Cosmic inhomogeneity and topology II

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https://cosmo.torun.pl/∼boud/Roukema20210727CIRM.pdf
https://cosmo.torun.pl/∼boud/Roukema20210729CIRM.pdf

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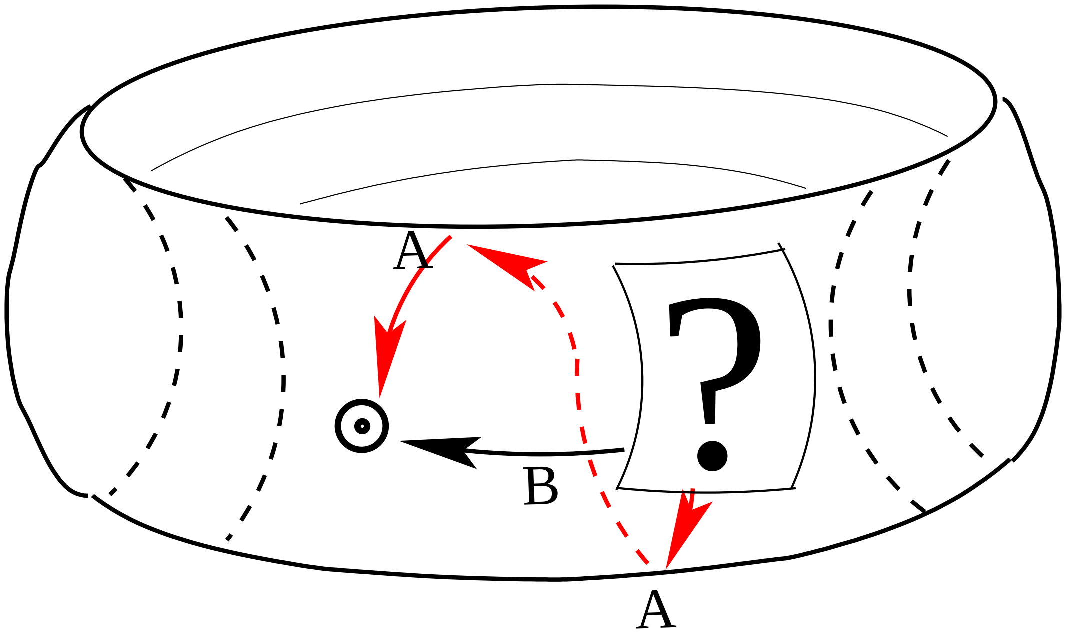

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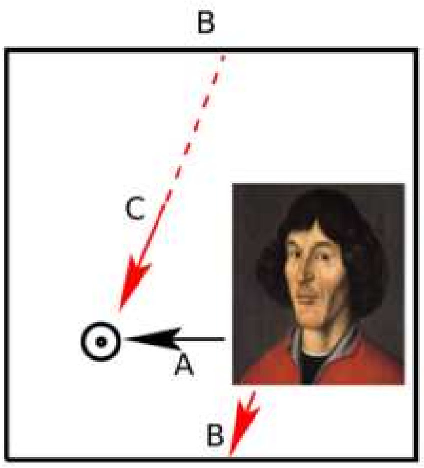

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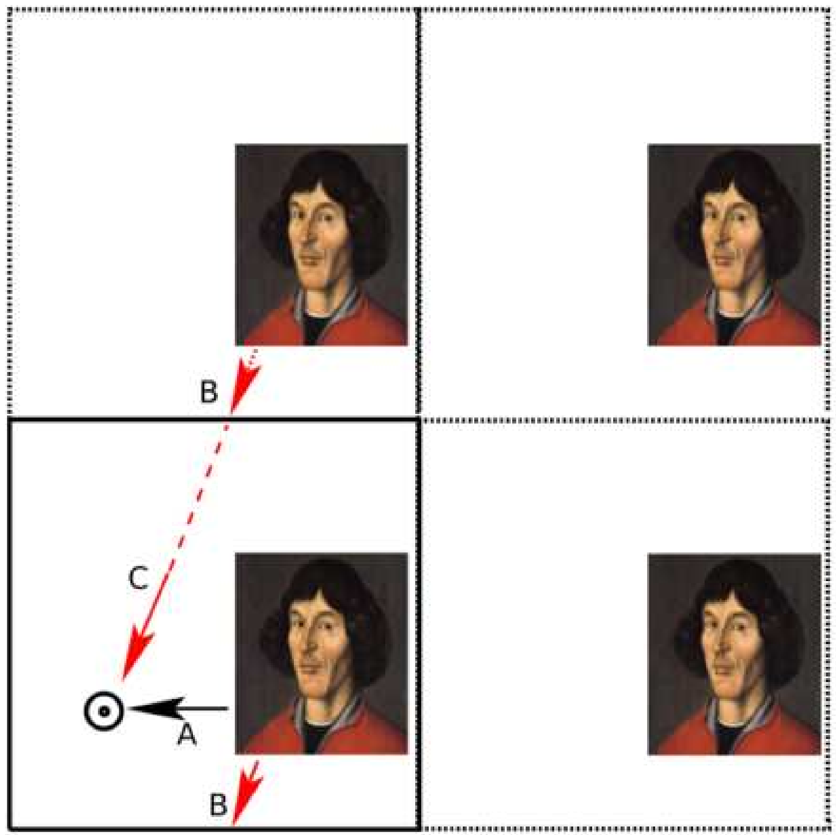

Non-radial spatial geodesics

- distances on the 2-sphere, embedded in $\mathbb{R}^3$

$$\begin{aligned} x_i &= R_\mathrm{C} \cos\delta_i \cos\alpha_i \\ y_i &= R_\mathrm{C} \cos\delta_i \sin\alpha_i \\ w_i &= R_\mathrm{C} \sin\delta_i \end{aligned}$$

$$\langle \mathbf{a}_1, \mathbf{a}_2 \rangle = x_1 x_2 + y_1 y_2 + w_1 w_2$$

- but also:

$$\langle \mathbf{a}_1, \mathbf{a}_2 \rangle = R^2 \cos\theta_{12}.$$

- a distance in S^2 = arc-length in $\mathbb{R}^3$:

$$\chi_{12} = R_\mathrm{C}\, \theta_{12} = R_\mathrm{C}\, \cos^{-1}\left[\langle \mathbf{a}_1, \mathbf{a}_2 \rangle / R_\mathrm{C}^2\right]$$

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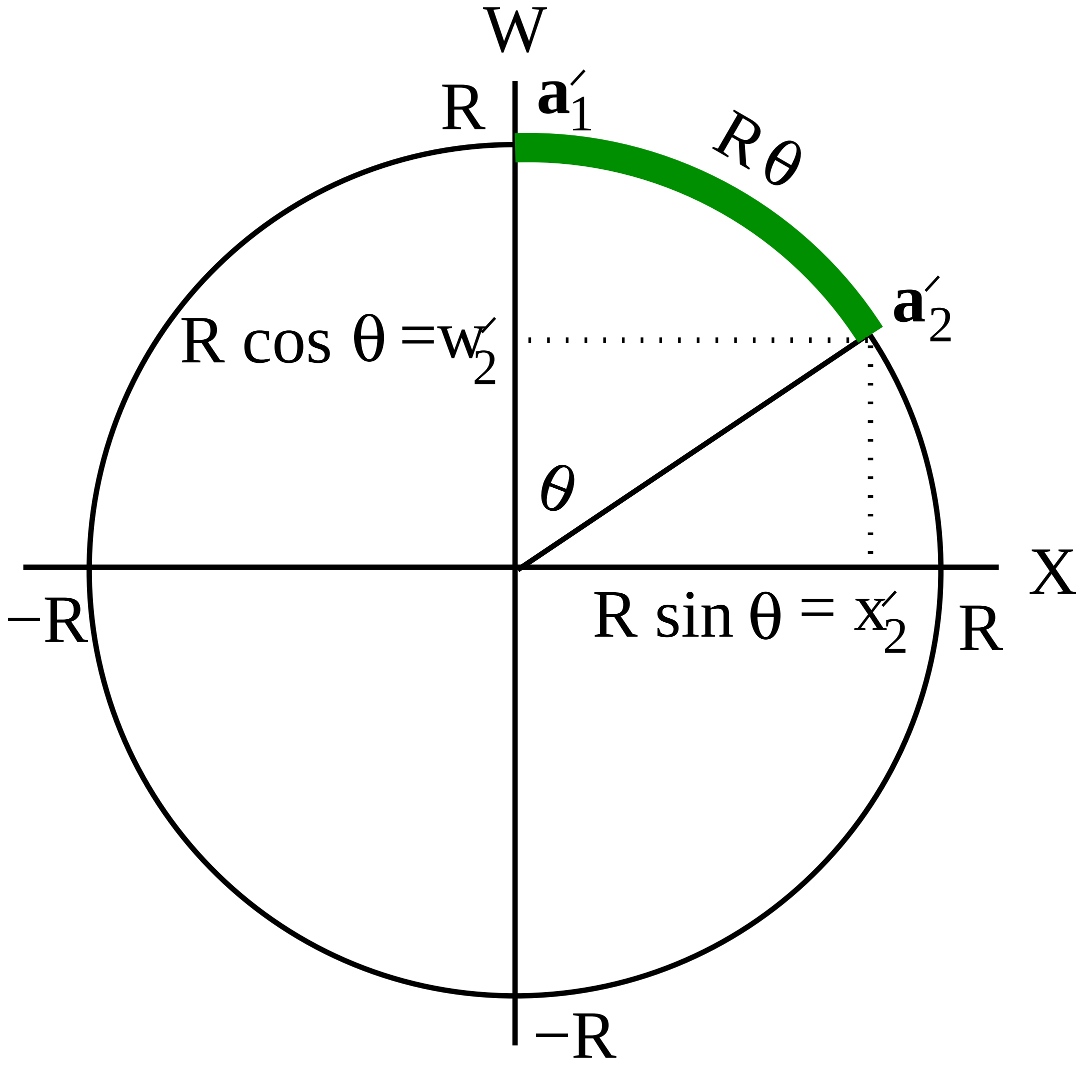

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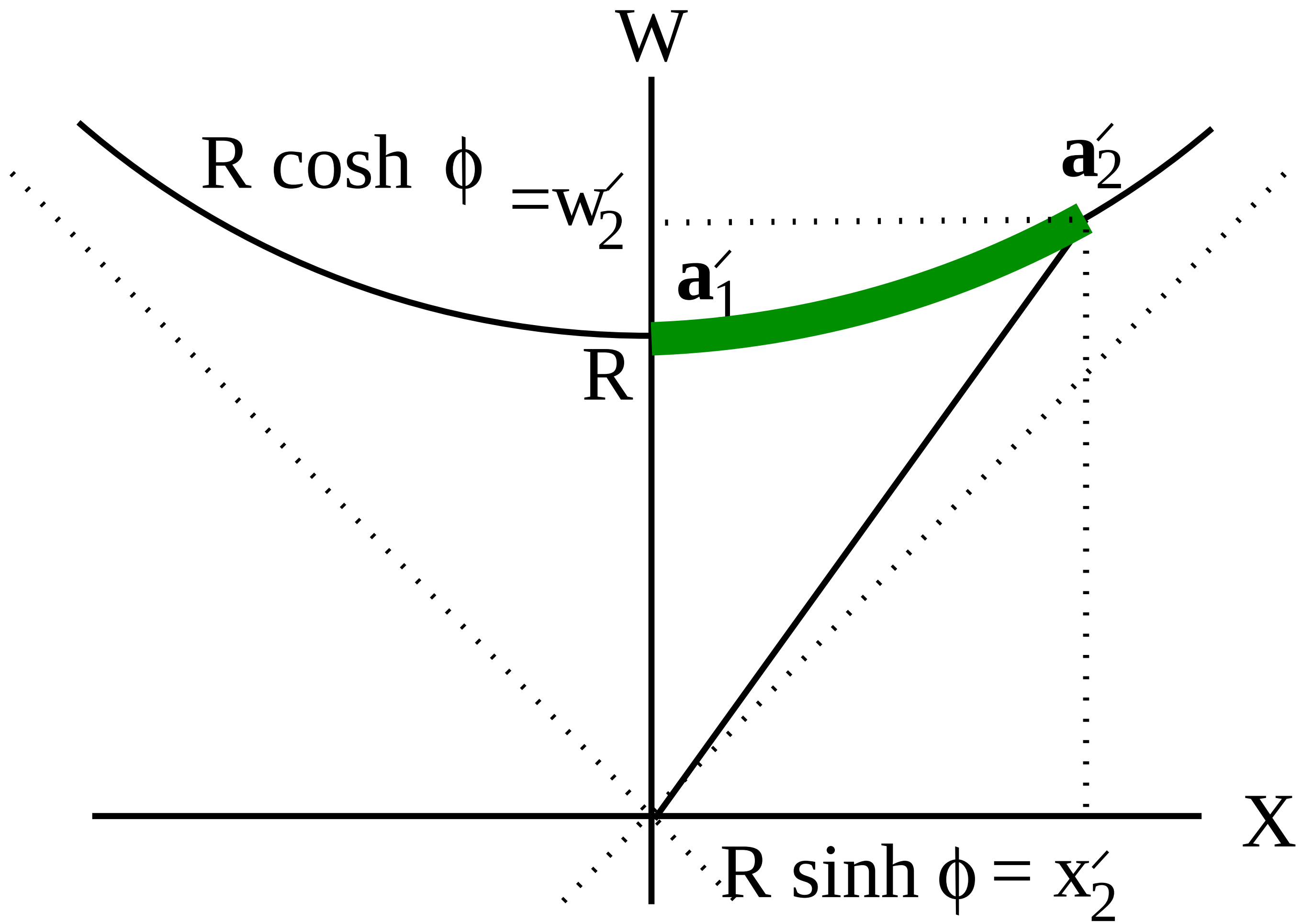

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distances on $\mathsf{S}^3 \subset \mathbb{R}^4$ or $\mathsf{H}^3 \subset \mathsf{M}^4$

$$\Sigma(\chi_i) := \begin{cases} R_{\mathrm{C}}\ \sinh(\chi_i/R_{\mathrm{C}}) & k < 0 \\ \chi_i & k = 0 \\ R_{\mathrm{C}}\ \sin(\chi_i/R_{\mathrm{C}}) & k > 0 \end{cases}$$

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- metric on S^3 (or $\mathbb{R}^3$ or H^3):
$$\mathrm{d}s^2 = \begin{cases} (k/|k|)\,(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) + \mathrm{d}w^2 & k \neq 0 \\ \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 & k = 0 \end{cases}$$

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- inner product:
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- fundamental domain (FD) is not unique; shape of FD may be non-unique

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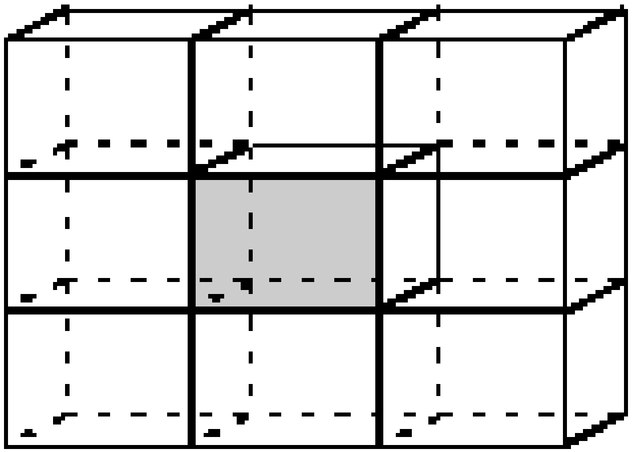

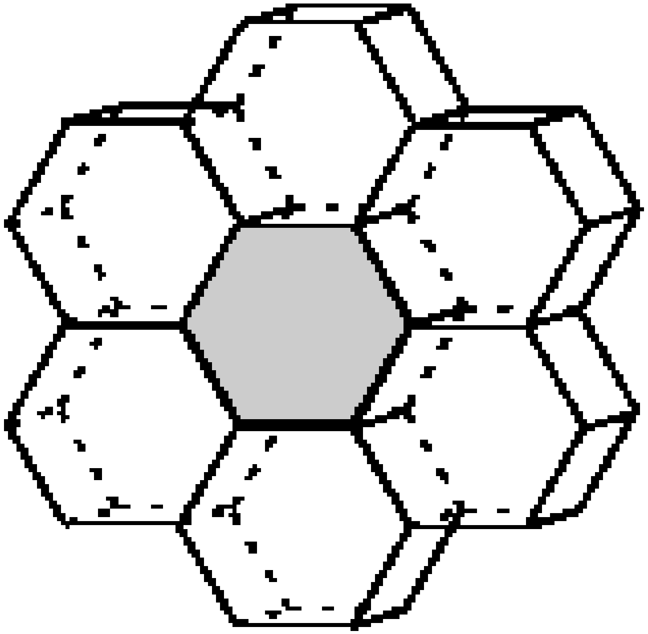

3D flat examples arXiv:astro-ph/9901364

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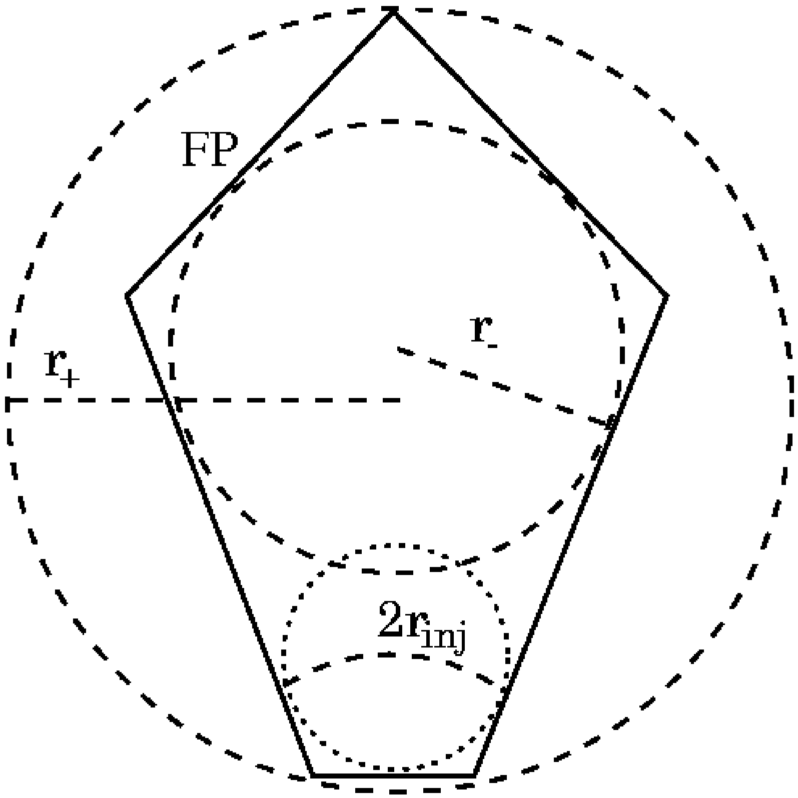

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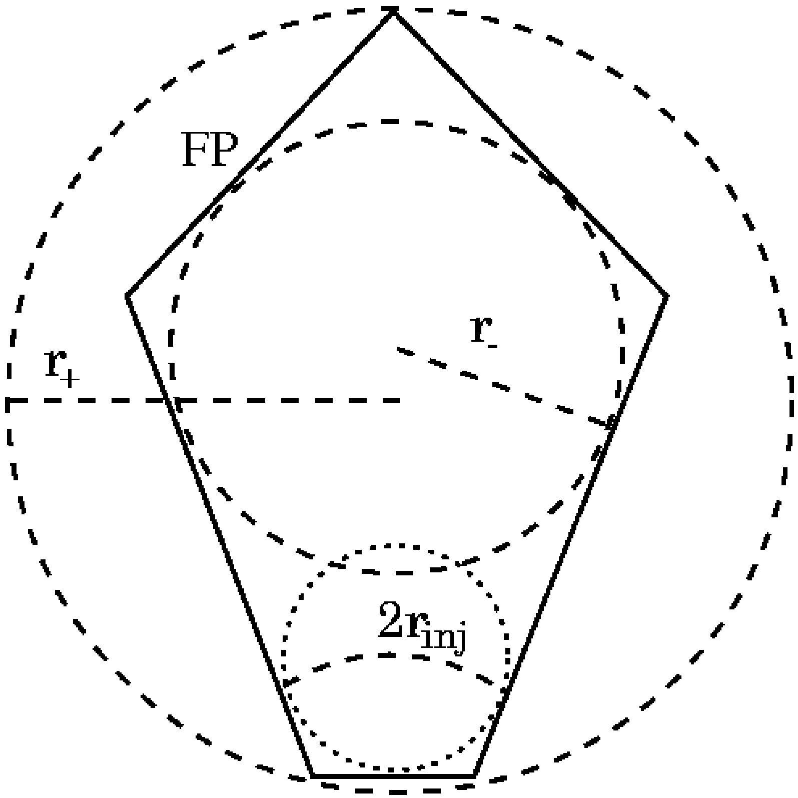

