Topological acceleration in the Universe

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https://cosmo.torun.pl/∼boud/Roukema20220203ASGRG.pdf

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- w:

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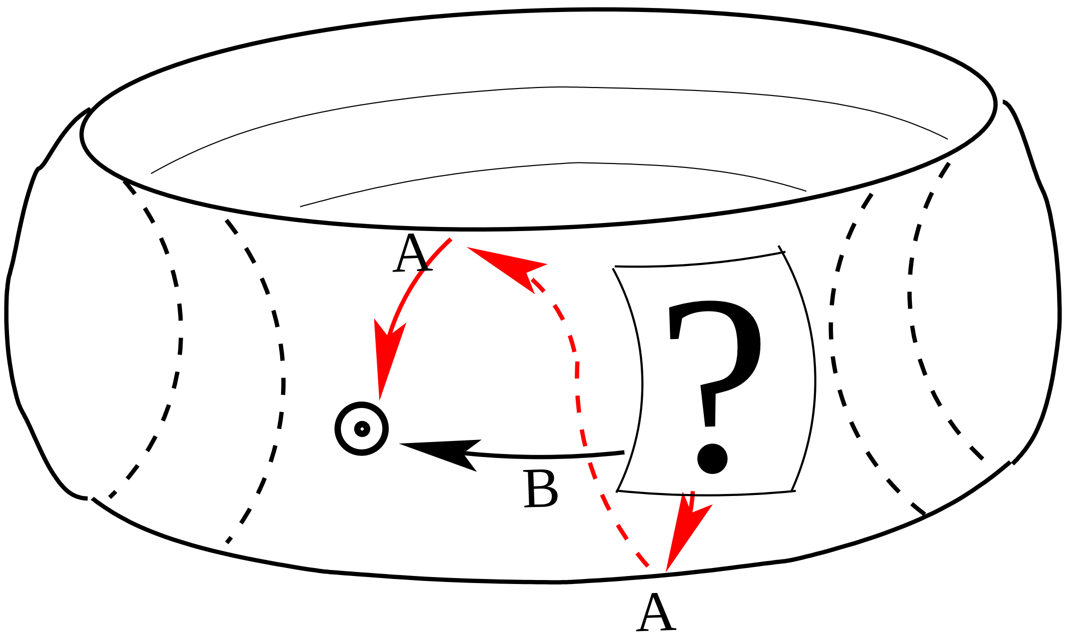

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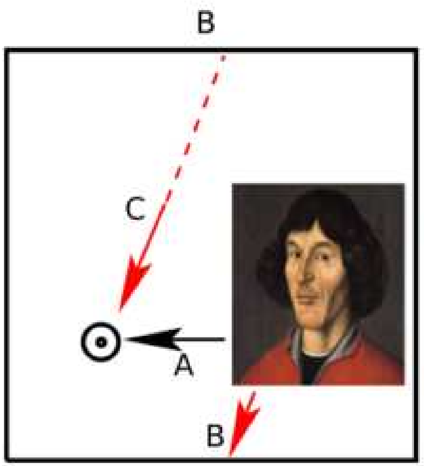

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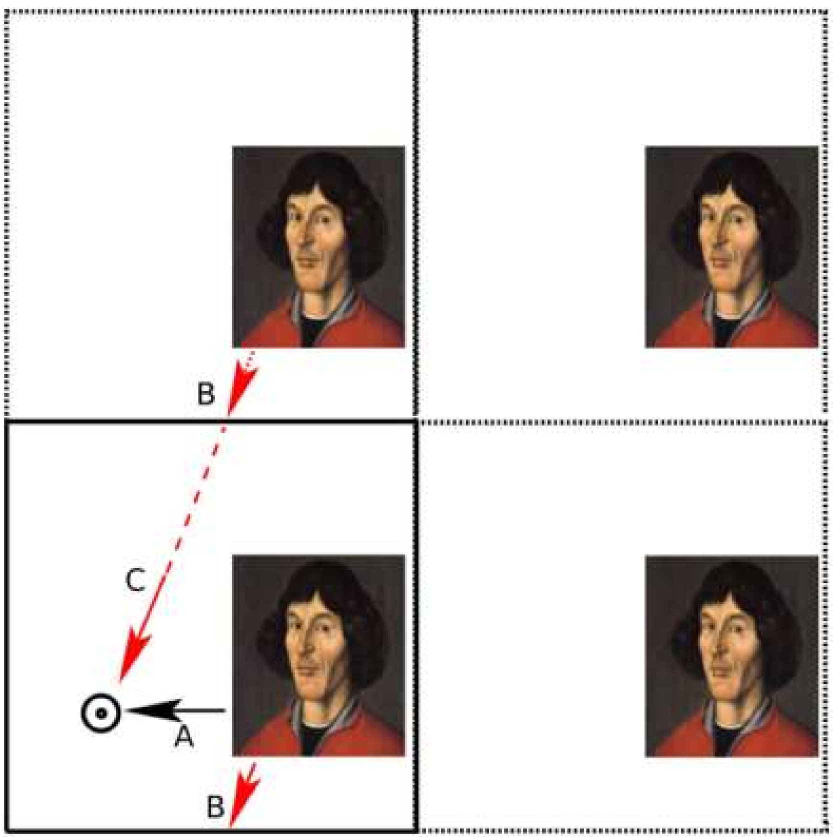

