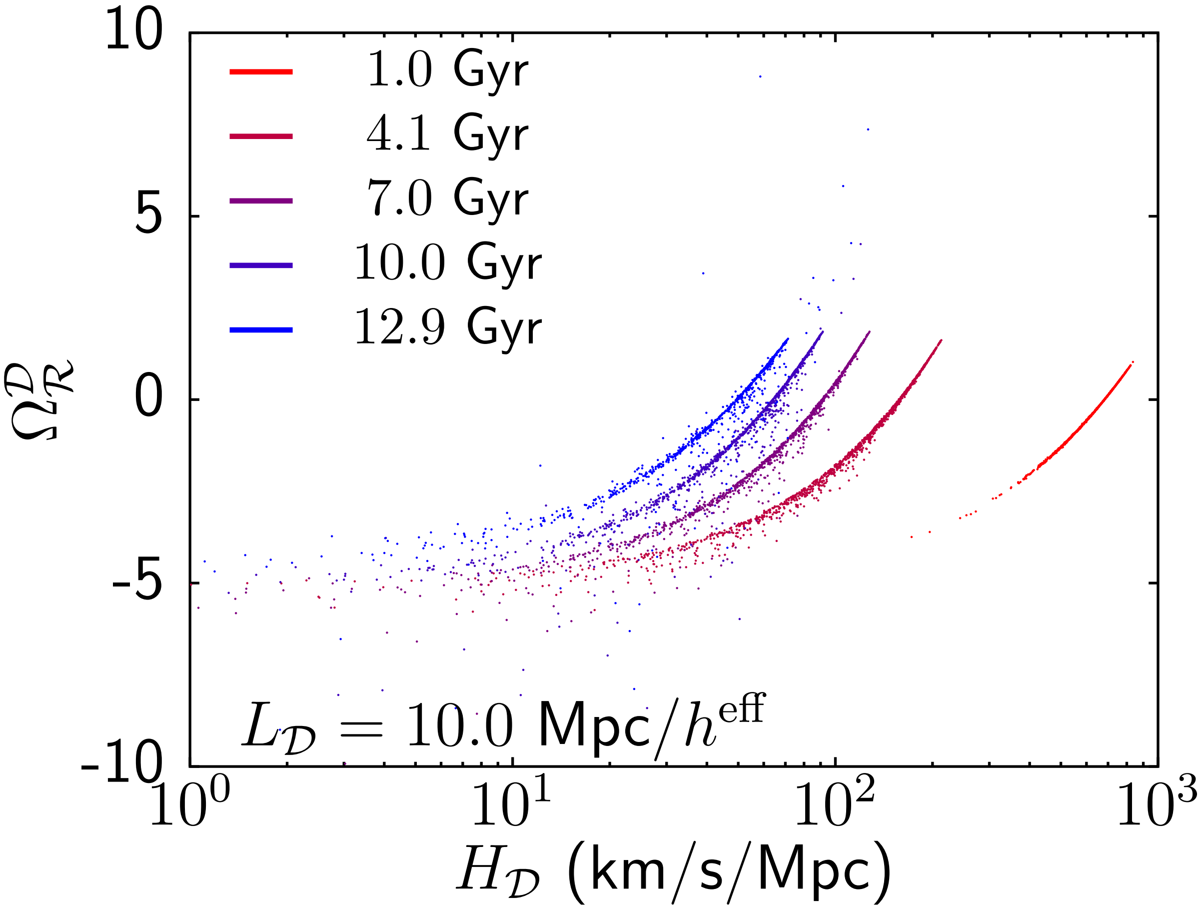
- Einstein eq: $\mathbf{G} = 8\pi\mathbf{T} \quad (\Lambda := 0)$
- $a_{\mathcal{D}}$: domain-averaged scale factor and Raychaudhuri equation for $\ddot{a}_{\mathcal{D}}$ ("silent" virialisation) on a spatial domain $\mathcal{D}$
- non-linear scales but no shell-crossing (gravitational collapse)
- Raychaudhuri + $\mathcal{Q}_{\mathcal{D}}$ analytical approximation (QZA)
- calculate global average scale factor: $a_{\mathrm{eff}}(t)$
- *expected result: generate* $a_{\mathrm{eff}}(t) \approx a_{\mathrm{EdS}}(t)$ *instead of assuming it*
- free-licensed numerical implementation with INHOMOG – https://codeberg.org/boud/inhomog – Roukema (2018) A&A, 610, A51