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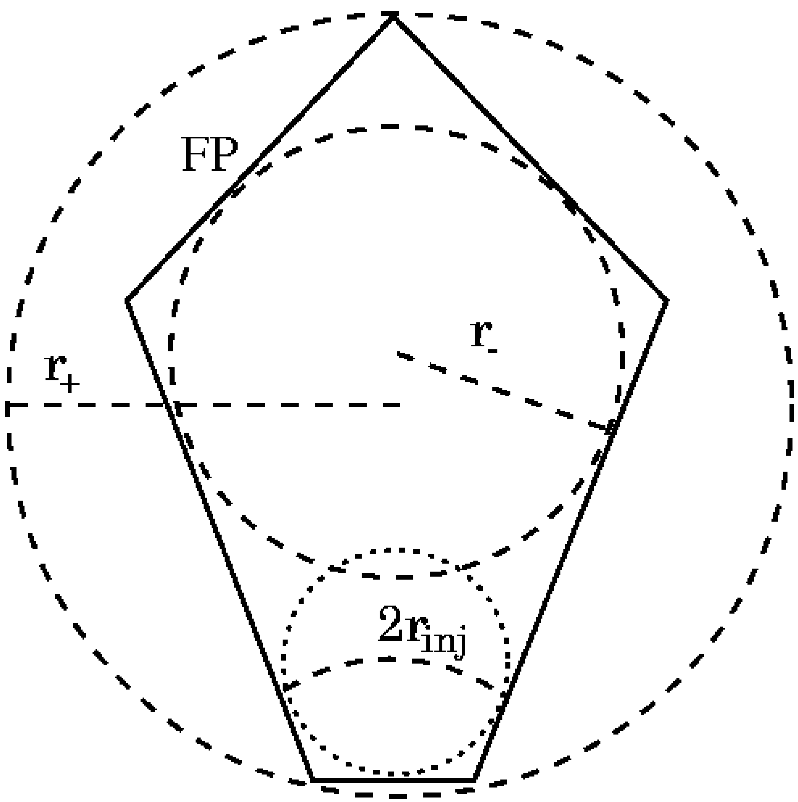

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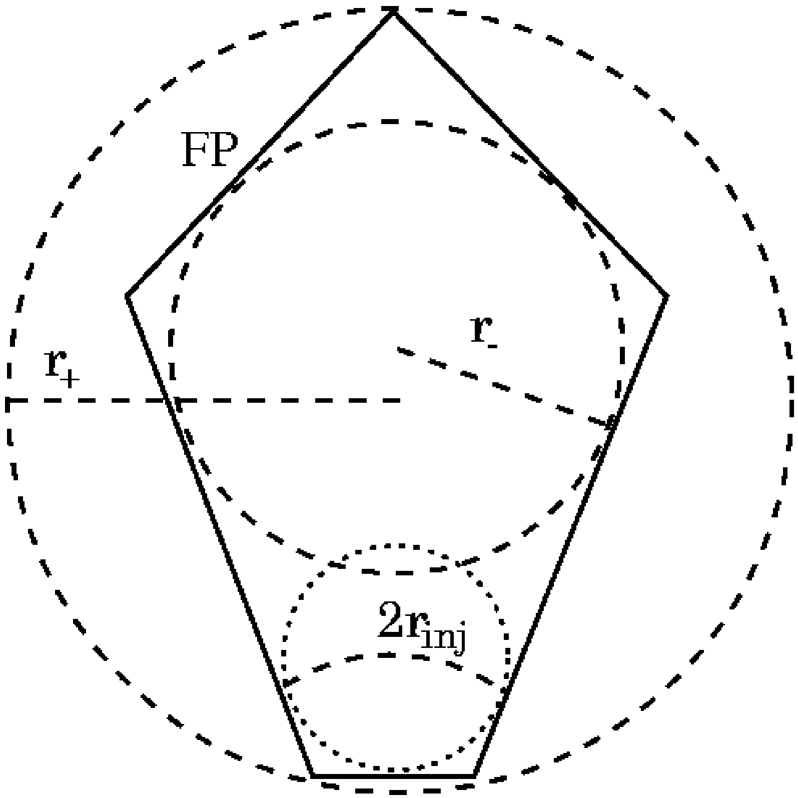

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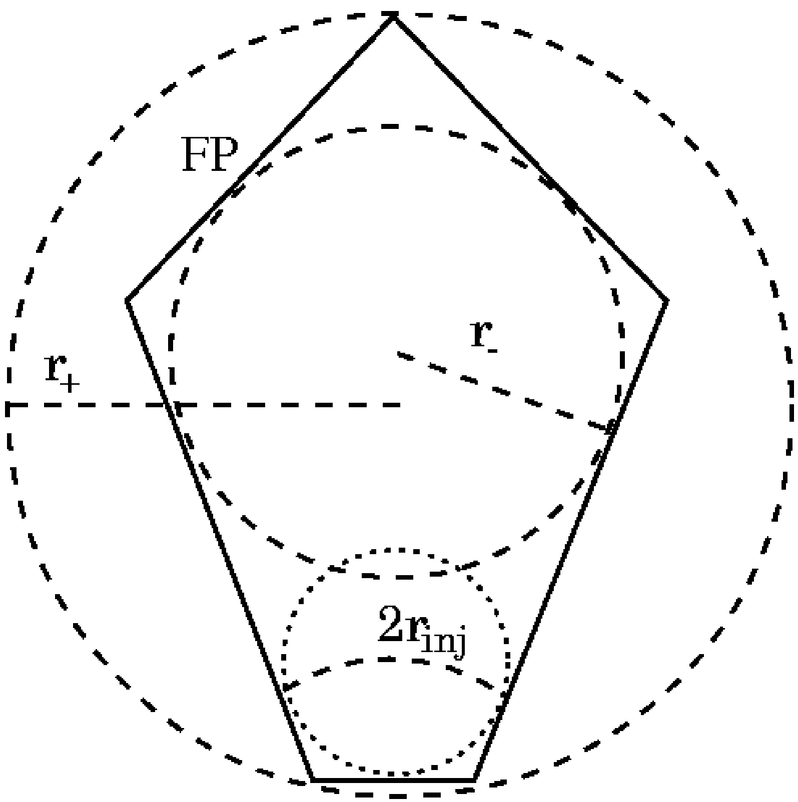

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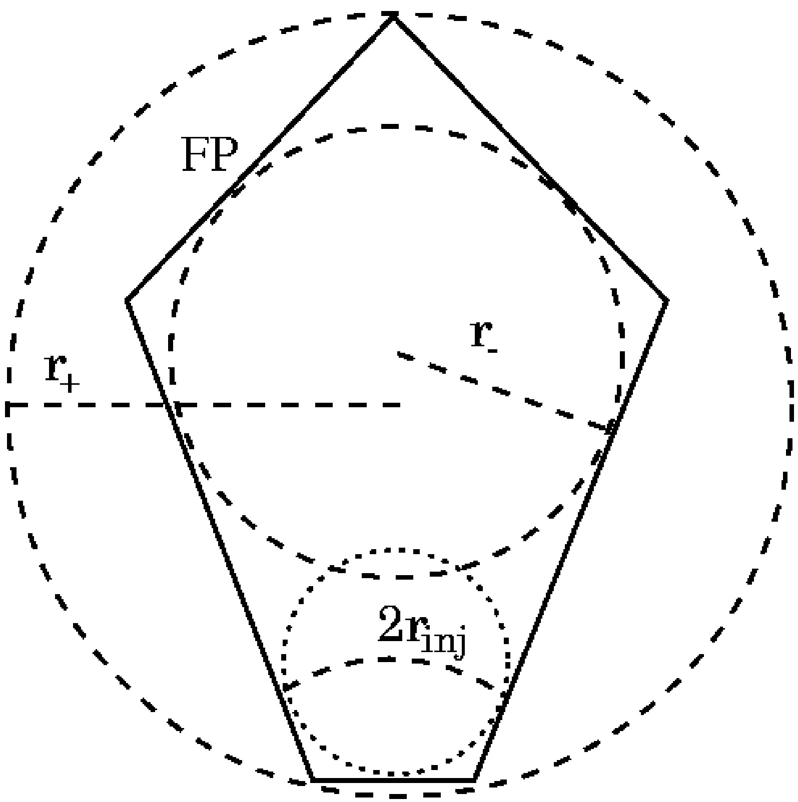

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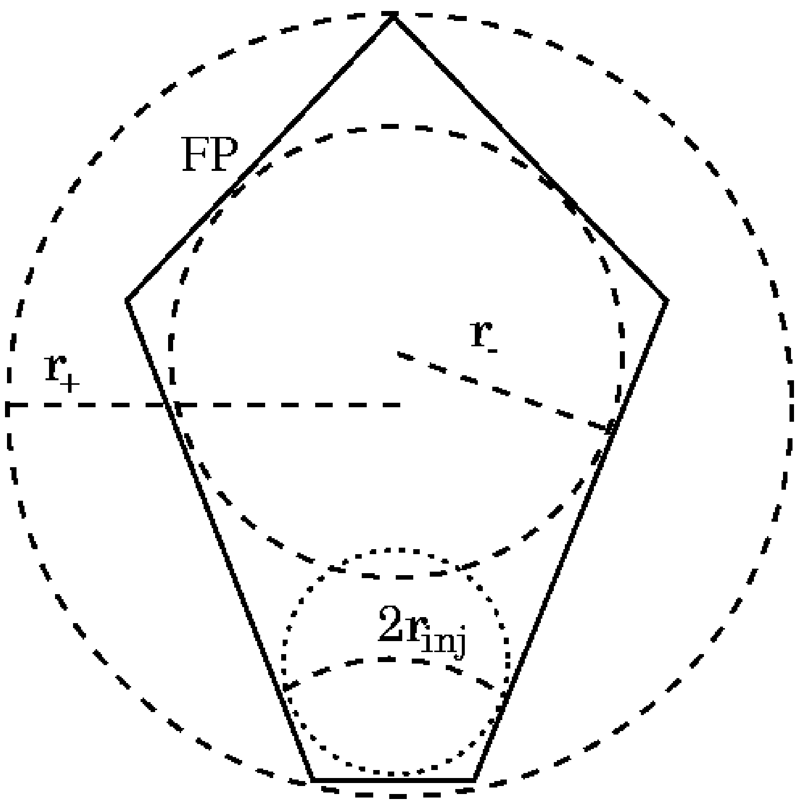

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- $r_{\rm inj} < r_-$ or $r_{\rm inj} \ll V_{\rm FD}^{1/3}$ possible

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  - w:Poincaré Conjecture *"Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere."* w:Grigori Perelman, arXiv:math/0211159 + arXiv:math/0303109 + arXiv:math/0307245

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  - active research area, e.g. arXiv:0705.4325 min. vol.

Cosmic topol: top. accel.

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- global spatial topology: topological acceleration (patching away black holes): Roukema+2007 (A&A 2007) arXiv:astro-ph/0602159

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cosmic topology theory:

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- scalar averaging and dynamical topology change (e.g. black holes): Brunswic & Buchert (CQG, 2020) arXiv:2002.08336

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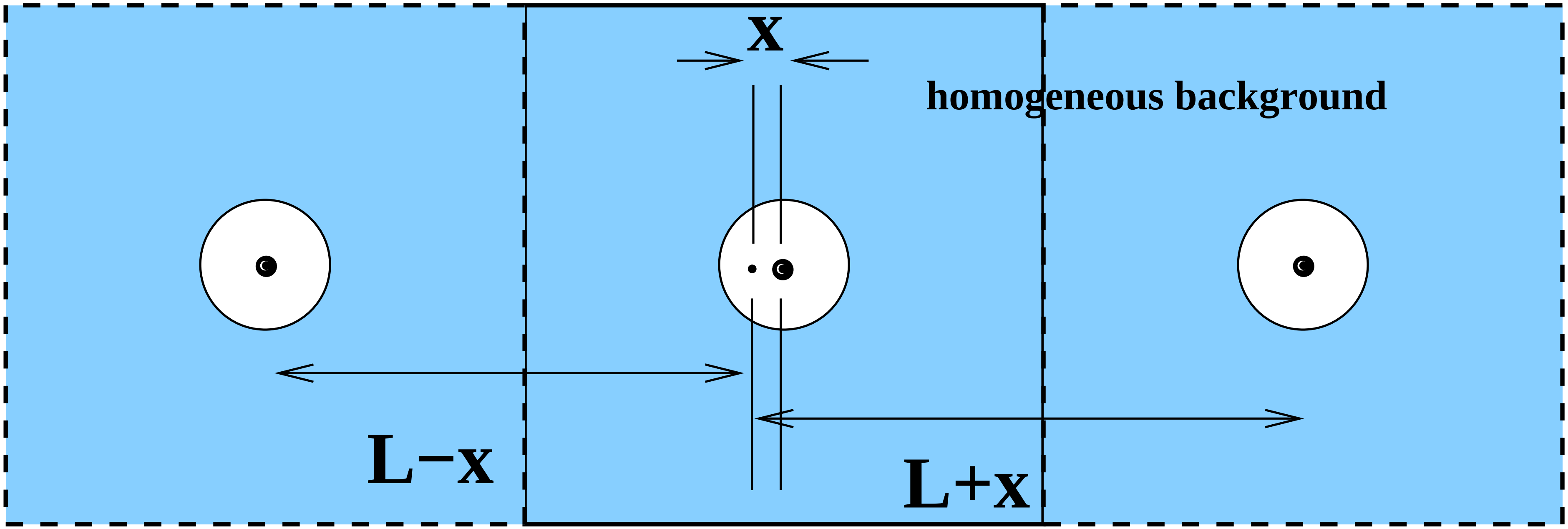

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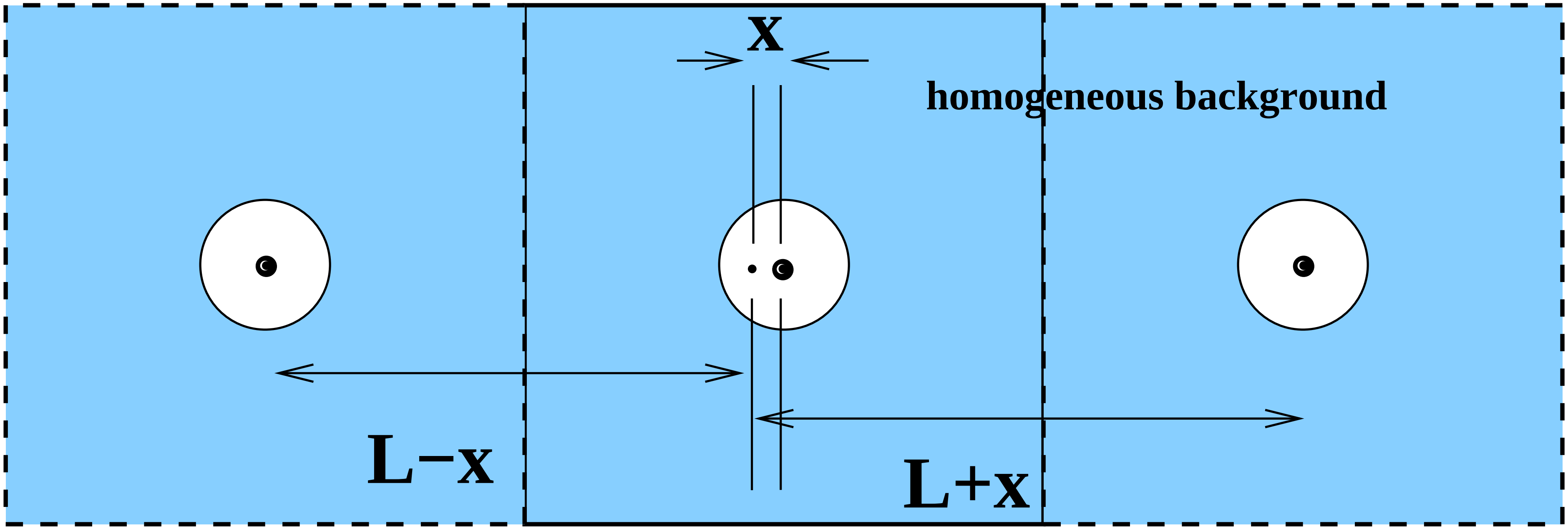

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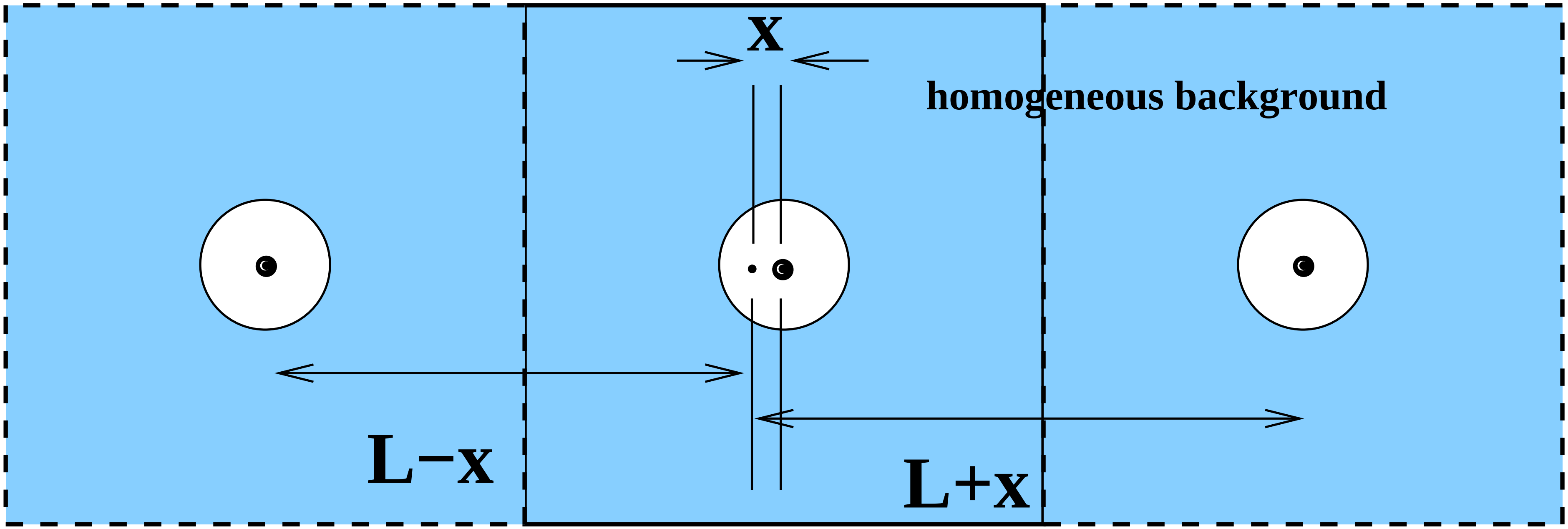