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  - w:Poincaré Conjecture *"Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere."* w:Grigori Perelman, arXiv:math/0211159 + arXiv:math/0303109 + arXiv:math/0307245

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  - active research area, e.g. arXiv:0705.4325 min. vol.
- 5 other Thurston classes – w:Geometrization conjecture

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- scalar averaging and dynamical topology change (e.g. black holes): Brunswic & Buchert (CQG, 2020) arXiv:2002.08336

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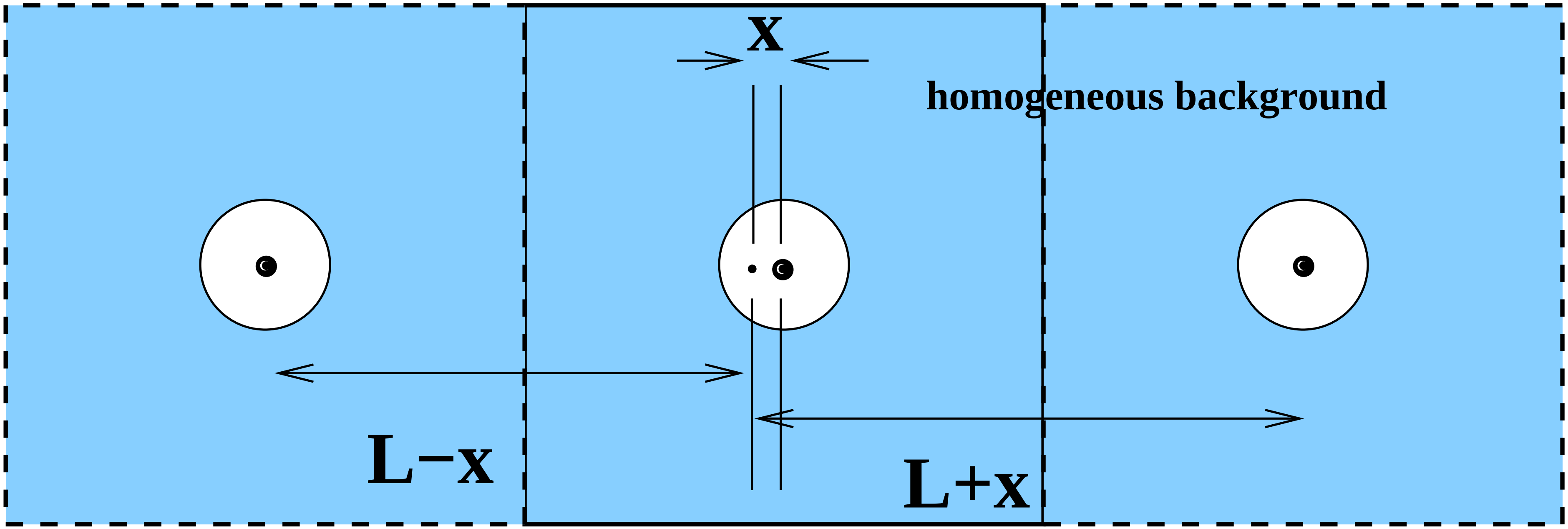