curvature–expansion-rate relation



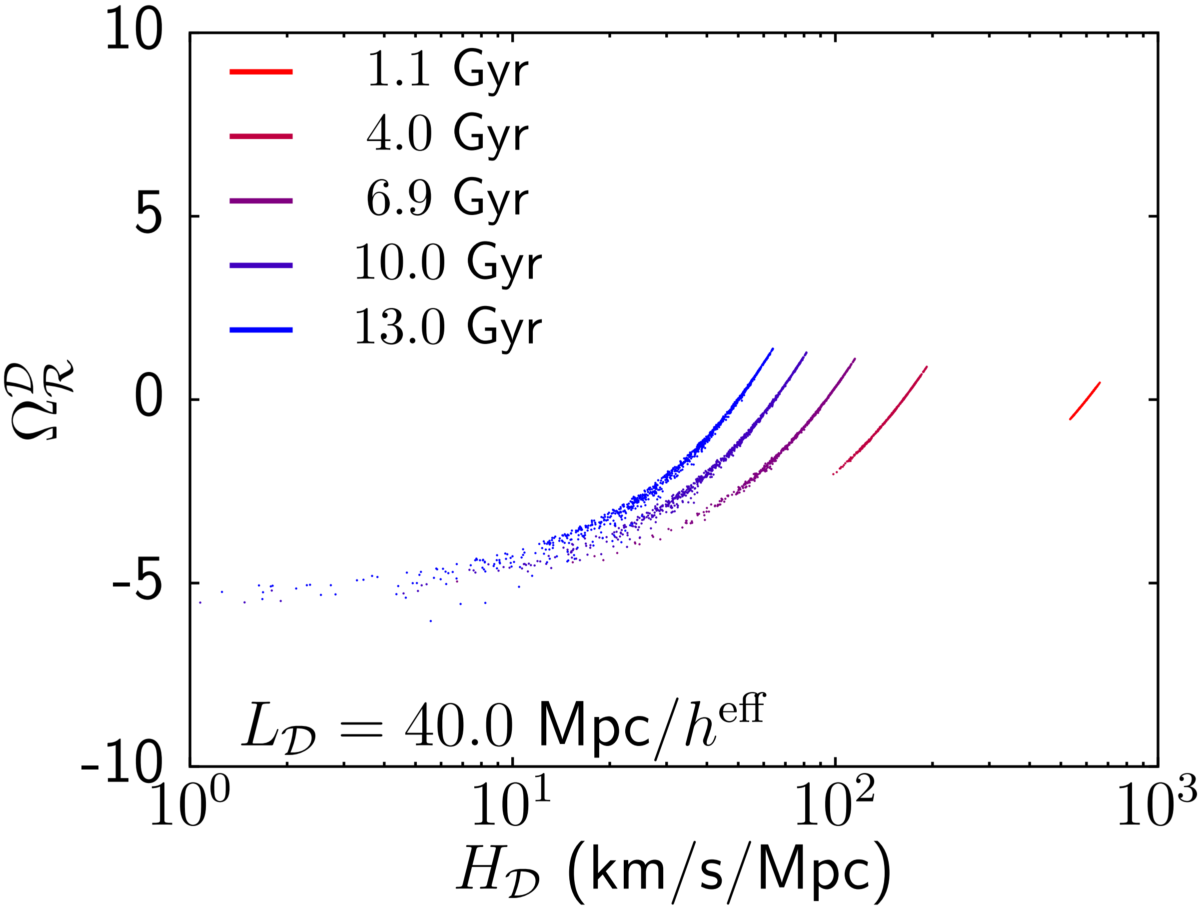
EdS

curvature–expansion-rate relation



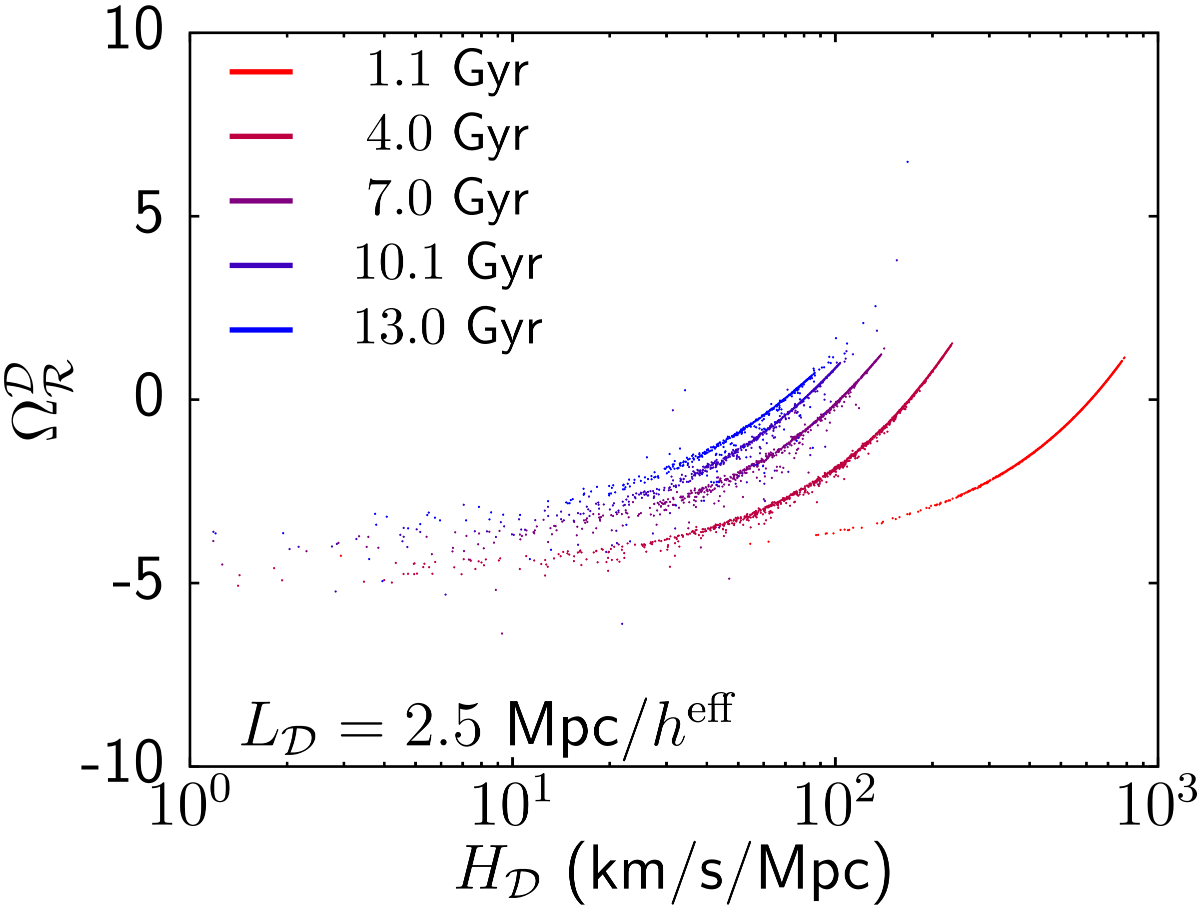
EdS

curvature–expansion-rate relation



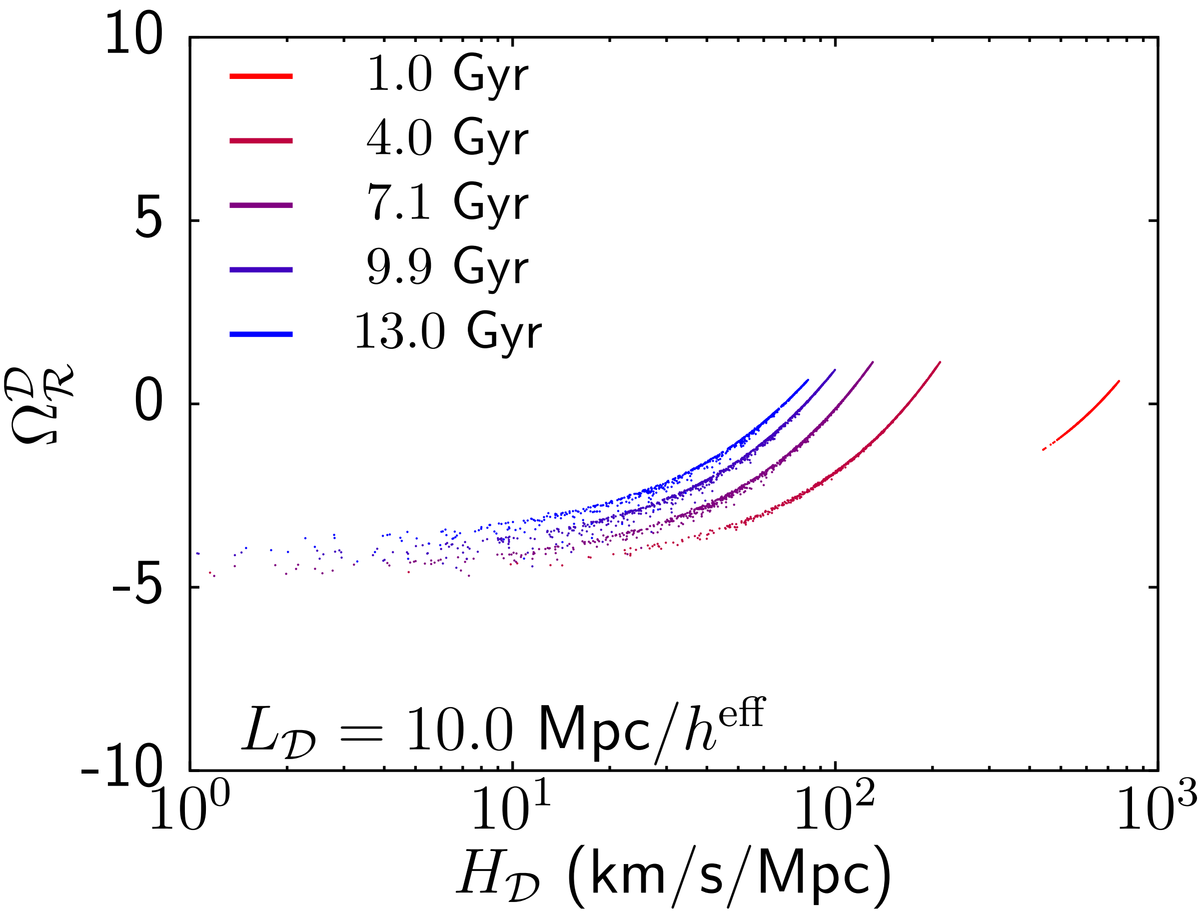
EdS

curvature–expansion-rate relation



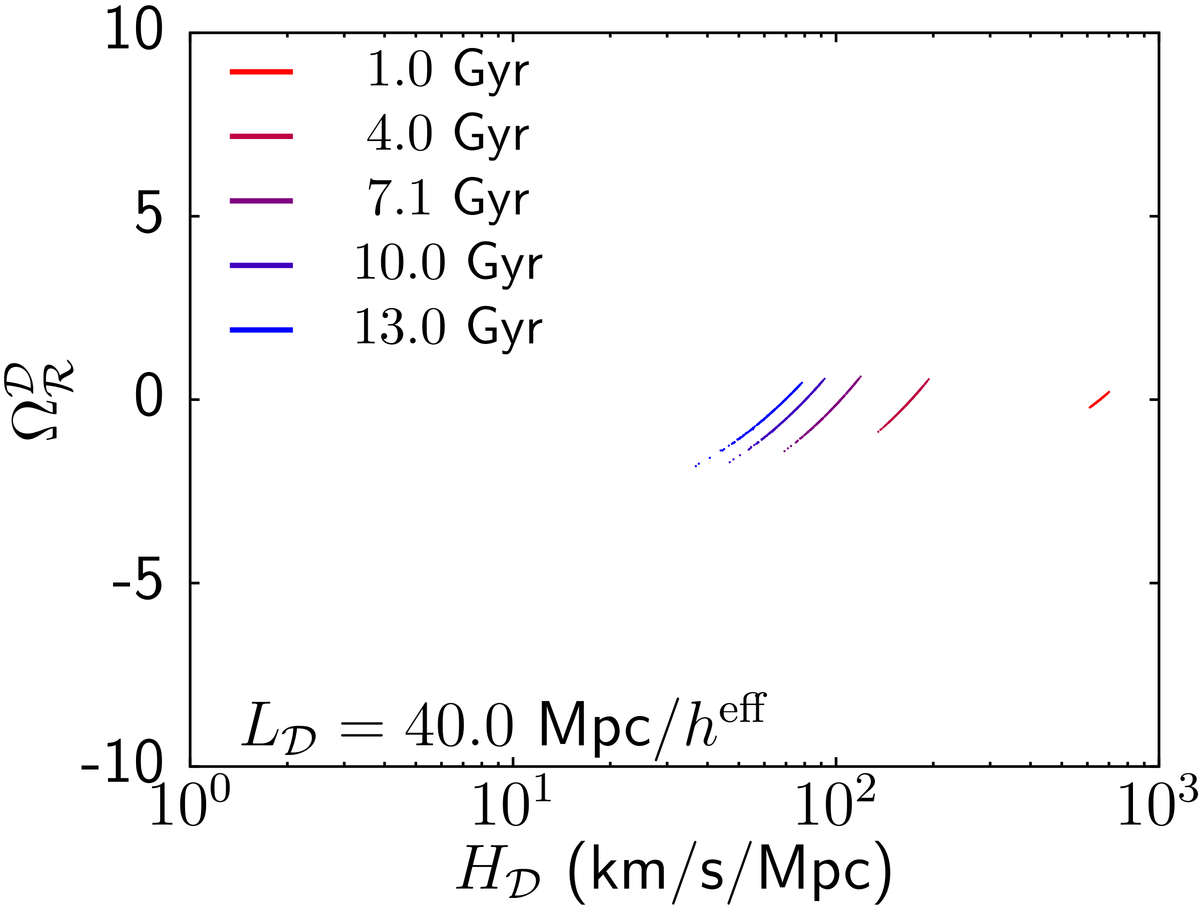
ΛCDM

curvature–expansion-rate relation



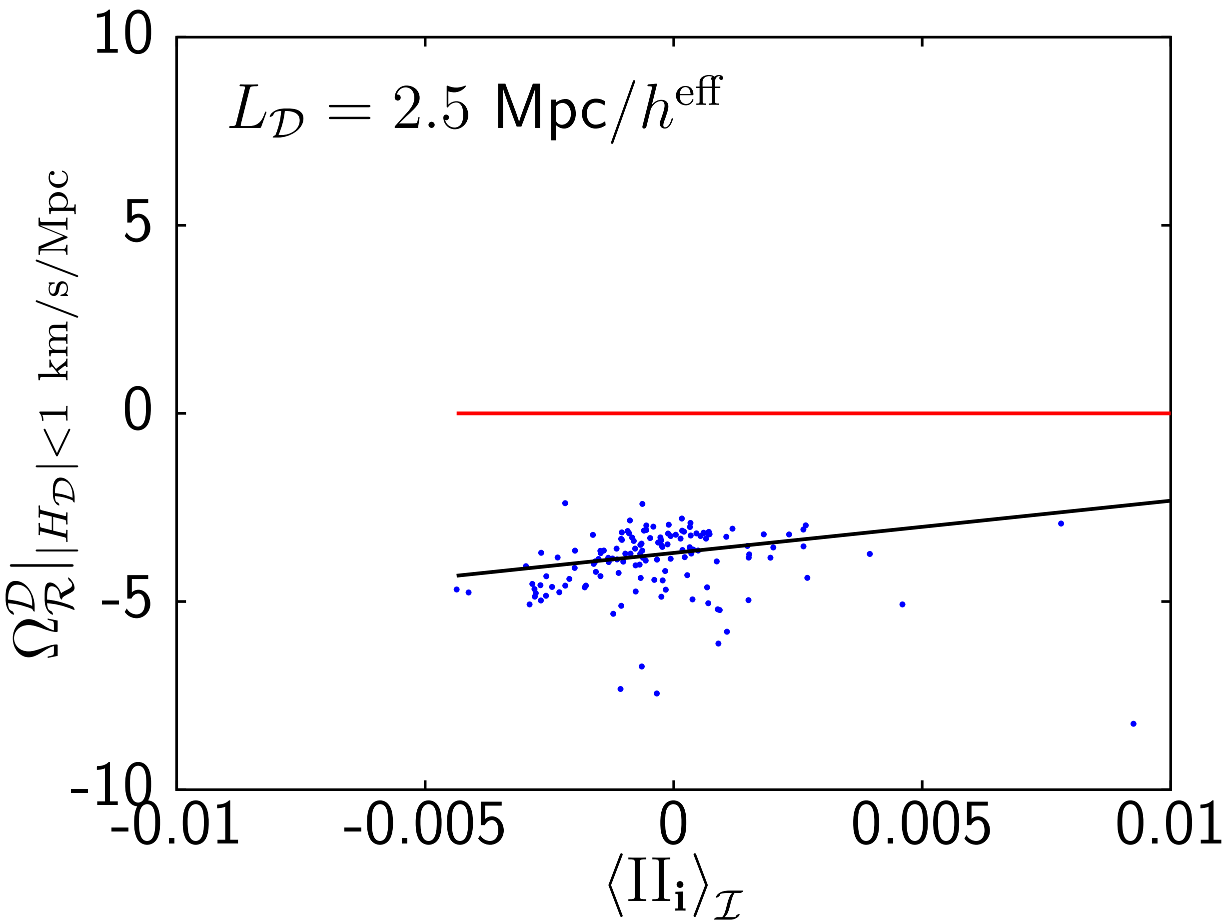
ΛCDM

curvature–expansion-rate relation



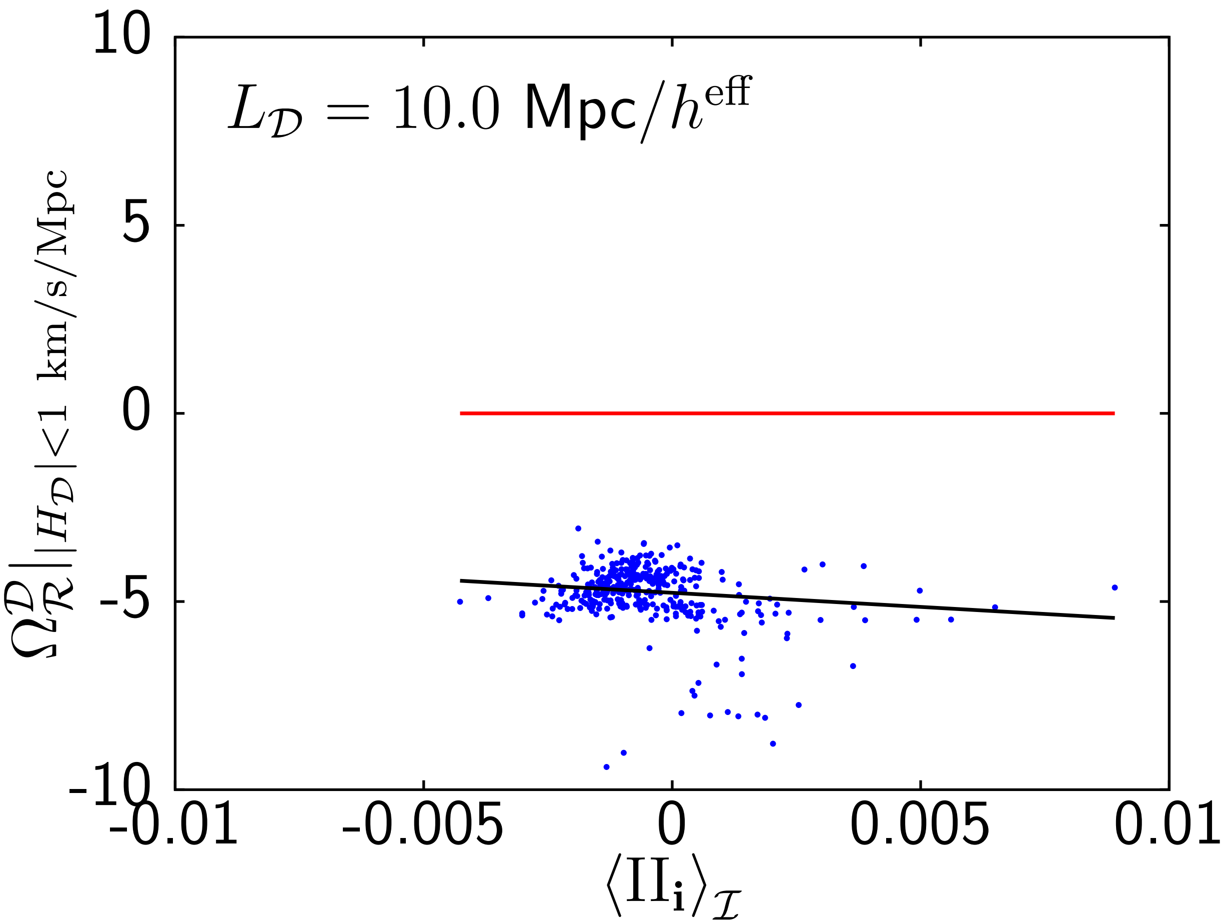
ΛCDM

critical $\Omega_{\mathcal{R}}^{\mathcal{D}}$ value at turnaround



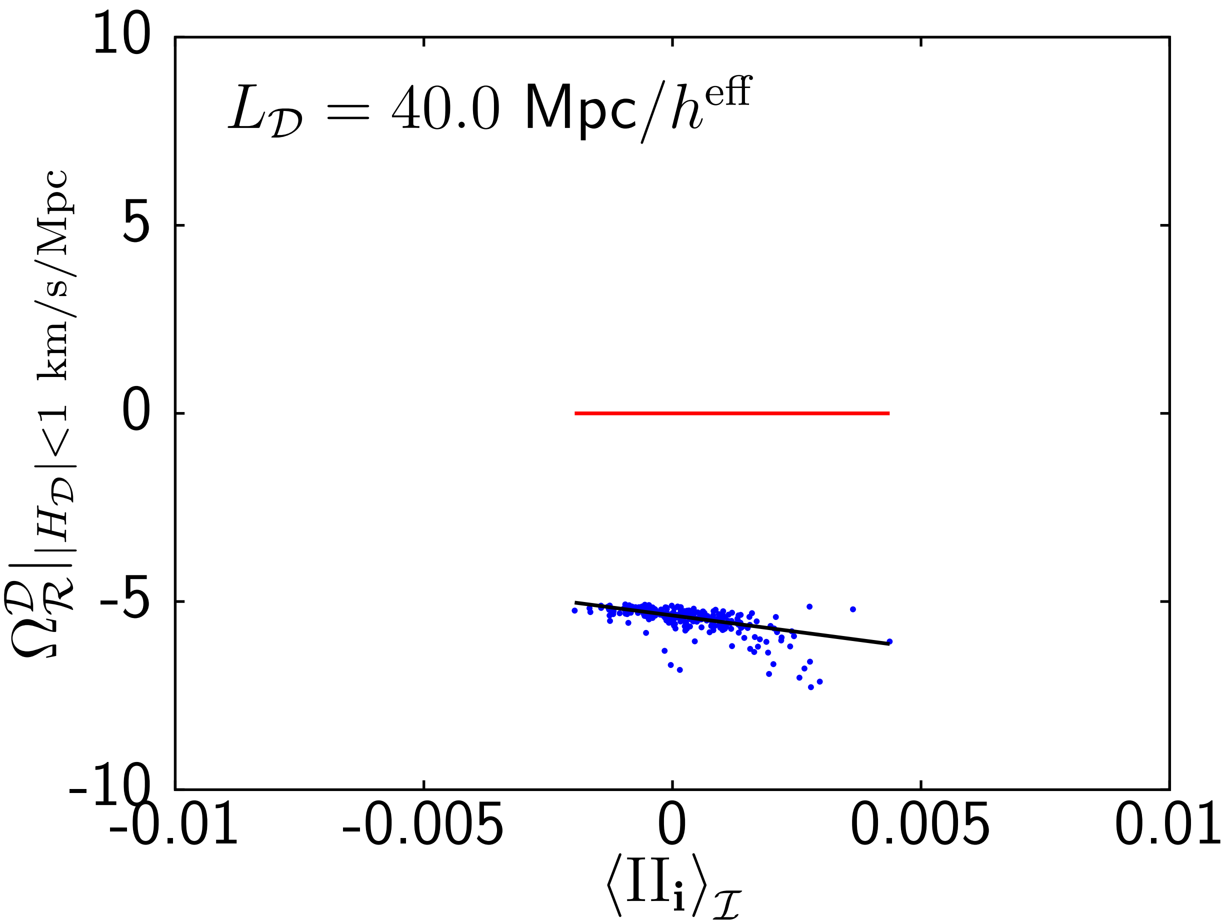
EdS

critical $\Omega^{\mathcal{D}}_{\mathcal{R}}$ value at turnaround



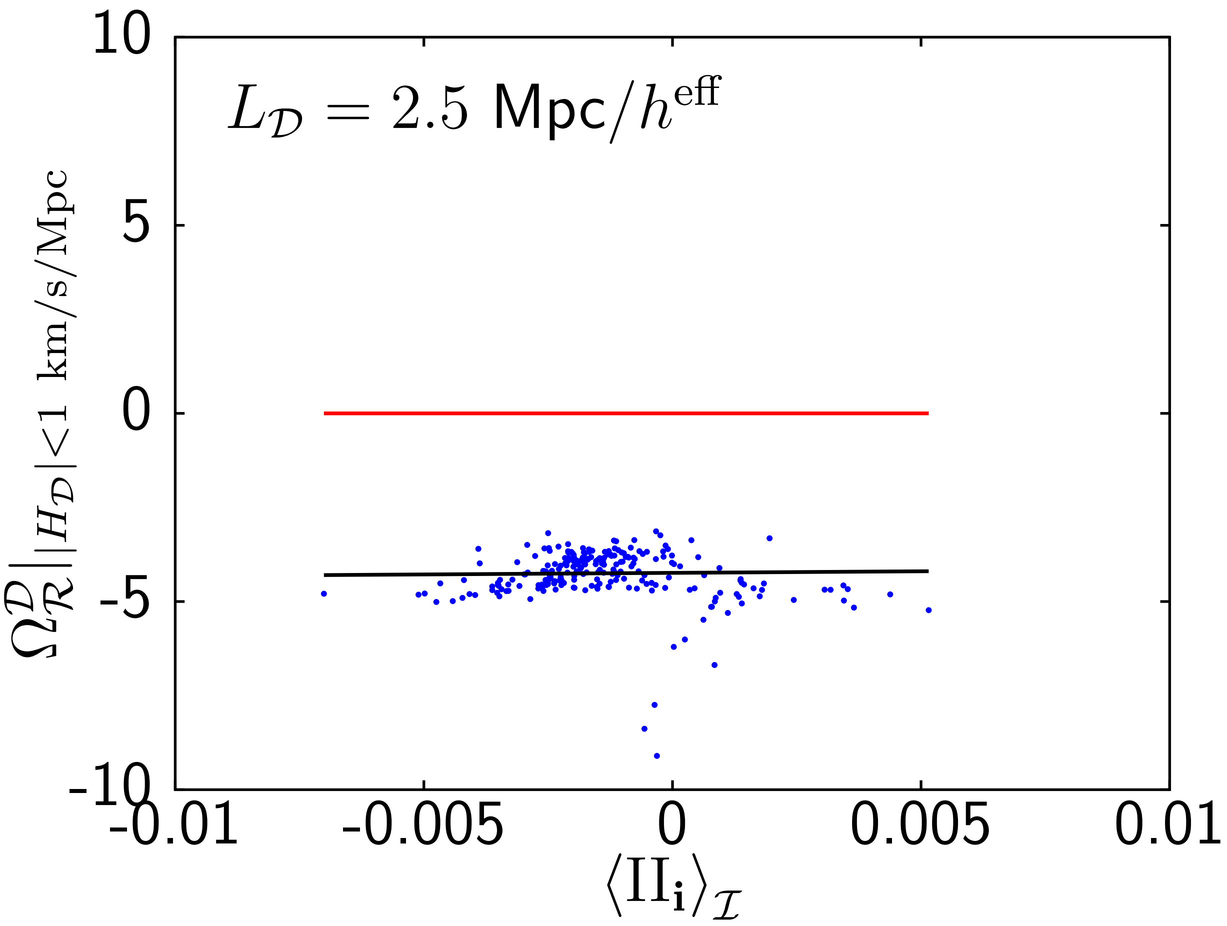
EdS

critical $\Omega_{\mathcal{R}}^{\mathcal{D}}$ value at turnaround



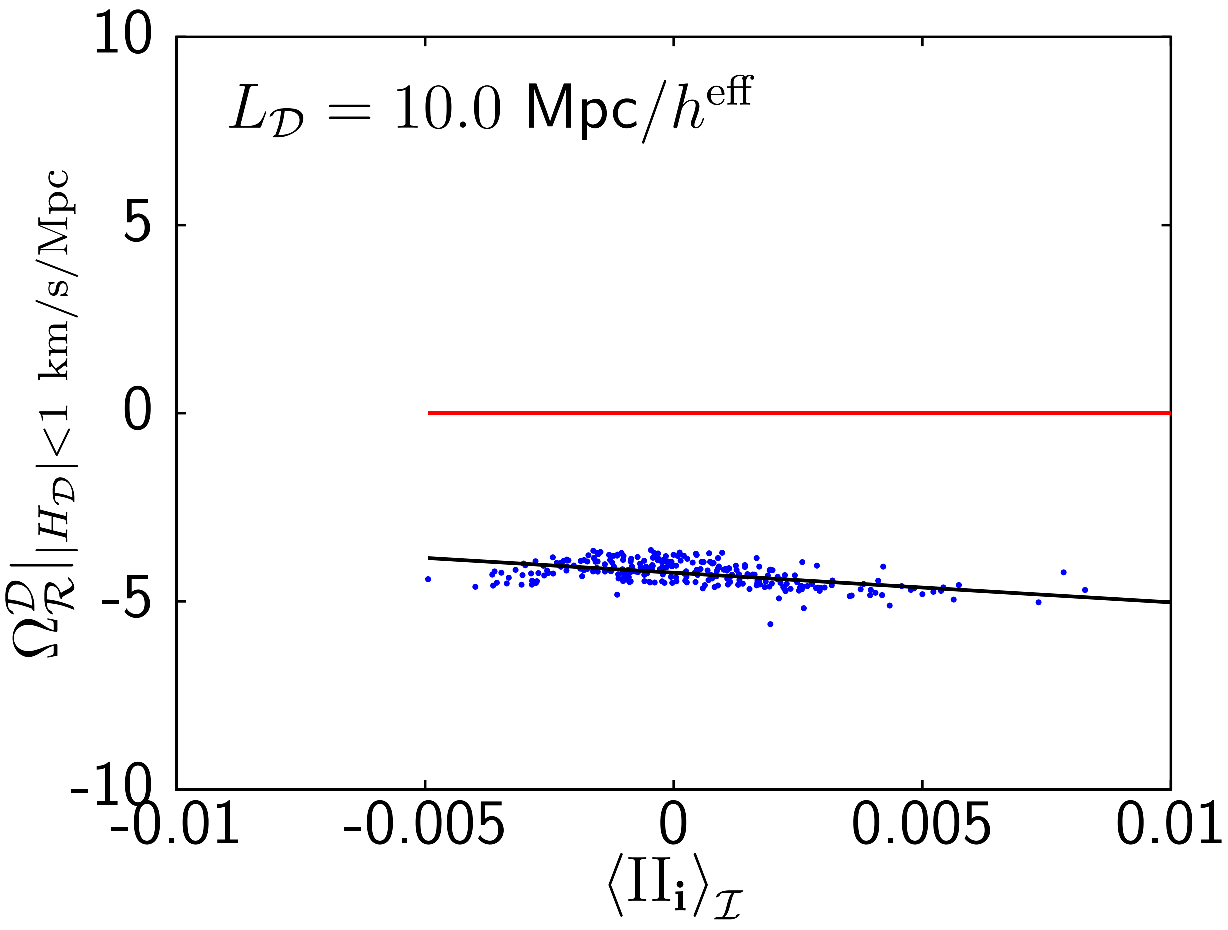
EdS

critical $\Omega_{\mathcal{R}}^{\mathcal{D}}$ value at turnaround



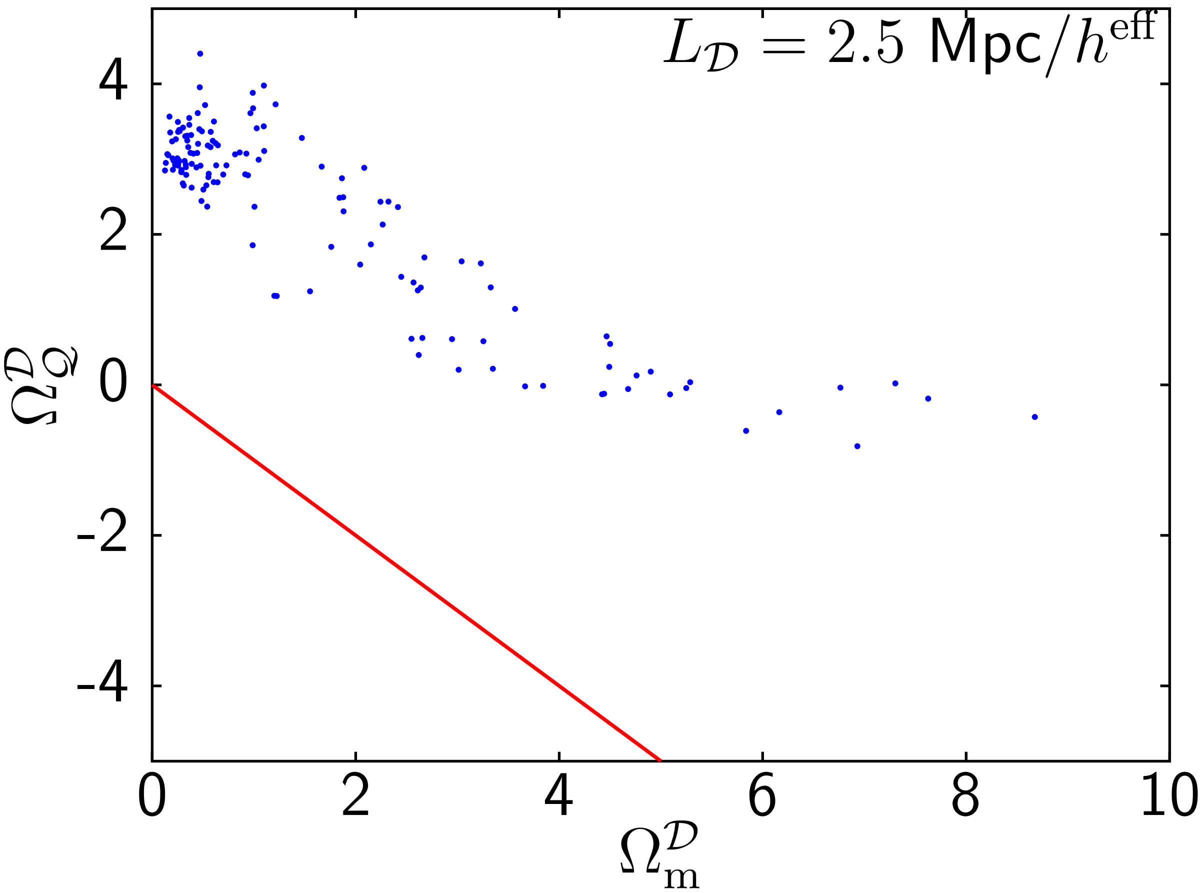
ΛCDM

critical $\Omega_{\mathcal{R}}^{\mathcal{D}}$ value at turnaround



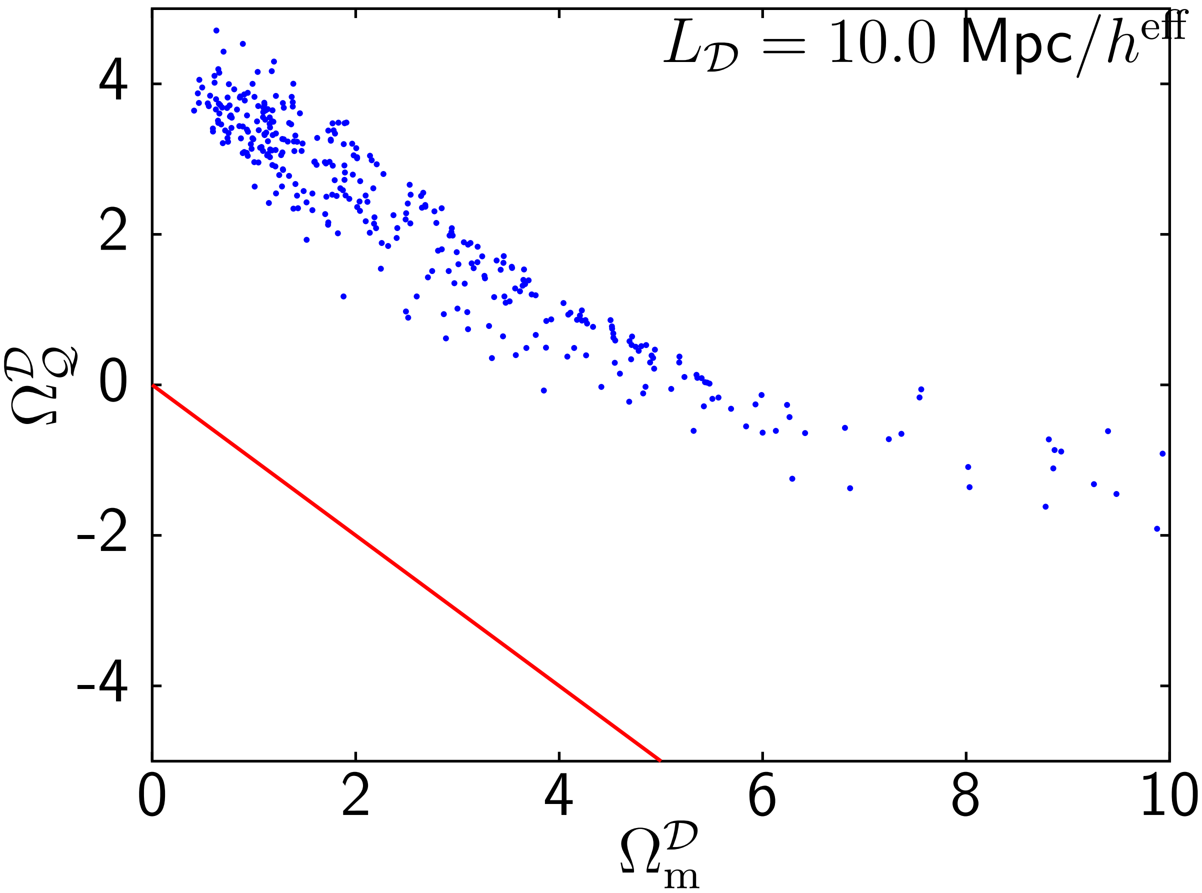
ΛCDM

Does $\Omega^{\mathcal{D}}_{\mathcal{Q}}$ allow negative ${}^3\mathcal{R}$ at TA?



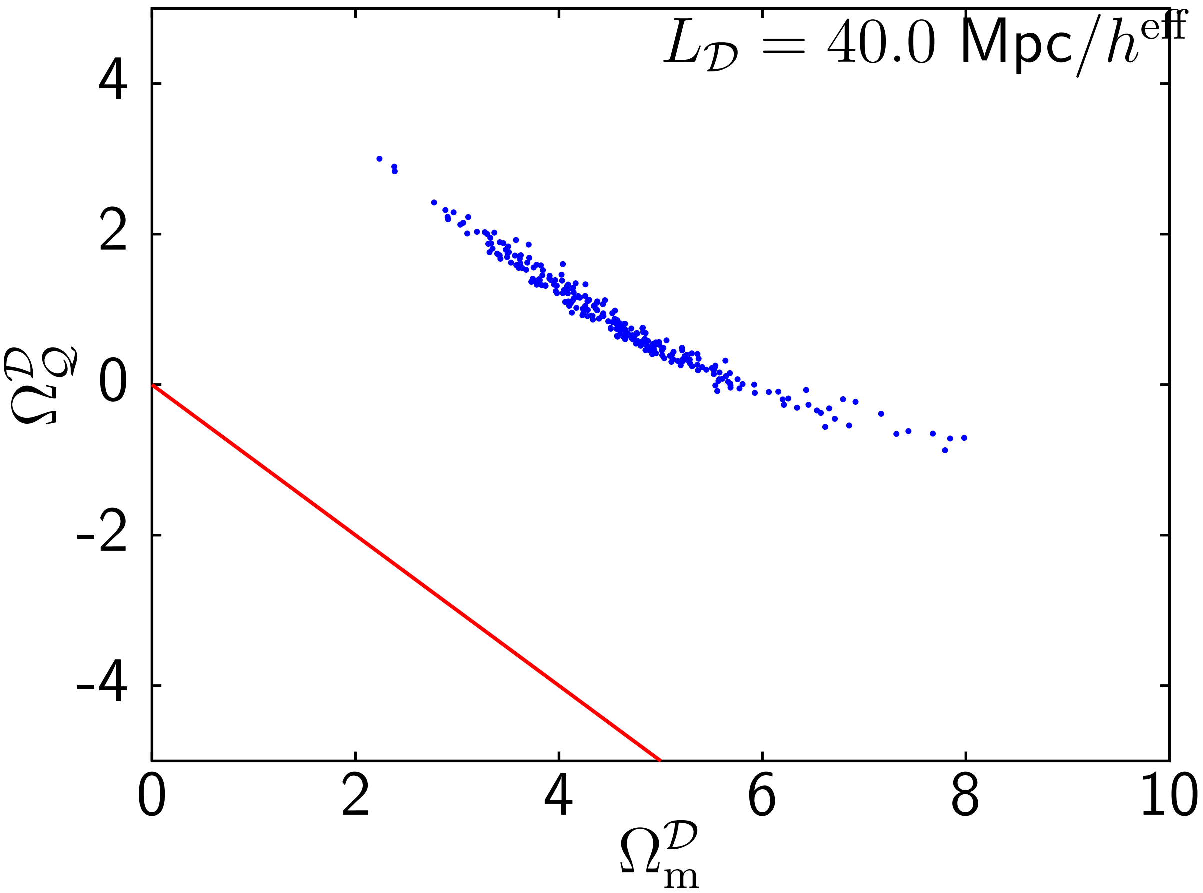
EdS

Does $\Omega_{\mathcal{Q}}^{\mathcal{D}}$ allow negative ${}^3\mathcal{R}$ at TA?



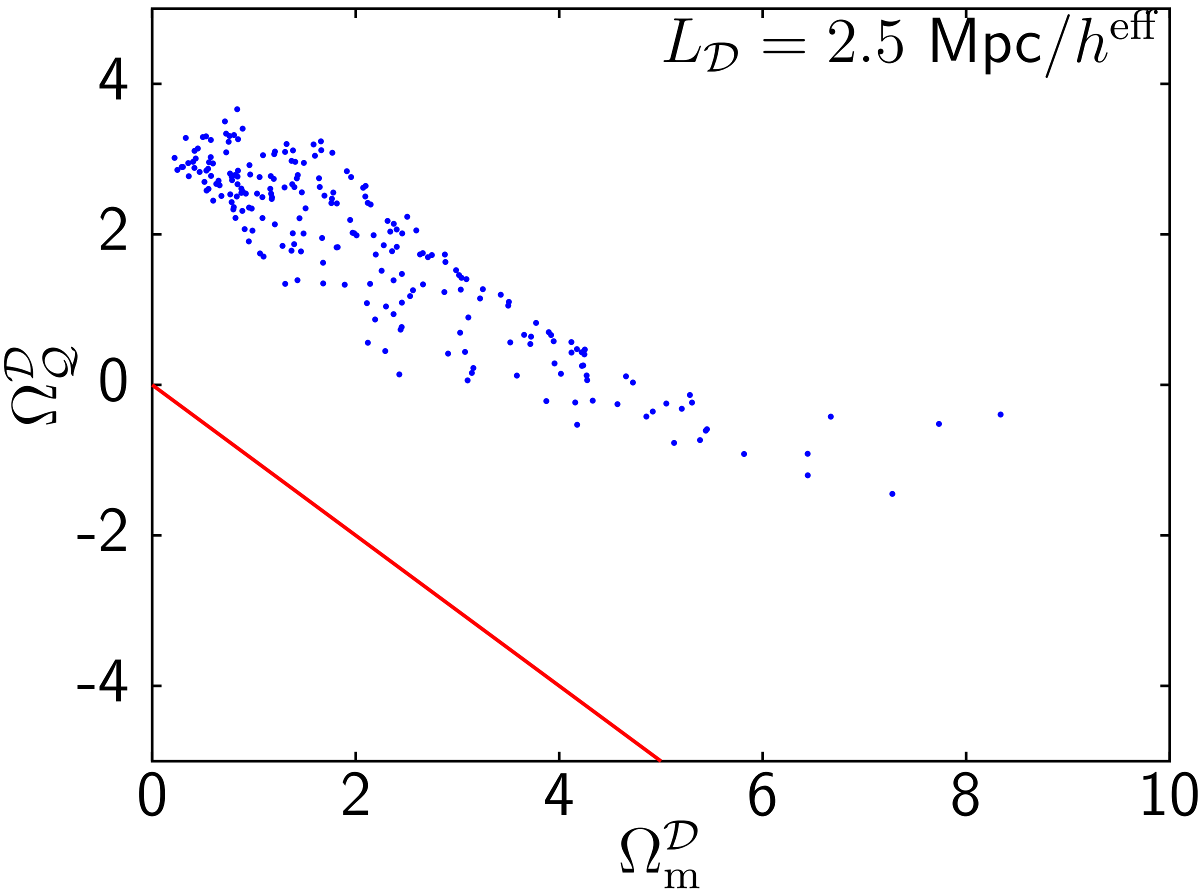
EdS

Does $\Omega_{\mathcal{Q}}^{\mathcal{D}}$ allow negative ${}^3\mathcal{R}$ at TA?



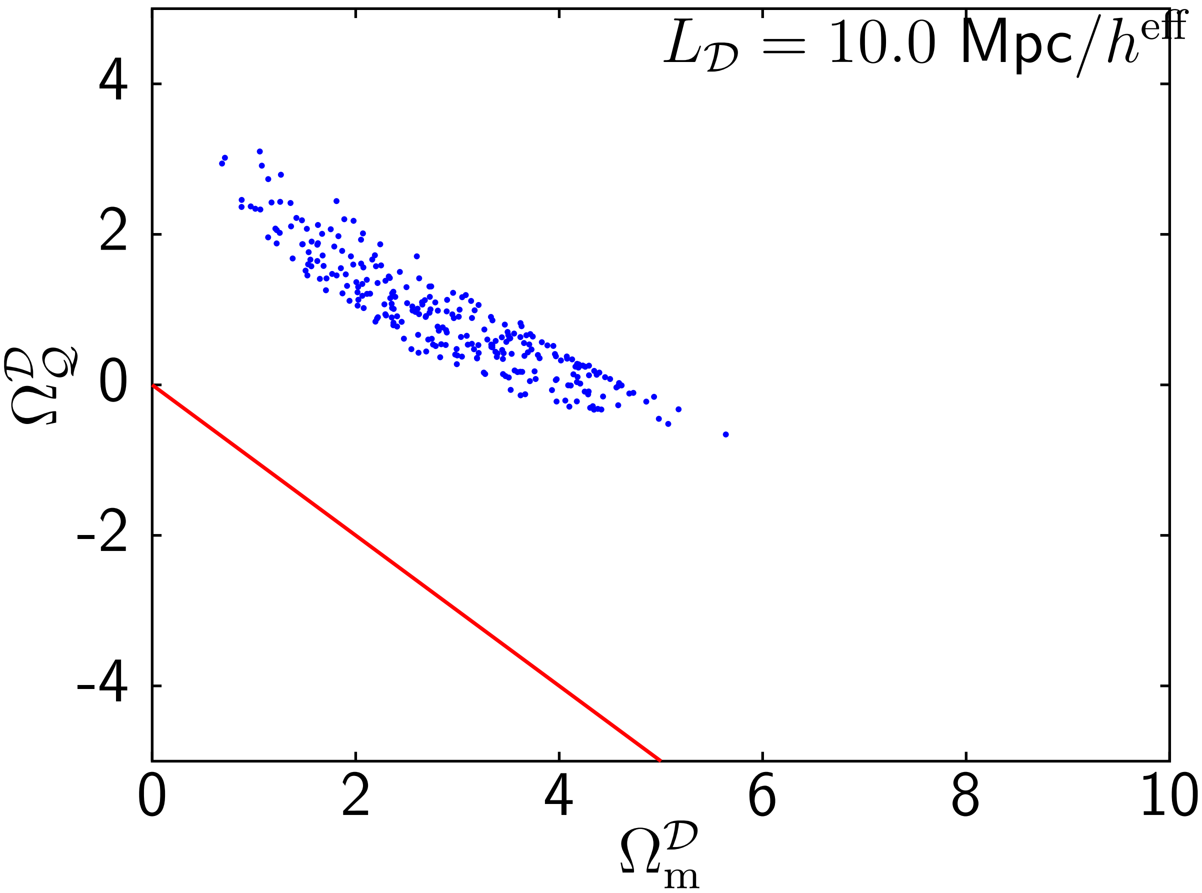
EdS

Does $\Omega_{\mathcal{Q}}^{\mathcal{D}}$ allow negative ${}^3\mathcal{R}$ at TA?



ΛCDM

Does $\Omega_{\mathcal{Q}}^{\mathcal{D}}$ allow negative ${}^3\mathcal{R}$ at TA?



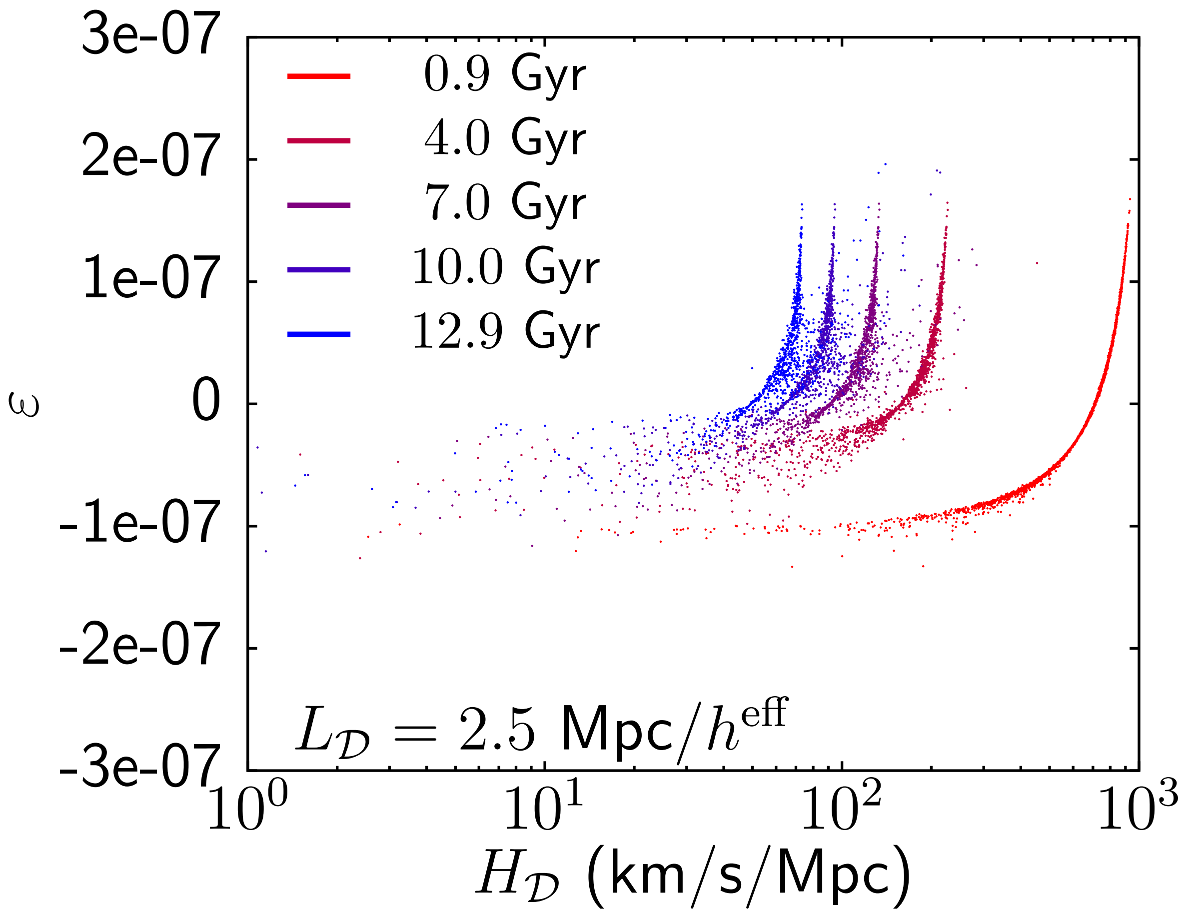
ΛCDM

Curvature-induced sp.geod. devn

Define:

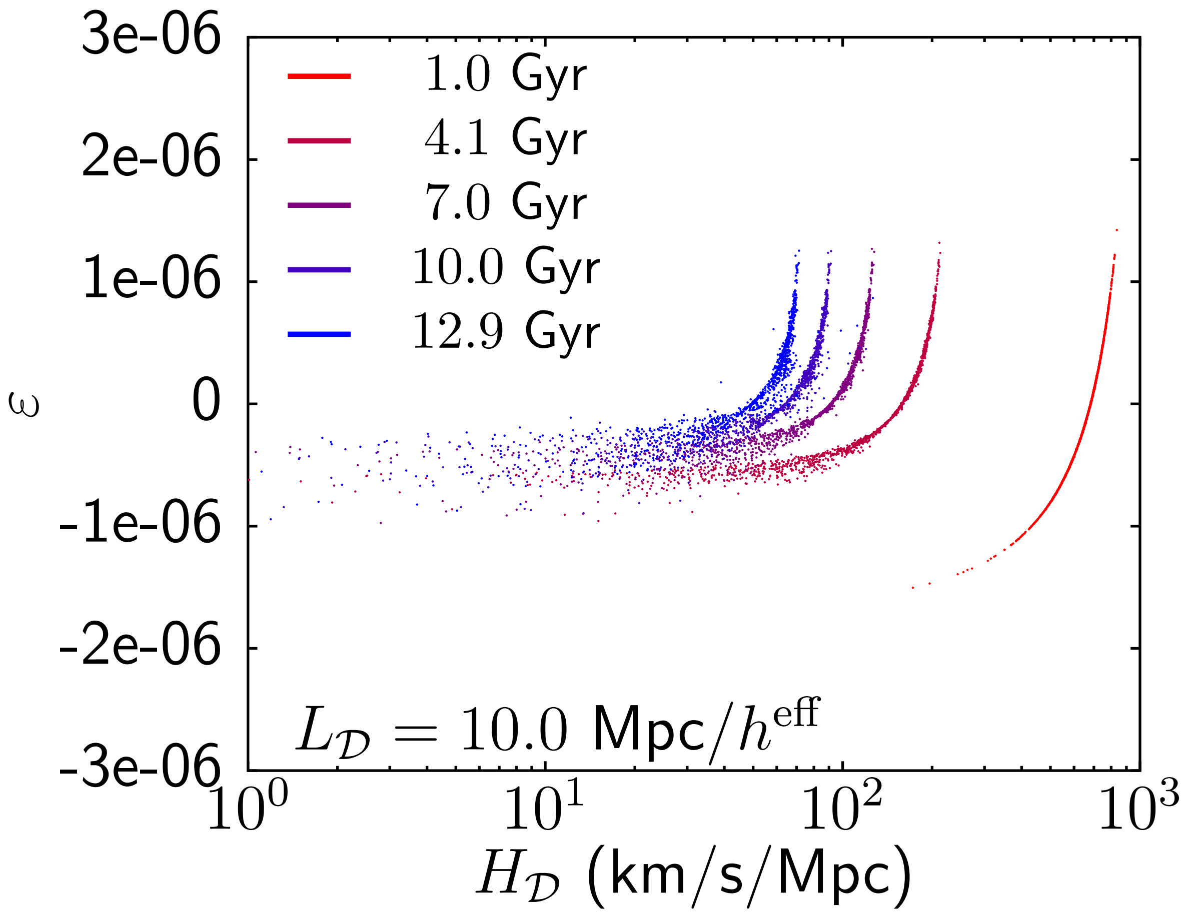
$$\varepsilon := \frac{1}{a_{\mathcal{D}}\, L_{\mathcal{D}}} \left(R_{\mathrm{C}}^{\mathrm{eff}} \sin \frac{a_{\mathcal{D}}\, L_{\mathcal{D}}}{R_{\mathrm{C}}^{\mathrm{eff}}} - a_{\mathcal{D}}\, L_{\mathcal{D}} \right)$$
$$= -\frac{1}{6}\, (a_{\mathcal{D}}\, H_{\mathrm{eff}}\, L_{\mathcal{D}})^2\, \Omega_{\mathcal{R}}^{\mathcal{D}} + \ldots\,,$$