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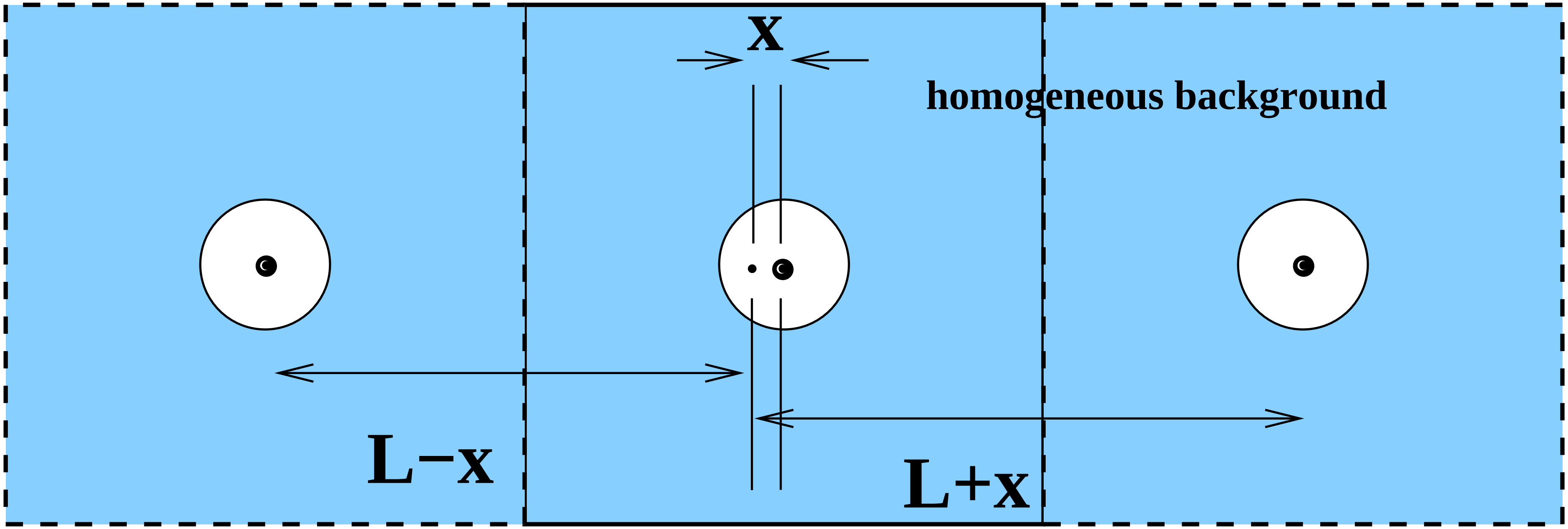

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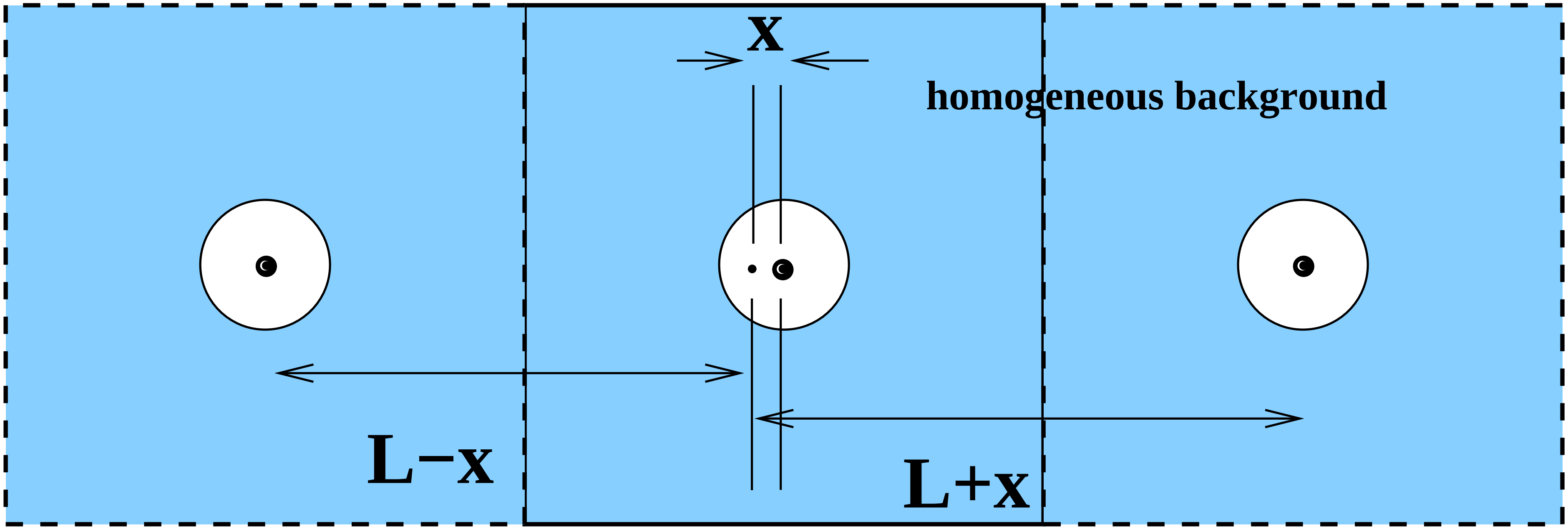

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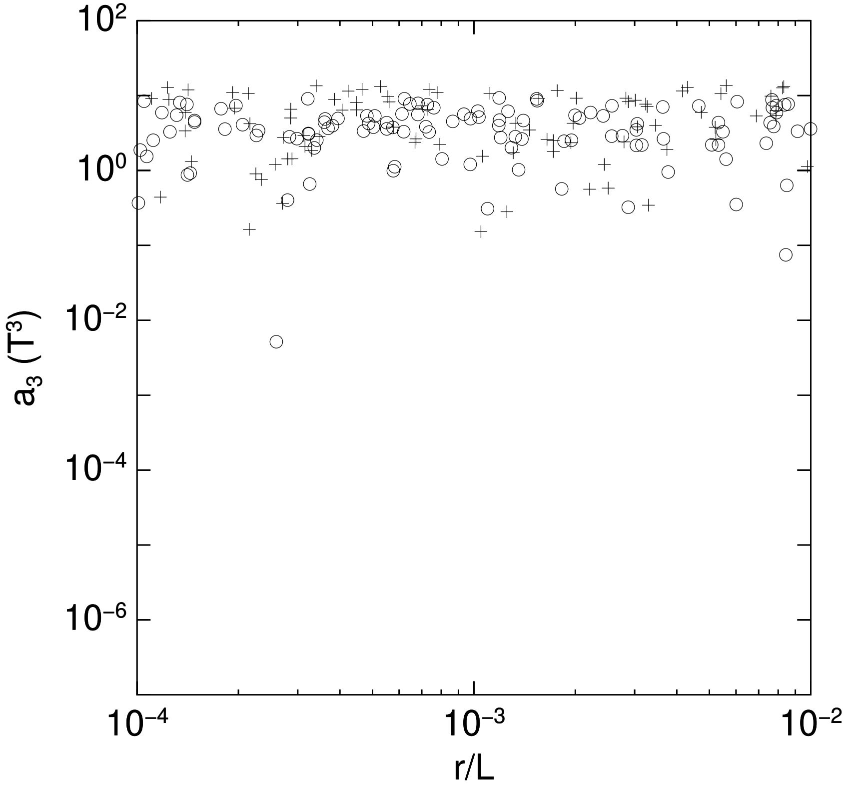
topological acceleration— arXiv:astro-ph/0602159

Heuristic top. accel.

- weak-field gravity of distant, multiple images
- covering space $\mathbb{R}^3$ or $\mathbb{S}^3$
- calculations made in covering space
- consider only first layer of topological images (e.g. particle horizon)

$T^3, S^3/\Gamma$



$$T^3 = \mathbb{R}^3/\mathbb{Z}^3 \quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/L)^3 + \ldots$$

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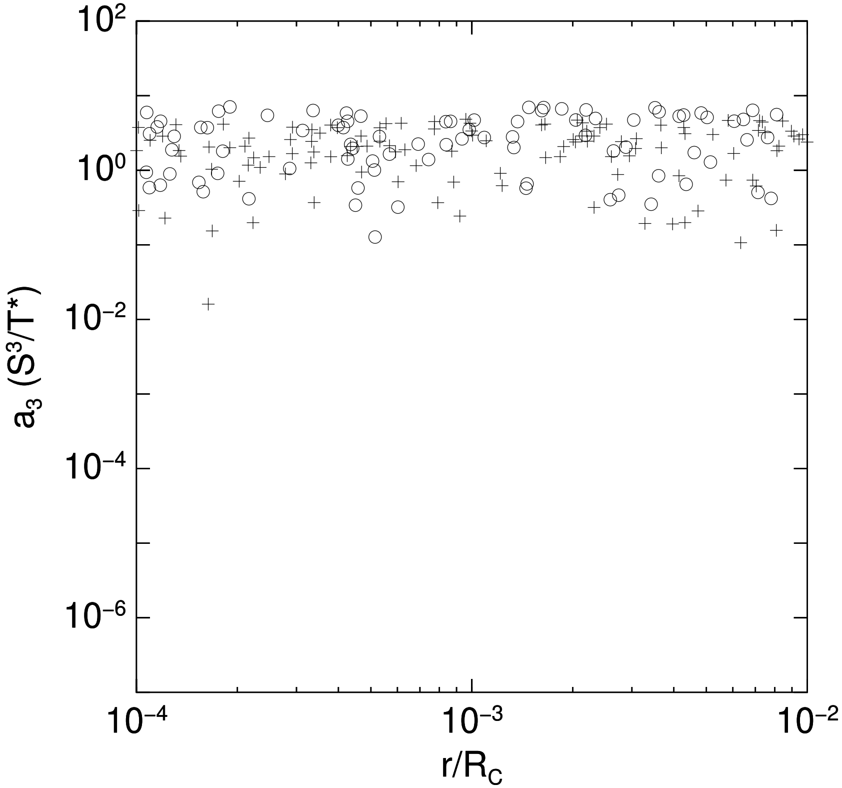

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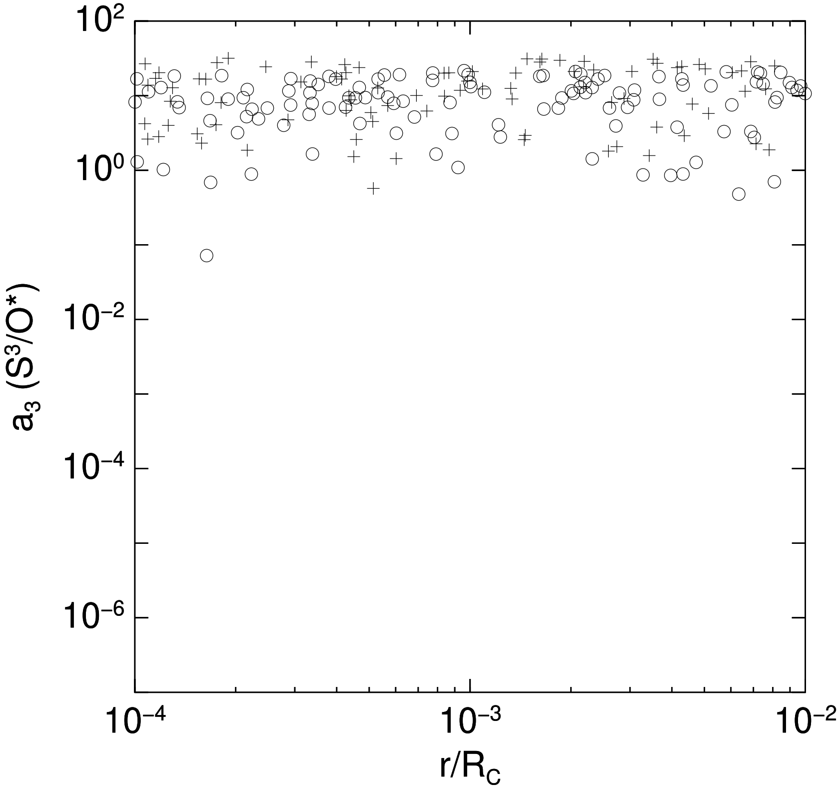

S^3/O^* (truncated cube space) $\Rightarrow$ $\ddot{x}_{\rm resid} \propto (x/R_C)^3 + \ldots$

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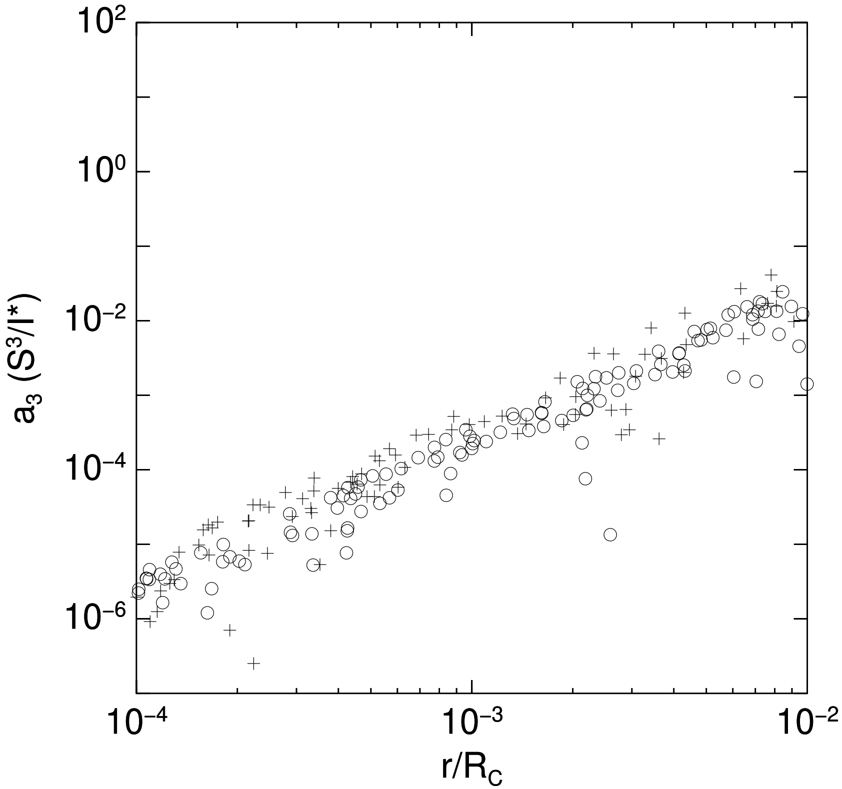

S^3/I^* (Poincaré dodecahedral space) $\Rightarrow$ $\ddot{x}_{\mathrm{resid}} \propto (x/R_C)^5 + \ldots$

$T^3, S^3/\Gamma$

- Some spaces are more equal than others.
- Roukema & Różański arXiv:0902.3402, A&A, 502, 27
- Newton–Cartan approach for preparing for full GR approach: Vigneron (2020) arXiv:2010.10247; Vigneron (2021) arXiv:2012.10213;

Cosmic topol: obs. strategies

empirical strategies: arXiv:astro-ph/0010189

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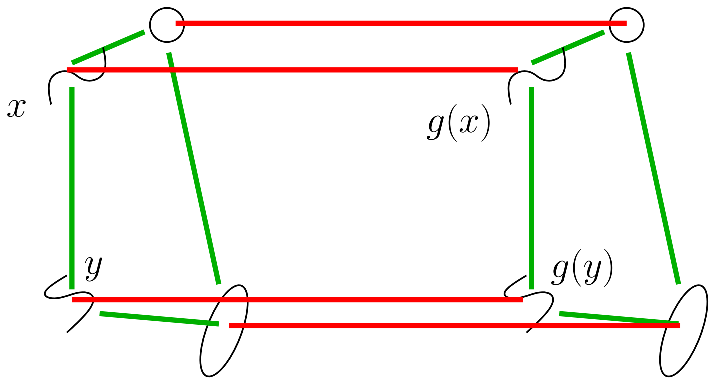
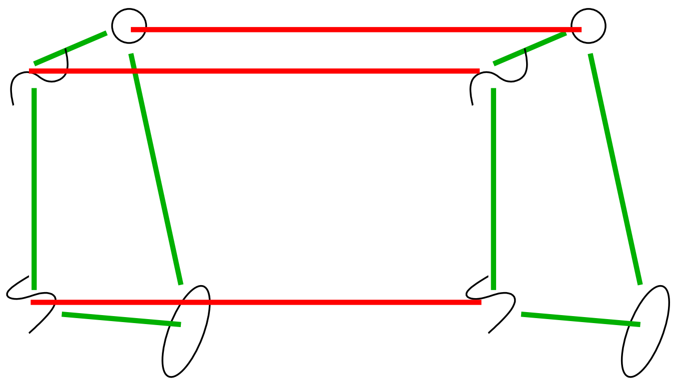
3D strategies—pair types





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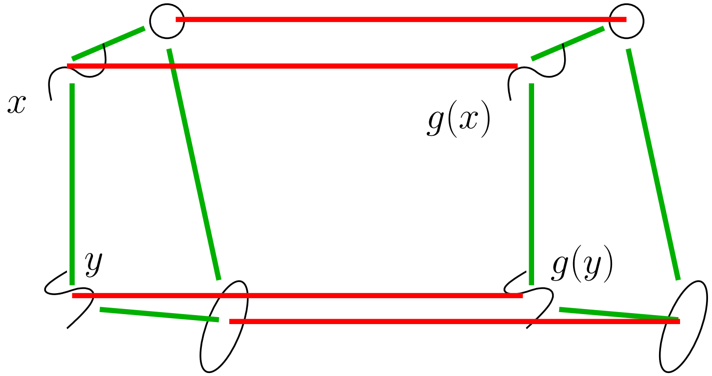

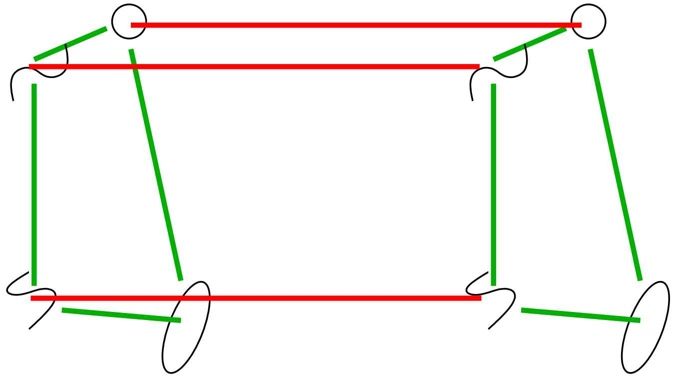

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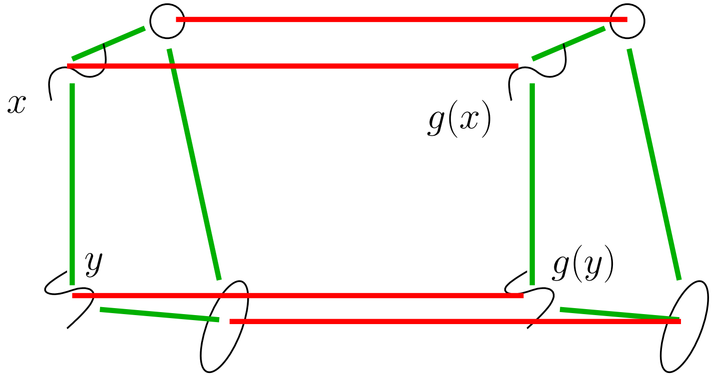