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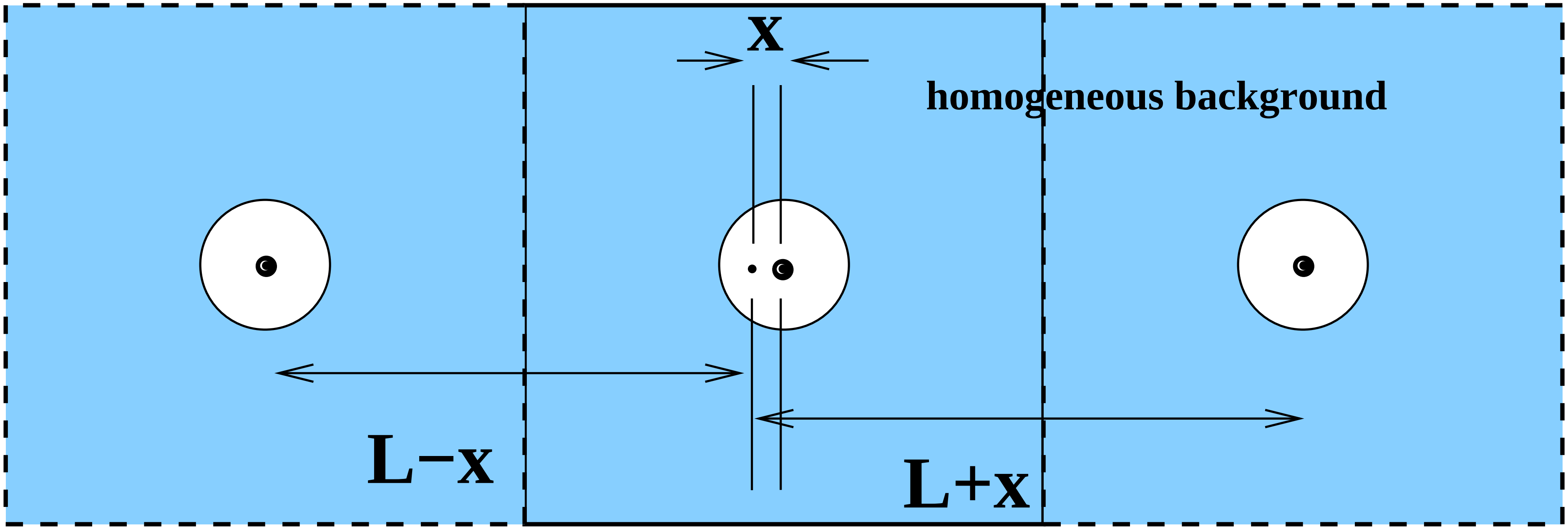

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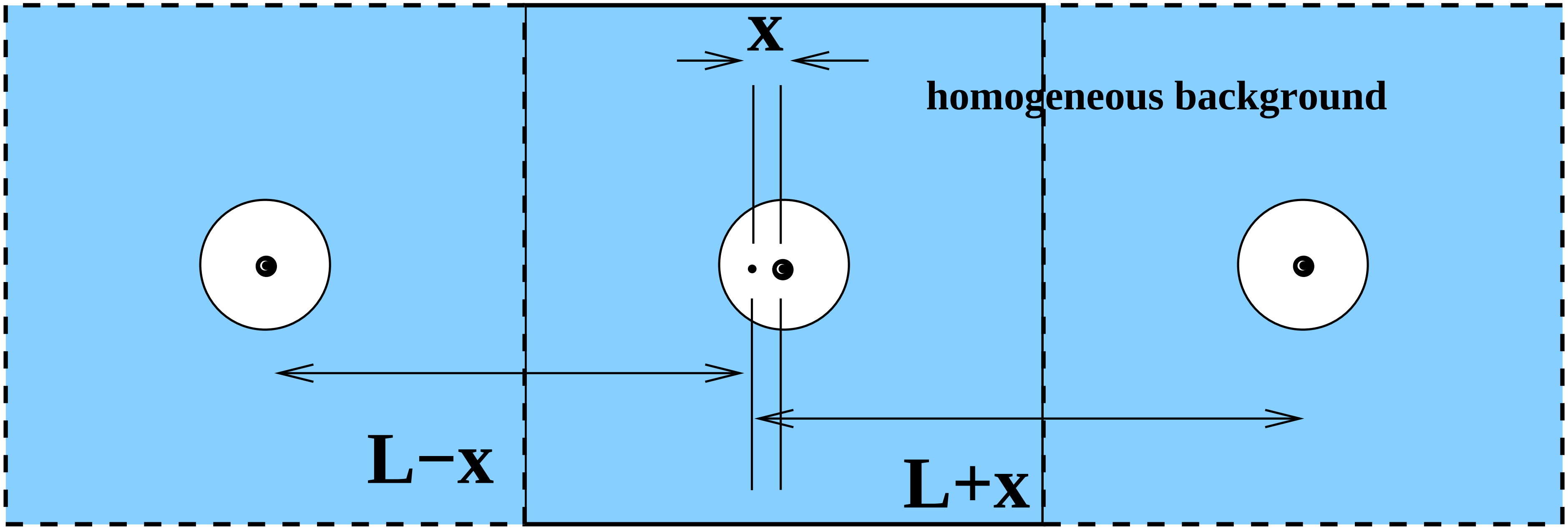