Curvature-induced sp.geod. devn



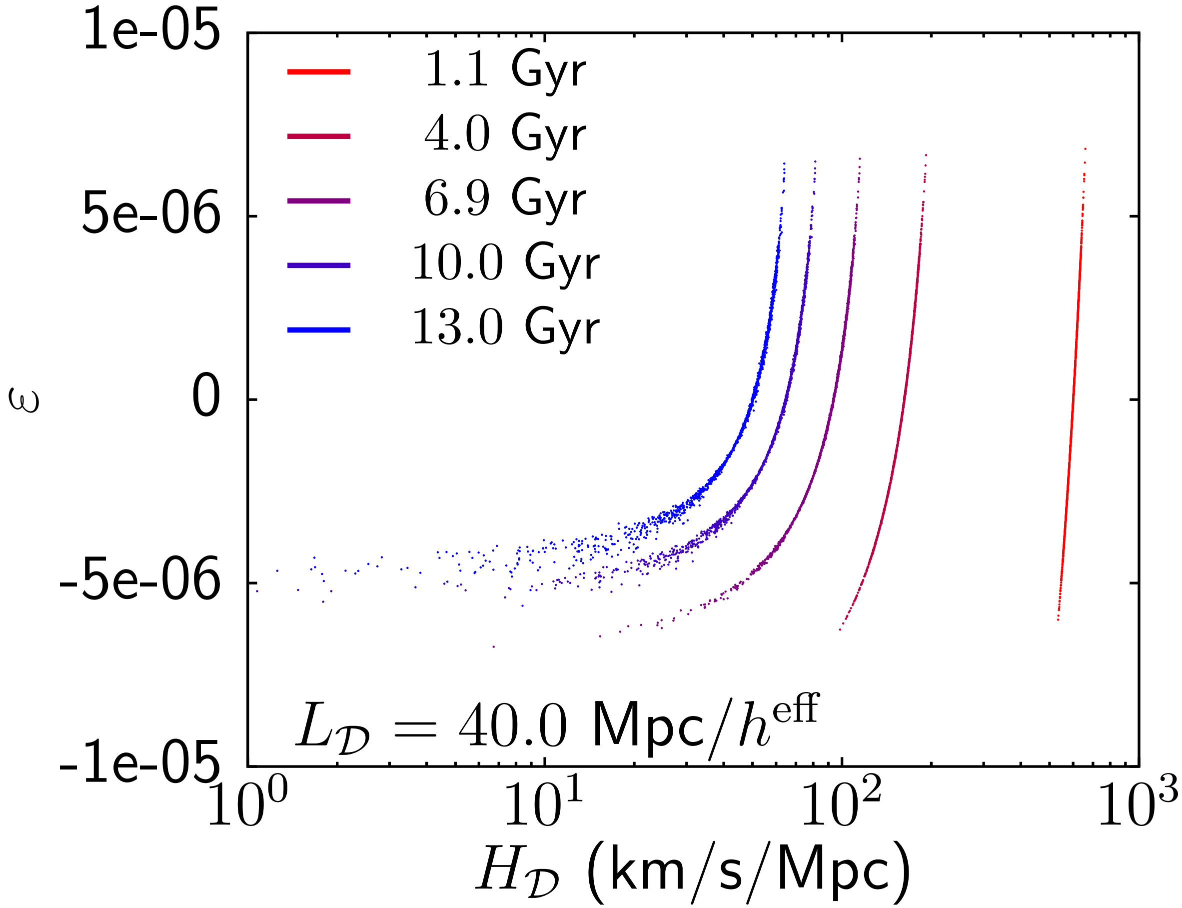
EdS

Curvature-induced sp.geod. devn



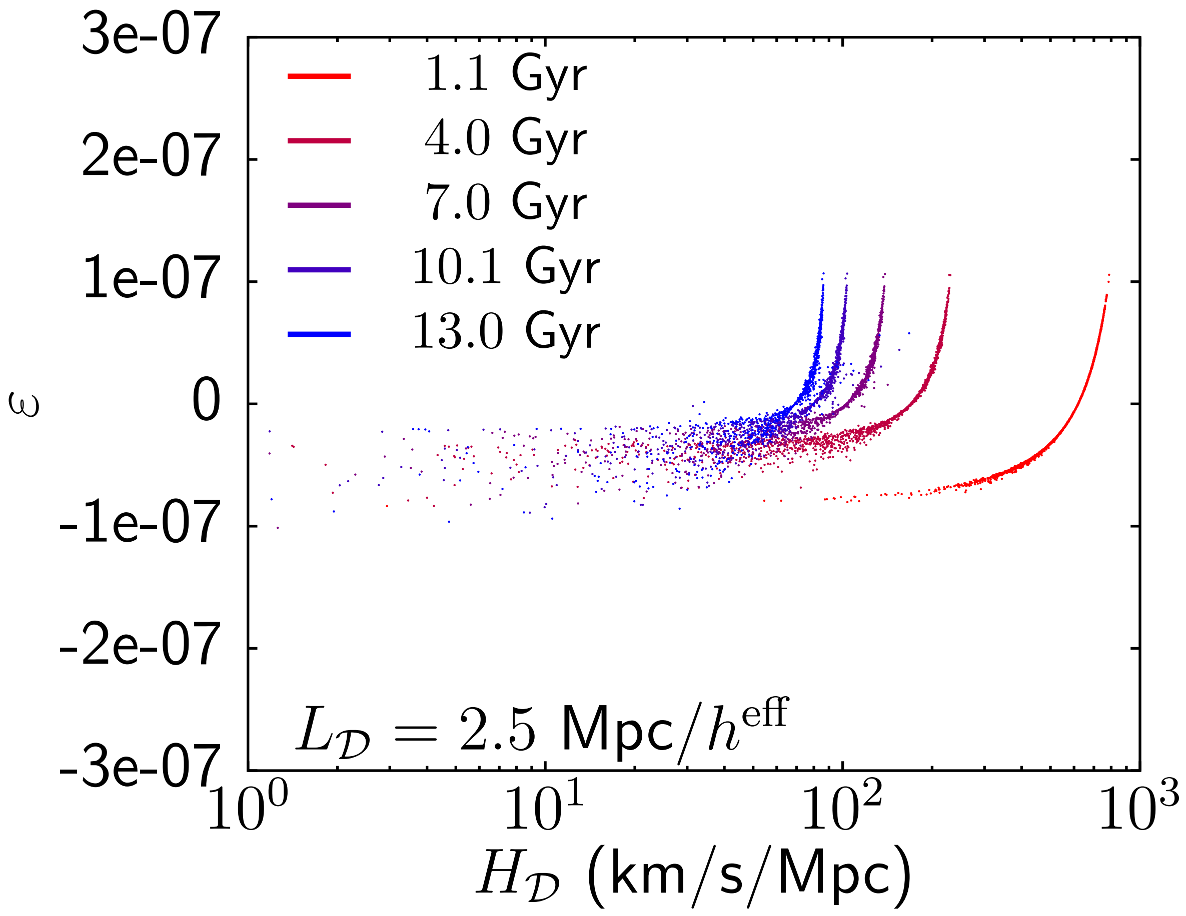
EdS

Curvature-induced sp.geod. devn



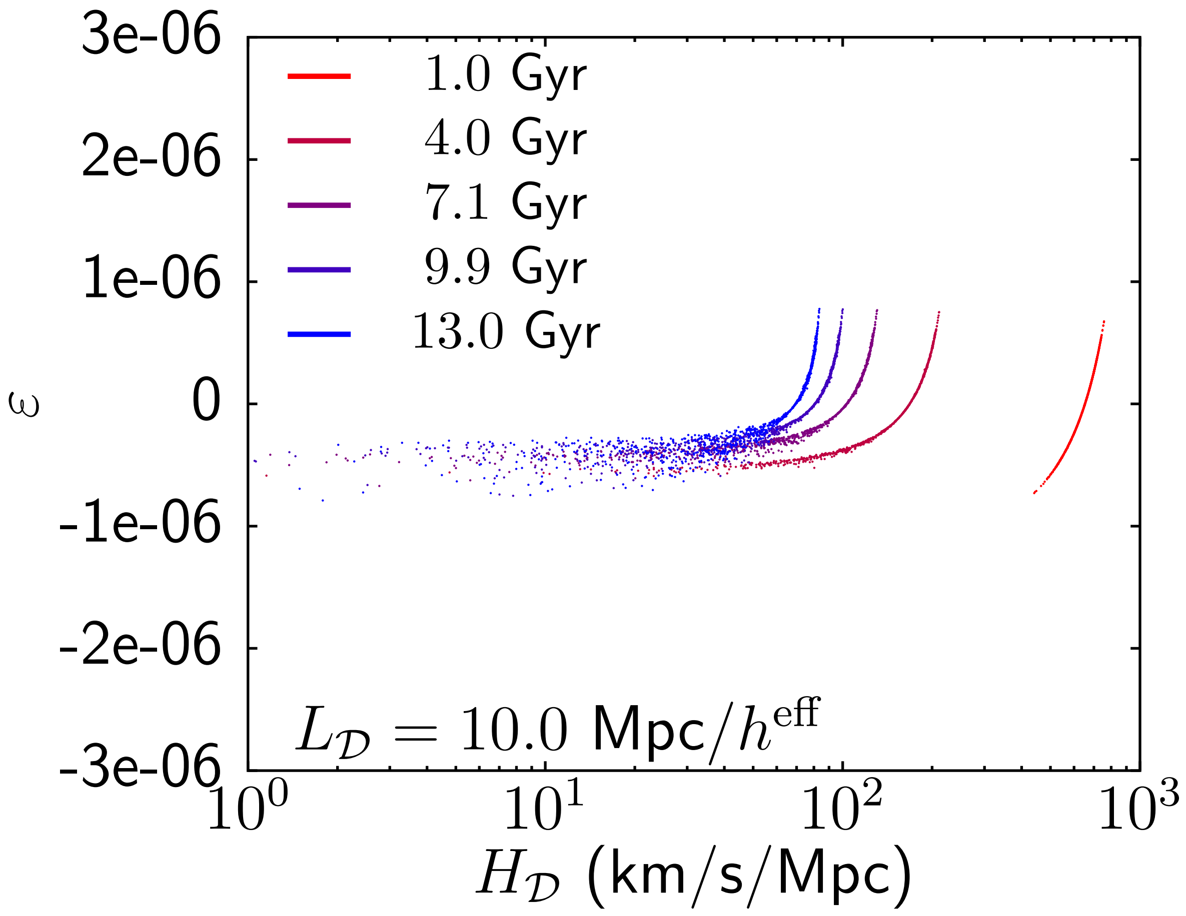
EdS

Curvature-induced sp.geod. devn



ΛCDM

Curvature-induced sp.geod. devn



ΛCDM

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information
- pointwise: spatial curvature must be positive at turnaround

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information
- pointwise: spatial curvature must be positive at turnaround
- plane-symmetric flat case: fundamental difference Newtonian vs GR

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information
- pointwise: spatial curvature must be positive at turnaround
- plane-symmetric flat case: fundamental difference Newtonian vs GR
- special case $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0, \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$ has critical values:

  $\Omega^{\mathcal{D}}_{\mathcal{R}} = -5\alpha^2, \;\; \Omega^{\mathcal{D}}_{\mathcal{Q}} = \alpha^2, \;\; \Omega^{\mathcal{D}}_{\mathrm{m}} = 4\alpha^2$

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information
- pointwise: spatial curvature must be positive at turnaround
- plane-symmetric flat case: fundamental difference Newtonian vs GR
- special case $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0, \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$ has critical values:
  $\Omega_{\mathcal{R}}^{\mathcal{D}} = -5\alpha^2, \;\; \Omega_{\mathcal{Q}}^{\mathcal{D}} = \alpha^2, \;\; \Omega_{\mathrm{m}}^{\mathcal{D}} = 4\alpha^2$
- averaged: spatial curvature at turnaround usually positive $(1 - P < 0.001)$

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information
- pointwise: spatial curvature must be positive at turnaround
- plane-symmetric flat case: fundamental difference Newtonian vs GR
- special case $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0, \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$ has critical values:
  $\Omega^{\mathcal{D}}_{\mathcal{R}} = -5\alpha^2,\ \ \Omega^{\mathcal{D}}_{\mathcal{Q}} = \alpha^2,\ \ \Omega^{\mathcal{D}}_{\mathrm{m}} = 4\alpha^2$
- averaged: spatial curvature at turnaround usually positive $(1 - P < 0.001)$
- effect on spatial geodesics is modest

Scalar averaging – QZA – curvature at turnaround

- turnaround argument gives non-perturbative, non–Newtonian-gauge curvature information
- pointwise: spatial curvature must be positive at turnaround
- plane-symmetric flat case: fundamental difference Newtonian vs GR
- special case $\langle \mathrm{II}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0, \langle \mathrm{III}_{\mathbf{i}} \rangle_{\mathcal{I}} = 0$ has critical values:

  $$\Omega^{\mathcal{D}}_{\mathcal{R}} = -5\alpha^2, \;\; \Omega^{\mathcal{D}}_{\mathcal{Q}} = \alpha^2, \;\; \Omega^{\mathcal{D}}_{\mathrm{m}} = 4\alpha^2$$
- averaged: spatial curvature at turnaround usually positive $(1 - P < 0.001)$
- effect on spatial geodesics is modest
- arXiv:1902.09064 (RO19) — special thanks Mourier + Vigneron
- *Scientific reproducibility — check all figures and tables yourself!*

Numerical relativity

- INHOMOG: QZA; fully free; fast arXiv:1706.06179 (R18) + arXiv:1902.09064 (RO19)
- GEVOLUTION: Poisson gauge; free if CLASS (non-free) excluded; fast arXiv:1604.06065, arXiv:1703.08585 (Adamek+16, 17)
- GRAMSES: waveless gravity; not yet public; licensing unclear — uses 2LPTic (non-free); arXiv:1905.08890 (Barrera–H+Li20)
- ET/FLRWSOLVER: full GR; fully free; arXiv:1611.05447 (Macpherson+17)
- COSMOGRAPH: full GR; unigrid version free, AMR version non-free; arXiv:1511.01106 (Mertens+16)
- comparison paper arXiv:2003.08014 (Adamek+20); the codes behave generally as expected; however...

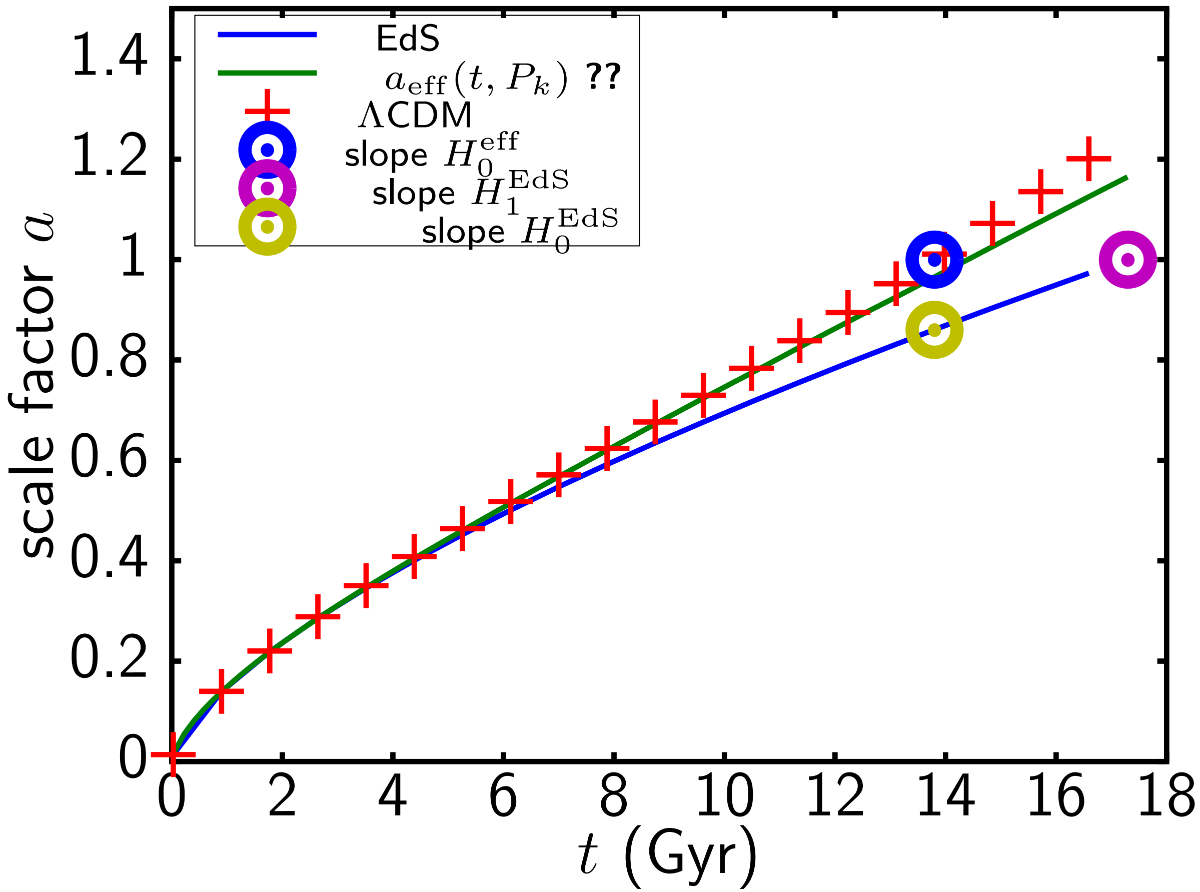
Numerical relativity

- INHOMOG: QZA; fully free; fast arXiv:1706.06179 (R18) + arXiv:1902.09064 (RO19)
- GEVOLUTION: Poisson gauge; free if CLASS (non-free) excluded; fast arXiv:1604.06065, arXiv:1703.08585 (Adamek+16, 17)
- GRAMSES: waveless gravity; not yet public; licensing unclear — uses 2LPTic (non-free); arXiv:1905.08890 (Barrera–H+Li20)
- ET/FLRWSOLVER: full GR; fully free; arXiv:1611.05447 (Macpherson+17)
- COSMOGRAPH: full GR; unigrid version free, AMR version non-free; arXiv:1511.01106 (Mertens+16)
- comparison paper arXiv:2003.08014 (Adamek+20); the codes behave generally as expected; however...

:(partially irreproducible results: non-free, faith-based package used for geodesic integration

initial conds: ΛCDM proxy

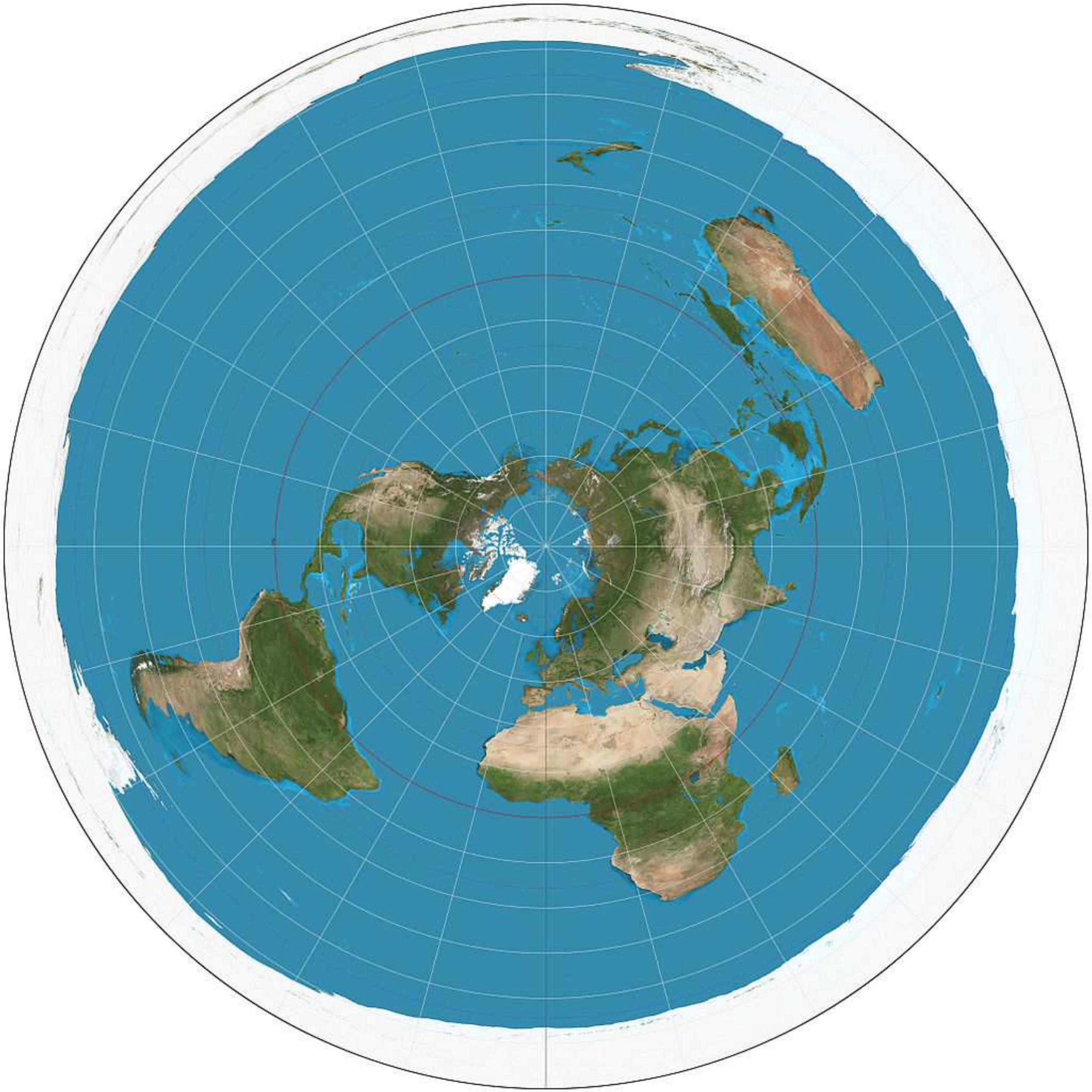
obsvns $\Rightarrow$ $H_0^{\rm eff}, H_1^{\rm EdS}, H_0^{\rm EdS}$ = 67.74, 37.7, 47.24 km/s/Mpc
(Roukema+2016 A&A arXiv:1608.06004)



EdS +
VQZA($P_k, L_{\mathcal{D}}$)
$\Rightarrow$
$\sim$ ΛCDM ?

RZA = relativistic Zel'dovich approximation (PRD arXiv:1303.6193)
VQZA: N-body init condns + RZA : A&A arXiv:1706.06179

the homogeneous model



the homogeneous model

- standard model: ΛCDM $(\Omega_{\mathrm{m0}} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous

the homogeneous model

- standard model: ΛCDM $(\Omega_{\mathrm{m}0} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous
- simpler model: EdS $(\Omega_{\mathrm{m}0} = 1, \Omega_{\Lambda 0} = 0)$ homogeneous

the homogeneous model

- standard model: ΛCDM $(\Omega_{\mathrm{m0}} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous
- simpler model: EdS $(\Omega_{\mathrm{m0}} = 1, \Omega_{\Lambda 0} = 0)$ homogeneous

$=$ background expands relativistically (Einstein)

the homogeneous model

- standard model: ΛCDM $(\Omega_{\mathrm{m0}} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous
- simpler model: EdS $(\Omega_{\mathrm{m0}} = 1, \Omega_{\Lambda 0} = 0)$ homogeneous

$=$ background expands relativistically (Einstein)

$+$ linear density perturbations $\delta := \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$, $|\delta| \ll 1$

the homogeneous model

- standard model: ΛCDM ($\Omega_{\mathrm{m0}} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68$) homogeneous
- simpler model: EdS ($\Omega_{\mathrm{m0}} = 1, \Omega_{\Lambda 0} = 0$) homogeneous

$=$ background expands relativistically (Einstein)

  $+$ linear density perturbations $\delta := \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$, $|\delta| \ll 1$

  $+$ large-scale structure, galaxy clusters, galaxies $\delta > \Delta_{\mathrm{vir}}$

the homogeneous model

- standard model: ΛCDM $(\Omega_{\rm m0} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous
- simpler model: EdS $(\Omega_{\rm m0} = 1, \Omega_{\Lambda 0} = 0)$ homogeneous

$=$ background expands relativistically (Einstein)

$+$ linear density perturbations $\delta := \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$, $|\delta| \ll 1$

$+$ large-scale structure, galaxy clusters, galaxies $\delta > \Delta_{\rm vir} \gg 1$ (Newton)

the homogeneous model

- standard model: ΛCDM ($\Omega_{\mathrm{m0}} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68$) homogeneous
- simpler model: EdS ($\Omega_{\mathrm{m0}} = 1, \Omega_{\Lambda 0} = 0$) homogeneous

$=$ background expands relativistically (Einstein)

$+$ linear density perturbations $\delta := \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$, $|\delta| \ll 1$

$+$ large-scale structure, galaxy clusters, galaxies $\delta > \Delta_{\mathrm{vir}} \gg 1$ (Newton)

$+$ voids $-1 \lesssim \delta \ll 0$ (Newton)

the homogeneous model

- standard model: ΛCDM $(\Omega_{\mathrm{m0}} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous
- simpler model: EdS $(\Omega_{\mathrm{m0}} = 1, \Omega_{\Lambda 0} = 0)$ homogeneous

$=$ background expands relativistically (Einstein)

$+$ linear density perturbations $\delta := \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$, $|\delta| \ll 1$

$+$ large-scale structure, galaxy clusters, galaxies $\delta > \Delta_{\mathrm{vir}} \gg 1$ (Newton)

$+$ voids $-1 \lesssim \delta \ll 0$ (Newton)

- when/where should ΛCDM, EdS fail?

the homogeneous model

- standard model: ΛCDM $(\Omega_{\rm m0} \approx 0.32, \Omega_{\Lambda 0} \approx 0.68)$ homogeneous
- simpler model: EdS $(\Omega_{\rm m0} = 1, \Omega_{\Lambda 0} = 0)$ homogeneous

$=$ background expands relativistically (Einstein)

$+$ linear density perturbations $\delta := \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$, $|\delta| \ll 1$

$+$ large-scale structure, galaxy clusters, galaxies $\delta > \Delta_{\rm vir} \gg 1$ (Newton)

$+$ voids $-1 \lesssim \delta \ll 0$ (Newton)

- Newton $\neq$ Einstein $\Rightarrow$ ΛCDM, EdS should fail at $\ll 3\ h^{-1}$ Gpc and $z < 3$

beyond the homogeneous model

- what parameter can measure the expected failure of the homogeneous models?