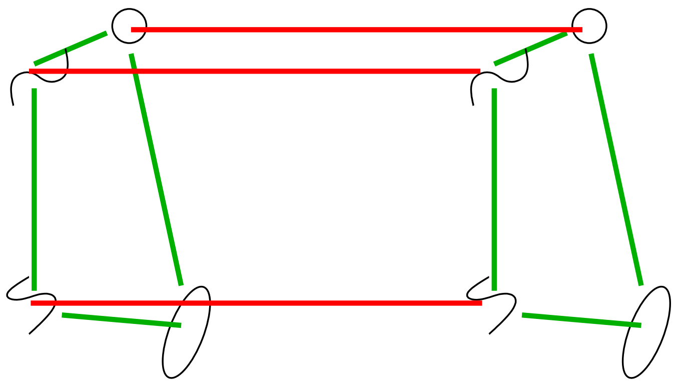

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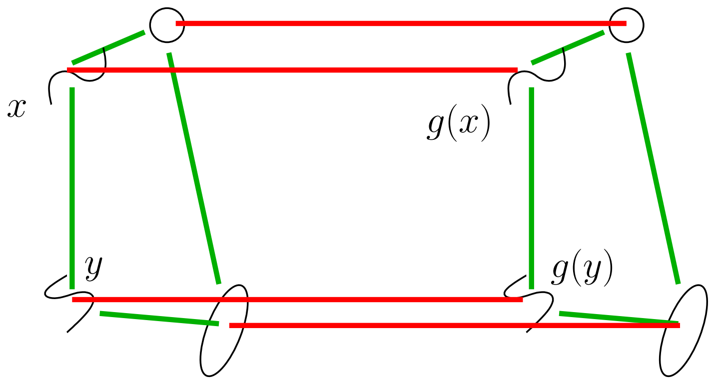

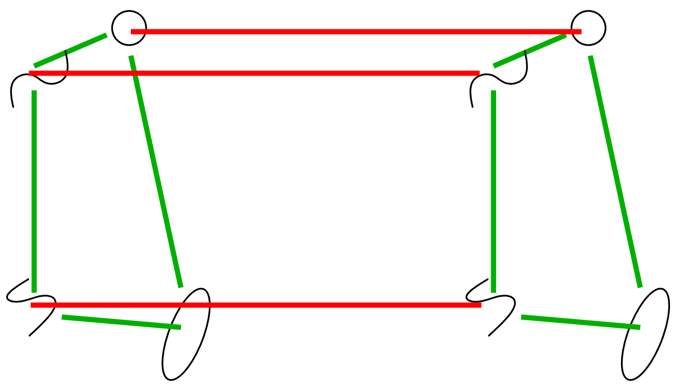

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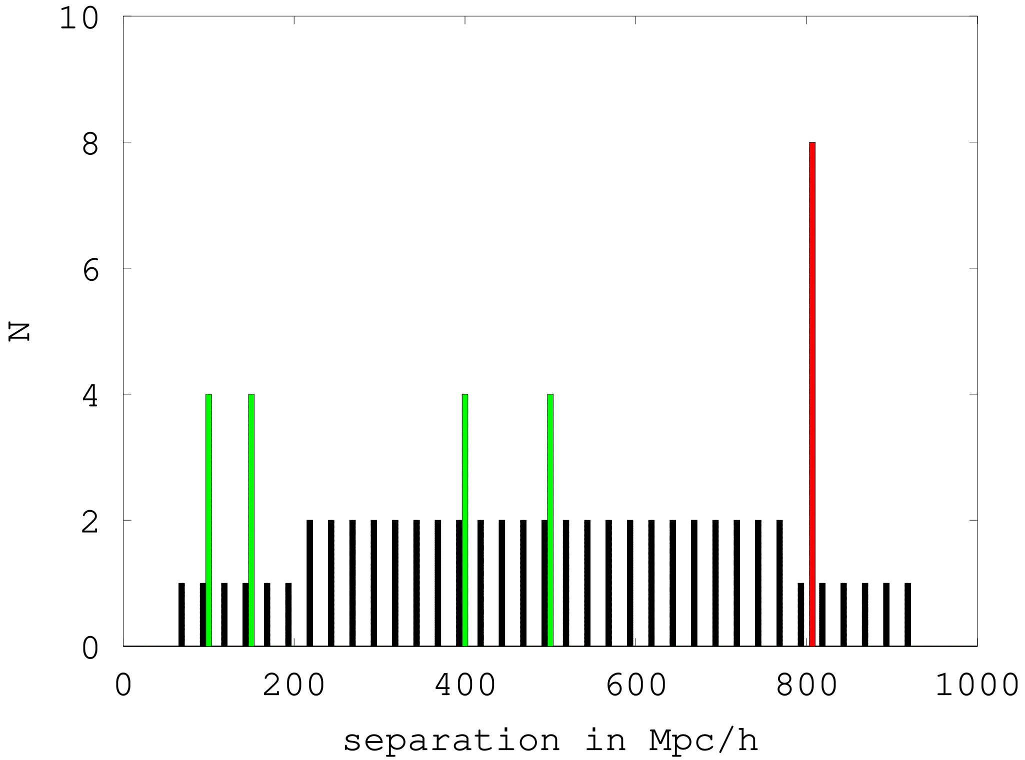

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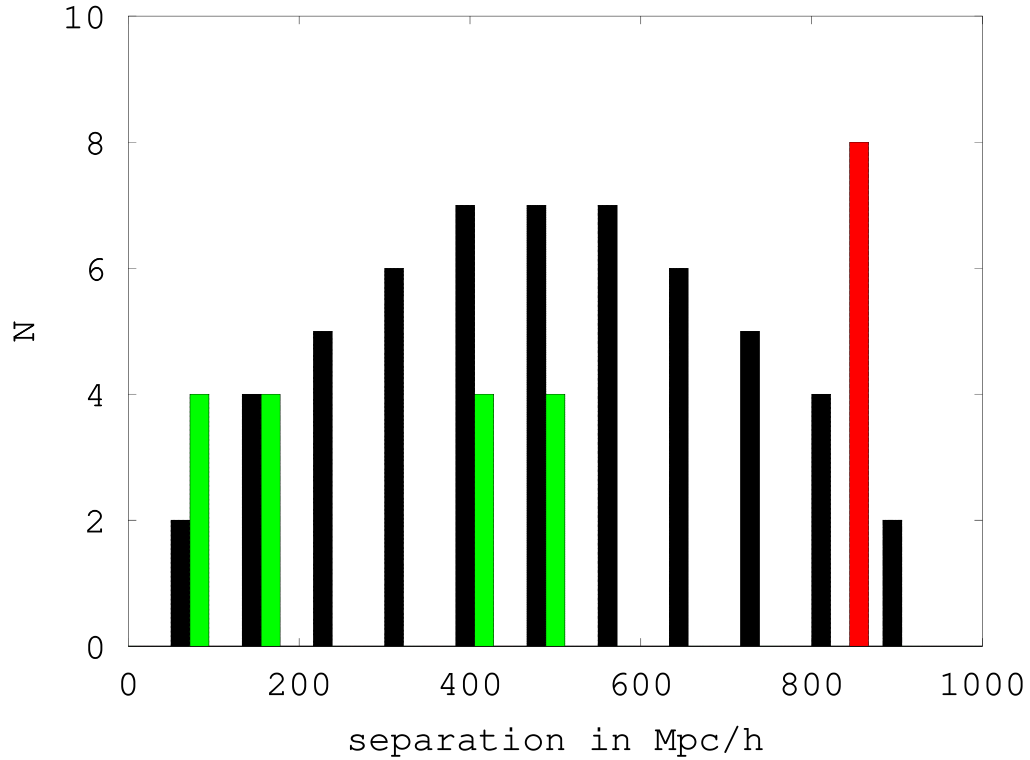

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- quadruples + successive filters + collect membership s of quadruples Fujii & Yoshii (2013)

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AGNs—successive filters

Marecki, Roukema, Bajtlik (2005) arXiv:astro-ph/0412181

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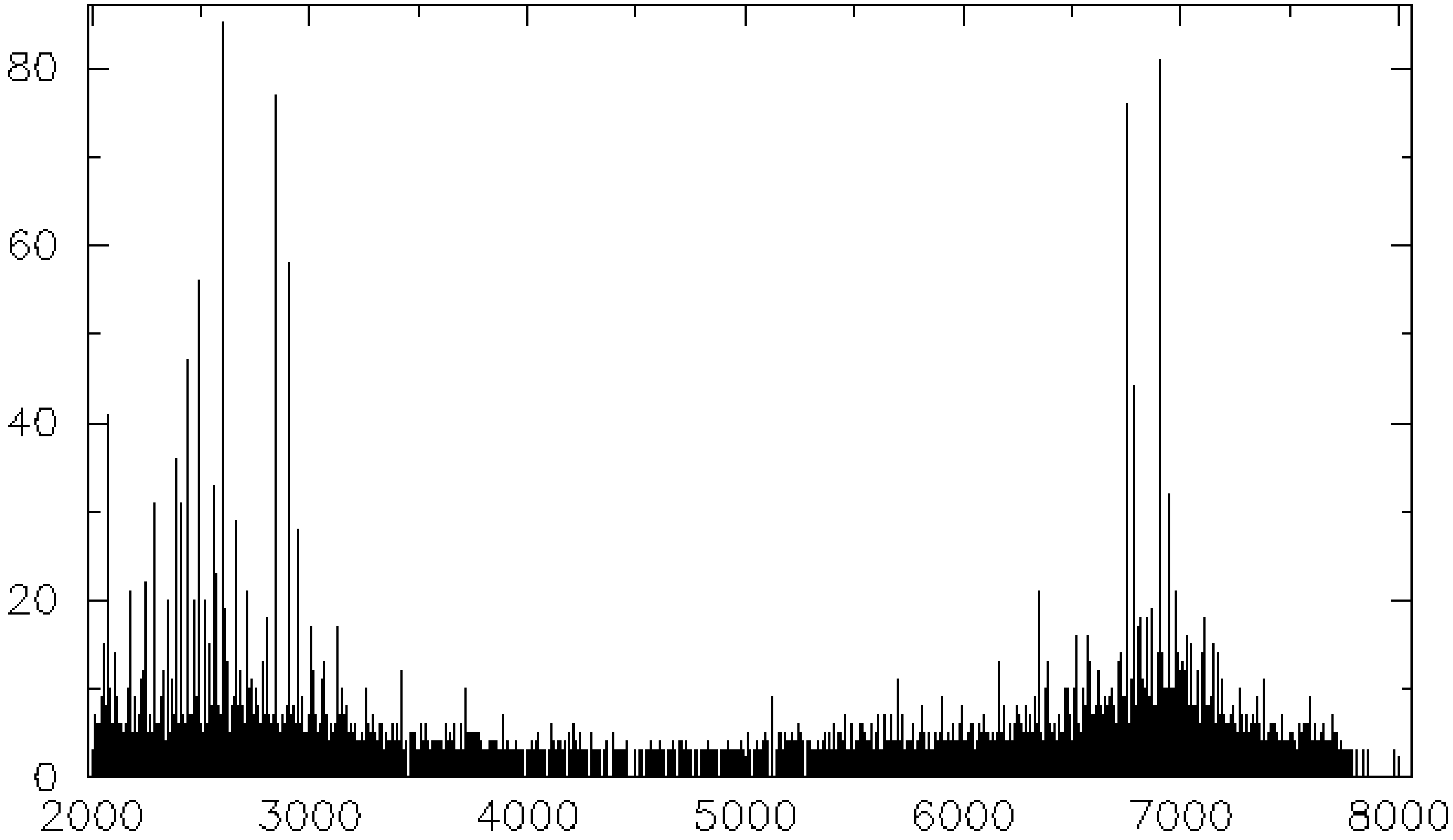
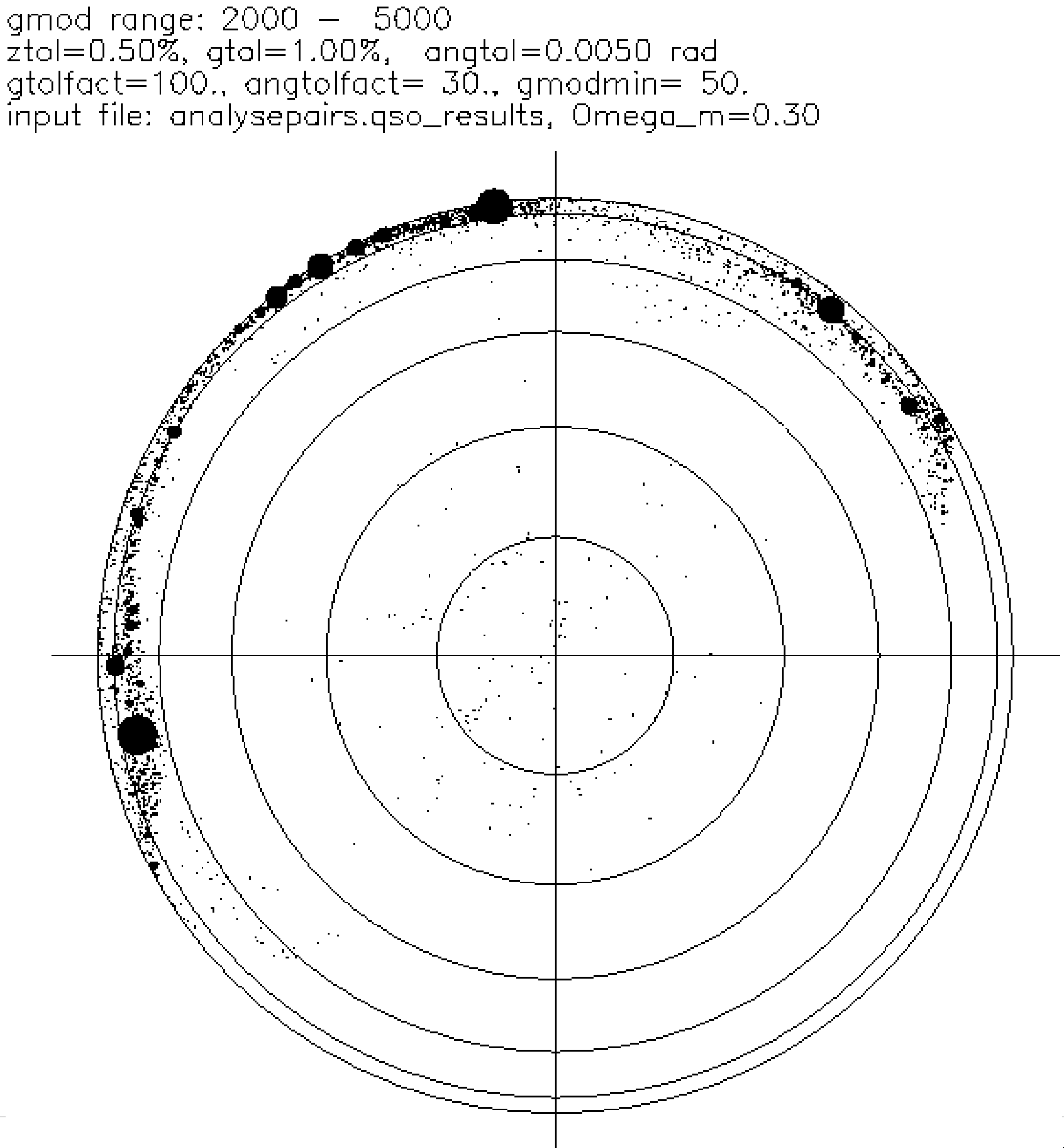


AGN Catalogues



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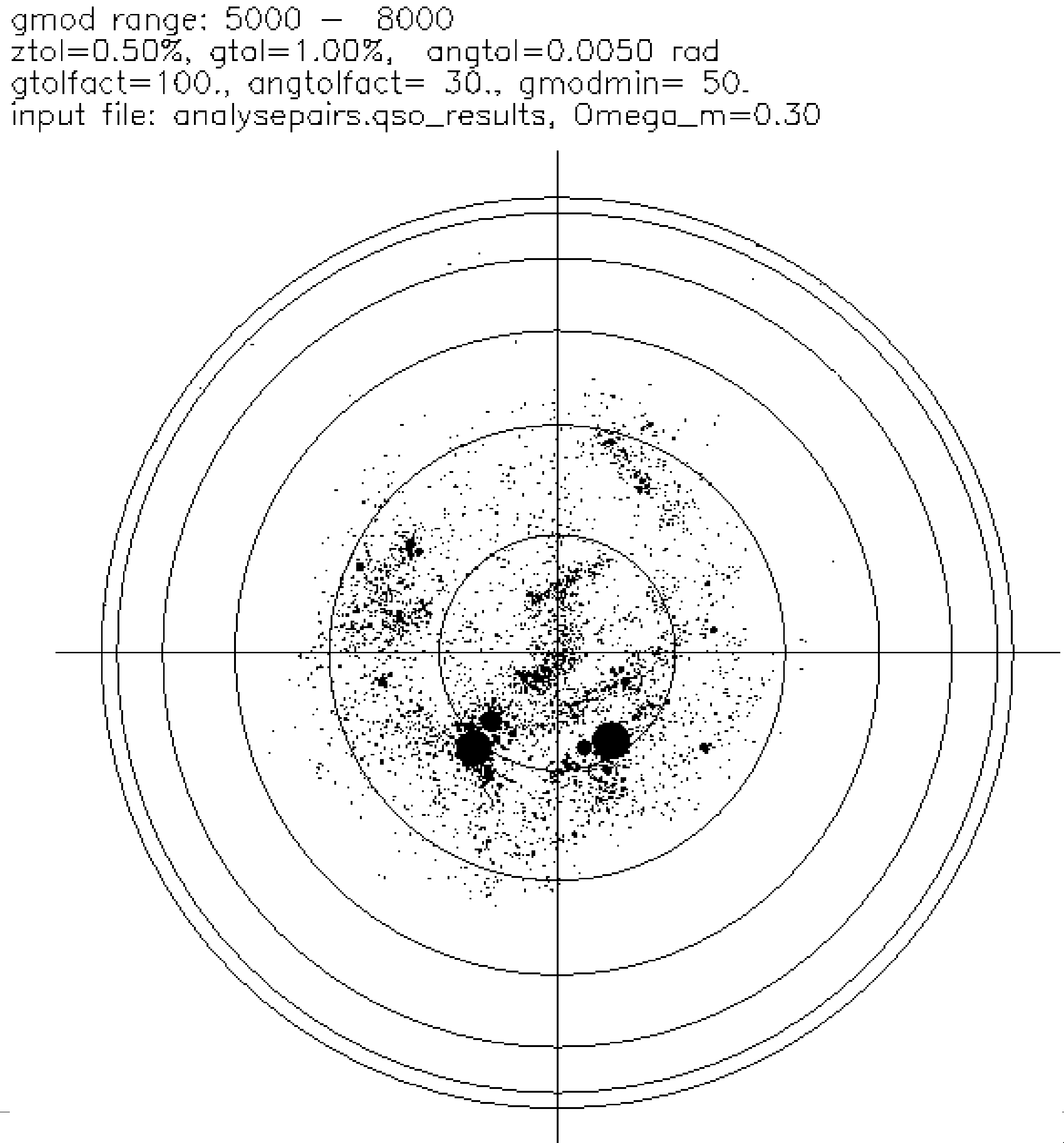

