Topological acceleration in the Universe

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https://cosmo.torun.pl/∼boud/Roukema20221001AWSwino.pdf

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- w:

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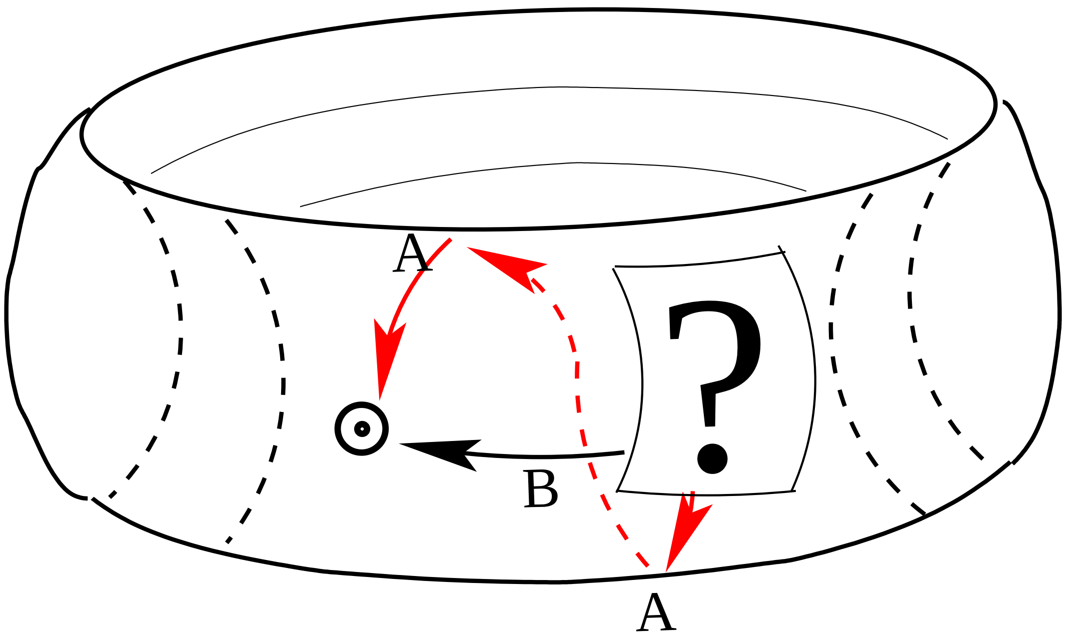

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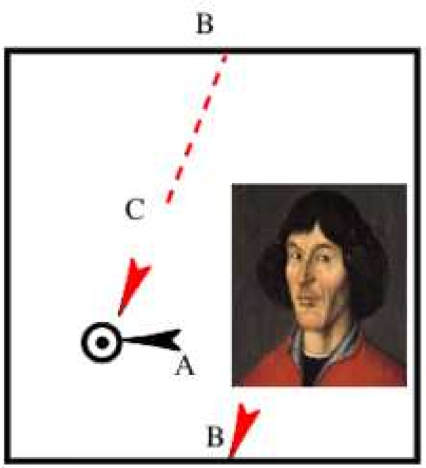

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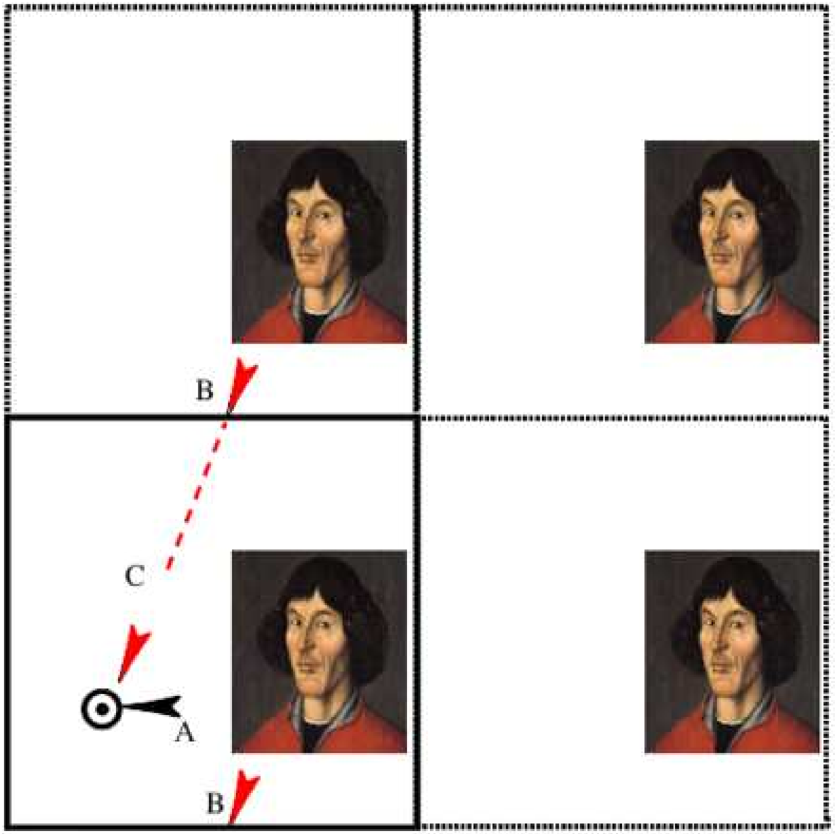