beyond the homogeneous model

- $f_{\rm vir}(z) :=$ fraction of virialised matter (large-scale structure, clusters, galaxies)

beyond the homogeneous model

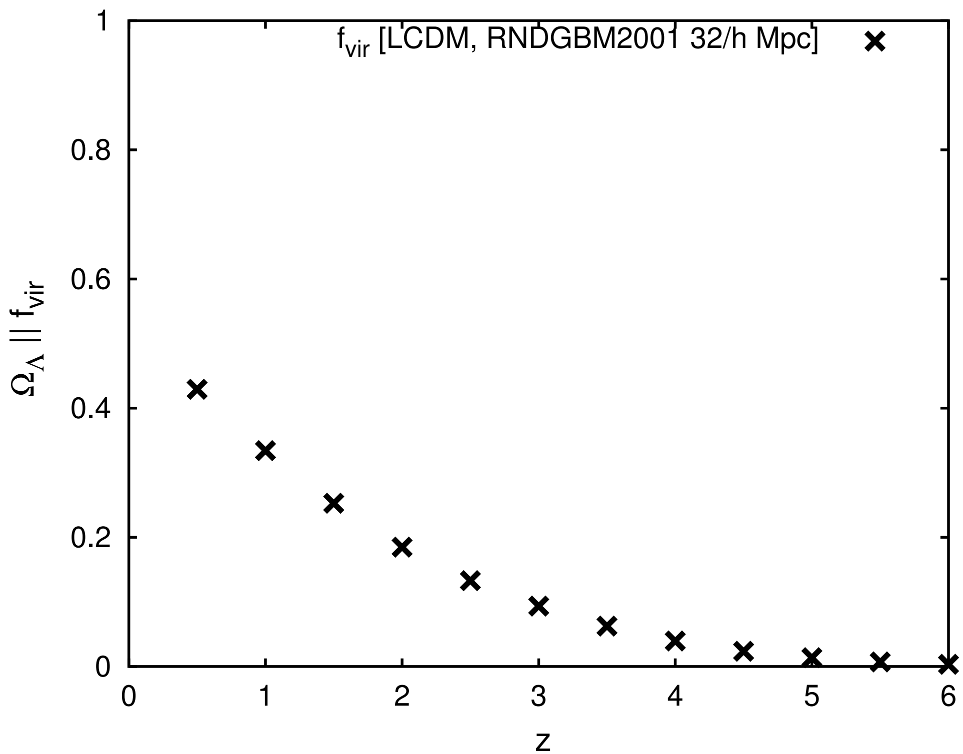
- $f_{\rm vir}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid

beyond the homogeneous model

- $f_{\mathrm{vir}}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid
- $f_{\mathrm{vir}}(z) \gg 0.01 \Rightarrow \Lambda$CDM, EdS $\sim$ fail

beyond the homogeneous model

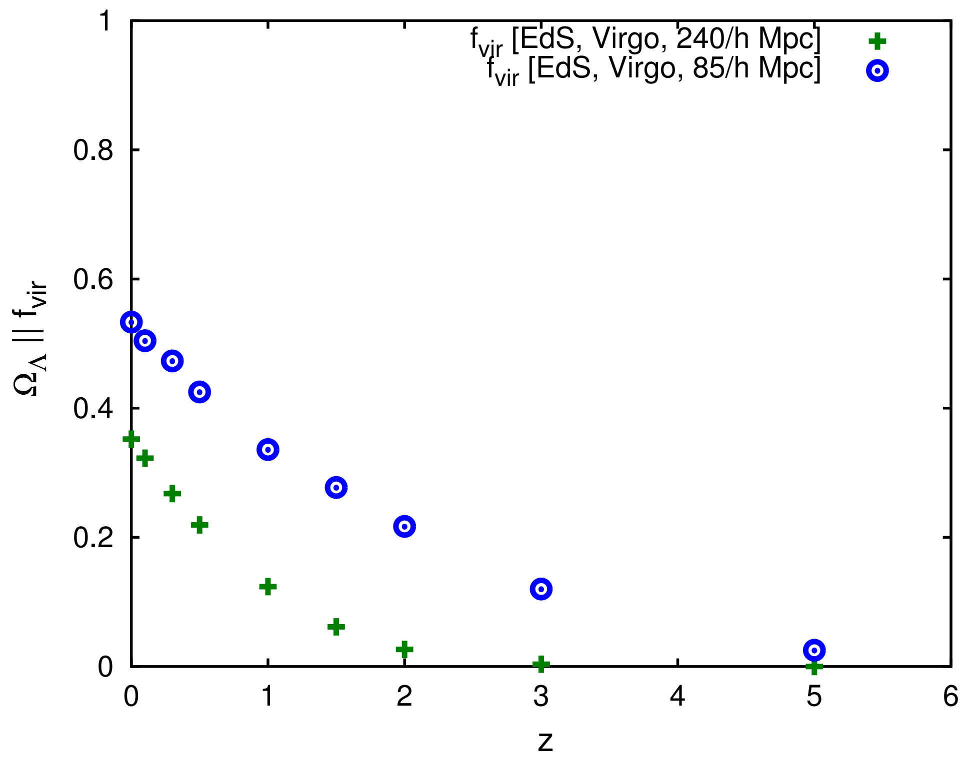
- $f_{\rm vir}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid
- $f_{\rm vir}(z) \gg 0.01 \Rightarrow \Lambda$CDM, EdS $\sim$ fail



Roukema, Ninin, Devriendt, Bouchet, Guiderdoni & Mamon (2001)

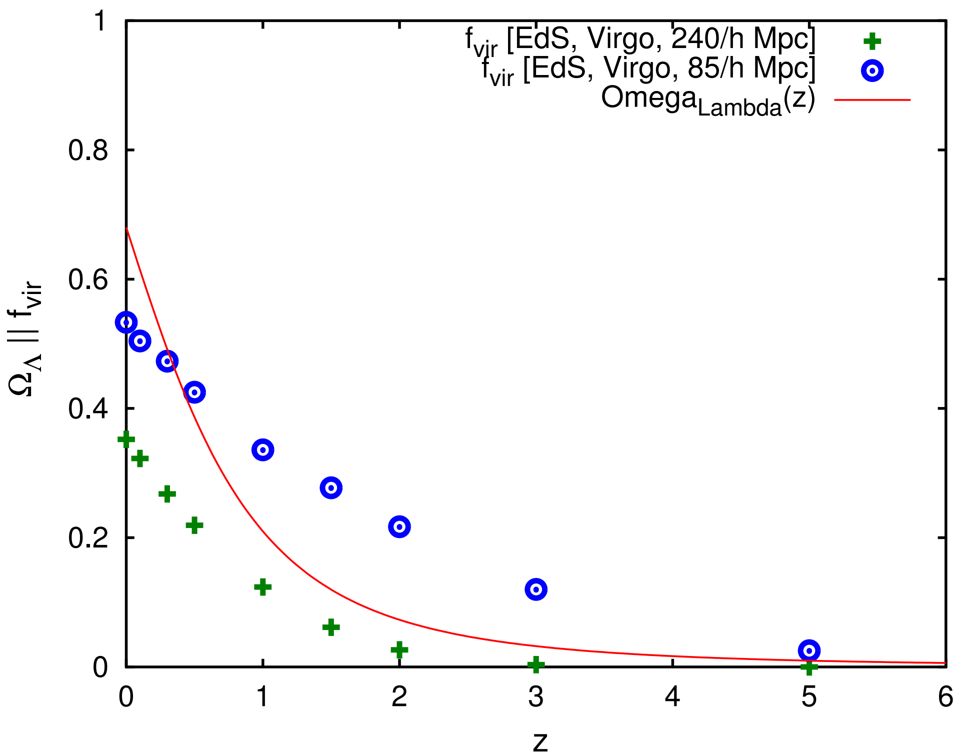
beyond the homogeneous model

- $f_{\mathrm{vir}}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid
- $f_{\mathrm{vir}}(z) \gg 0.01 \Rightarrow \Lambda$CDM, EdS $\sim$ fail



beyond the homogeneous model

- $f_{\rm vir}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid
- $f_{\rm vir}(z) \gg 0.01 \Rightarrow \Lambda$CDM, EdS $\sim$ fail



Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043

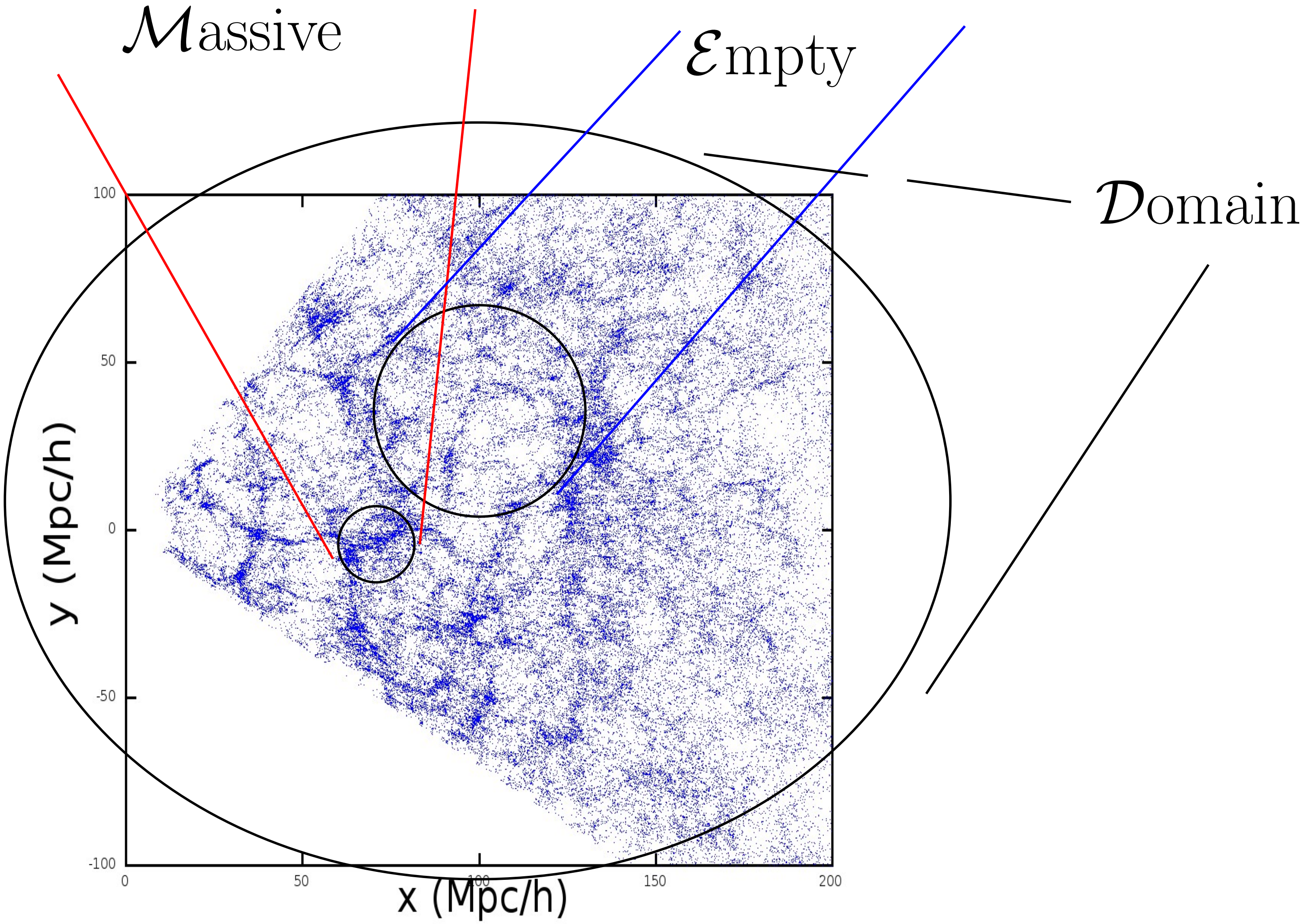
beyond the homogeneous model

- $f_{\rm vir}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid
- $f_{\rm vir}(z) \gg 0.01 \Rightarrow \Lambda$CDM, EdS $\sim$ fail
- $f_{\rm vir}(z)$ strongly correlates with $\Omega_\Lambda(z)$
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043;
  Roukema 2013, IJMPD, 22, 1341018;

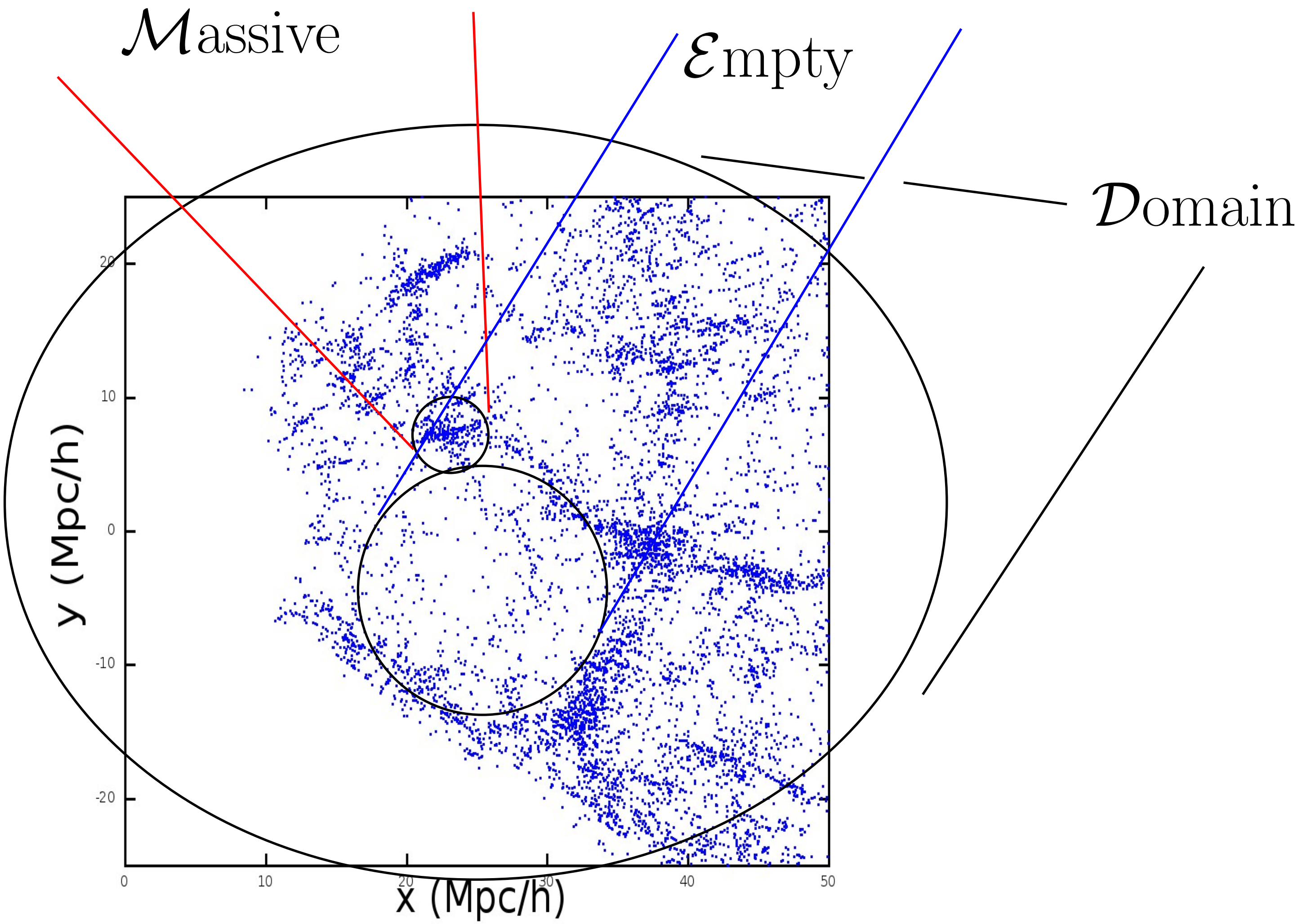
beyond the homogeneous model

- $f_{\rm vir}(z) \ll 1 \Rightarrow \Lambda$CDM, EdS $\sim$ valid
- $f_{\rm vir}(z) \gg 0.01 \Rightarrow \Lambda$CDM, EdS $\sim$ fail
- $f_{\rm vir}(z)$ strongly correlates with $\Omega_\Lambda(z)$
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043;
  Roukema 2013, IJMPD, 22, 1341018;
- Is $\Omega_\Lambda(z)$ an artefact of ignoring virialisation?

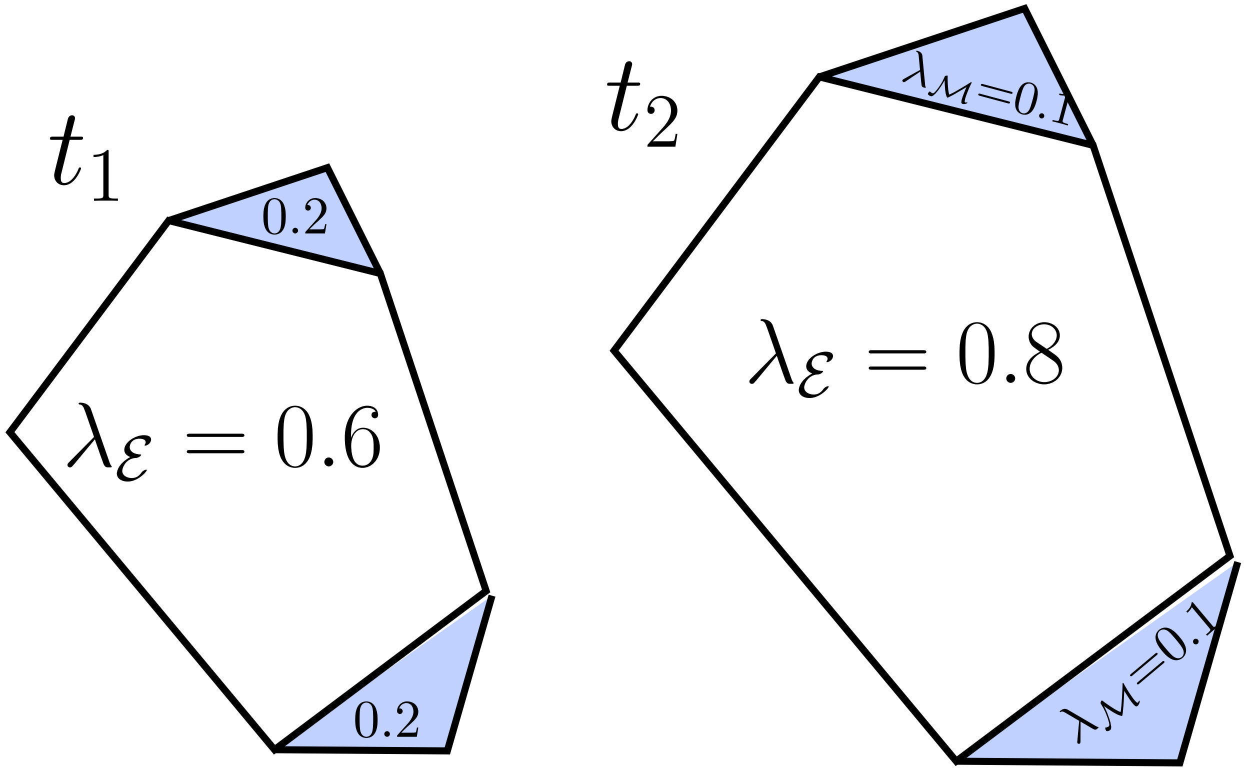
Intuition: projections



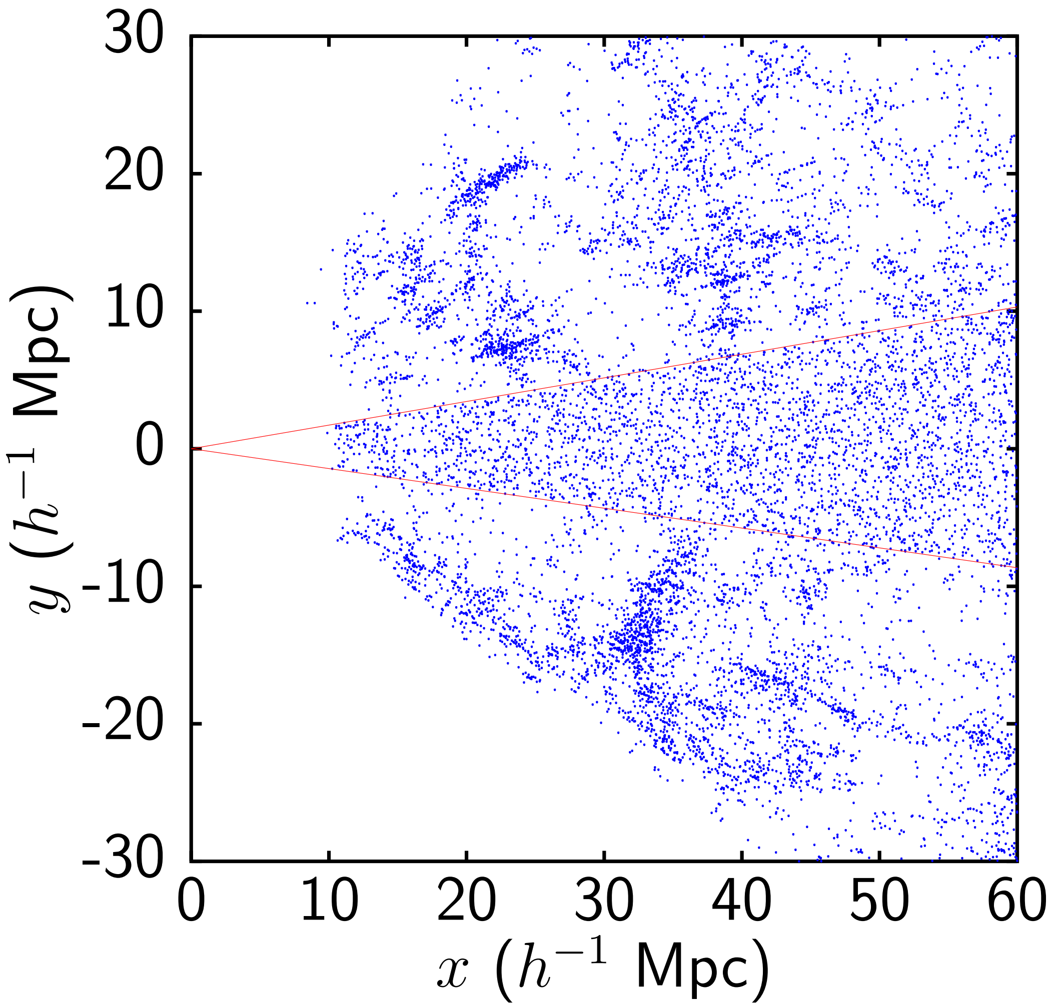
Intuition: projections



Intuition: projections



Intuition: extrapolation



EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043
- Newton : constant spatial curvature of structures + voids

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043

- Newton : constant spatial curvature of structures + voids Friedmann equation:
  $$\Omega_{\rm m} + \Omega_\Lambda + \Omega_{\rm k} = 1.$$

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043
- Newton : constant spatial curvature of structures + voids Friedmann equation:
  $$\Omega_{\mathrm{m}} + \Omega_\Lambda + \Omega_{\mathrm{k}} = 1.$$
- Einstein : hyperbolic voids
  Buchert : volume-weighted means

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043
- Newton : constant spatial curvature of structures + voids Friedmann equation:
  $$\Omega_{\mathrm{m}} + \Omega_\Lambda + \Omega_{\mathrm{k}} = 1.$$
- Einstein : hyperbolic voids
  Buchert : volume-weighted means
  generalised Friedmann equation (Hamiltonian constraint):
  $$\Omega_{\mathrm{m}}^{\mathcal{F}} + \Omega_\Lambda^{\mathcal{F}} + \Omega_{\mathcal{R}}^{\mathcal{F}} + \Omega_{\mathcal{Q}}^{\mathcal{F}} = \frac{H_{\mathcal{F}}^2}{H_{\mathcal{D}}^2} \, , \quad \text{where } \mathcal{F} \in \{\mathcal{M}, \mathcal{E}, \mathcal{D}\}$$

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043
- Newton : constant spatial curvature of structures + voids Friedmann equation:
  $$\Omega_{\mathrm{m}} + \Omega_\Lambda + \Omega_{\mathrm{k}} = 1.$$
- Einstein : hyperbolic voids
  Buchert : volume-weighted means on domains $\mathcal{D} := \mathcal{M} \cup \mathcal{E}$
  generalised Friedmann equation (Hamiltonian constraint):
  $$\Omega_{\mathrm{m}}^{\mathcal{F}} + \Omega_\Lambda^{\mathcal{F}} + \Omega_{\mathcal{R}}^{\mathcal{F}} + \Omega_{\mathcal{Q}}^{\mathcal{F}} = \frac{H_{\mathcal{F}}^2}{H_{\mathcal{D}}^2}\,, \quad \text{where } \mathcal{F} \in \{\mathcal{M}, \mathcal{E}, \mathcal{D}\}$$
- assume $\Omega_\Lambda^{\mathcal{F}} = 0$

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043
- Newton : constant spatial curvature of structures + voids Friedmann equation:
  $$\Omega_{\mathrm{m}} + \Omega_{\Lambda} + \Omega_{\mathrm{k}} = 1.$$
- Einstein : hyperbolic voids
  Buchert : volume-weighted means on domains $\mathcal{D} := \mathcal{M} \cup \mathcal{E}$
  generalised Friedmann equation (Hamiltonian constraint):
  $$\Omega_{\mathrm{m}}^{\mathcal{F}} + \Omega_{\mathcal{R}}^{\mathcal{F}} + \Omega_{\mathcal{Q}}^{\mathcal{F}} = \frac{H_{\mathcal{F}}^2}{H_{\mathcal{D}}^2}\,, \quad \text{where } \mathcal{F} \in \{\mathcal{M}, \mathcal{E}, \mathcal{D}\}$$
- assume $\Omega_{\Lambda}^{\mathcal{F}} = 0$
- assume $|\Omega_{\mathcal{Q}}^{\mathcal{F}}| \ll 1$

EdS + virialisation approx

- EdS + virialisation approximation:
  Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043
- Newton : constant spatial curvature of structures + voids Friedmann equation:
  $$\Omega_{\mathrm{m}} + \Omega_\Lambda + \Omega_{\mathrm{k}} = 1.$$
- Einstein : hyperbolic voids
  Buchert : volume-weighted means on domains $\mathcal{D} := \mathcal{M} \cup \mathcal{E}$
  generalised Friedmann equation (Hamiltonian constraint):
  $$\Omega_{\mathrm{m}}^{\mathcal{F}} + \Omega_{\mathcal{R}}^{\mathcal{F}} = \frac{H_{\mathcal{F}}^2}{H_{\mathcal{D}}^2}\,, \quad \text{where } \mathcal{F} \in \{\mathcal{M}, \mathcal{E}, \mathcal{D}\}$$
- assume $\Omega_\Lambda^{\mathcal{F}} = 0$
- assume $|\Omega_{\mathcal{Q}}^{\mathcal{F}}| \ll 1$

EdS + virialisation

- early: V(voids) : V(structures) $\sim 1 - f_{\mathrm{vir}} : f_{\mathrm{vir}}$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir}$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} : f_{\mathrm{vir}}/\Delta_{\mathrm{vir}}$
- define $\lambda_{\mathcal{M}} := f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} \ll 1$

  $\Rightarrow$ voids dominate any volume average

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\mathcal{D}} = \lambda_{\mathcal{M}}\Omega_{\rm m}^{\mathcal{M}} + (1-\lambda_{\mathcal{M}})\,\Omega_{\rm m}^{\mathcal{E}}$$
$$H^{\mathcal{D}} = \lambda_{\mathcal{M}}H^{\mathcal{M}} + (1-\lambda_{\mathcal{M}})\,H^{\mathcal{E}}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} : f_{\mathrm{vir}}/\Delta_{\mathrm{vir}}$
- define $\lambda_{\mathcal{M}} := f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} \ll 1$

  $\Rightarrow$ voids dominate any volume average

$$\Omega_{\mathrm{m}}^{\mathcal{E}}(z) \approx \frac{(1 - f_{\mathrm{vir}})\, \Omega_{\mathrm{m}}^{\mathrm{bg}}}{(H_{\mathrm{eff}}/H^{\mathrm{bg}})^2}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} : f_{\mathrm{vir}}/\Delta_{\mathrm{vir}}$
- define $\lambda_{\mathcal{M}} := f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} \ll 1$

  $\Rightarrow$ voids dominate any volume average

$$\Omega_{\mathrm{m}}^{\mathcal{M}}(z) \approx \frac{\Delta_{\mathrm{vir}}\, \Omega_{\mathrm{m}}^{\mathrm{bg}}}{(H_{\mathrm{eff}}/H^{\mathrm{bg}})^2}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} : f_{\mathrm{vir}}/\Delta_{\mathrm{vir}}$
- define $\lambda_{\mathcal{M}} := f_{\mathrm{vir}}/\Delta_{\mathrm{vir}} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\mathrm{m}}^{\mathrm{eff}}(z) := \Omega_{\mathrm{m}}^{\mathcal{D}}(z) \approx \qquad \Omega_{\mathrm{m}}^{\mathrm{bg}}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

  $\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

  $\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}(z)$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}^{\rm com}(z)\, a^{-1}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}^{\rm com}(0)\,\frac{f_{\rm vir}(z)}{f_{\rm vir}(0)}\,a^{-1}$$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}^{\rm com}(0)\,\frac{f_{\rm vir}(z)}{f_{\rm vir}(0)}\,a^{-1}$$

$$H_{\rm pec}^{\rm com}(0) := \frac{2v_{\rm infall}}{D_{\rm void}} \approx \frac{4\sigma_v}{D_{\rm void}} = 36 \pm 3 \text{ km/s/Mpc}$$

$\Rightarrow$

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}^{\rm com}(0)\,\frac{f_{\rm vir}(z)}{f_{\rm vir}(0)}\,a^{-1}$$

$$H_{\rm pec}^{\rm com}(0) := \frac{2v_{\rm infall}}{D_{\rm void}} \approx \frac{4\sigma_v}{D_{\rm void}} = 36 \pm 3 \text{ km/s/Mpc}$$

$\Rightarrow \Omega_{\rm m}^{\rm eff}(z)$ drops when $f_{\rm vir}(z)$ grows

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}^{\rm com}(0)\,\frac{f_{\rm vir}(z)}{f_{\rm vir}(0)}\,a^{-1}$$

$$H_{\rm pec}^{\rm com}(0) := \frac{2v_{\rm infall}}{D_{\rm void}} \approx \frac{4\sigma_v}{D_{\rm void}} = 36 \pm 3 \text{ km/s/Mpc}$$

$\Rightarrow \Omega_{\rm m}^{\rm eff}(z)$ drops when z drops

EdS + virialisation

- late : V(voids) : V(structures) $\sim 1 - f_{\rm vir}/\Delta_{\rm vir} : f_{\rm vir}/\Delta_{\rm vir}$
- define $\lambda_{\mathcal{M}} := f_{\rm vir}/\Delta_{\rm vir} \ll 1$