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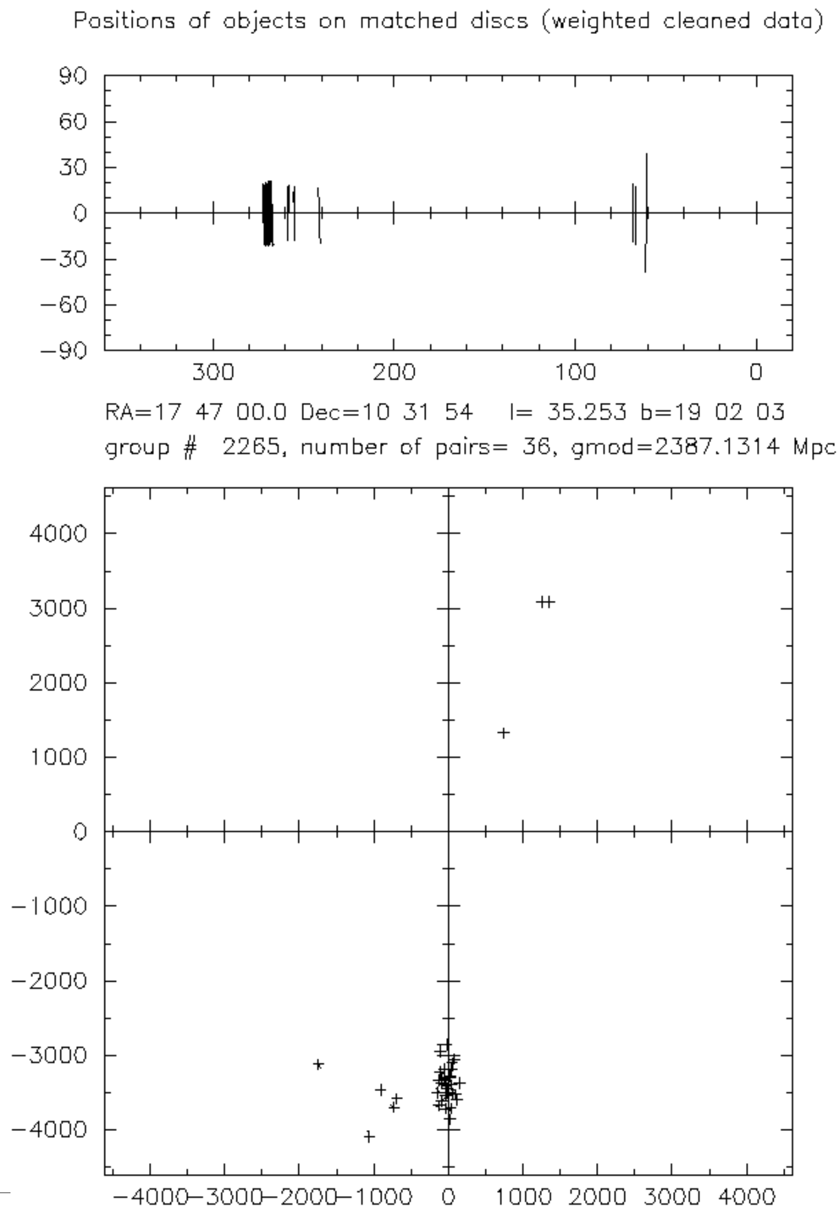

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Fujii & Yoshii (2013) arXiv:1103.1466

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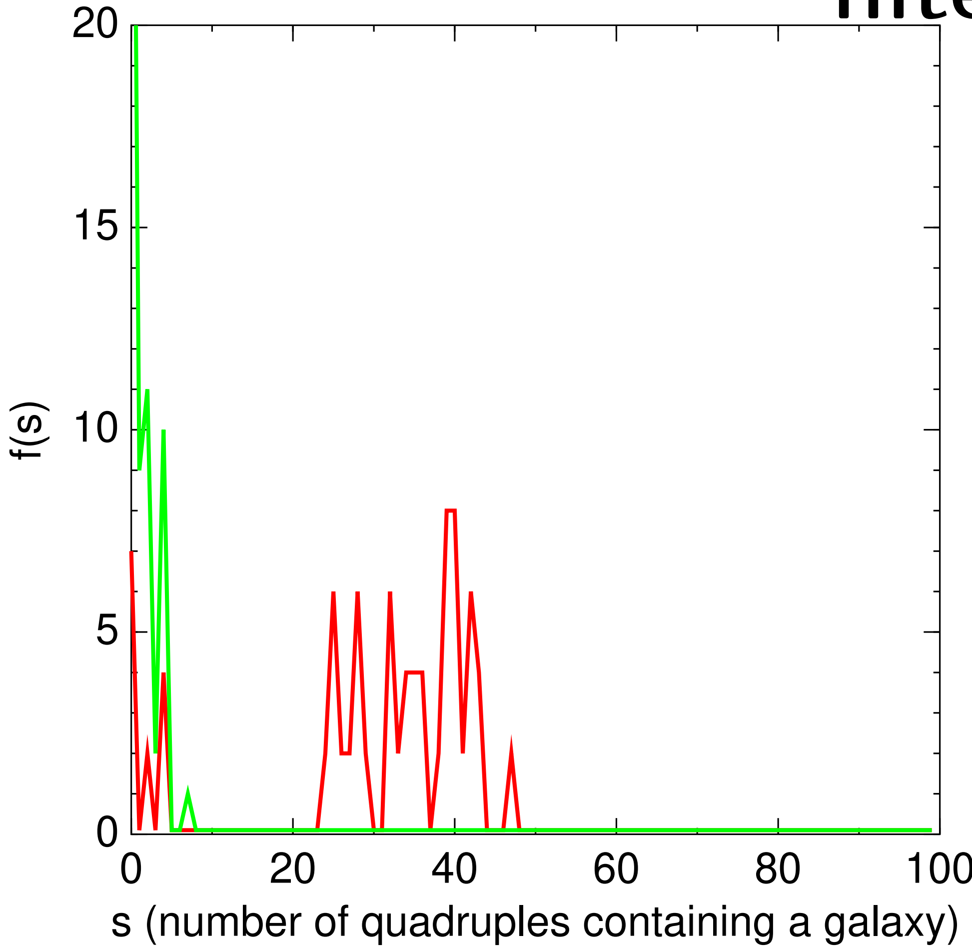
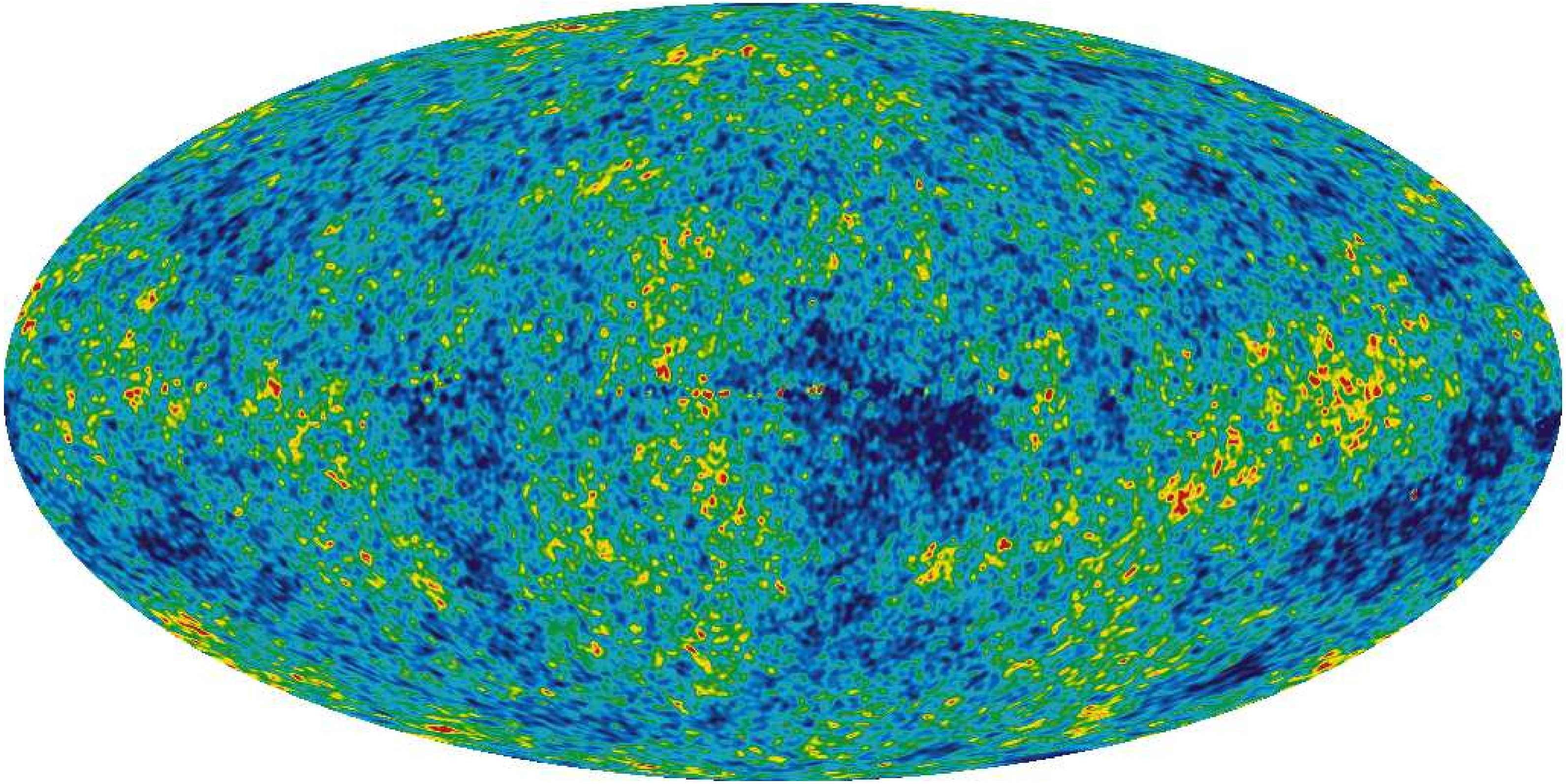


simulation of s histogram for Lyman break galaxies (LBGs) at $z \approx 6$

green: simply connected; red: T^3

ADS:2014MNRAS.437.1096R

2D methods: structure cutoff



WMAP 5yr ILC (internal linear combination)

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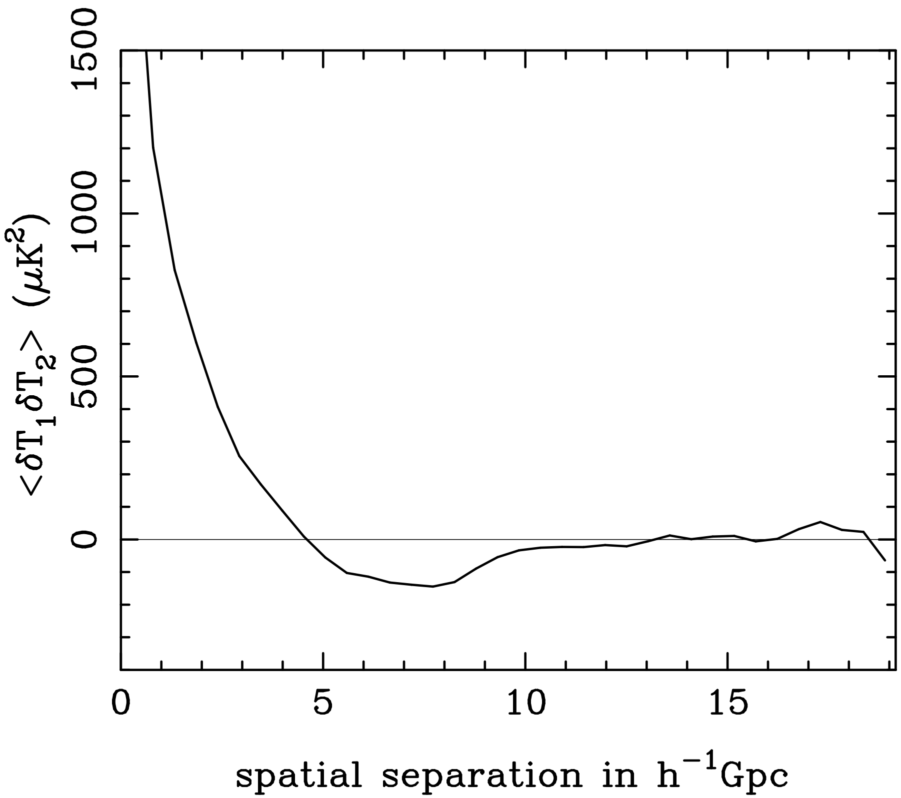

3D: structures bigger than FD cannot exist

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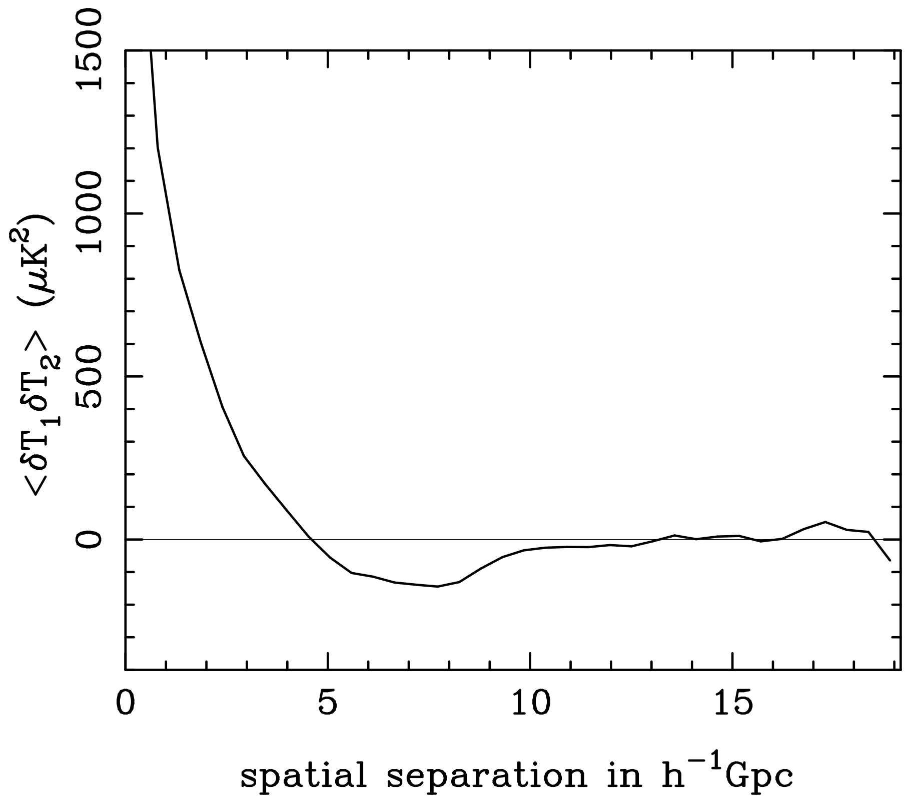

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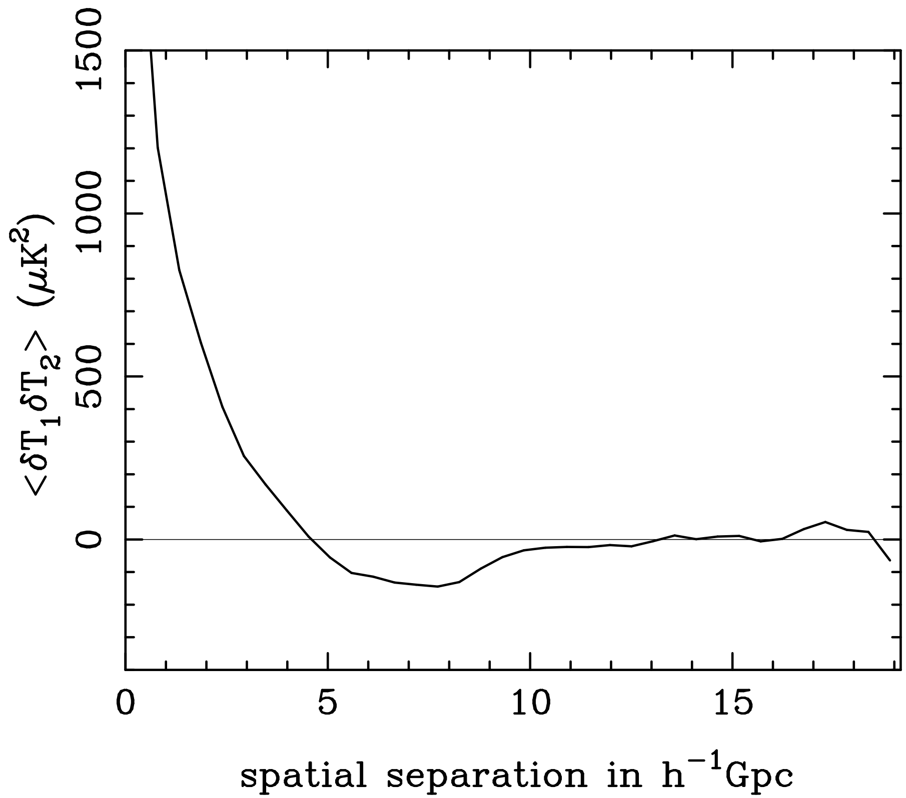

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Starobinsky (1993); Stevens et al. (1993)