Cosmic topology: definitions

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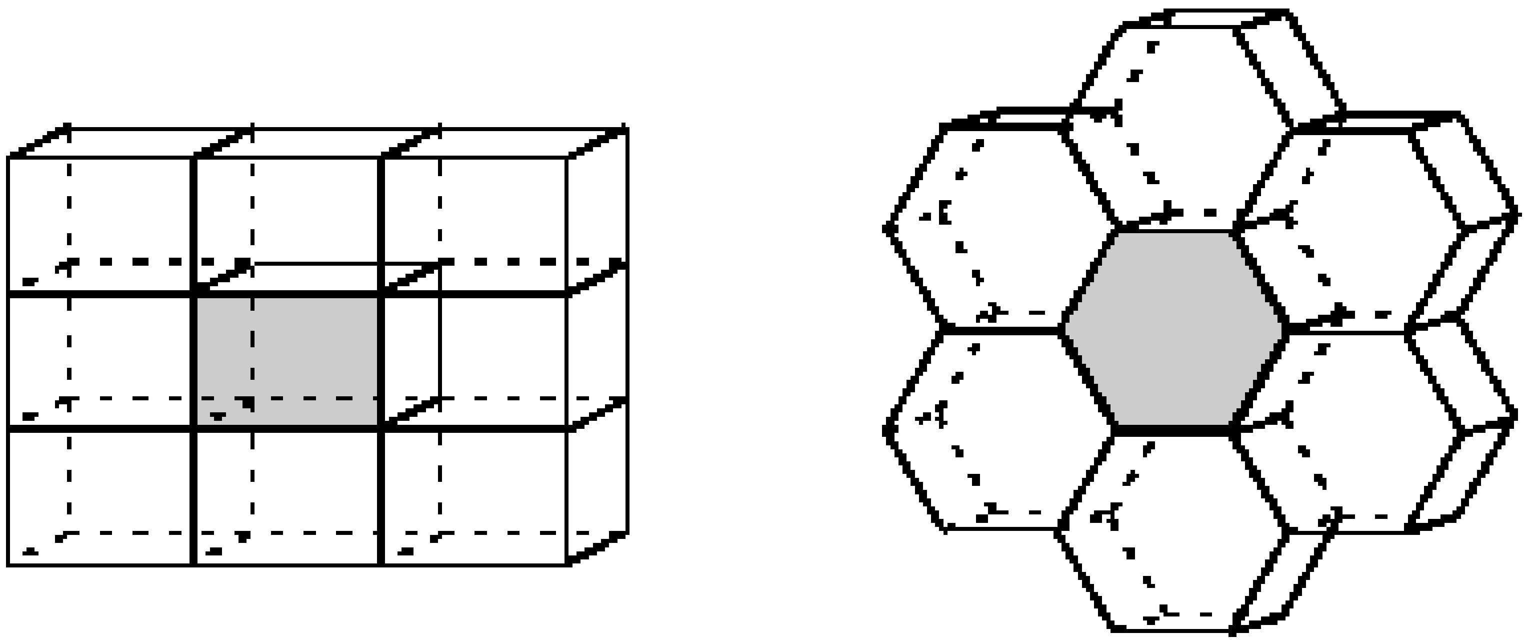