$\Rightarrow$ voids dominate any volume average

$$\Omega_{\rm m}^{\rm eff}(z) := \Omega_{\rm m}^{\mathcal{D}}(z) \approx \frac{\Omega_{\rm m}^{\rm bg}}{(H_{\rm eff}/H^{\rm bg})^2}$$

$$H_{\rm eff}(z) \approx H(z) + H_{\rm pec}^{\rm com}(0)\,\frac{f_{\rm vir}(z)}{f_{\rm vir}(0)}\,a^{-1}$$

$$H_{\rm pec}^{\rm com}(0) := \frac{2v_{\rm infall}}{D_{\rm void}} \approx \frac{4\sigma_v}{D_{\rm void}} = 36 \pm 3 \text{ km/s/Mpc}$$

$\Rightarrow$ curvature more negative when z drops

effective parameters

$$\Omega_{\mathcal{R}}^{\mathrm{eff}}(z) = 1 - \Omega_{\mathrm{m}}^{\mathrm{eff}}(z)$$

effective parameters

$$\Omega_{\mathcal{R}}^{\rm eff}(z) = 1 - \Omega_{\rm m}^{\rm eff}(z)$$

$$H_{\rm eff}(0) = \ 74.0 \pm 1.6\,{\rm km/s/Mpc}$$

(Riess et al. 2011; Freedman et al. 2012)

effective parameters

$$\Omega_{\mathcal{R}}^{\mathrm{eff}}(z) = 1 - \Omega_{\mathrm{m}}^{\mathrm{eff}}(z)$$

$$H_{\mathrm{eff}}(0) = \; 74.0 \pm 1.6\,\mathrm{km/s/Mpc}$$

(Riess et al. 2011; Freedman et al. 2012)

$$R_{\mathrm{C}}^{\mathrm{eff}}(z) = \frac{c}{a H_{\mathrm{eff}}(z)\,\sqrt{\Omega_{\mathcal{R}}^{\mathrm{eff}}(z)}}$$

effective metric

$$ds^2 = -dt^2 + a^2(t)\left[d\chi^{\mathrm{eff}\,2} + R_{\mathrm{C}}^{\mathrm{eff}\,2}\left(\sinh^2\frac{\chi^{\mathrm{eff}}}{R_{\mathrm{C}}^{\mathrm{eff}}}\right)\left(d\theta^2 + \cos^2\theta\, d\phi^2\right)\right],$$

effective metric

$$\mathrm{d}s^2 \;=\; -\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}{\chi^{\mathrm{eff}}}^2 + {R_{\mathrm{C}}^{\mathrm{eff}}}^2 \left(\sinh^2 \frac{\chi^{\mathrm{eff}}}{R_{\mathrm{C}}^{\mathrm{eff}}}\right)(\mathrm{d}\theta^2 + \cos^2\theta\,\mathrm{d}\phi^2)\right],$$

$$\mathrm{d}\chi^{\mathrm{eff}}(z) := \frac{c}{a^2\,H_{\mathrm{eff}}(z)}\,\mathrm{d}a$$

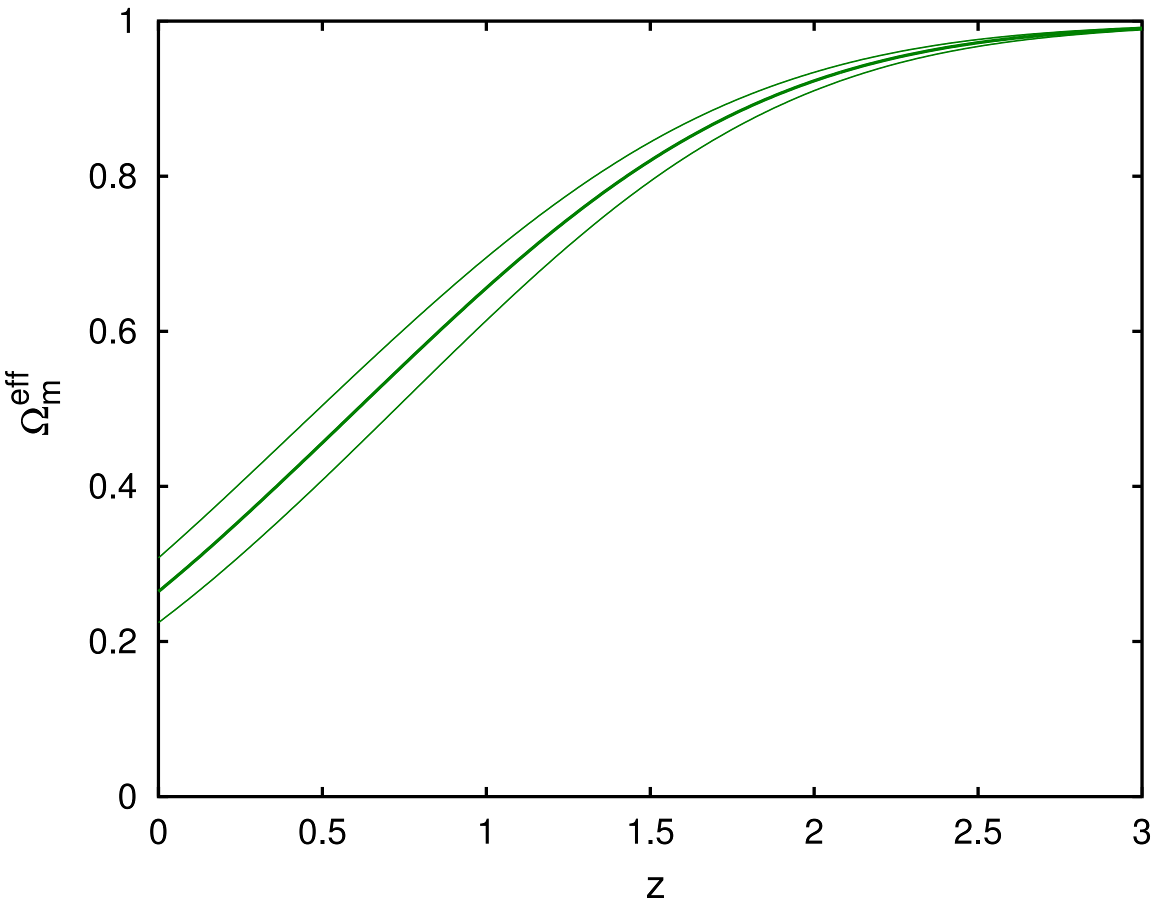
effective metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}\chi^{\mathrm{eff}\,2} + R_{\mathrm{C}}^{\mathrm{eff}\,2}\left(\sinh^2\frac{\chi^{\mathrm{eff}}}{R_{\mathrm{C}}^{\mathrm{eff}}}\right)(\mathrm{d}\theta^2 + \cos^2\theta\,\mathrm{d}\phi^2)\right],$$

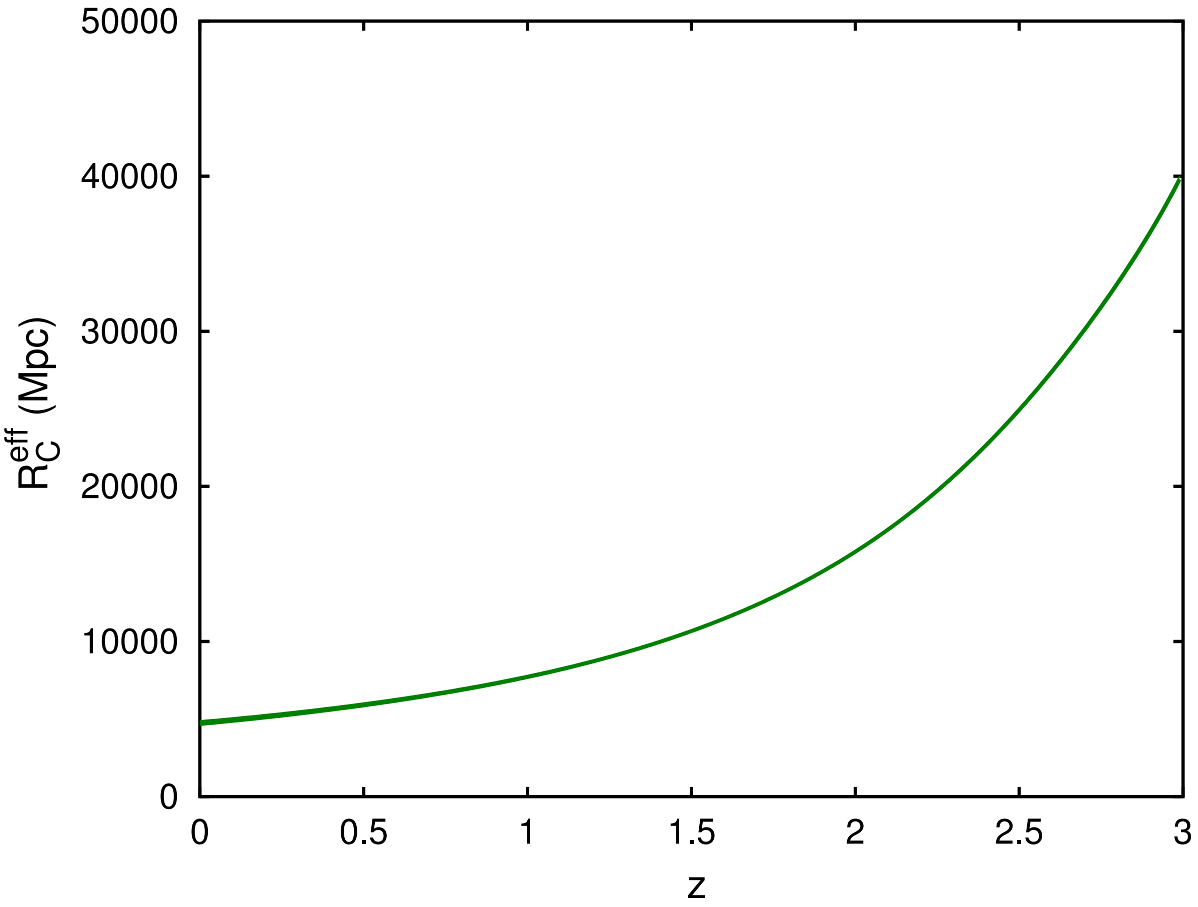
$$\mathrm{d}\chi^{\mathrm{eff}}(z) := \frac{c}{a^2\,H_{\mathrm{eff}}(z)}\,\mathrm{d}a$$

$$d_L^{\mathrm{eff}} = (1+z)R_{\mathrm{C}}^{\mathrm{eff}}\sinh\frac{\chi^{\mathrm{eff}}}{R_{\mathrm{C}}^{\mathrm{eff}}}$$

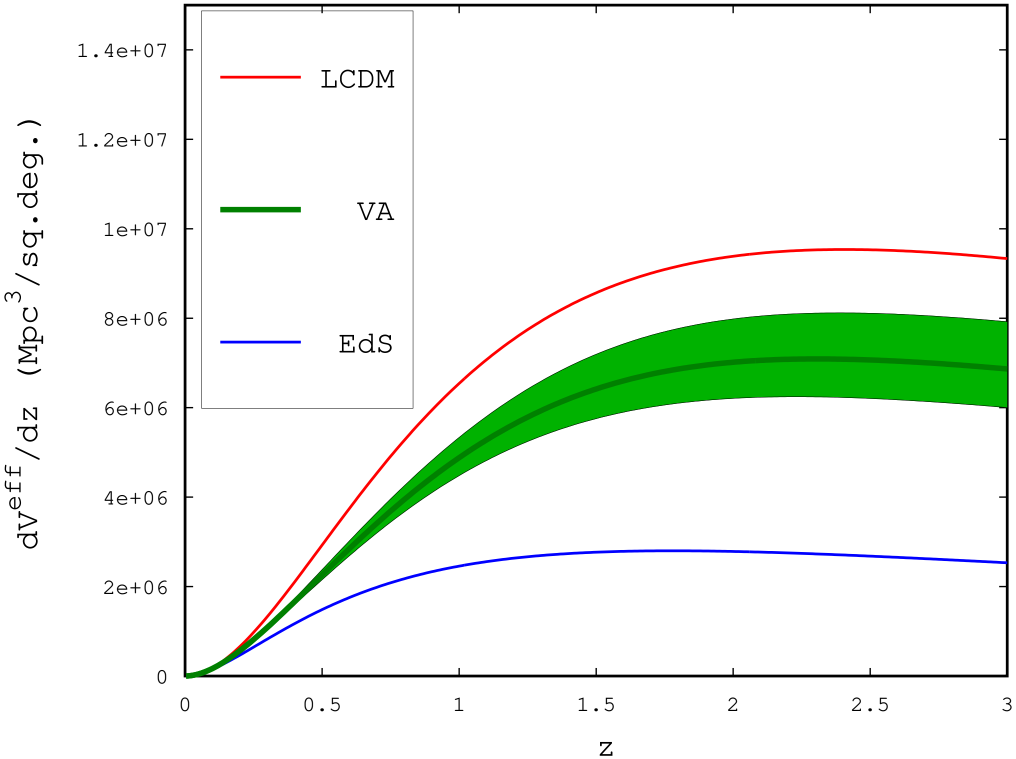
EdS + virialisation



EdS + virialisation

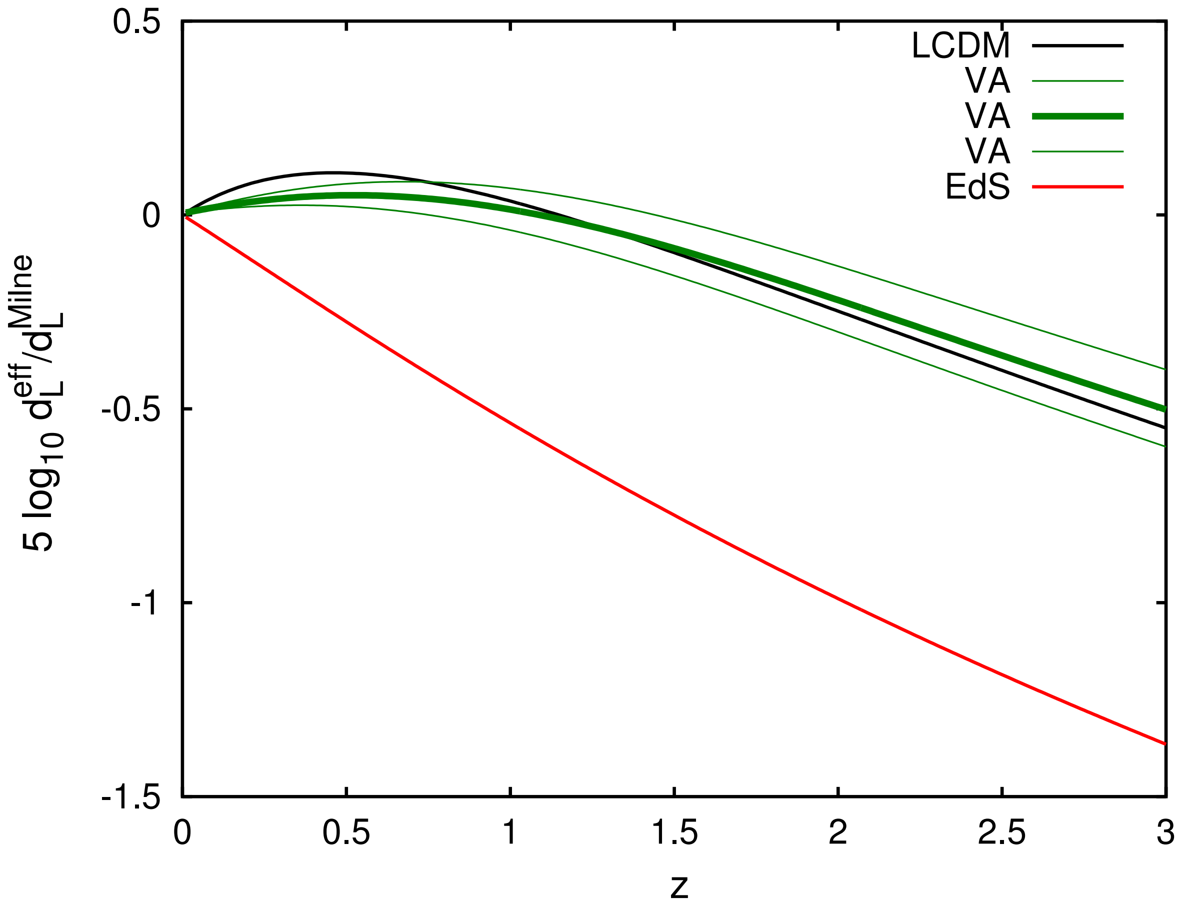


EdS + virialisation



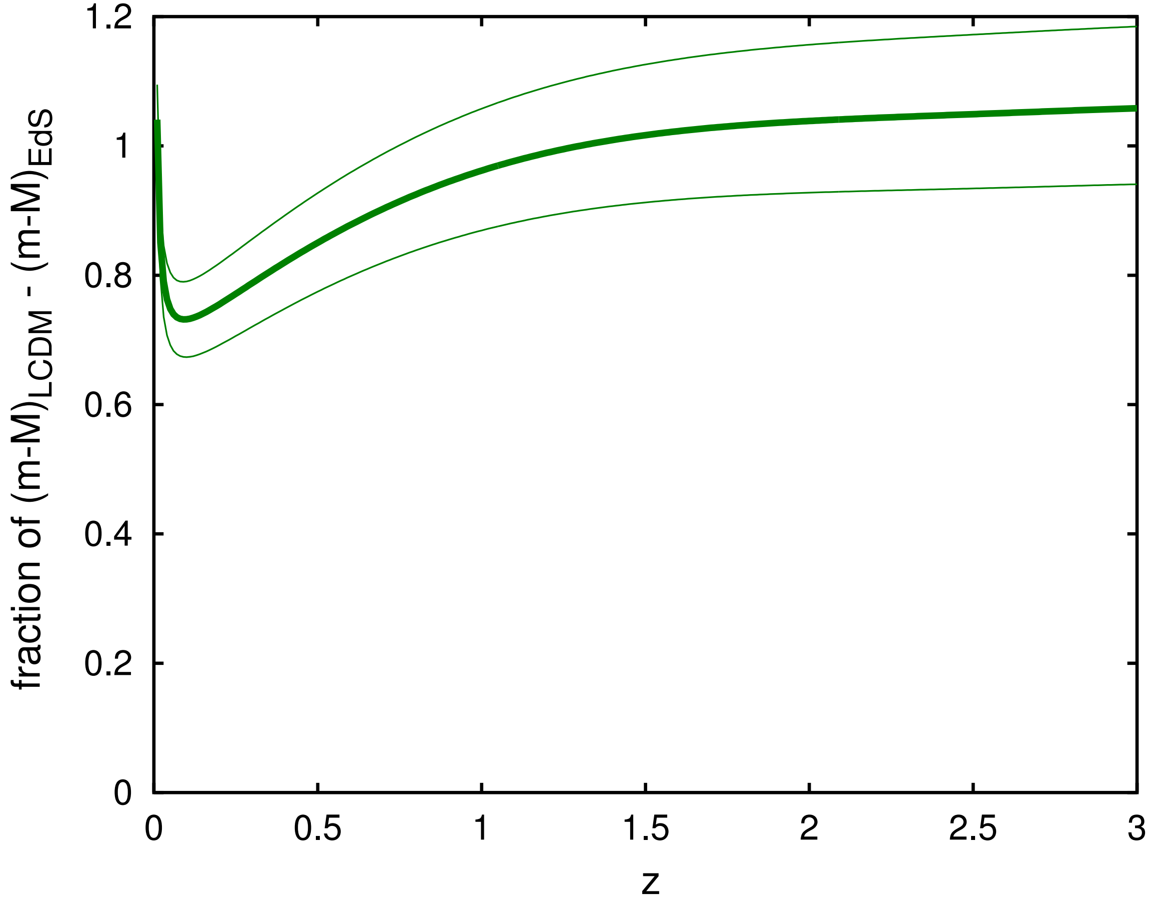
VA model: volume element for
$H_0^{\mathrm{eff}}(\mathrm{Riess}+2018) - H_0^{\mathrm{EdS}} = 74 - 47 = 27$ km/s/Mpc
assume ± 3 km/s/Mpc;
cf. Fig. 7, Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043

EdS + virialisation



Roukema, Ostrowski, Buchert, 2013, JCAP, 10, 043

EdS + virialisation



metric template

- $f_{\rm vir}(z) \sim \Omega_\Lambda(z)$

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$
- remove free param $(\Omega_{\Lambda 0})$; assume EdS background $(\Omega_\mathrm{m} = 1, \Omega_\Lambda = 0)$ at early times $(z \gg 3)$

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$
- remove free param $(\Omega_{\Lambda 0})$; assume EdS background $(\Omega_\mathrm{m} = 1, \Omega_\Lambda = 0)$ at early times $(z \gg 3)$
- add GR (scalar averaging): virialisation approximation:
  2 obs inputs: $H_\mathrm{eff}(z=0) = 74 \pm 1.6$ km/s/Mpc;
  $H_\mathrm{pec}^\mathrm{com}(z=0) = 36 \pm 3$ km/s/Mpc;

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$
- remove free param $(\Omega_{\Lambda 0})$; assume EdS background $(\Omega_\mathrm{m} = 1, \Omega_\Lambda = 0)$ at early times $(z \gg 3)$
- add GR (scalar averaging): virialisation approximation:
  2 obs inputs: $H_\mathrm{eff}(z=0) = 74 \pm 1.6$ km/s/Mpc;
  $H^\mathrm{com}_\mathrm{pec}(z=0) = 36 \pm 3$ km/s/Mpc;
- background model + virialisation
  $\Rightarrow$ void-dominated neg. curvature + inhomogeneous expansion $\Rightarrow$
  $\Omega^\mathrm{eff}_\mathrm{m}(z=0) \sim 0.3$

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$
- remove free param $(\Omega_{\Lambda 0})$; assume EdS background $(\Omega_\mathrm{m} = 1, \Omega_\Lambda = 0)$ at early times $(z \gg 3)$
- add GR (scalar averaging): virialisation approximation:
  2 obs inputs: $H_\mathrm{eff}(z=0) = 74 \pm 1.6$ km/s/Mpc;
  $H_\mathrm{pec}^\mathrm{com}(z=0) = 36 \pm 3$ km/s/Mpc;
- background model + virialisation
  $\Rightarrow$ void-dominated neg. curvature + inhomogeneous expansion $\Rightarrow$
  $\Omega_\mathrm{m}^\mathrm{eff}(z=0) \sim 0.3$
  distance modulus $m - M$ vs z : EdS+VA $\sim \Lambda$CDM

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$
- remove free param $(\Omega_{\Lambda 0})$; assume EdS background $(\Omega_\mathrm{m} = 1, \Omega_\Lambda = 0)$ at early times $(z \gg 3)$
- add GR (scalar averaging): virialisation approximation:
  2 obs inputs: $H_\mathrm{eff}(z=0) = 74 \pm 1.6$ km/s/Mpc;
  $H_\mathrm{pec}^\mathrm{com}(z=0) = 36 \pm 3$ km/s/Mpc;
- background model + virialisation
  $\Rightarrow$ void-dominated neg. curvature + inhomogeneous expansion $\Rightarrow$
  $\Omega_\mathrm{m}^\mathrm{eff}(z=0) \sim 0.3$
  distance modulus $m - M$ vs z : EdS+VA $\sim$ ΛCDM
- dark energy $\sim$ virialisation-epoch negative curvature

metric template

- expected homogeneous model failure $\sim \Omega_\Lambda(z)$
- remove free param $(\Omega_{\Lambda 0})$; assume EdS background $(\Omega_\mathrm{m} = 1, \Omega_\Lambda = 0)$ at early times $(z \gg 3)$
- add GR (scalar averaging): virialisation approximation:
  2 obs inputs: $H_\mathrm{eff}(z = 0) = 74 \pm 1.6$ km/s/Mpc;
  $H^\mathrm{com}_\mathrm{pec}(z = 0) = 36 \pm 3$ km/s/Mpc;
- background model + virialisation
  $\Rightarrow$ void-dominated neg. curvature + inhomogeneous expansion $\Rightarrow$
  $\Omega^\mathrm{eff}_\mathrm{m}(z = 0) \sim 0.3$
  distance modulus $m - M$ vs z : EdS+VA $\sim$ ΛCDM
- dark energy $\sim$ virialisation-epoch negative curvature
  Roukema, Ostrowski, Buchert 2013 JCAP, 10, 043 (arXiv:1303.4444);
  Roukema 2013, IJMPD, 22, 1341018 (arXiv:1305.4415)

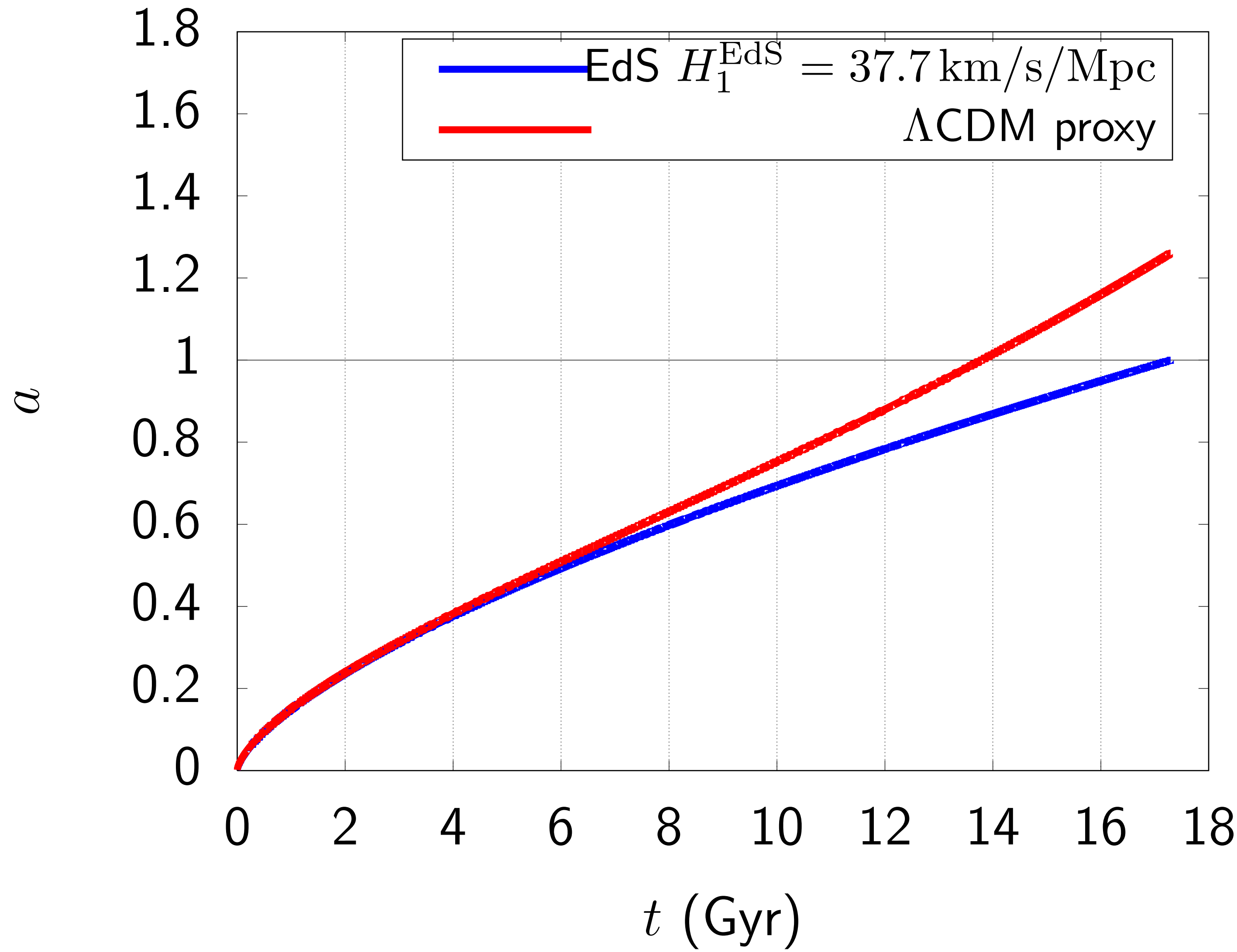
DE from inhomog? basic numbers

- early-time EdS "background"; extrapolate to present:
$$a_{\mathrm{EdS}} := (3H_1^{\mathrm{EdS}} t/2)^{2/3}\,, \quad H_{\mathrm{EdS}} := \dot{a}_{\mathrm{bg}}/a_{\mathrm{EdS}} = 2/(3t)$$

- $a_{\mathrm{eff}} = 1$ at $t_0 \equiv t_{a_{\mathrm{eff}}=1}$
- $a_{\mathrm{eff}} \approx a_{\mathrm{EdS}}$ at $t \ll t_0$
- zero cosmological constant/dark energy, i.e. $\Lambda := 0$
- bi-domain scalar averaging
- virialisation of collapsed (overdense) regions

$$H_0^{\mathrm{EdS}} := H_{\mathrm{EdS}}(a_{\mathrm{eff}} = 1)$$

key relations



key relations

- $$H_{\text{eff}}(t) \approx H_{\text{EdS}}(t) + H_{\text{pec}}(t)$$

- $$H_{\text{eff}}(t \ll t_0) \approx H_{\text{EdS}} = H_1^{\text{EdS}} a_{\text{EdS}}^{-3/2}$$

- $$H_0^{\text{eff}} \approx H_0^{\text{EdS}} + H_{\text{pec},0}^{\text{void}}$$

key relations

- $$\Omega_{\mathrm{m0}}^{\mathrm{eff}} = \frac{\Omega_{\mathrm{m0}}^{\mathrm{bg}}}{\left(H_0^{\mathrm{eff}}/H_0^{\mathrm{EdS}}\right)^2} \left(\frac{a_{\mathrm{EdS0}}}{a_{\mathrm{eff0}}}\right)^3 = a_{\mathrm{EdS0}}^3 \left(\frac{H_0^{\mathrm{EdS}}}{H_0^{\mathrm{eff}}}\right)^2$$