The Identified Circles Principle

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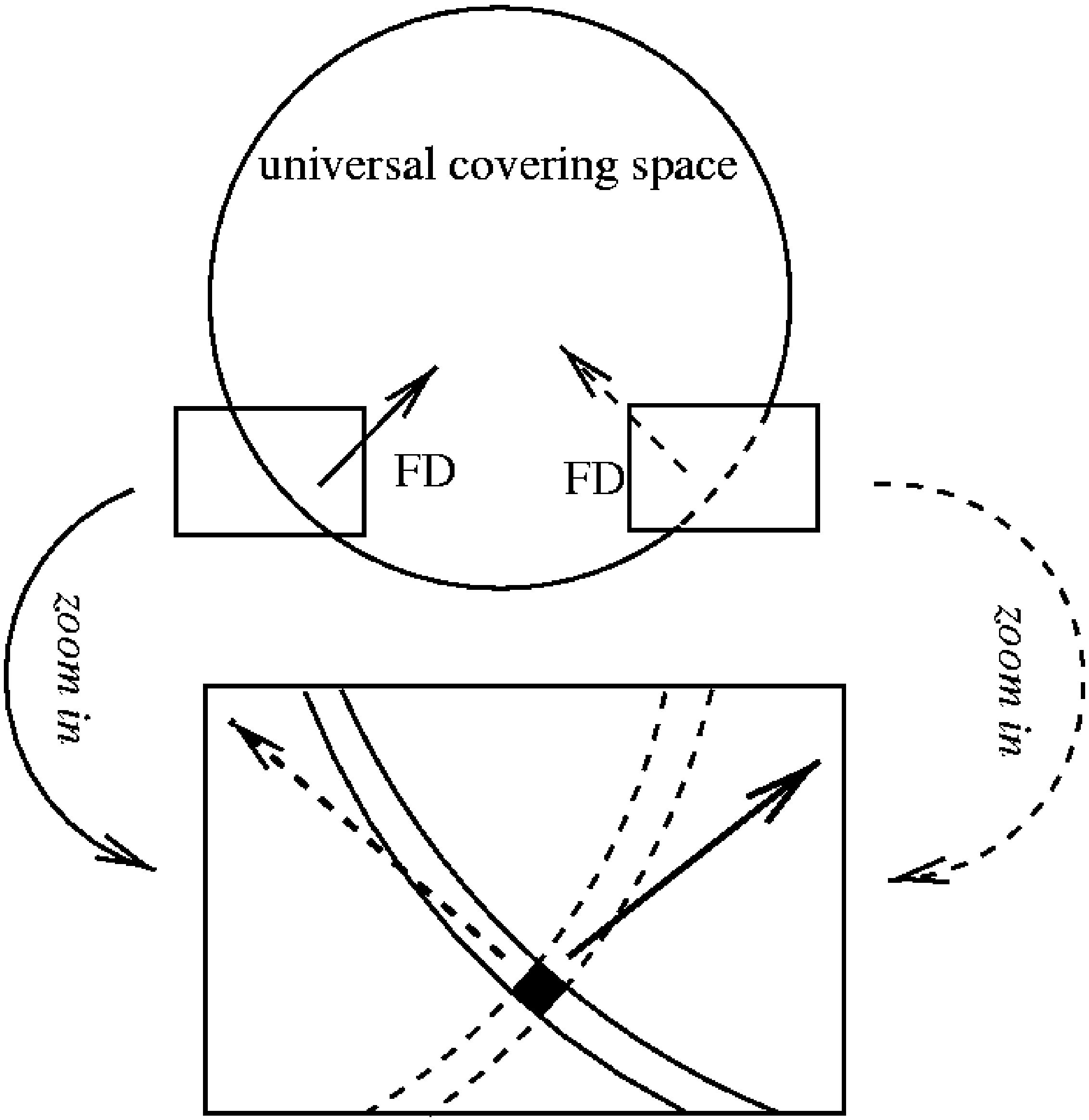
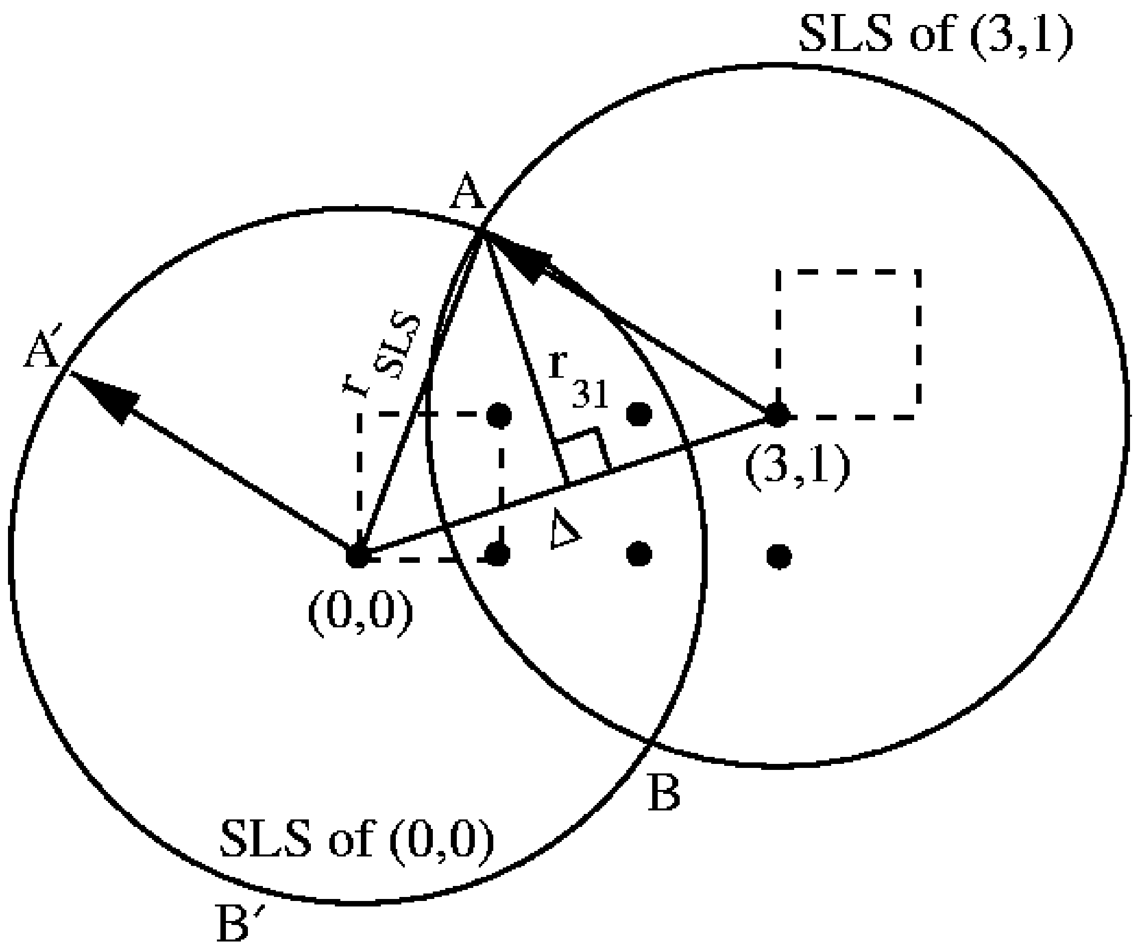
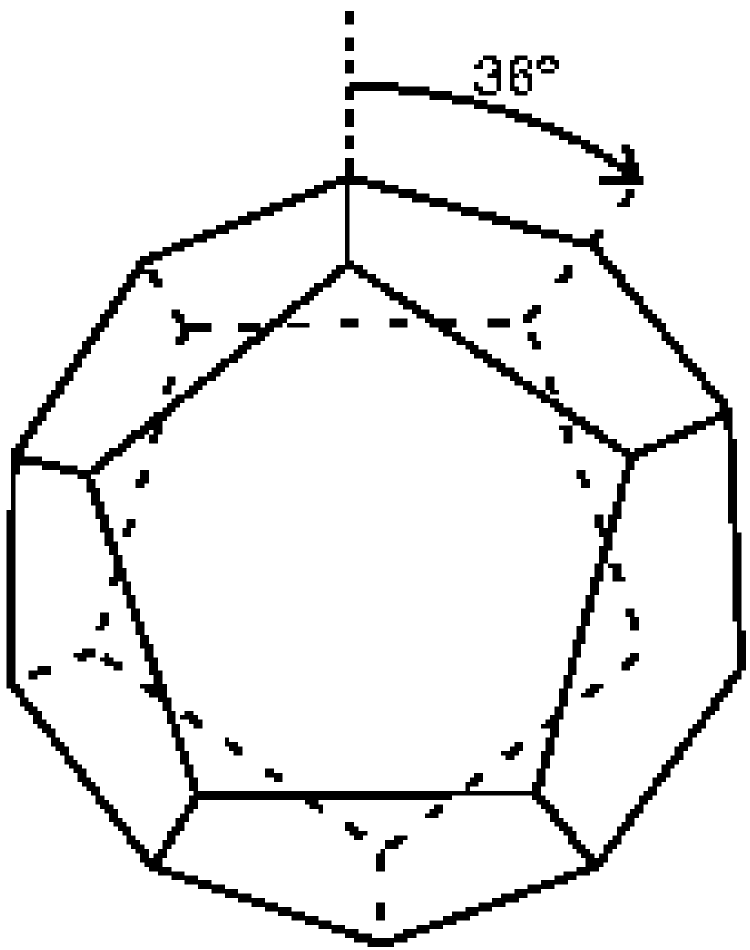


FD (fundamental domain)

The Identified Circles Principle



The Poincaré Dodecahedral 3-Manifold



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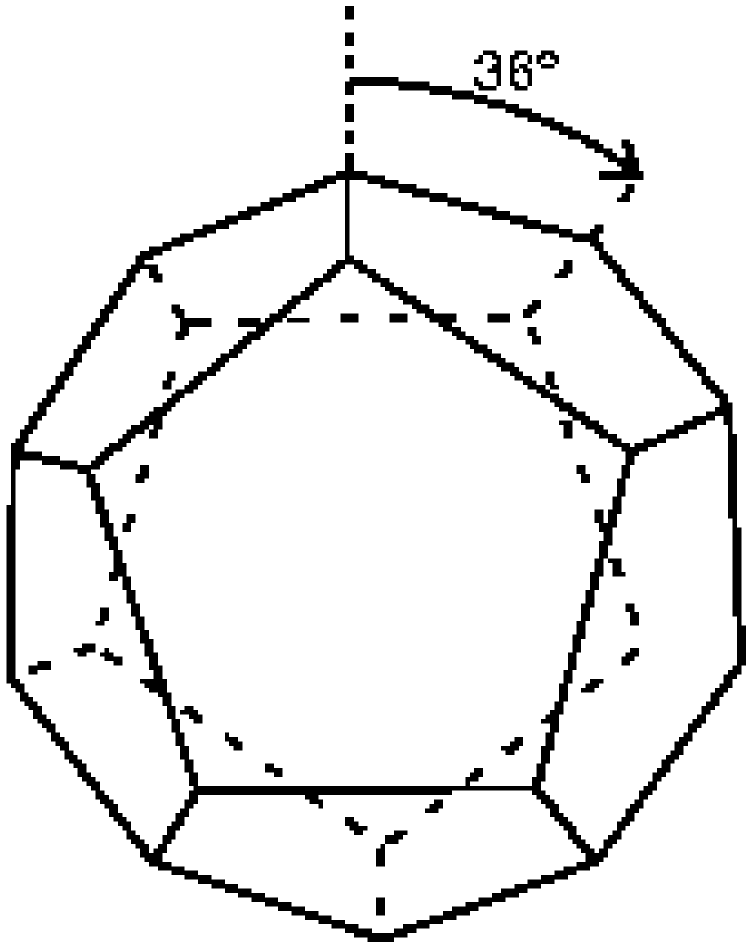

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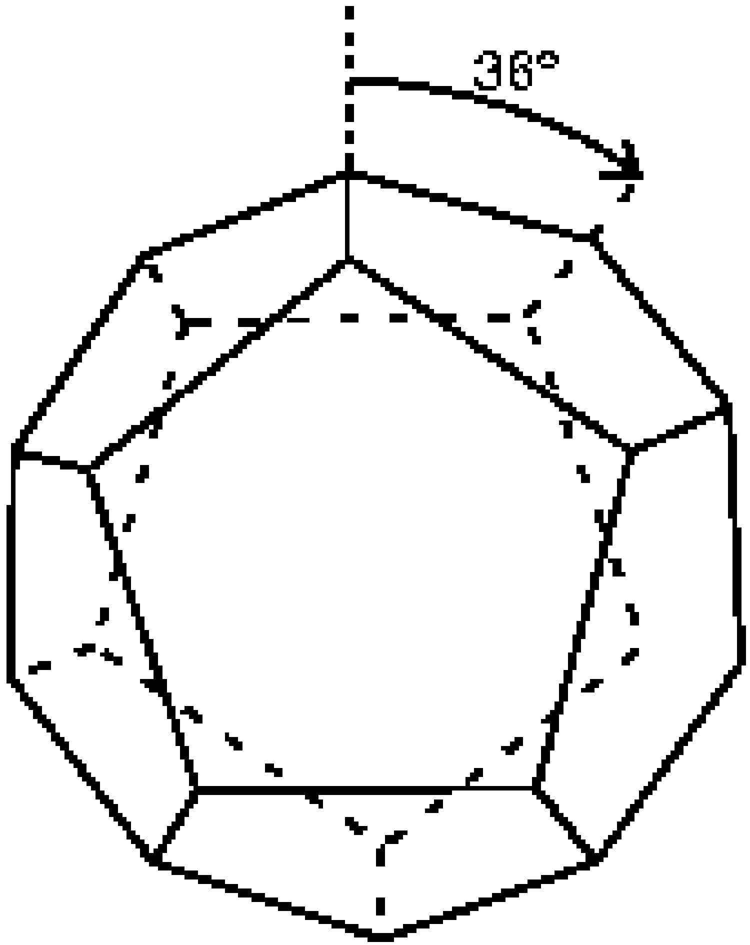

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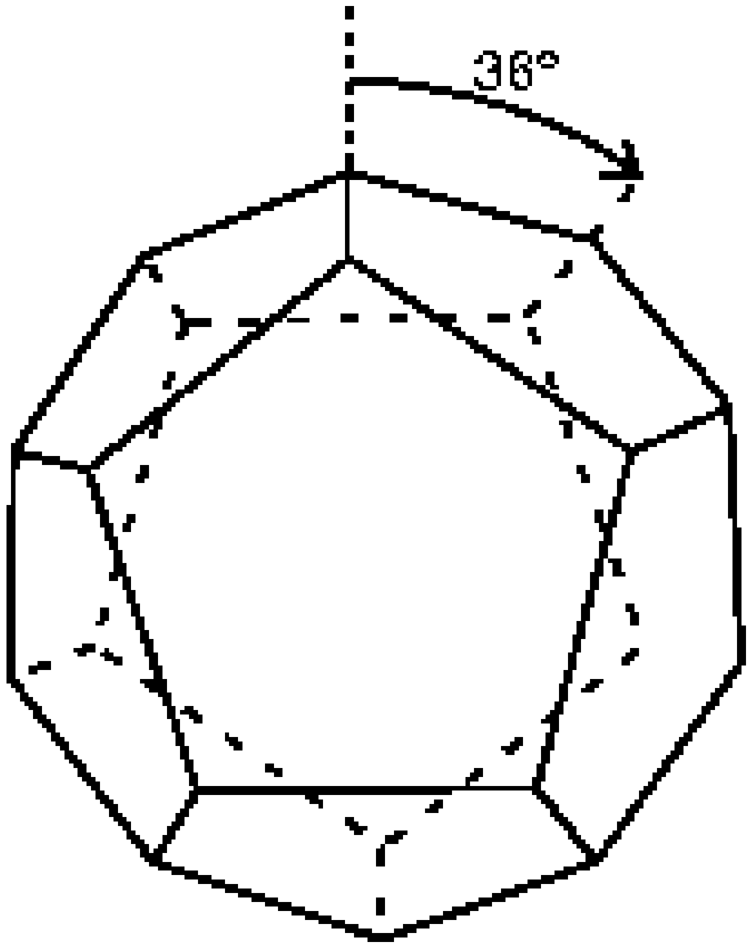

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- Luminet et al. (2003): S^3/I^* favoured by WMAP statistics

Optimal cross-correlation method

- extension to identified circles principle:

Optimal cross-correlation method

- for a given manifold, e.g. S^3/I^*:

Optimal cross-correlation method

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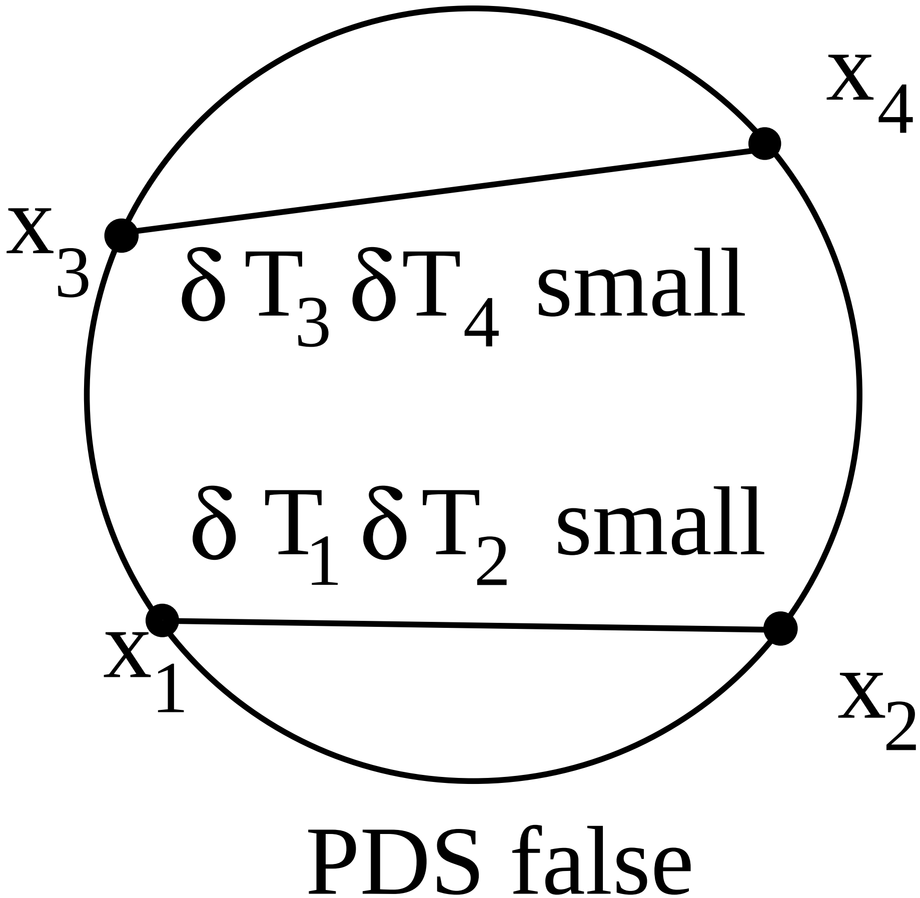

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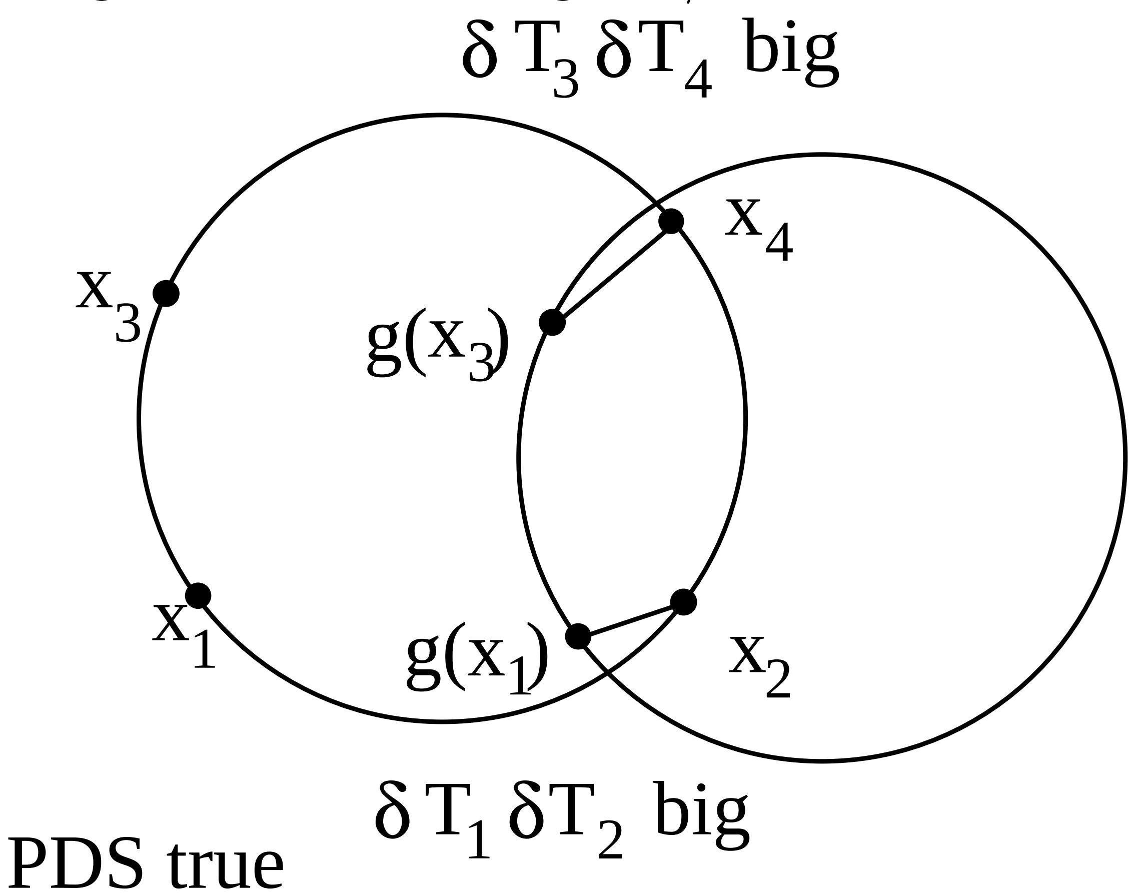

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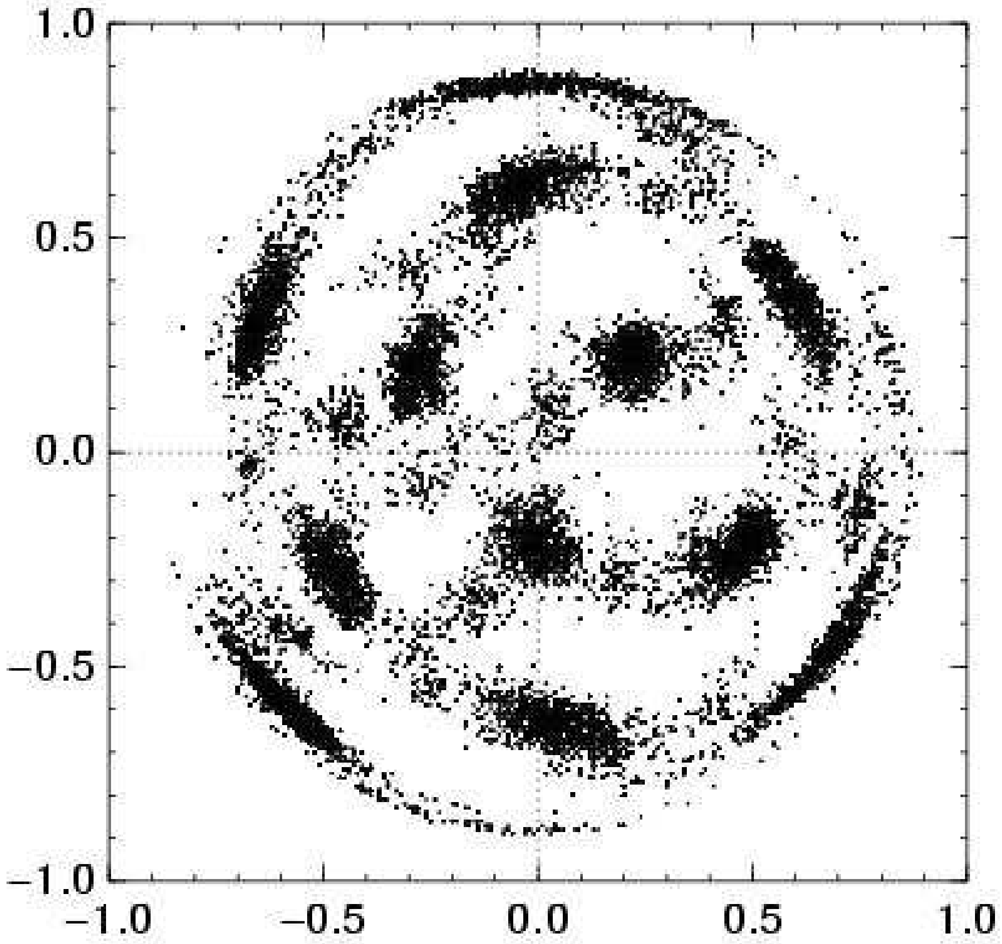
- allow arbitrary ϕ so that accidental correlations are likely to give an invalid value $\phi \neq \pm 36^\circ$