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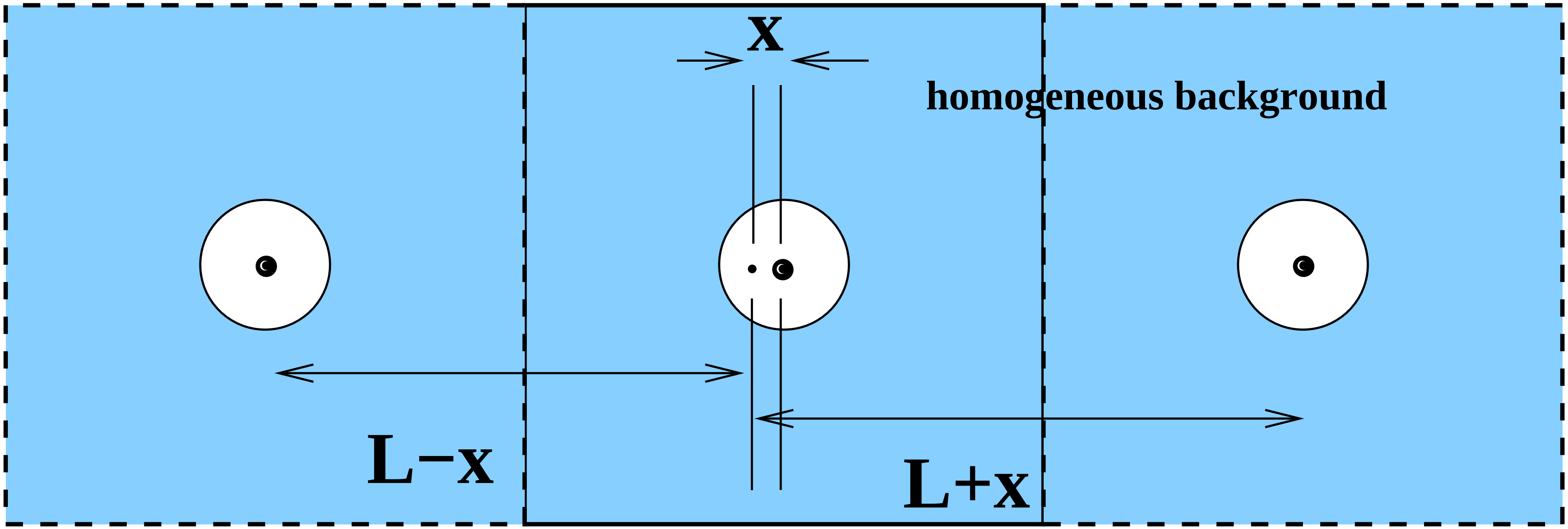

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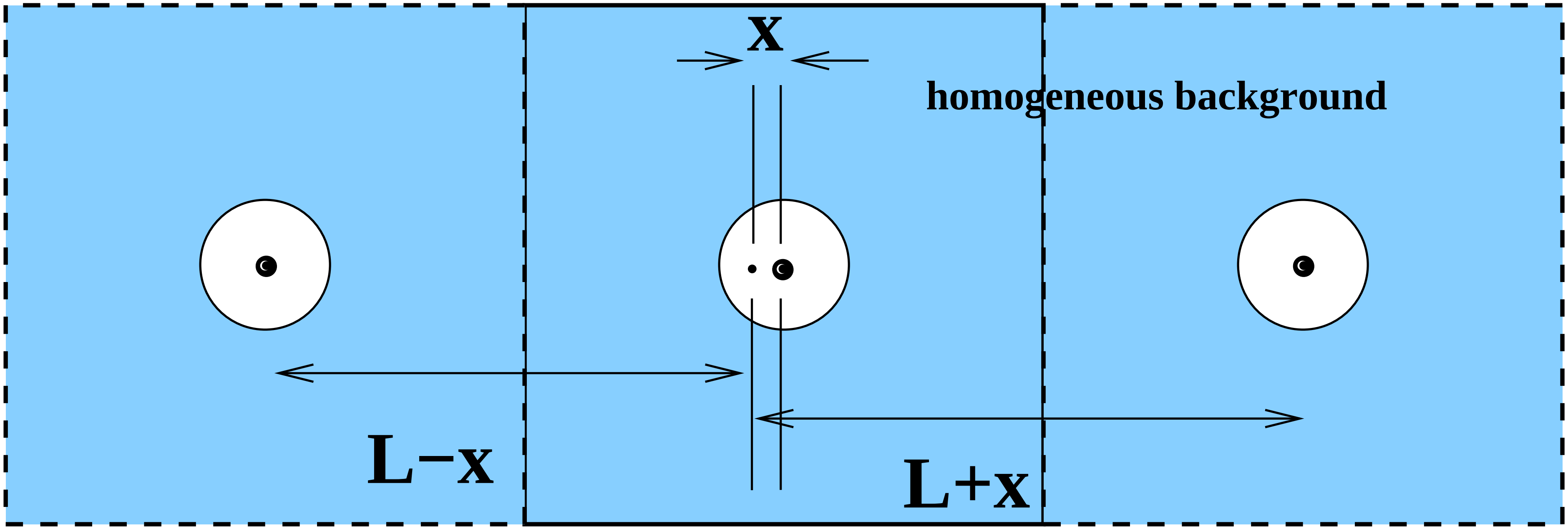