- $$H_0^{\mathrm{EdS}} = H_0^{\mathrm{eff}} \sqrt{\Omega_{\mathrm{m0}}^{\mathrm{eff}}/a_{\mathrm{EdS0}}^3}$$

- $$\Omega_{\mathcal{R}0}^{\mathrm{eff}} = 1 - \Omega_{\mathrm{m0}}^{\mathrm{eff}} - \Omega_{\mathcal{Q}0}^{\mathrm{eff}}$$

ΛCDM as an observational proxy

- $$H_0^{\mathrm{EdS}} = 2/(3t_0)\,, \qquad H_1^{\mathrm{EdS}} = H_0^{\mathrm{eff}}\sqrt{\Omega_{\mathrm{m0}}^{\mathrm{eff}}}$$

- ΛCDM proxy: $\Omega_{\mathrm{m0}} = 0.309 \pm 0.006$, $H_0 = 67.74 \pm 0.46\,\mathrm{km/s/Mpc}$ $\Rightarrow$
$$H_1^{\mathrm{EdS}} = 37.7 \pm 0.4\,\mathrm{km/s/Mpc}$$

- ΛCDM proxy: $t_0^{\Lambda\mathrm{CDM}} = 13.80 \pm 0.02$ Gyr $\Rightarrow$
$$H_0^{\mathrm{EdS}} = 47.24 \pm 0.07\,\mathrm{km/s/Mpc}$$

- $$a_{\mathrm{EdS0}} = \left(H_1^{\mathrm{EdS}}/H_0^{\mathrm{EdS}}\right)^{2/3} = 0.860 \pm 0.007$$

DE-free cosmo params

- key parameters of GR, DE-free cosmology are observationally realistic:

$$\frac{2}{3} \approx \frac{H_0^{\rm EdS}}{H_0^{\rm eff}} \gtrsim \frac{H_1^{\rm EdS}}{H_0^{\rm eff}} \approx \sqrt{\Omega_{\rm m0}^{\rm eff}} = \sqrt{1 - \Omega_{\mathcal{R}0}^{\rm eff} - \Omega_{\mathcal{Q}0}^{\rm eff}} \approx \frac{1}{2} \gtrsim \frac{H_{\rm pec,0}^{\rm void}}{H_0^{\rm eff}} \approx \frac{1}{3}$$

- ΛCDM proxy: $a_{\rm EdS0} = 0.860 \pm 0.007$
  today's scale factor only needs to be 16% super-EdS

- Roukema, Mourier, Buchert & Ostrowski (2017) A&A 598, A111
  Virialisation approximation (VA)
  Roukema, Ostrowski, Buchert (2013) JCAP, 10, 043

- cf. Timescape Nazer+Wiltshire PRD 2015;
  Tardis Lavinto+ JCAP 2013;
  Simsilun Bolejko PRD 2018

BAO peak—SDSS DR7

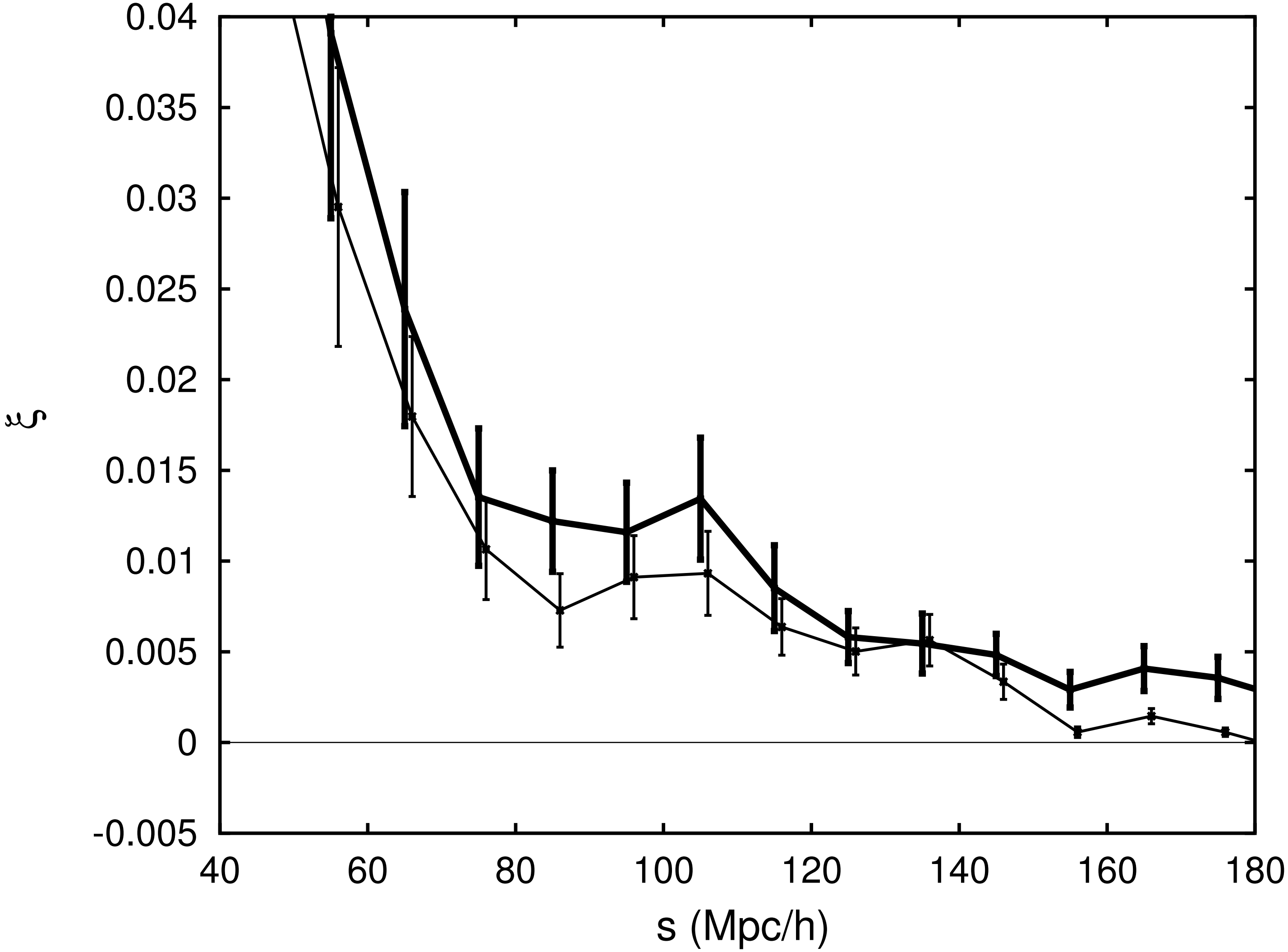
subset	D	R	ref
LRGs:			
dim	61899	3082871	Kazin2010 arXiv:0908.2598
bright	30272	1521736	Kazin2010
superclusters:			
dim + bright	235		NH2013 arXiv:1310.2791
$z < 0.6$	2701		Liivamägi arXiv:1012.1989
voids:			
dim + bright	83		NH2013

BAO peak—SDSS DR7

subset	D	R	ref
LRGs:			
dim	61899	3082871	Kazin2010 arXiv:0908.2598
bright	30272	1521736	Kazin2010
superclusters:			
dim + bright	235		NH2013 arXiv:1310.2791
$z < 0.6$	2701		Liivamägi arXiv:1012.1989
voids:			
dim + bright	83		NH2013

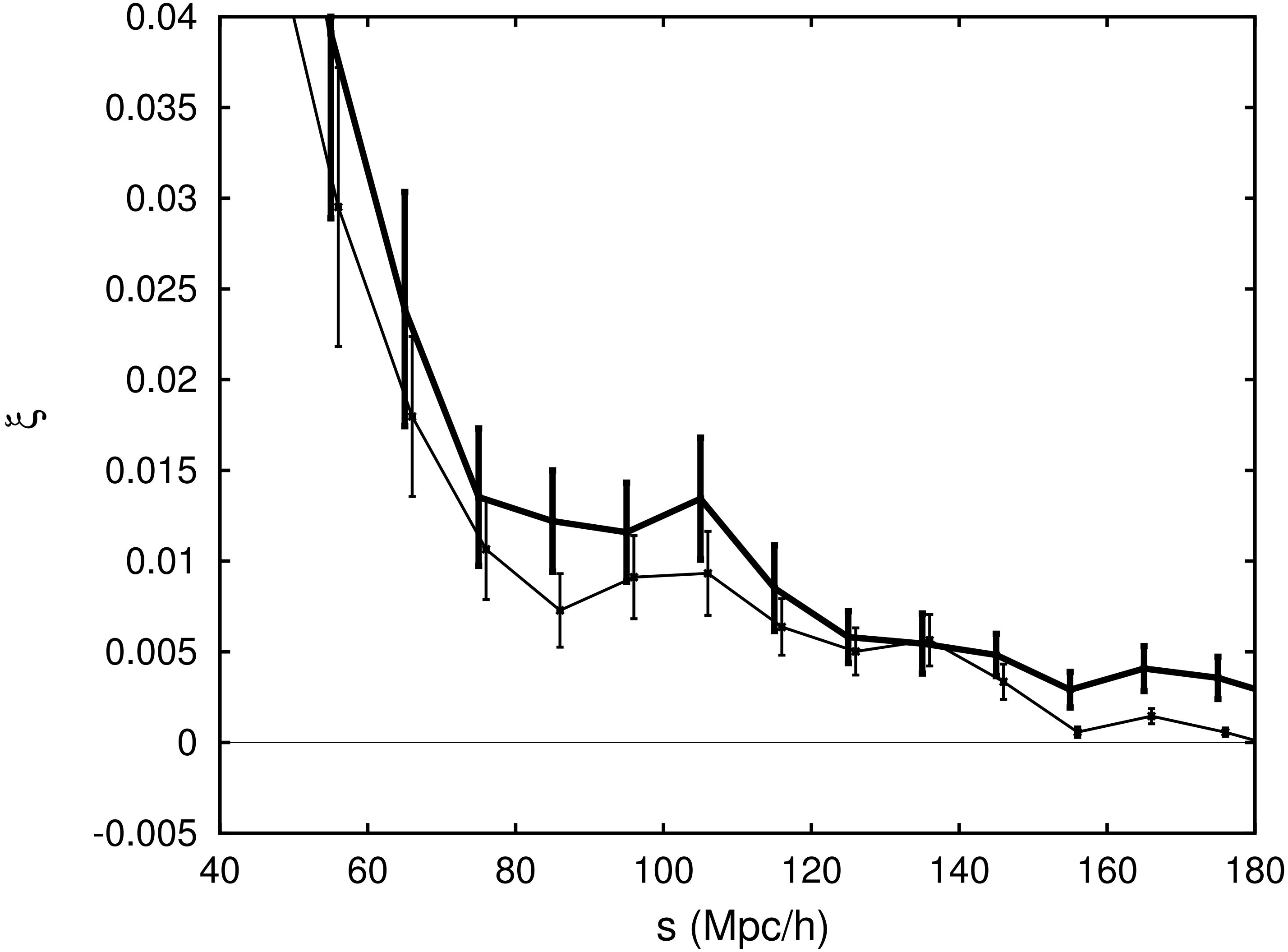
$$\xi(s) = \frac{\mathrm{DD}(s)/N_{\mathrm{DD}} - 2\mathrm{DR}(s)/N_{\mathrm{DR}} + \mathrm{RR}(s)/N_{\mathrm{RR}}}{\mathrm{RR}(s)/N_{\mathrm{RR}}}$$

BAO peak—SDSS DR7



full br/dim

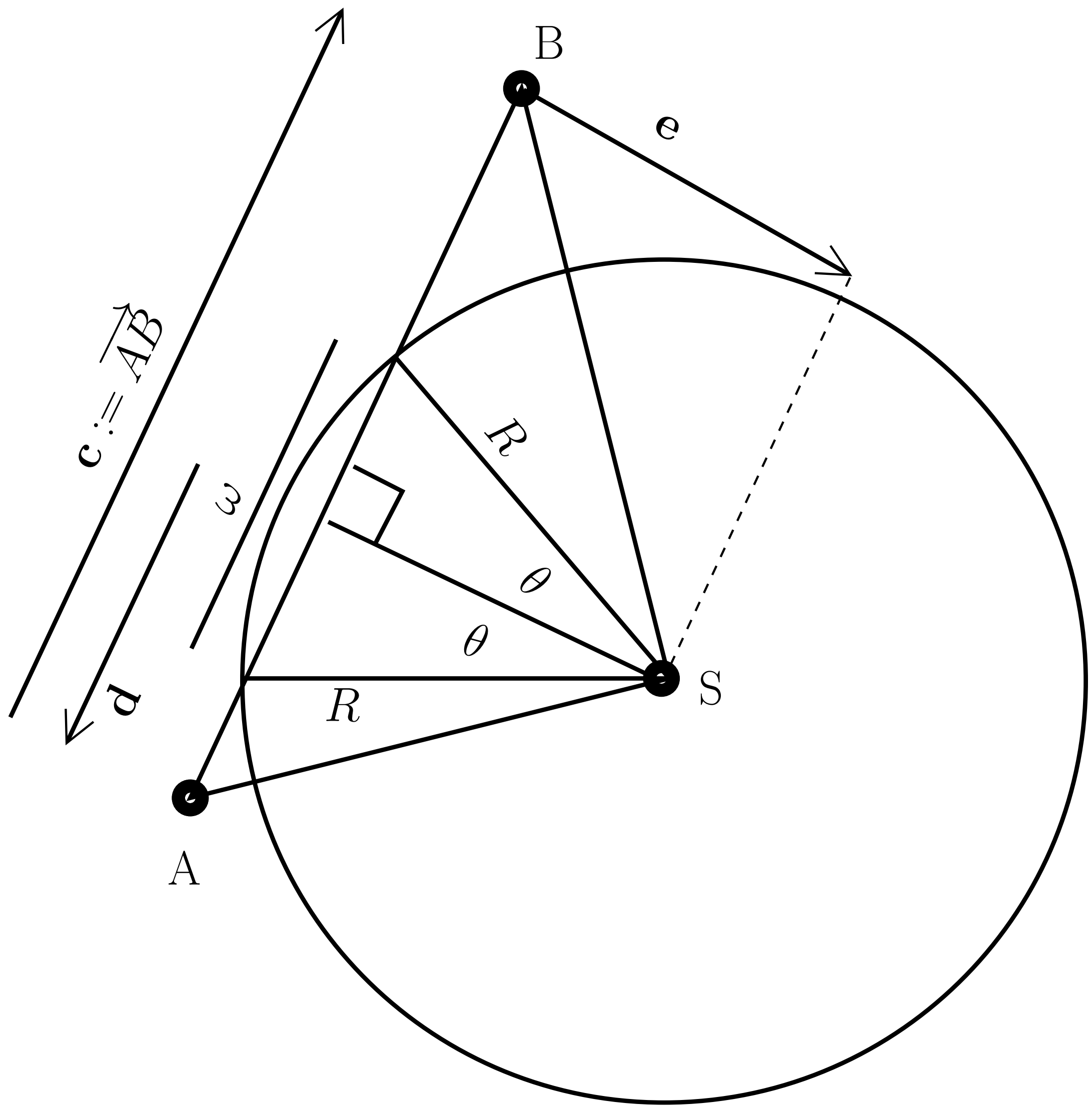
BAO peak—SDSS DR7



full br/dim

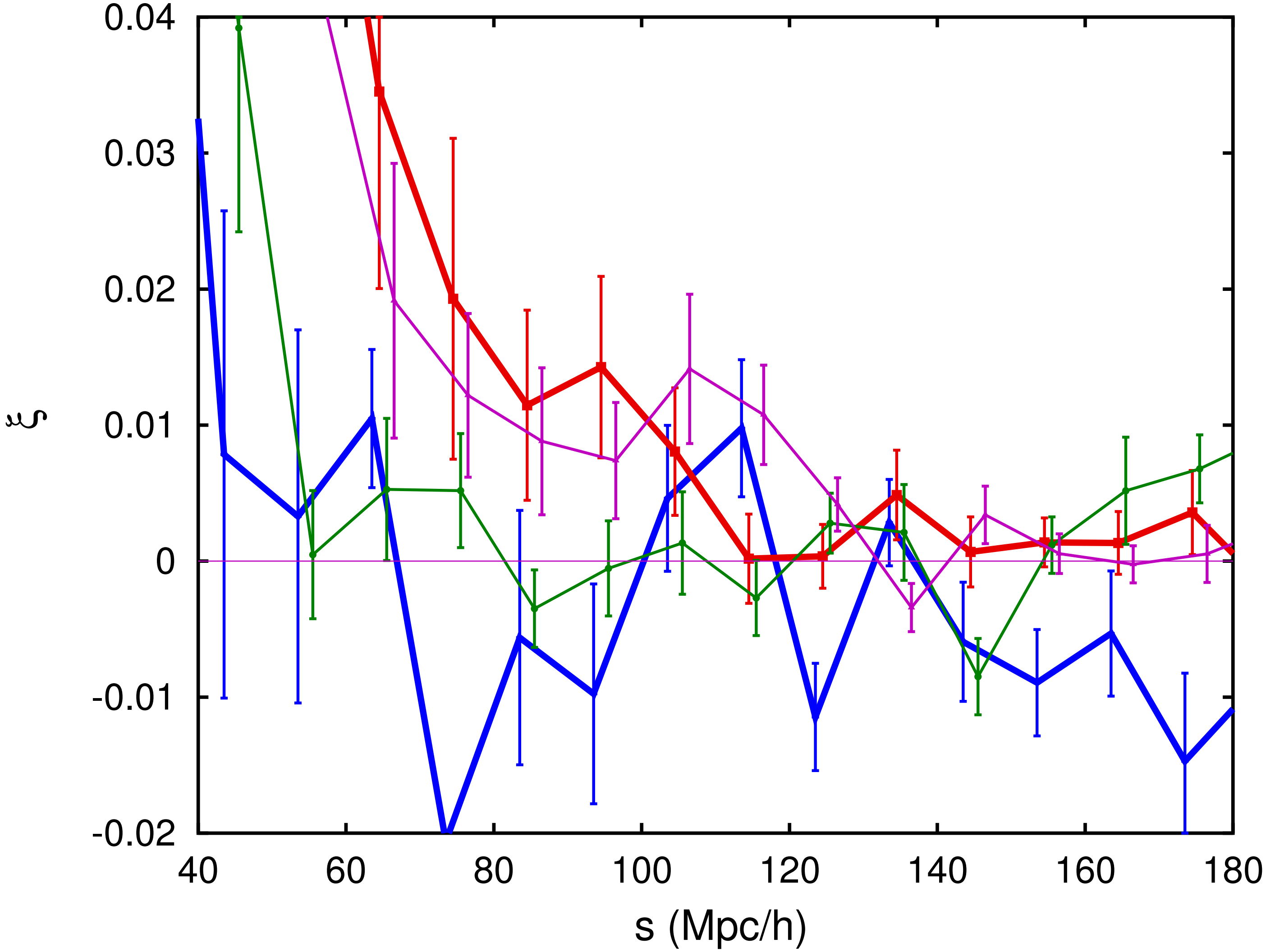
peak better defined in bright (bigger scale) sample