WMAP + Poincaré S^3/I^*



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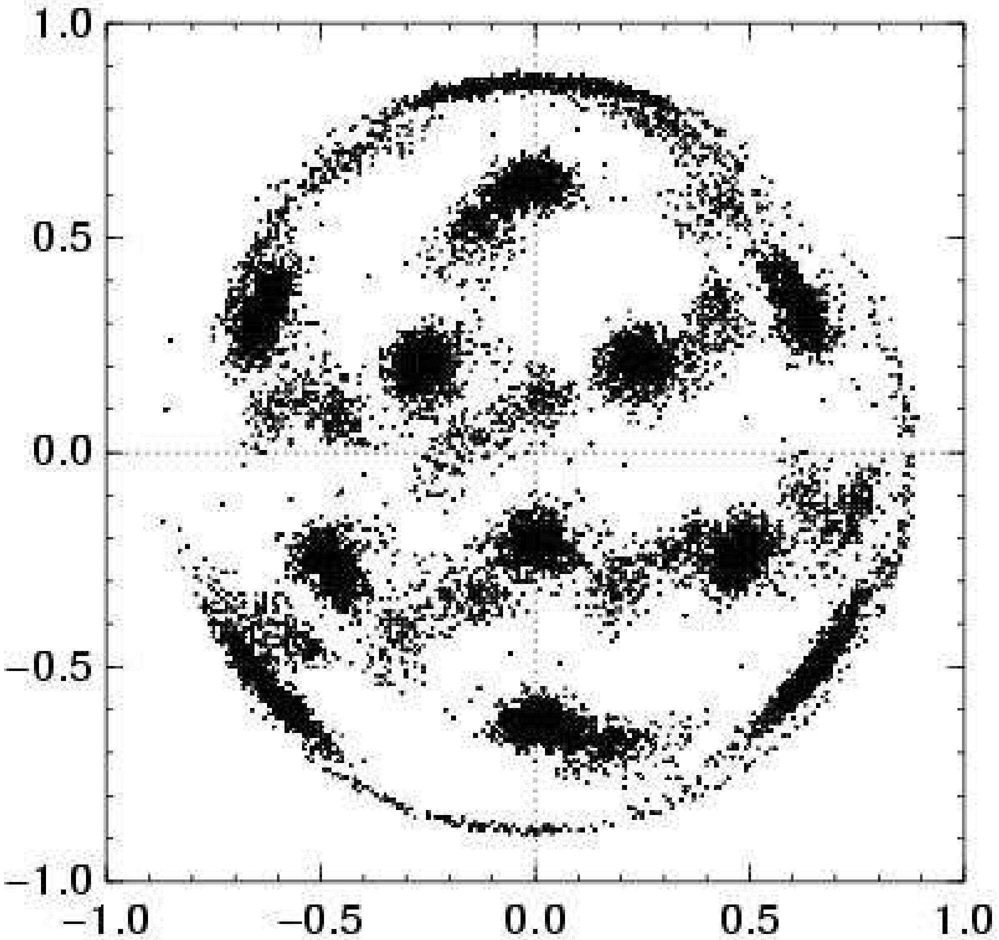

ILC + nomask

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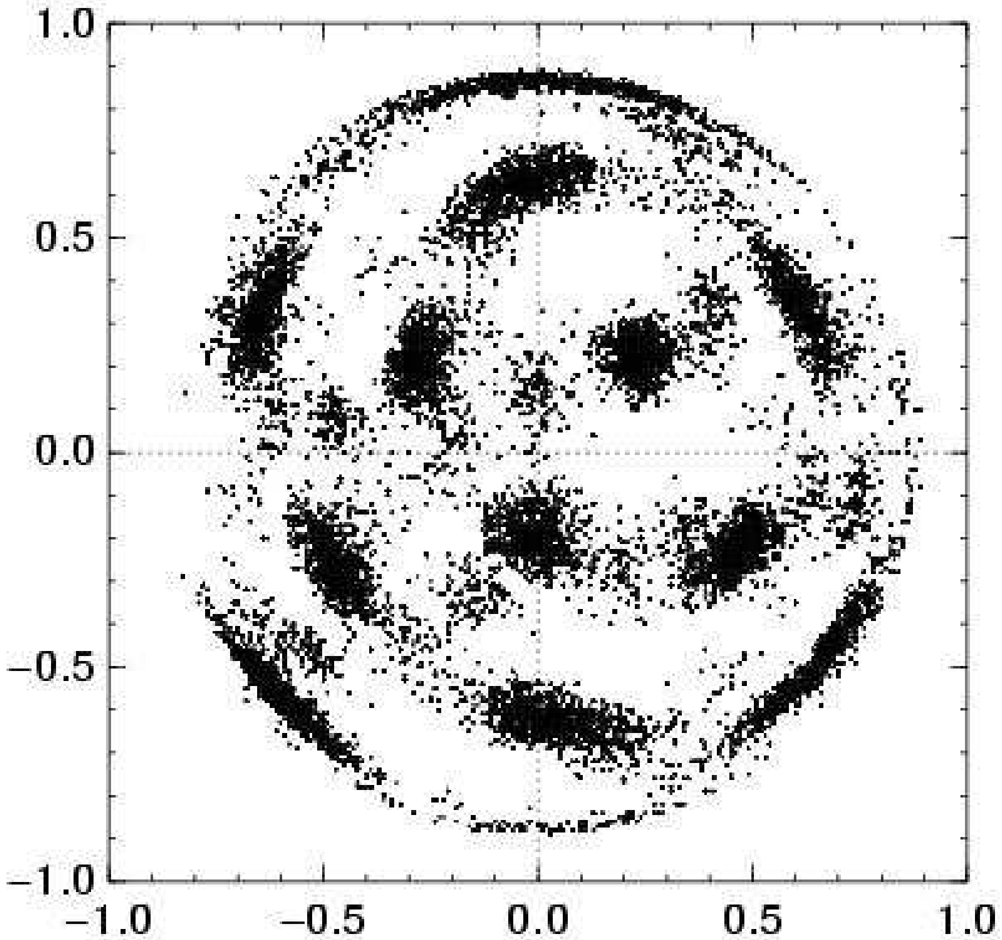

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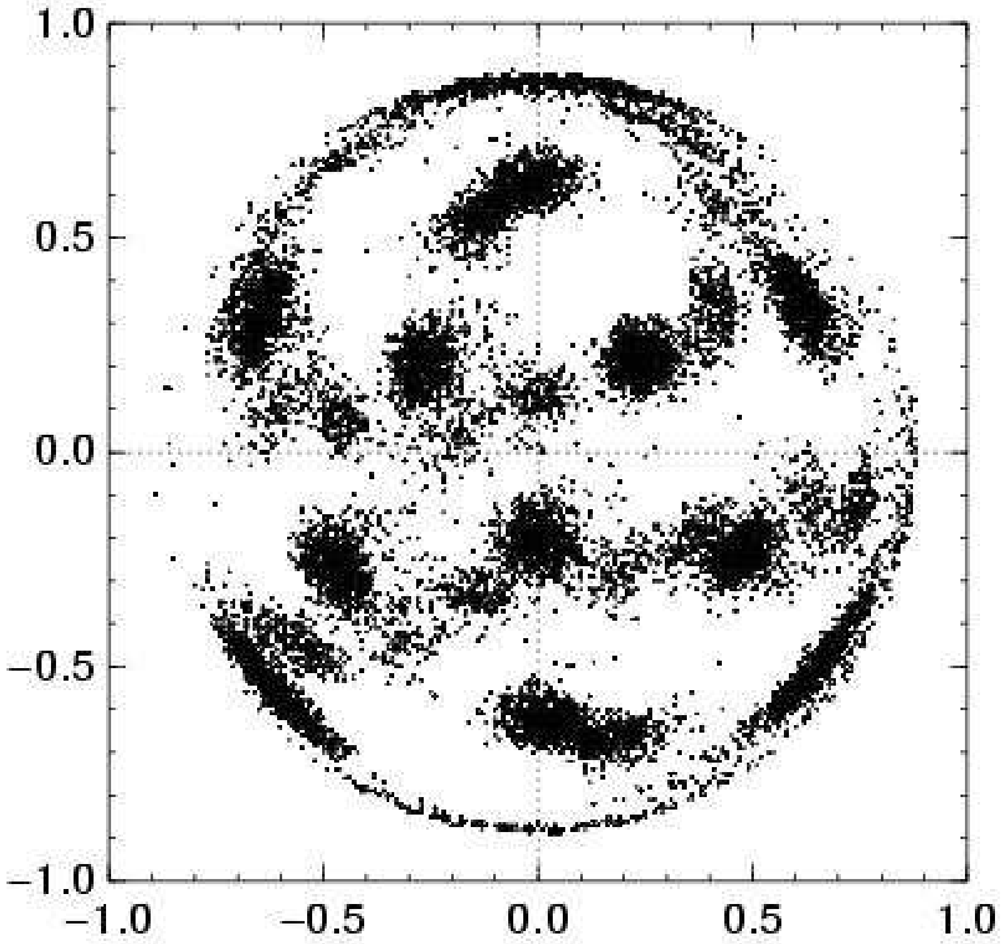

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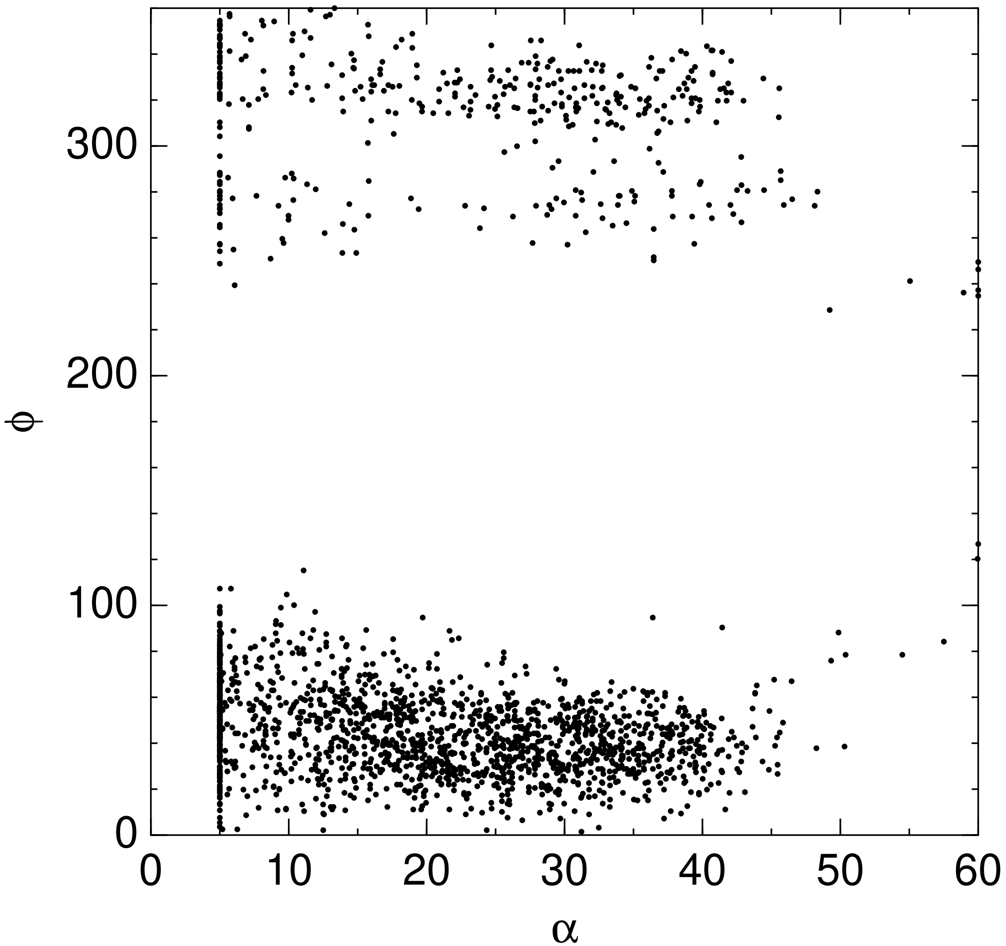
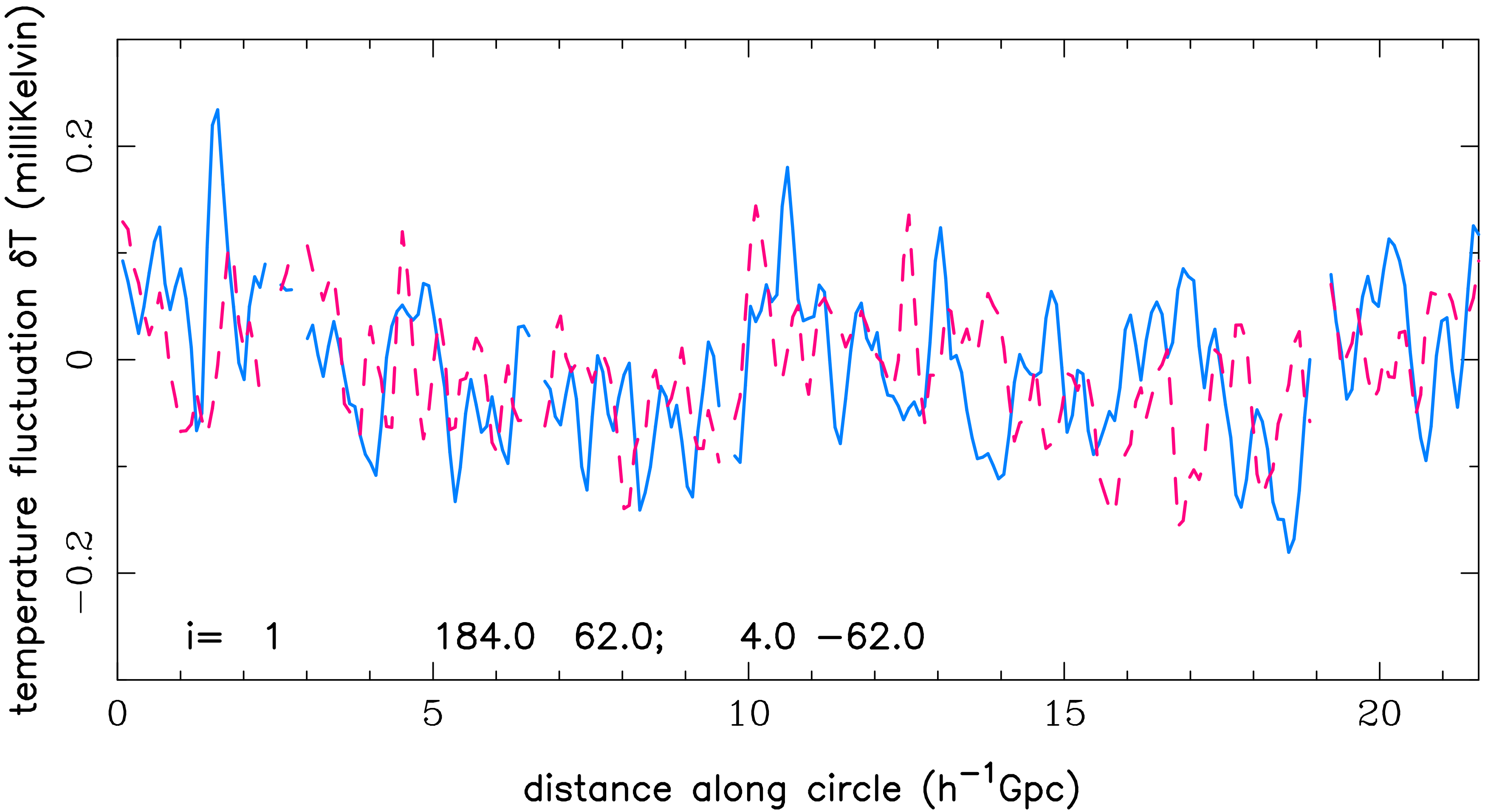


ϕ vs matched circle size α

WMAP + Poincaré S^3/I^*

$P_{\min}$	n	α°	$\sigma^\circ_{\langle\alpha\rangle}$	ϕ°	$\sigma^\circ_{\langle\phi\rangle}$
0.4	12589.0	20.6	0.6	**39.0**	**2.4**
0.5	6537.5	20.8	0.7	**38.7**	**2.2**
0.6	2961.0	22.1	0.5	**37.4**	**2.1**

RBSG08 matched circles



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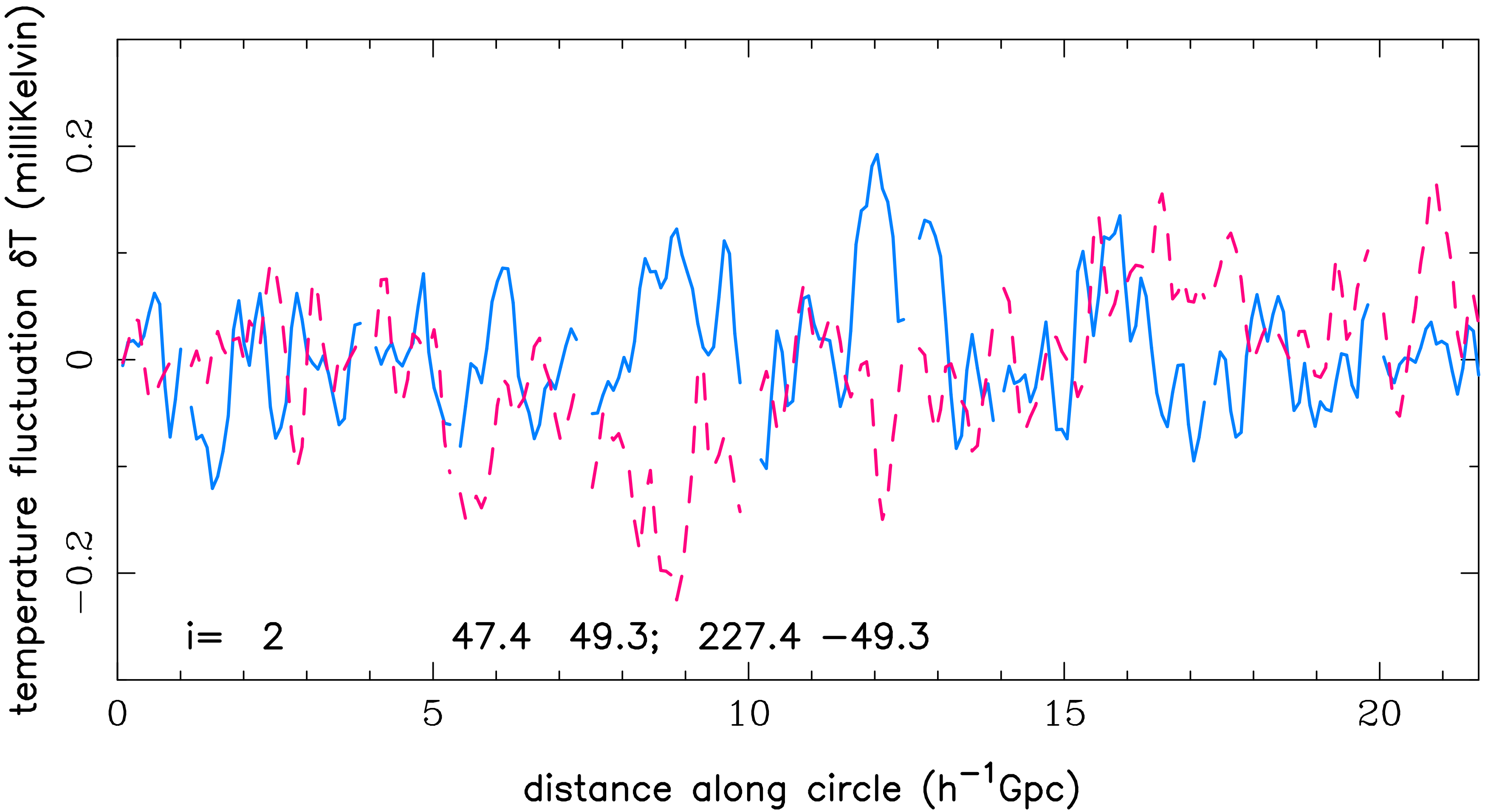