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topological acceleration— arXiv:astro-ph/0602159

Is topolog. acceleration relativistic?

- Korotkin & Nikolai (1994) arXiv:gr-qc/9403029 solution: Schwarzschild-like BH in $S^1 \times E^2$ (slab space $= T^1$)
- outside event horizon, inside topology scale:

$$\ddot{x} = 4\zeta(3) G \frac{M}{L^3} x \propto x$$

Ostrowski, Roukema & Buliński (2012) arXiv:1109.1596

$\Rightarrow$ Yes.

Heuristic top. accel.

original heuristic — Roukema+2007 A&A arXiv:astro-ph/0602159

- weak-field gravity of distant, multiple images
- covering space $\mathbb{E}^3$ or $\mathbb{S}^3$
- calculations made in covering space
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- $\mathbb{T}^3 = \mathbb{E}^3/\mathbb{Z}^3 \quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/L)^3 + \ldots$
- $\mathbb{S}^3/T^* \equiv M_6$ (octahedral space) $\quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/R_C)^3 + \ldots$
- $\mathbb{S}^3/O^*$ (truncated cube space) $\quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/R_C)^3 + \ldots$
- $\mathbb{S}^3/I^* \equiv M_8$ (Poincaré dodecahedral space)
  $\Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/R_C)^5 + \ldots$

- *topological acceleration is manifold-dependent*
  Roukema & Różański arXiv:0902.3402, A&A, 502, 27

Newt. non-Euclid. top.accel.

NEN: $\Phi_{\mathbb{S}}(\xi) \propto -\cot\xi\,(1-\xi/\pi) + A$

Topology	N_Σ	Φ_{-1}	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5
Euclidean (infinite or Thurston-type)							
$\mathbb{E}^3$		-1	0	0	0	0	0
$\mathbb{T}^3$		-1	0	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	0	–	0
Spherical							
$\mathbb{S}^3$	1	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	$-\frac{2\pi}{45}\frac{\mathcal{R}/6}{V_\Sigma}$	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
M_3	8	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	–	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
M_6	24	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	–	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
M_7	120	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	$-\frac{2\pi}{45}\frac{\mathcal{R}/6}{V_\Sigma}$	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
Hyperbolic (infinite)							
$\mathbb{H}^3$		-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	0	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	0	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$

even terms $\Rightarrow$ closed; odd terms $\Rightarrow$ curved

Vigneron & Roukema (2022) arXiv:2201.09102

$T^3, S^3/\Gamma$

- Some spaces are more equal than others.
- Roukema & Różański arXiv:0902.3402, A&A, 502, 27
- Newton–Cartan approach for preparing for full GR approach: Vigneron (2020, PRD) arXiv:2010.10247; Vigneron (2021, PRD) arXiv:2012.10213; Vigneron (2022a, PRD) arXiv:2109.10336; Vigneron (2022b) arXiv:2201.02112; Vigneron & Roukema (2022) arXiv:2201.09102

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- patterns of time-integrated effects of topological acceleration should exist at ~ 10–$1000h^{-1}$ Mpc
  - very difficult to separate from artefacts
  - need detailed GR modelling
  - need excellent quality surveys
- numerical simulations — Buliński 2015 PhD thesis NCU