BAO peak—environment



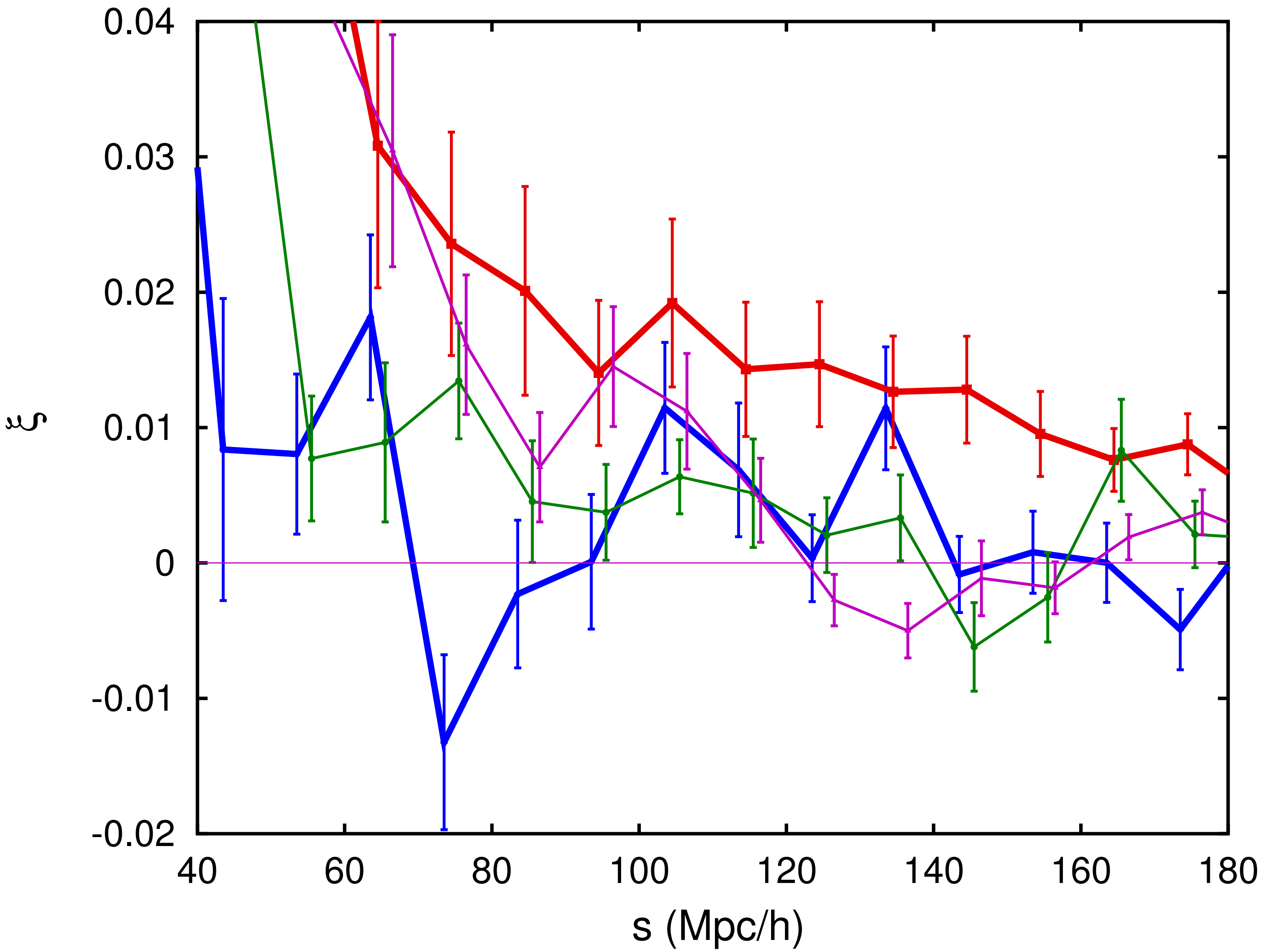
overlap defn

BAO peak—environment



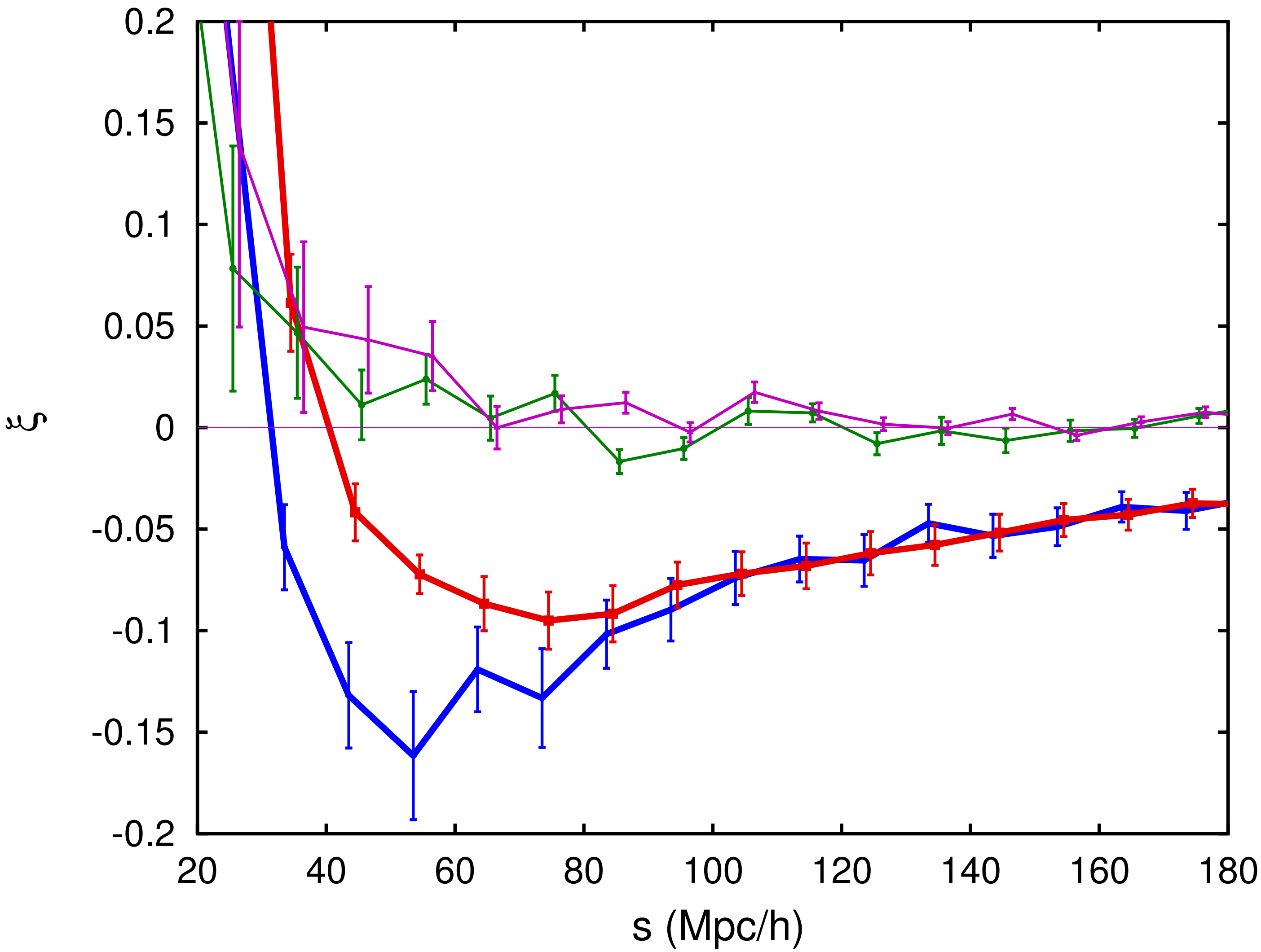
sc NH

BAO peak—environment



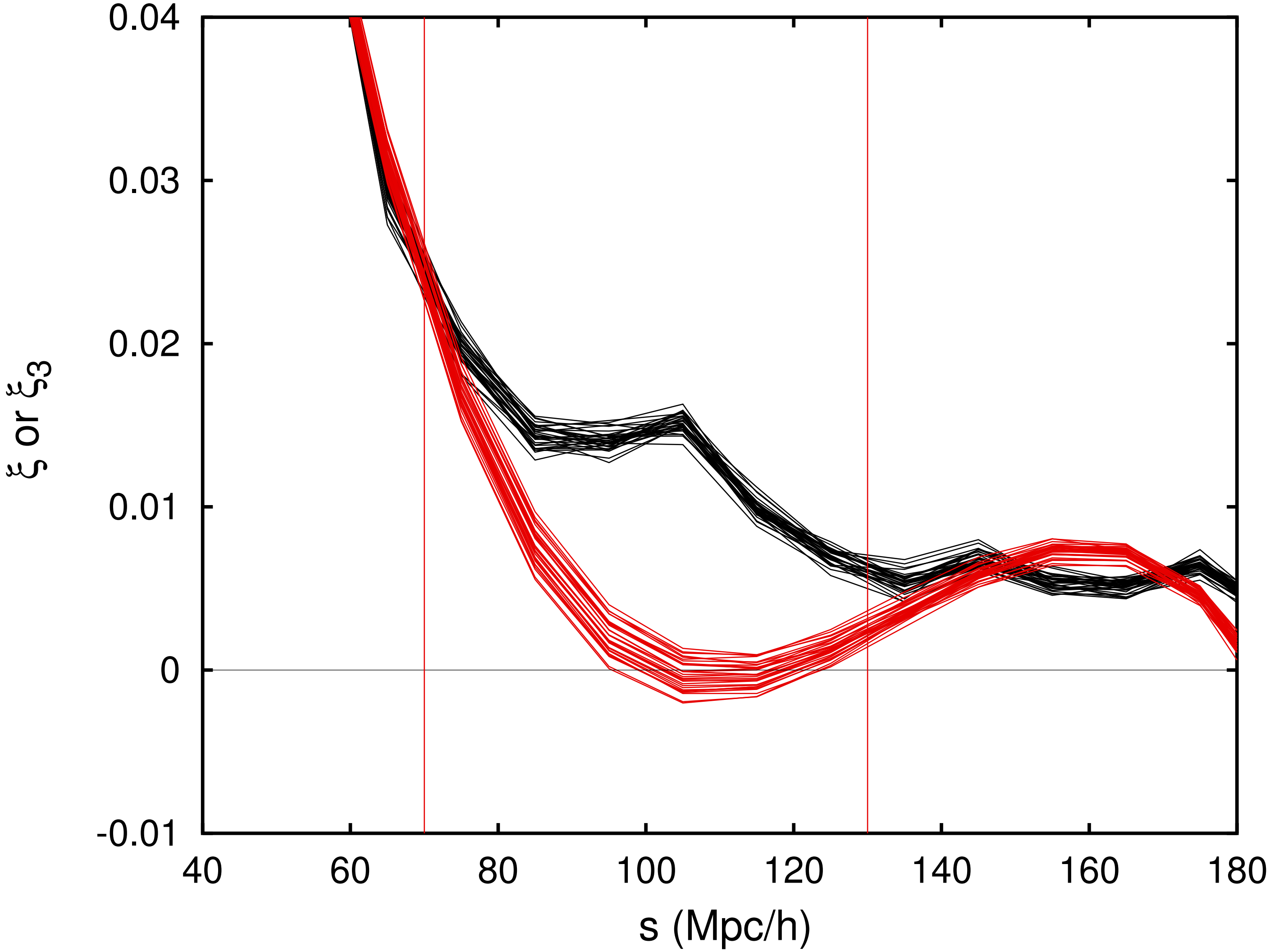
void NH

BAO peak—environment



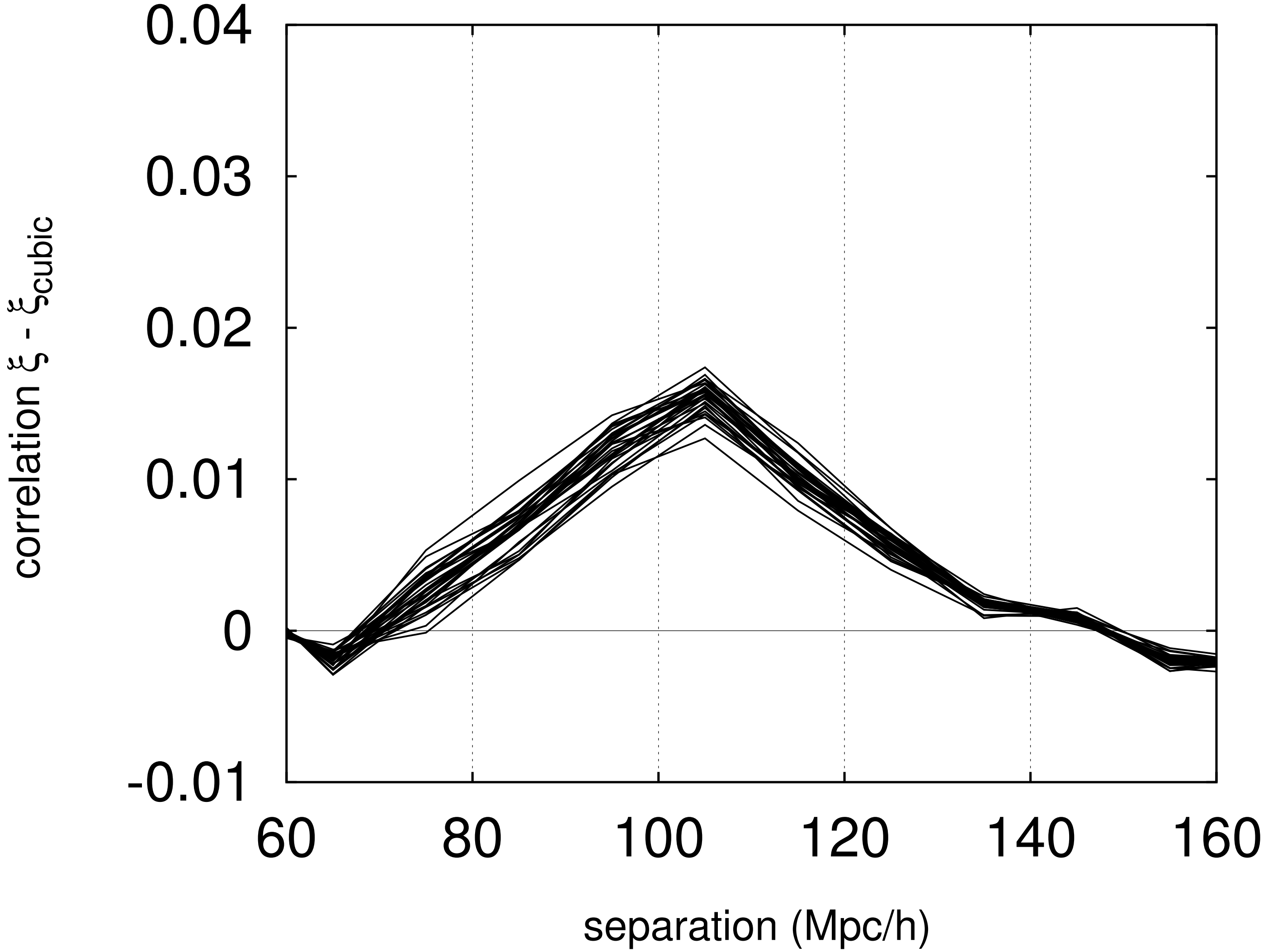
sc Liiva

BAO peak: NH superclusters



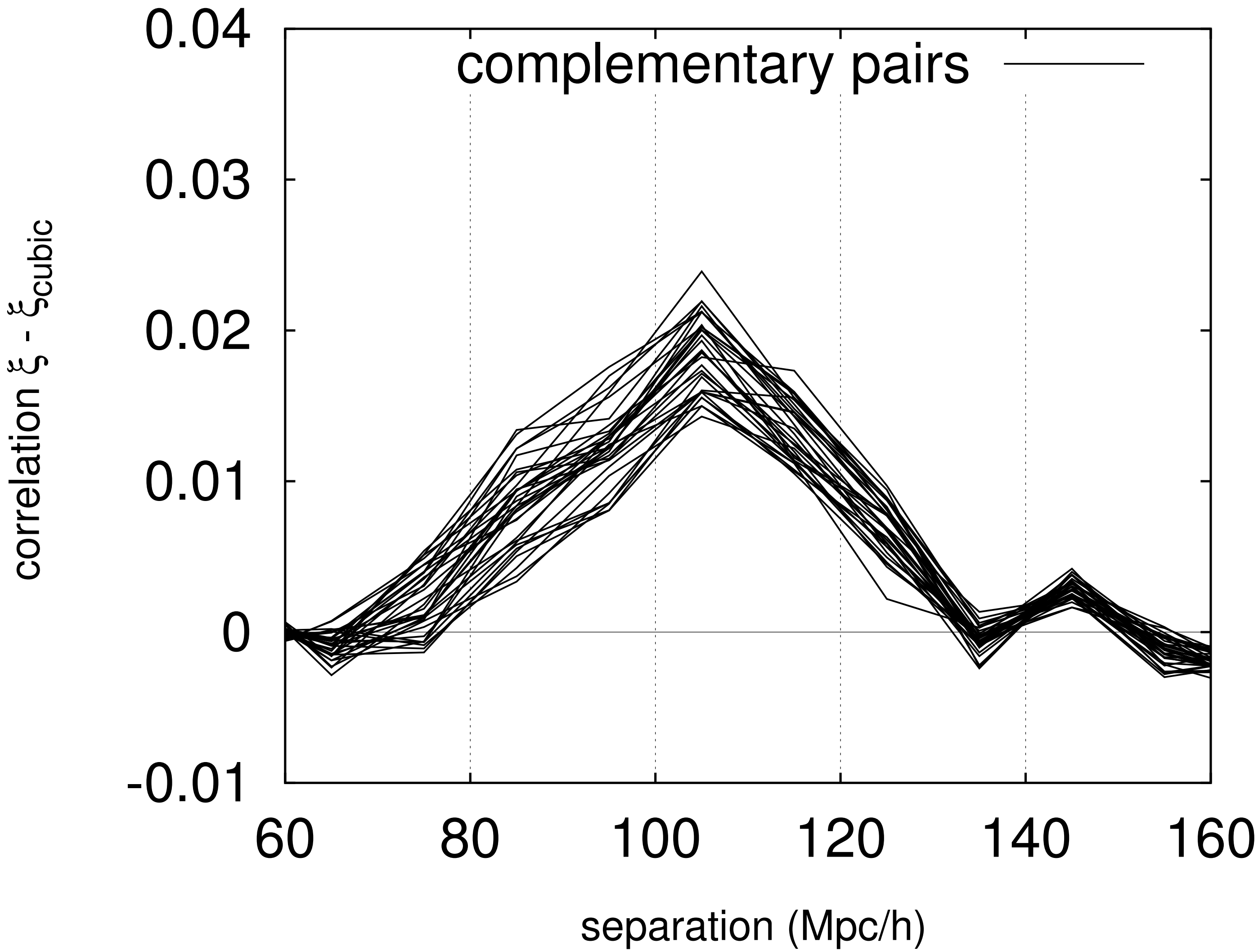
cubical fit $(< 70h^{-1}$ Mpc$) \cup (> 130h^{-1}$ Mpc$)$

BAO peak: NH superclusters



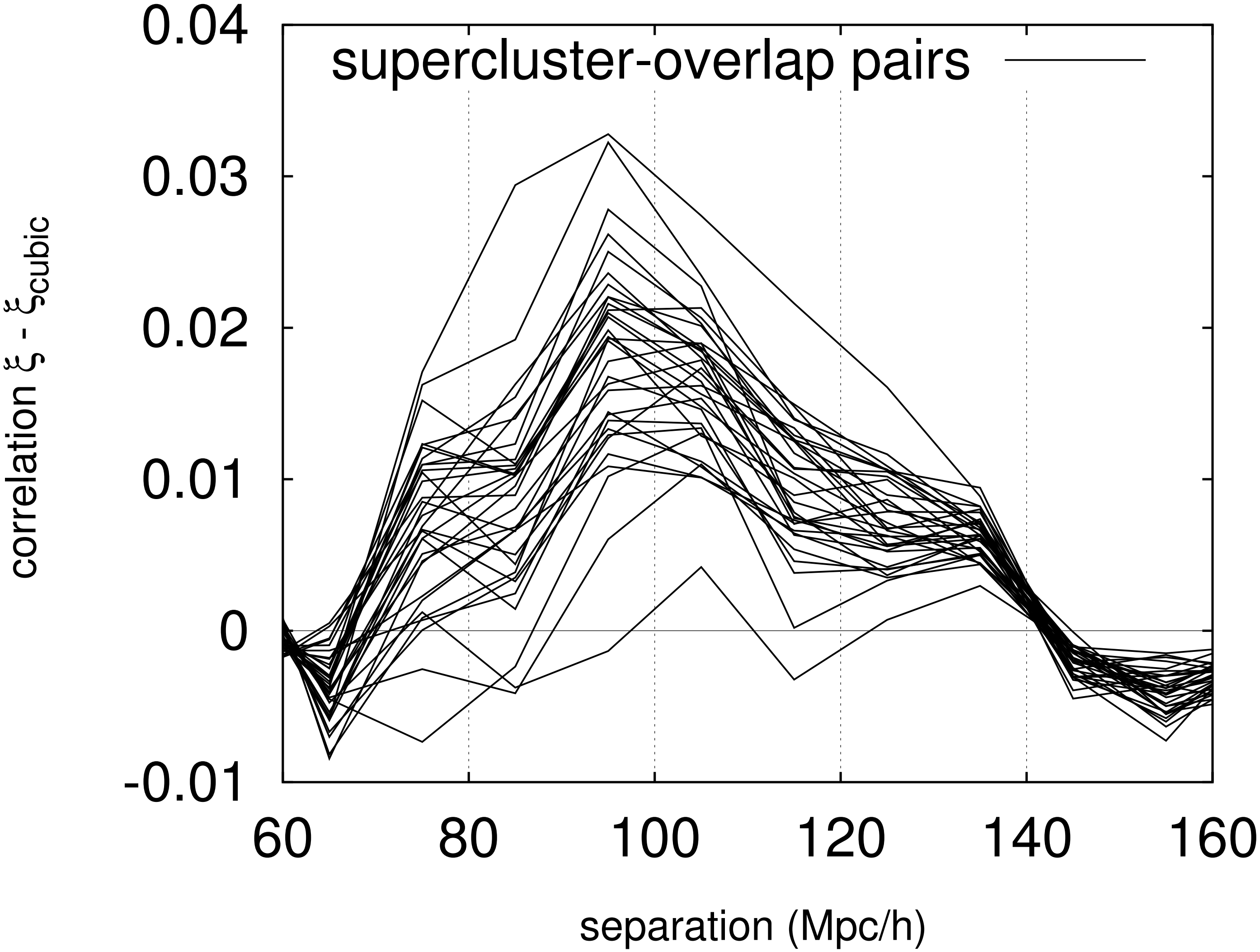
full

BAO peak: NH superclusters



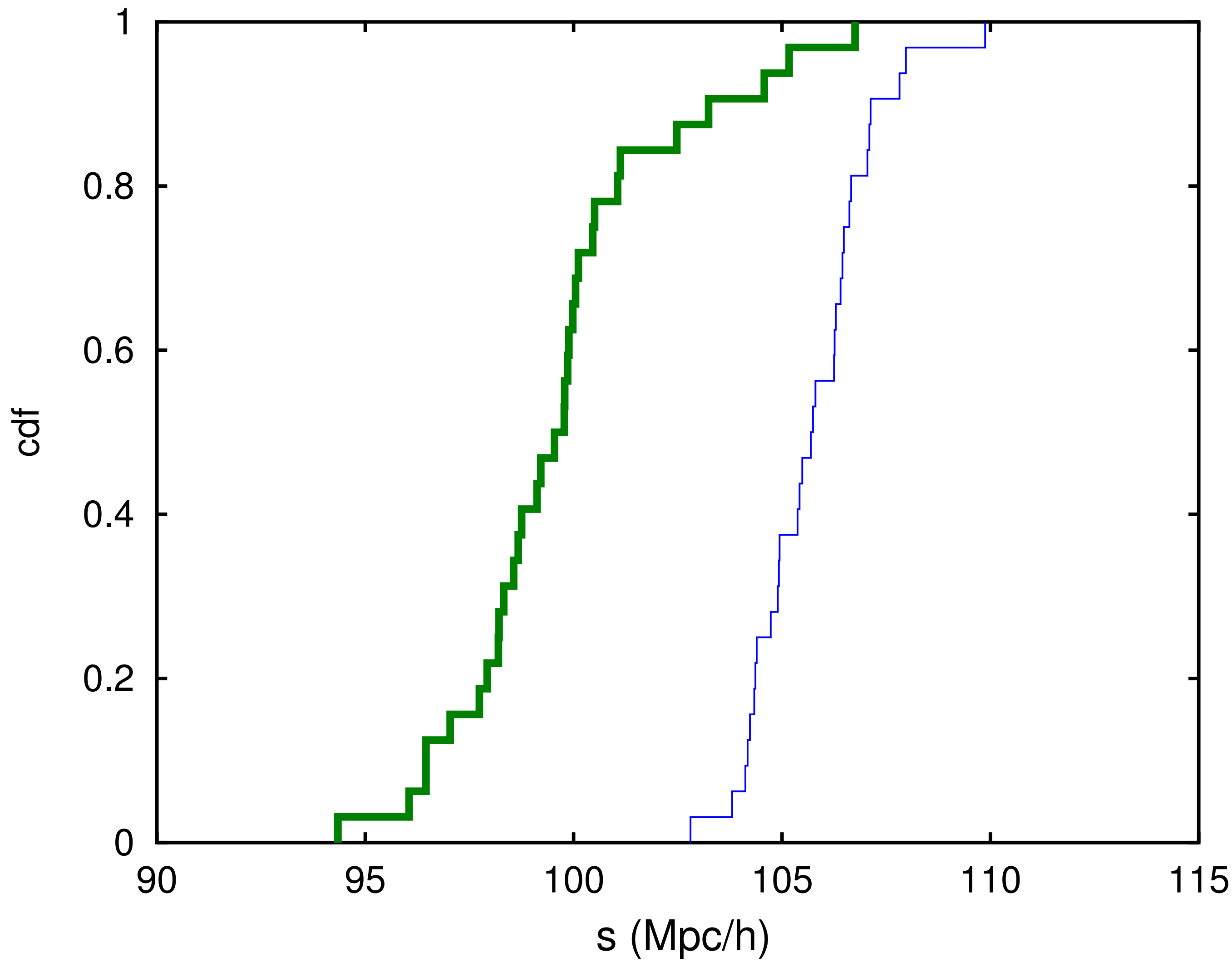
subset: LRG pairs that do **not** overlap with superclusters

BAO peak: NH superclusters

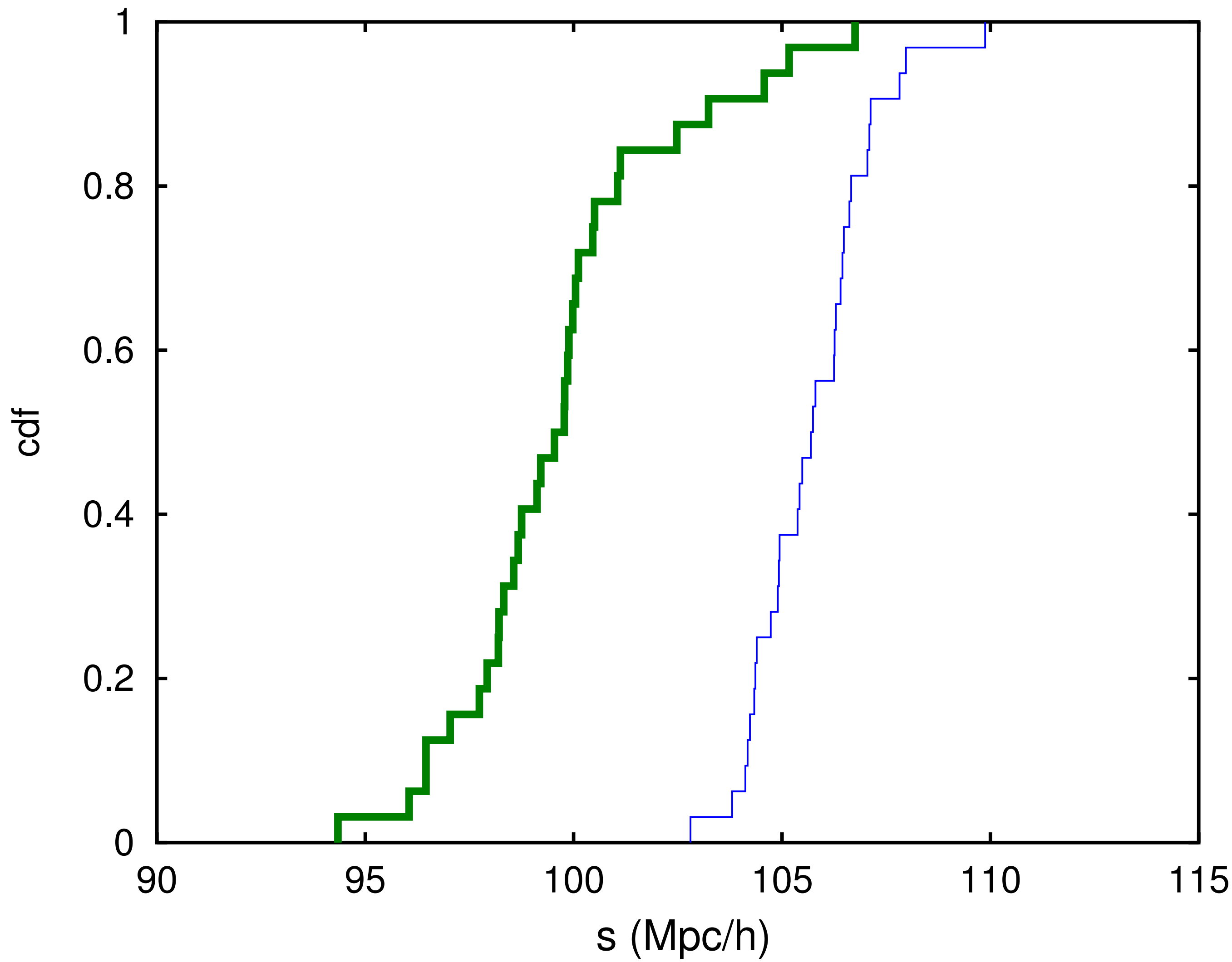


subset: LRG pairs that **overlap** with superclusters

BAO peak: NH superclusters



BAO peak: NH superclusters

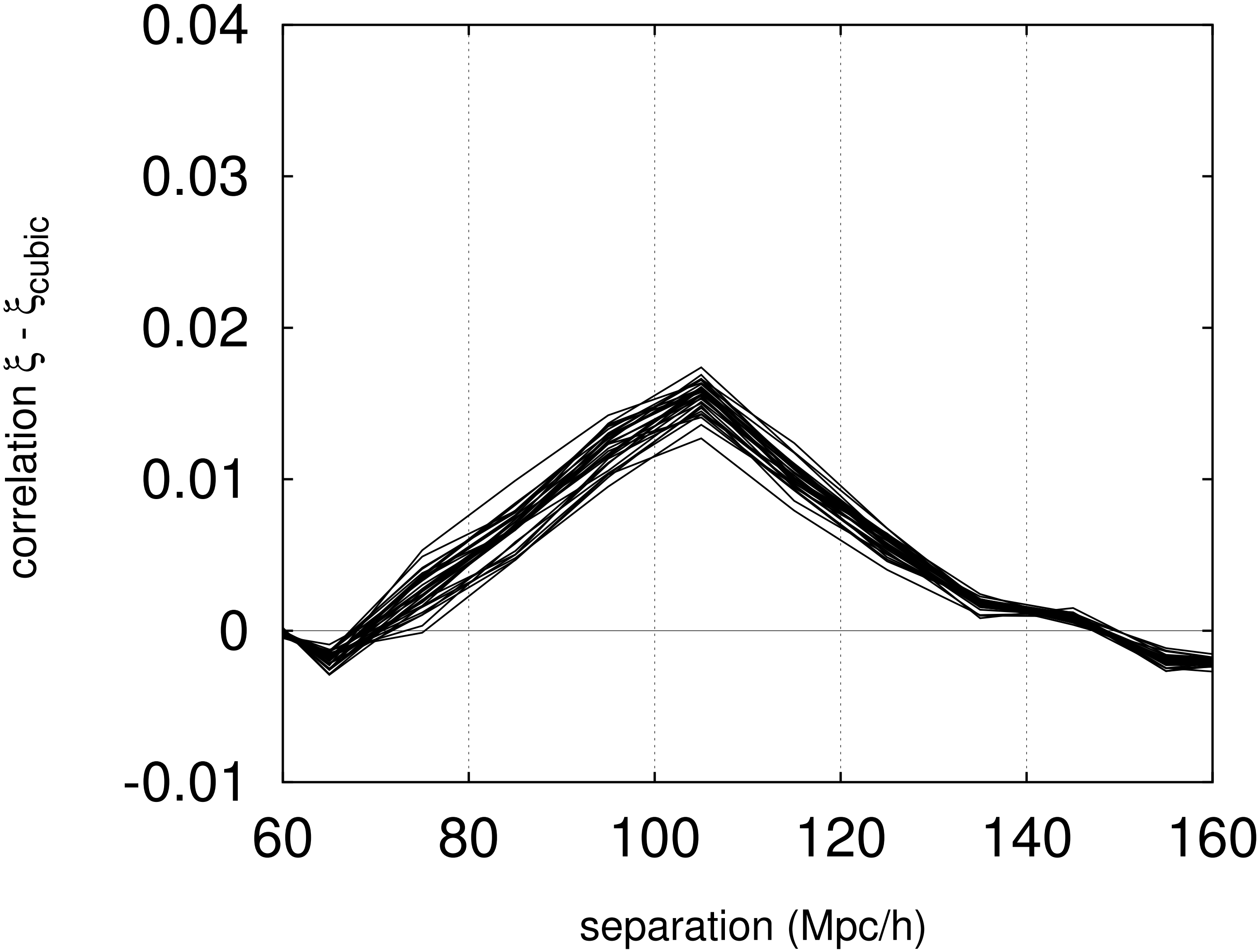


$p_{\mathrm{KS}} = 5 \times 10^{-11}$

environment-dependent BAO peak shift: 6% for SDSS DR7 LRGs

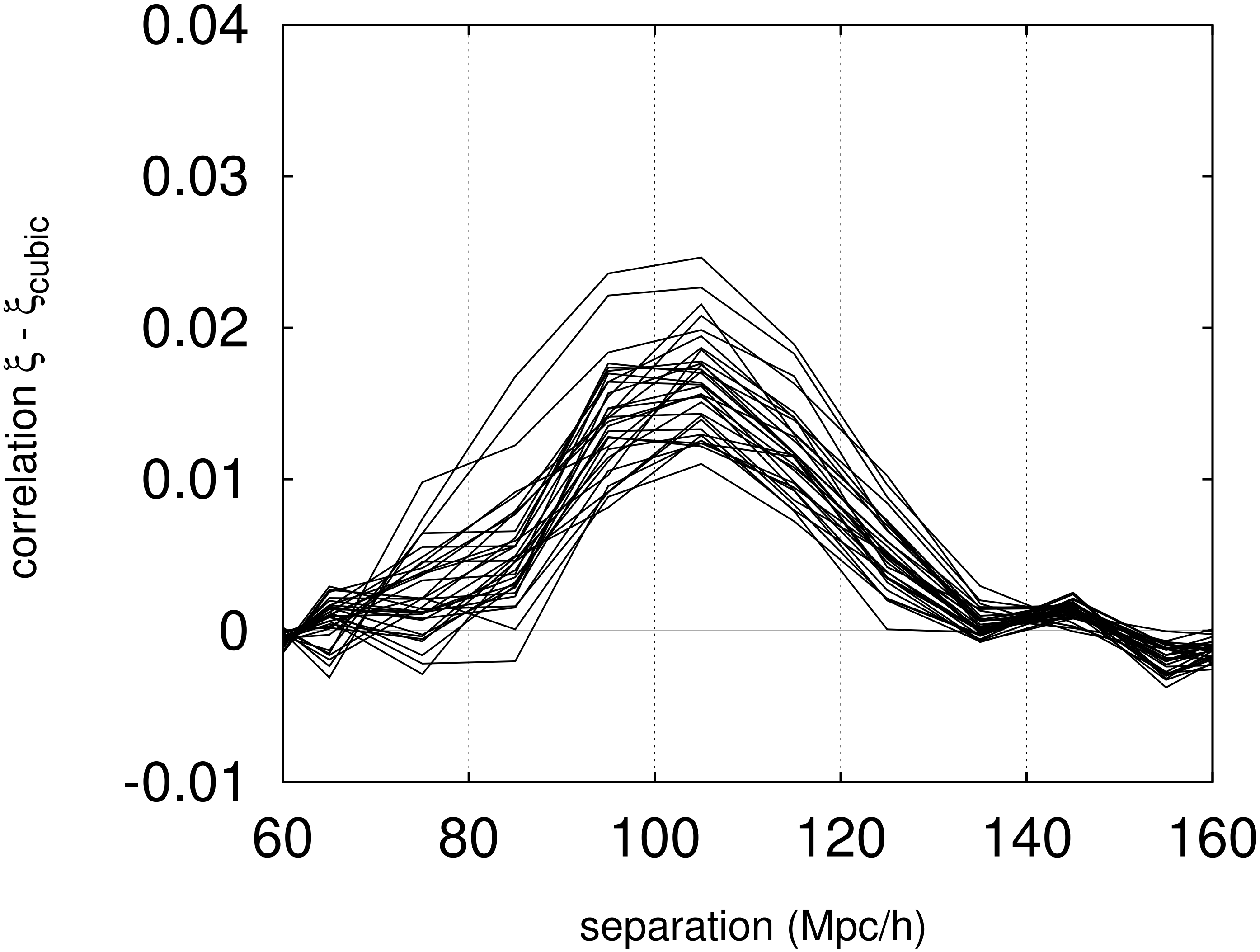
Roukema, Buchert, Ostrowski & France 2015 MNRAS, 448, 1660