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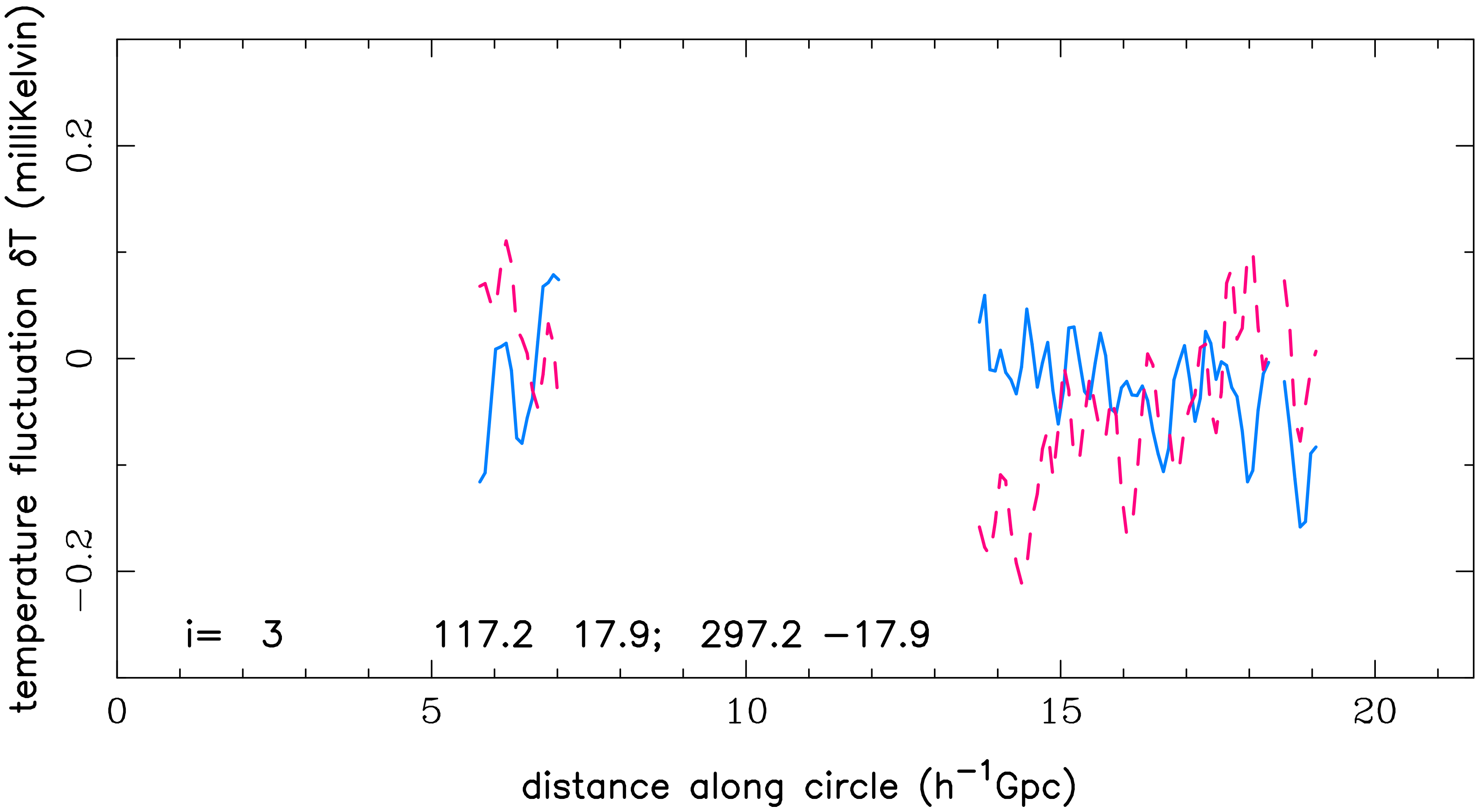

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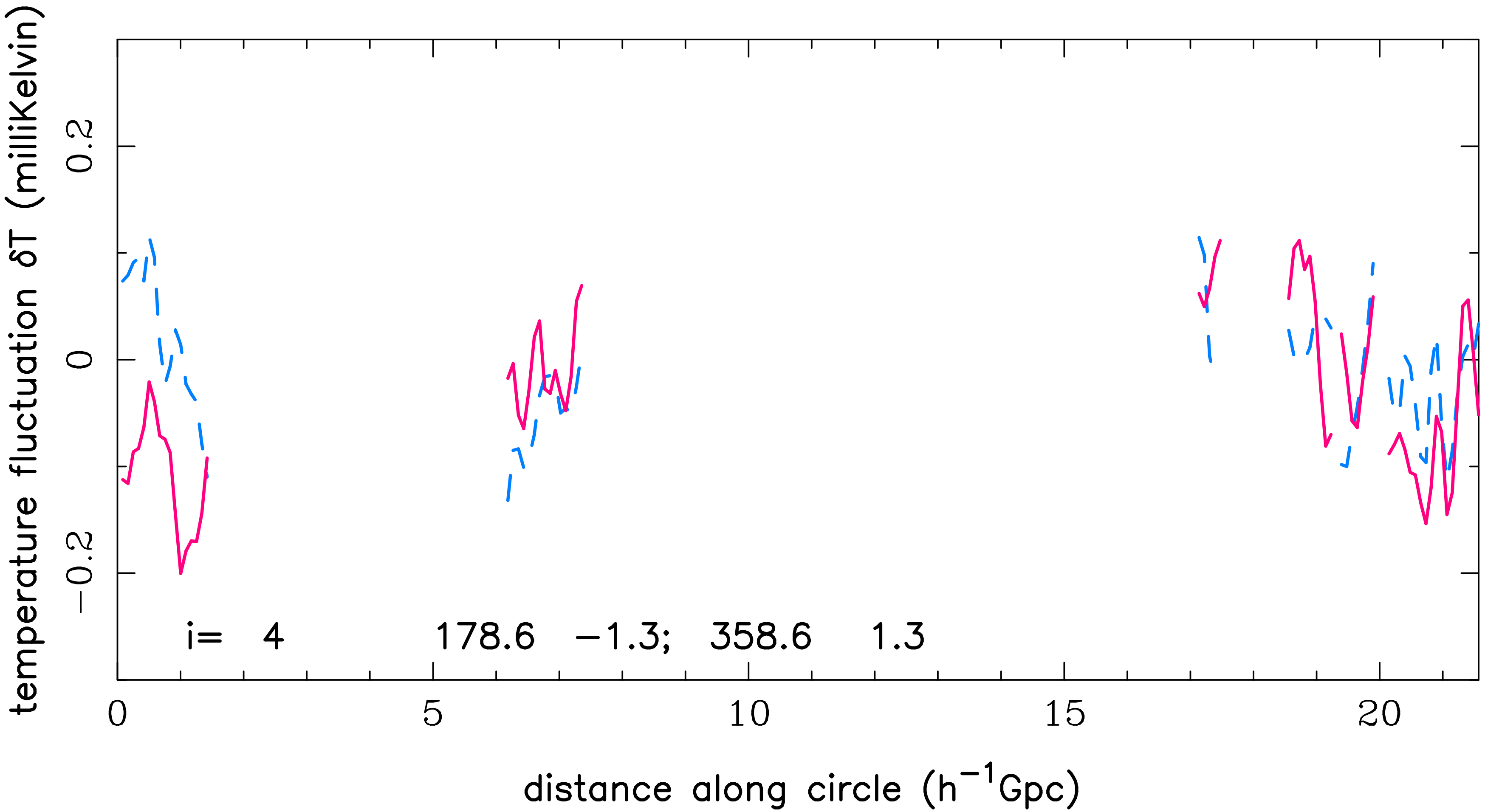

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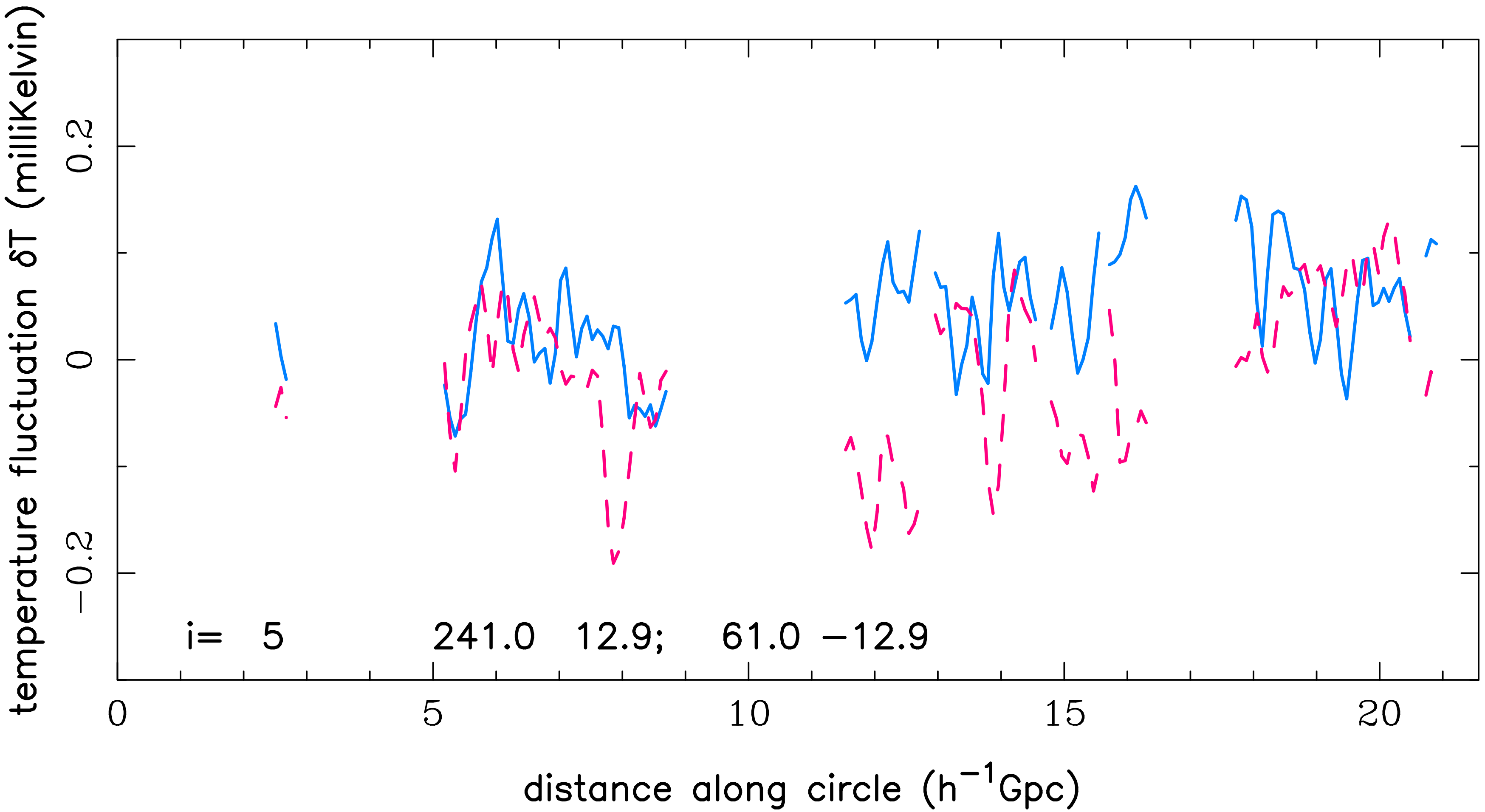

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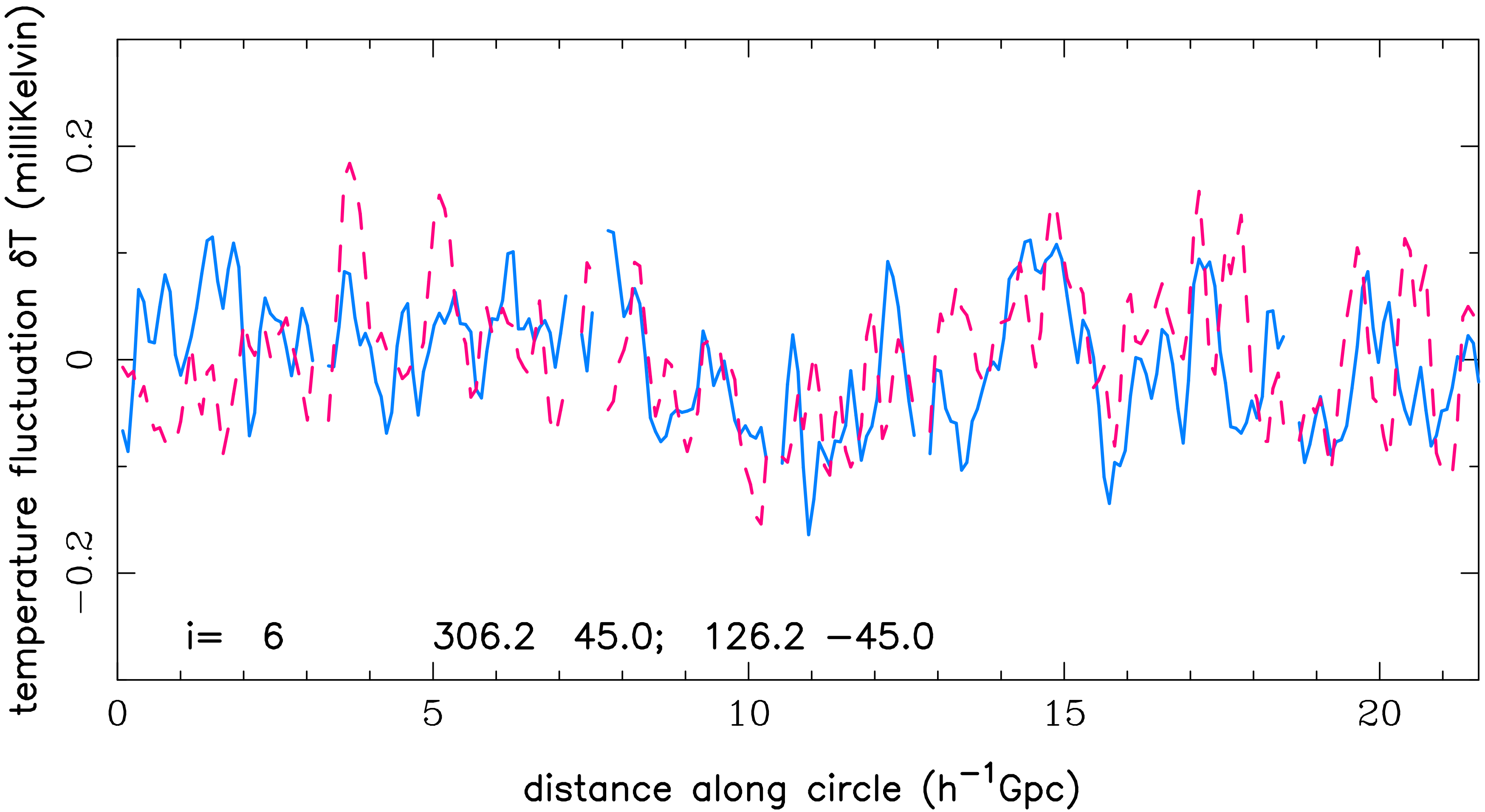

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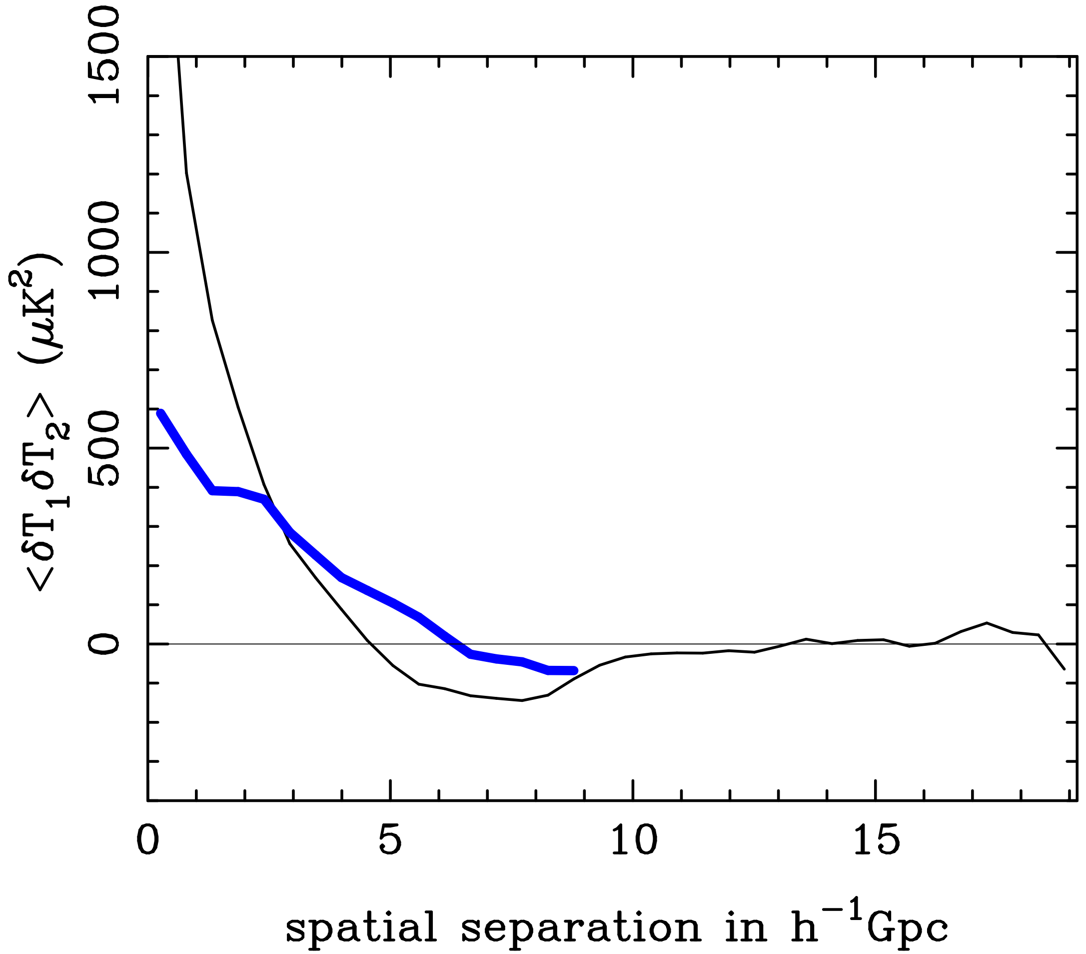

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- Planck (2013): (i) perturbation statististics assumption method; + (ii) identified circles: small correlation signal from S^3/I^* and other well-proportioned spaces, but consistent with noise

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