BAO peak: NH voids



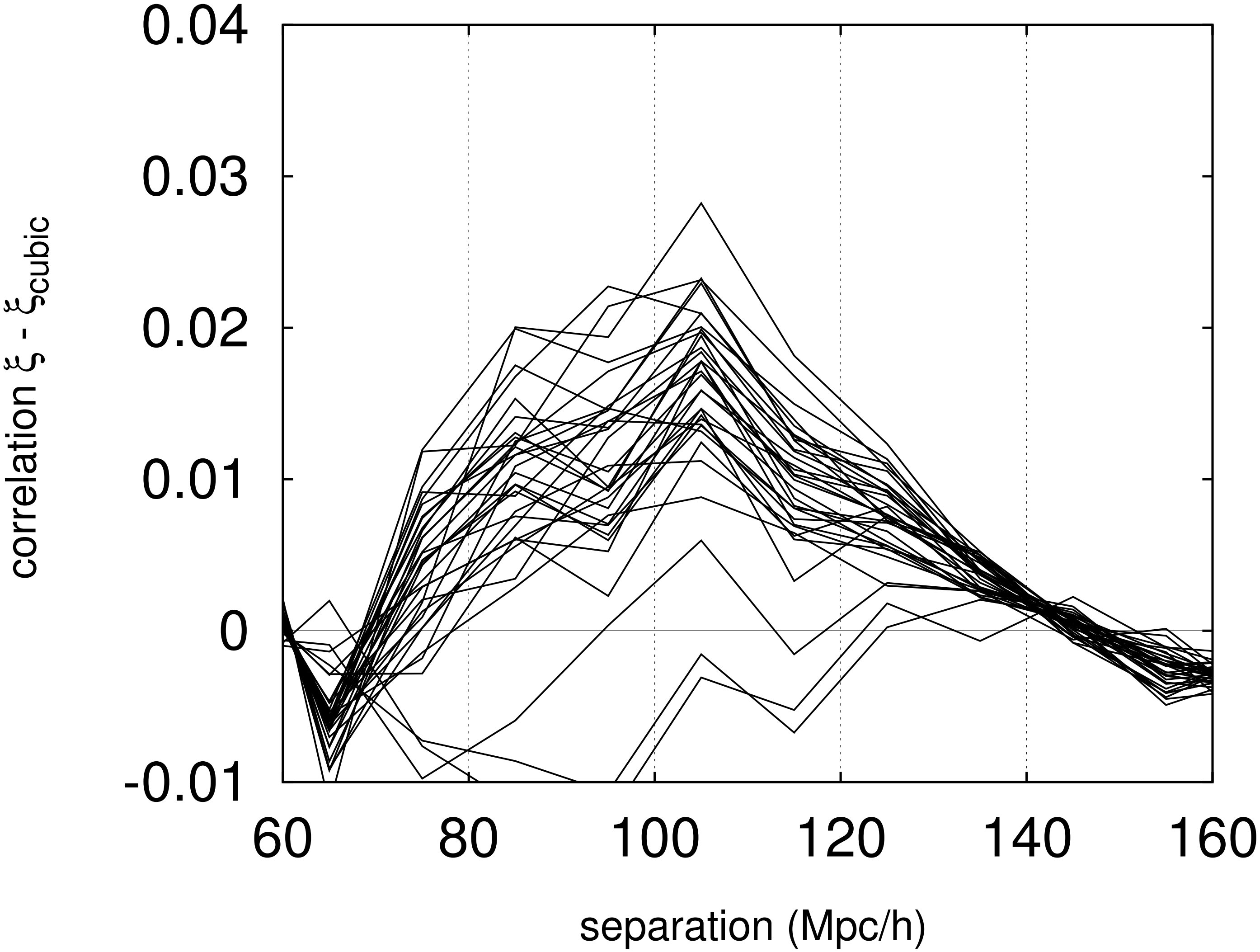
full

BAO peak: NH voids



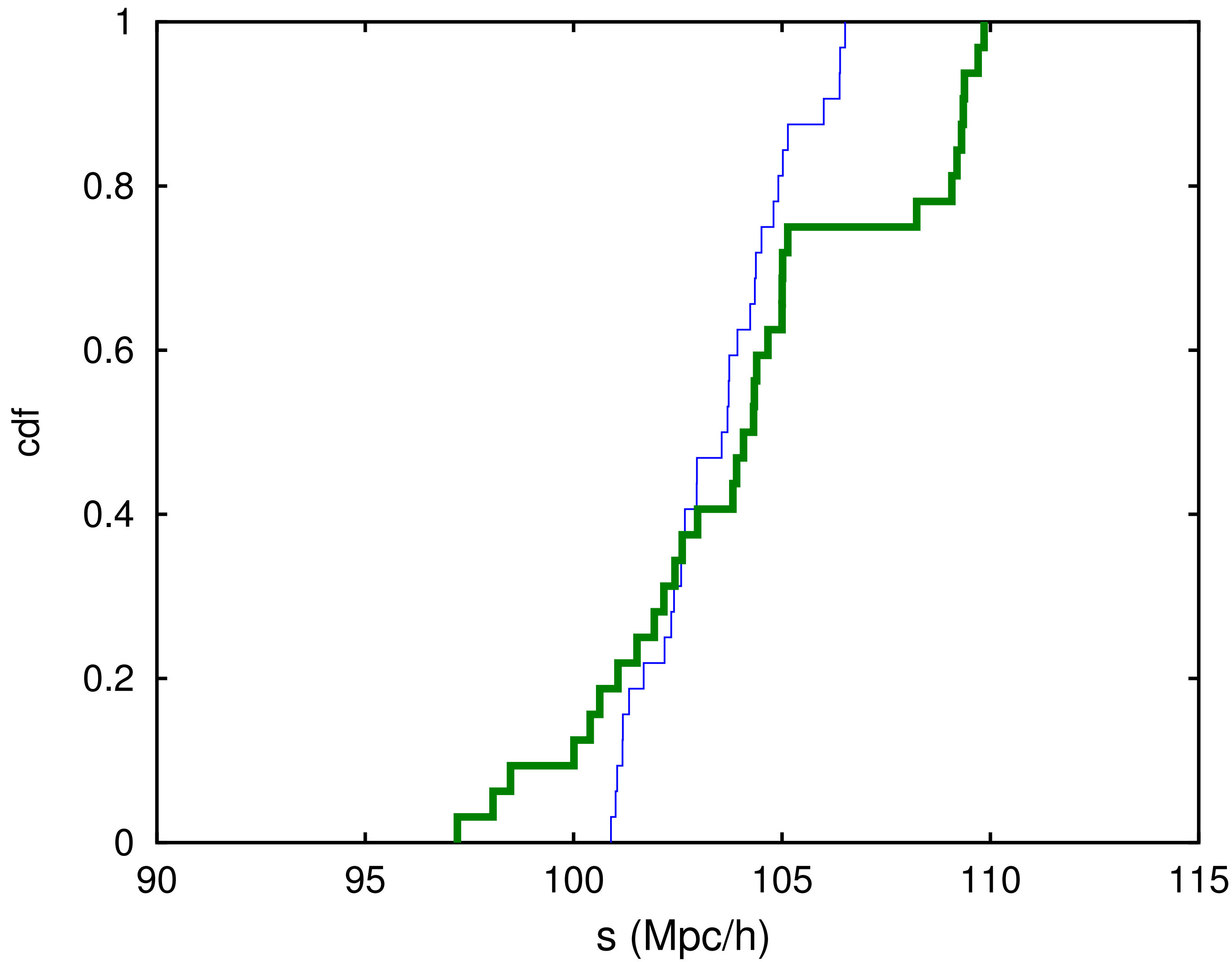
NH non-voids

BAO peak: NH voids



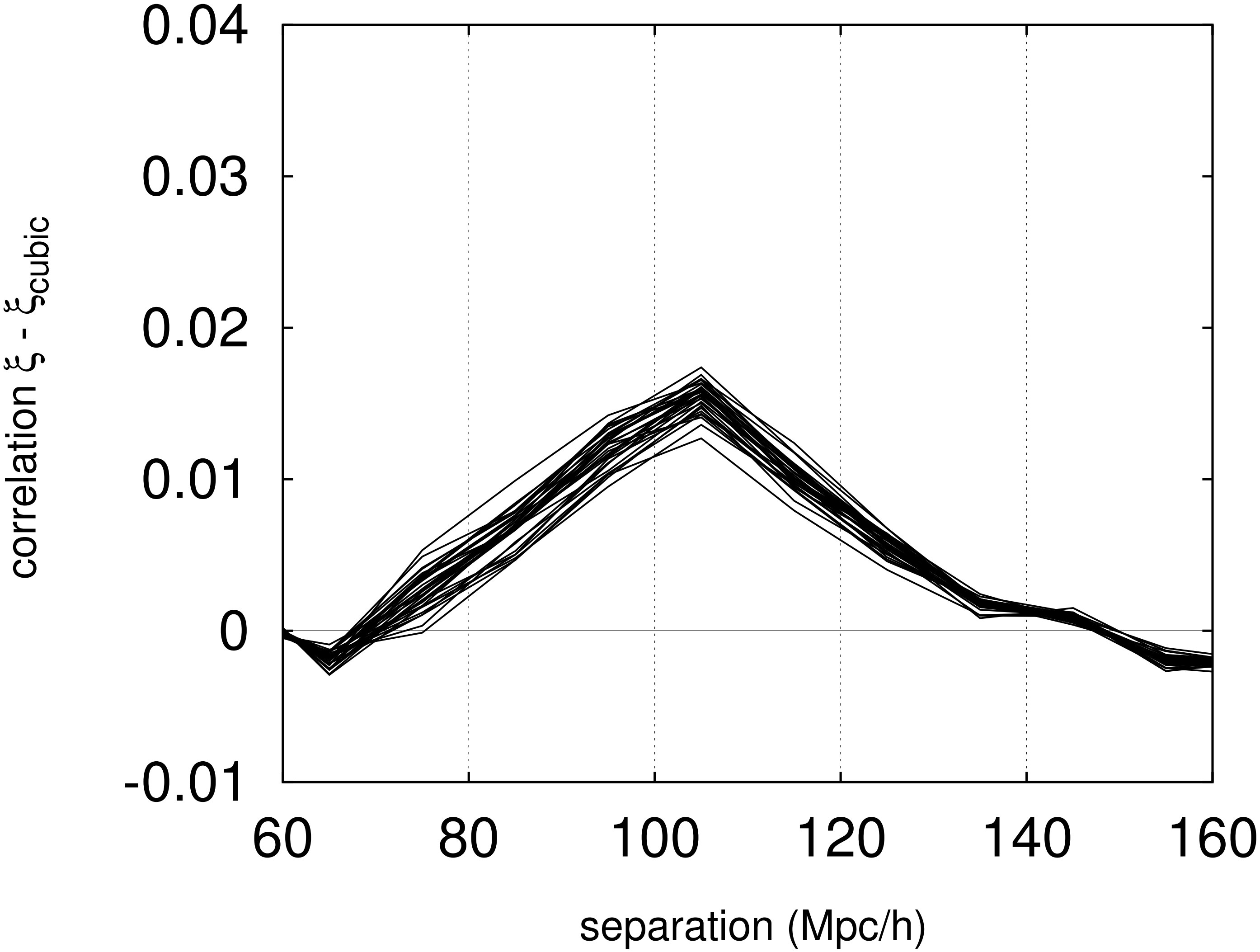
NH voids

BAO peak: NH voids



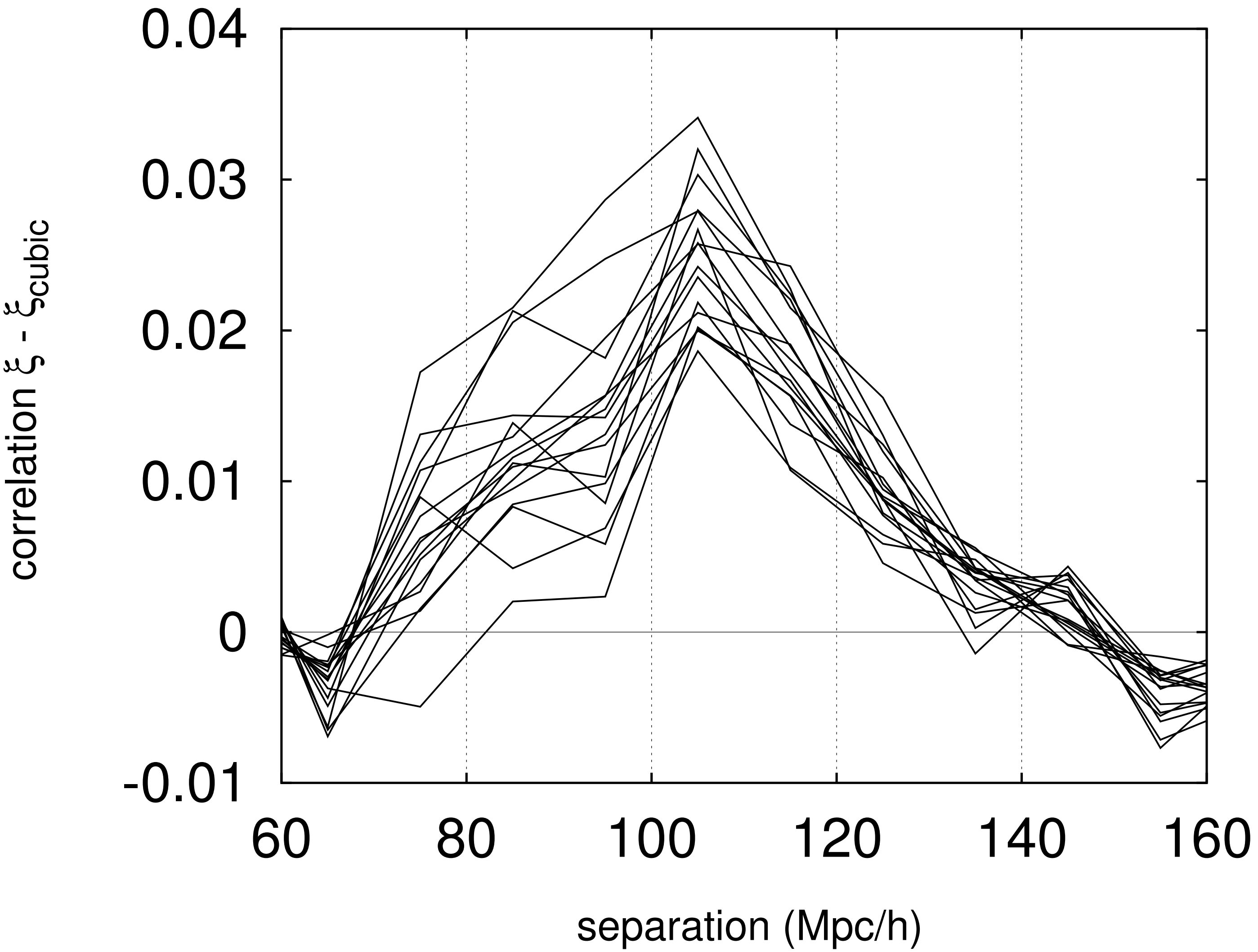
$p_{\mathrm{KS}} = 0.3$

BAO peak: Liivamägi sc's



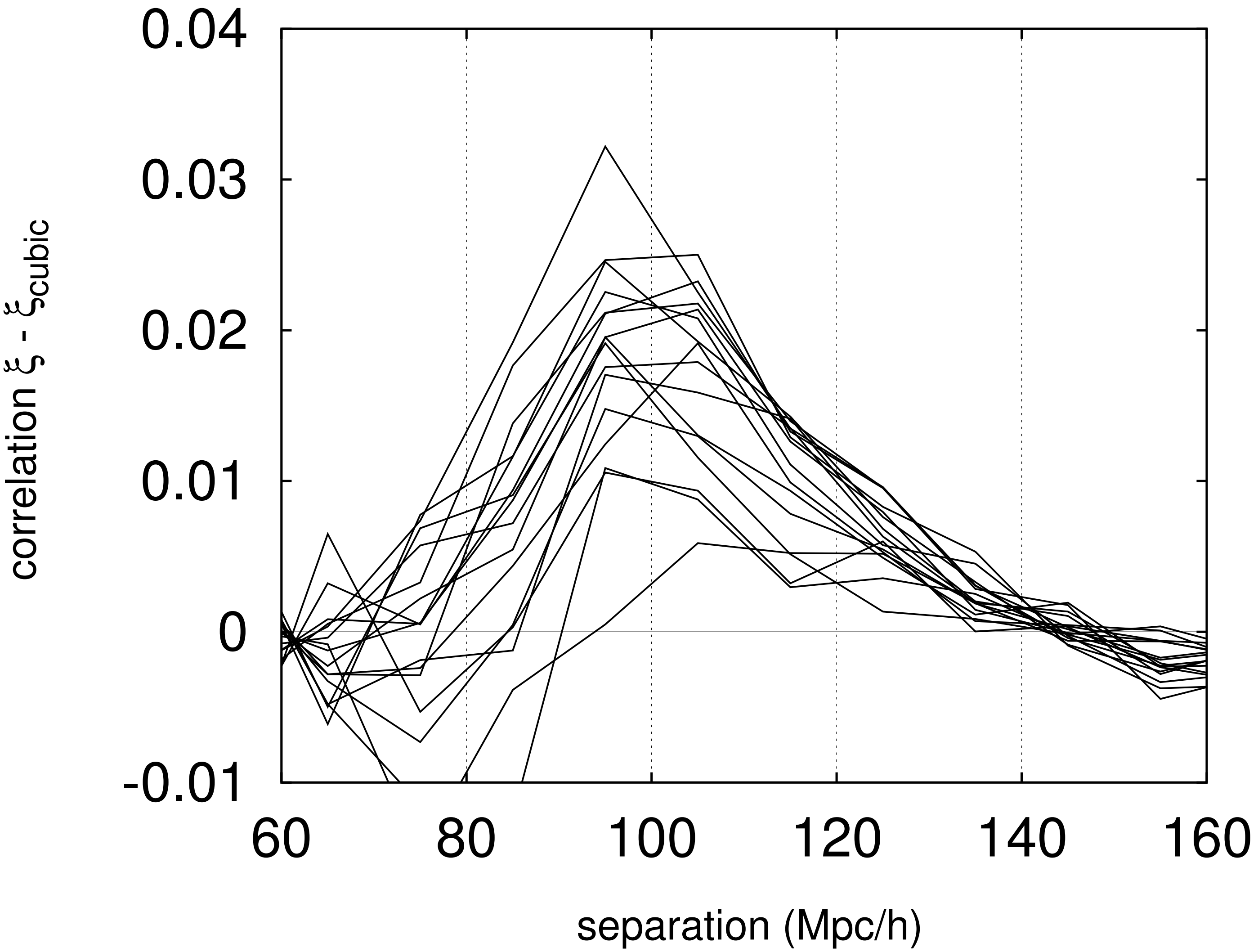
full

BAO peak: Liivamägi sc's



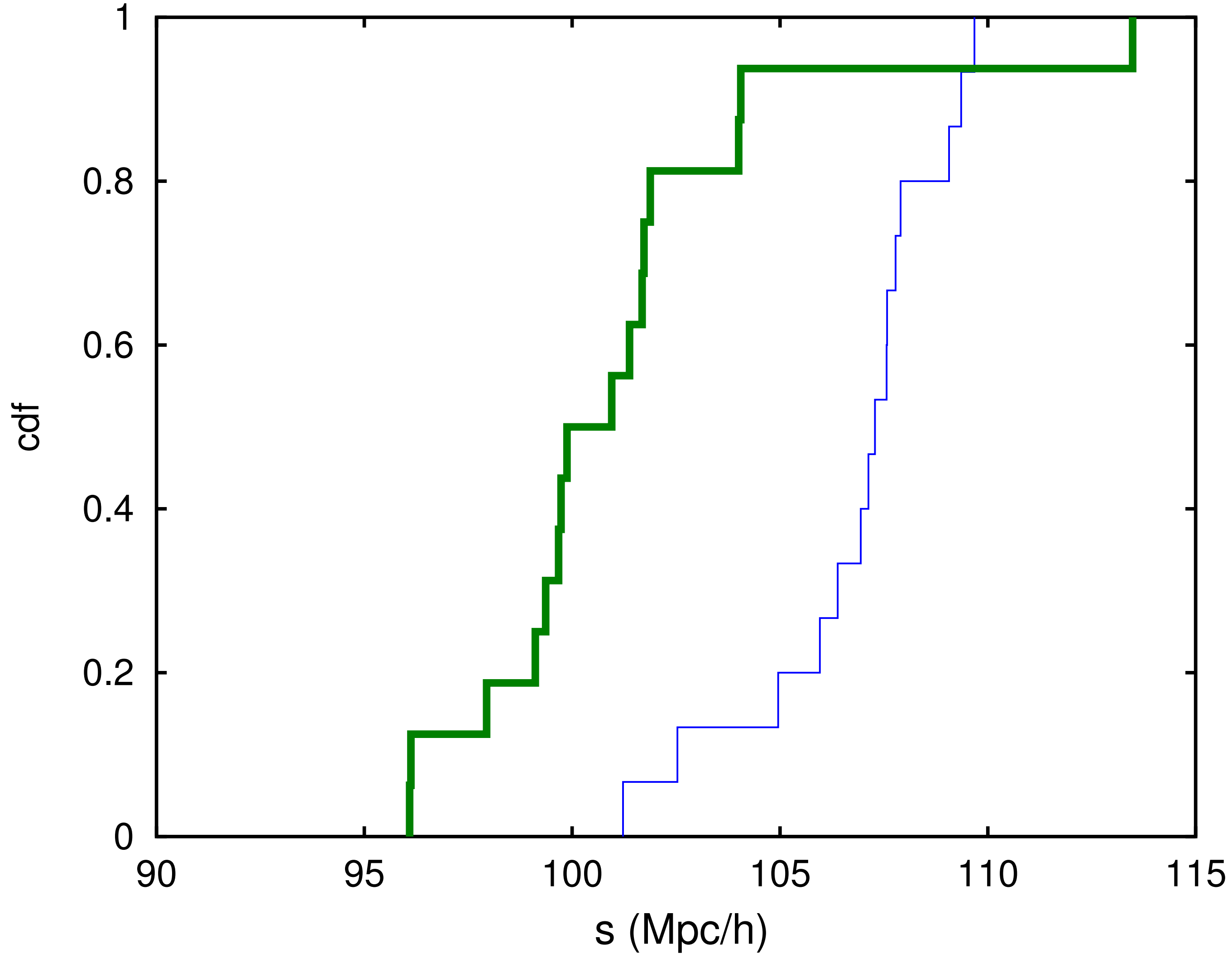
Liiv non-sc

BAO peak: Liivamägi sc's



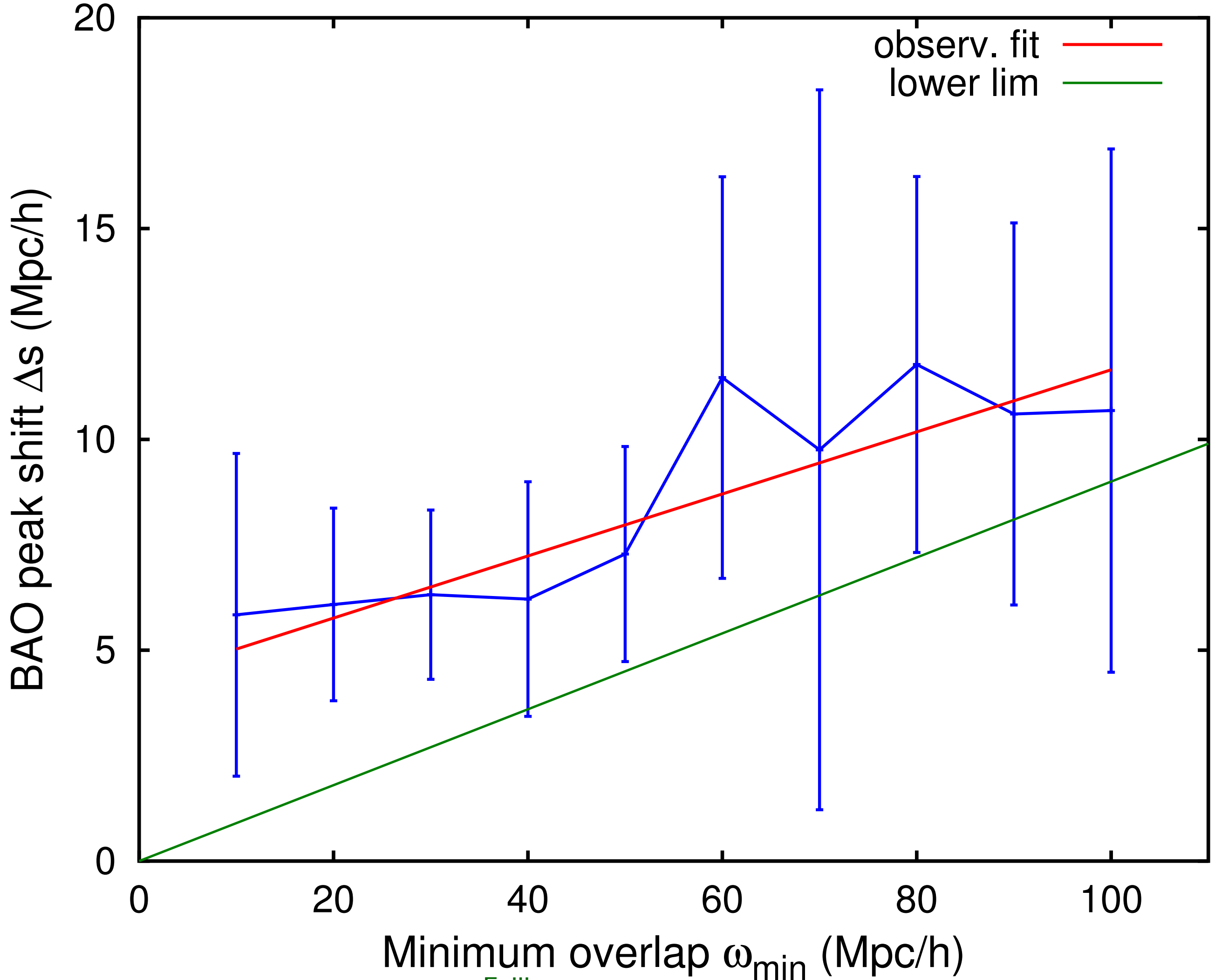
Liiv sc

BAO peak: Liivamägi sc's



$p_{\mathrm{KS}} = 3 \times 10^{-5}$

$\Delta s(\omega_{\min})$ relation



Roukema, Buchert, 藤井 (Fujii) & Ostrowski 2015 MNRAS

BAO results

catalogue	$r_\perp^0 - r_\perp^{\rm sc}$	$r_\perp^{\rm non-sc} - r_\perp^{\rm sc}$	$r_\perp^0 - r_\perp^{\rm void}$	$r_\perp^{\rm non-void} - r_\perp^{\rm void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

BAO results

catalogue	$r_\perp^0 - r_\perp^{\rm sc}$	$r_\perp^{\rm non-sc} - r_\perp^{\rm sc}$	$r_\perp^0 - r_\perp^{\rm void}$	$r_\perp^{\rm non-void} - r_\perp^{\rm void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx$ 6% for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc

BAO results

catalogue	$r^0_\perp - r^{\rm sc}_\perp$	$r^{\rm non-sc}_\perp - r^{\rm sc}_\perp$	$r^0_\perp - r^{\rm void}_\perp$	$r^{\rm non-void}_\perp - r^{\rm void}_\perp$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx 6\%$ for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues

BAO results

catalogue	$r_\perp^0 - r_\perp^{\rm sc}$	$r_\perp^{\rm non-sc} - r_\perp^{\rm sc}$	$r_\perp^0 - r_\perp^{\rm void}$	$r_\perp^{\rm non-void} - r_\perp^{\rm void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx 6\%$ for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues
- Δs increases with $\omega_{\rm min}$ to $\sim 10\%$

BAO results

catalogue	$r_\perp^0 - r_\perp^{\rm sc}$	$r_\perp^{\rm non-sc} - r_\perp^{\rm sc}$	$r_\perp^0 - r_\perp^{\rm void}$	$r_\perp^{\rm non-void} - r_\perp^{\rm void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx$ 6% for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues
- Δs increases with $\omega_{\rm min}$ to $\sim 10\%$; P(no increase) $= 0.0008$

BAO results

catalogue	$r_\perp^0 - r_\perp^{sc}$	$r_\perp^{non-sc} - r_\perp^{sc}$	$r_\perp^0 - r_\perp^{void}$	$r_\perp^{non-void} - r_\perp^{void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx$ 6% for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues
- Δs increases with $\omega_{\rm min}$ to $\sim 10\%$; P(no increase) $= 0.0008$
- effect too noisy for void-overlapping pairs

BAO results

catalogue	$r_\perp^0 - r_\perp^{\rm sc}$	$r_\perp^{\rm non-sc} - r_\perp^{\rm sc}$	$r_\perp^0 - r_\perp^{\rm void}$	$r_\perp^{\rm non-void} - r_\perp^{\rm void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx$ 6% for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues
- Δs increases with $\omega_{\rm min}$ to ~ 10%; P(no increase) $= 0.0008$
- effect too noisy for void-overlapping pairs
- unpredicted in FLRW; expected from scalar averaging

BAO results

catalogue	$r_\perp^0 - r_\perp^{\rm sc}$	$r_\perp^{\rm non-sc} - r_\perp^{\rm sc}$	$r_\perp^0 - r_\perp^{\rm void}$	$r_\perp^{\rm non-void} - r_\perp^{\rm void}$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx$ 6% for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues
- Δs increases with $\omega_{\rm min}$ to $\sim 10\%$; P(no increase) $= 0.0008$
- effect too noisy for void-overlapping pairs
- unpredicted in FLRW; expected from scalar averaging
- Roukema, Buchert, Ostrowski & France (2015) MNRAS, 448, 1660

BAO results

catalogue	$r^0_\perp - r^{\rm sc}_\perp$	$r^{\rm non-sc}_\perp - r^{\rm sc}_\perp$	$r^0_\perp - r^{\rm void}_\perp$	$r^{\rm non-void}_\perp - r^{\rm void}_\perp$
N&H	4.3 ± 1.6	6.6 ± 2.8	-0.2 ± 4.0	-1.1 ± 5.5
LTS	3.7 ± 2.9	6.3 ± 2.6	all in h^{-1} Mpc	

- BAO comoving ruler compressed by $\approx$ 6% for supercluster-overlapping pairs for $\omega_{\rm min} = 1\ h^{-1}$ Mpc
- similar result for both NH2013 and Liivamägi2012 supercluster catalogues
- Δs increases with $\omega_{\rm min}$ to $\sim 10\%$; P(no increase) $= 0.0008$
- effect too noisy for void-overlapping pairs
- unpredicted in FLRW; expected from scalar averaging
- Roukema, Buchert, Ostrowski & France (2015) MNRAS, 448, 1660; Roukema, Buchert, 藤井 (Fujii) & Ostrowski (2015) MNRAS, 456, L45 *[cf Cold Spot C_l shift Chiang (2018) ApJ arXiv:1805.06636]*

Inhom. cosmo. community resources

inhomogeneous cosmology meetings:
https://cosmo.torun.pl/cosmotorun19_hist.html

newsletter (read archives, subscribe, or unsubscribe):
https://cosmo.torun.pl/listinfo/inhom

ADSinhom
(redirect: https://cosmo.torun.pl/ADSinhom)
431 articles listed as of 2021-07-24

Inhomogeneous cosmology conclusion I

- exact solutions: still many prospects for qualitative inferences about general case
- scalar averaging: formalisms continuing to develop; many specific applications (turnaround curvature) | $a_{\mathcal{D}}$
- numerical relativity: developing intensively, reproducibility needs attention | N
- observational applications: DE from emerging negative average curvature is viable at the order-of-magnitude level; several tests proposed | H_0's | $\mathcal{O}$ | BAO

Inhomogeneous cosmology conclusion I

- exact solutions: still many prospects for qualitative inferences about general case
- scalar averaging: formalisms continuing to develop; many specific applications (turnaround curvature) | $a_{\mathcal{D}}$
- numerical relativity: developing intensively, reproducibility needs attention | N
- observational applications: DE from emerging negative average curvature is viable at the order-of-magnitude level; several tests proposed | H_0's | $\mathcal{O}$ | BAO
- *relation between inhomogeneity and cosmic topology — topological acceleration — lecture II*