3D flat examples arXiv:astro-ph/9901364

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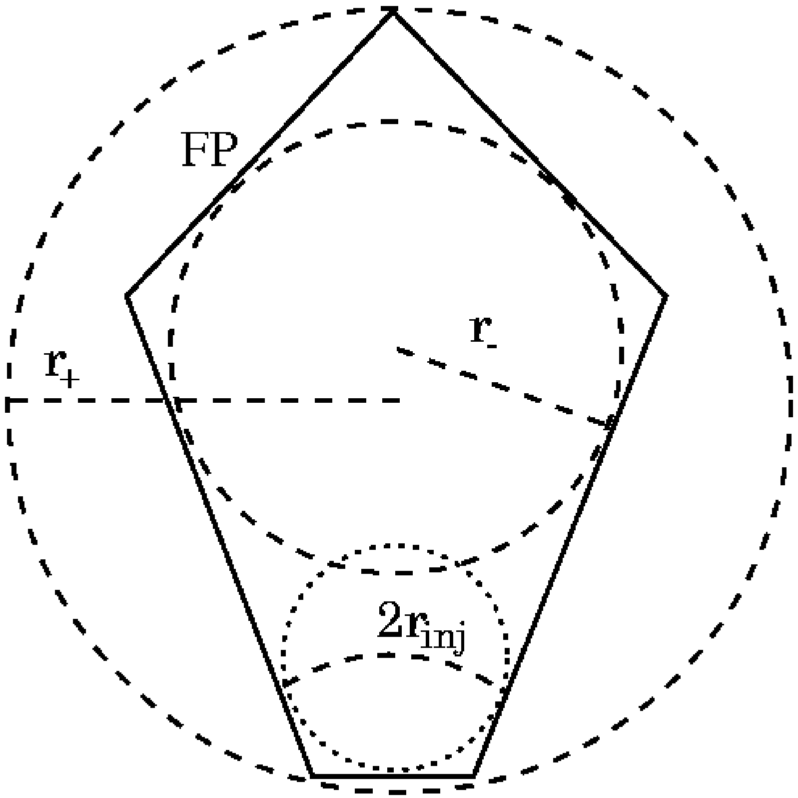

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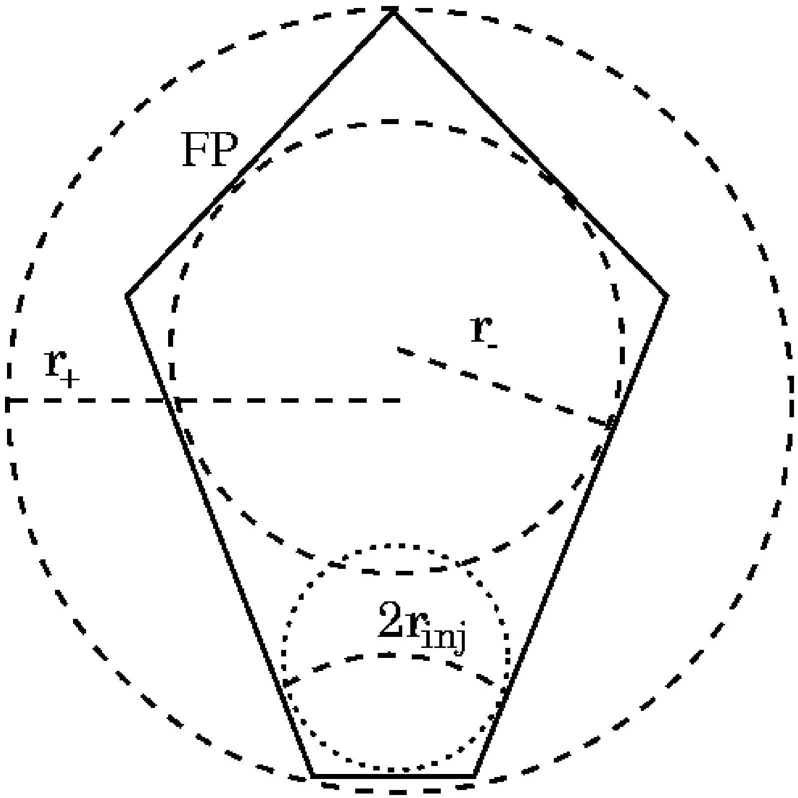

size of universe:

- r_- : biggest sphere *inside* FD

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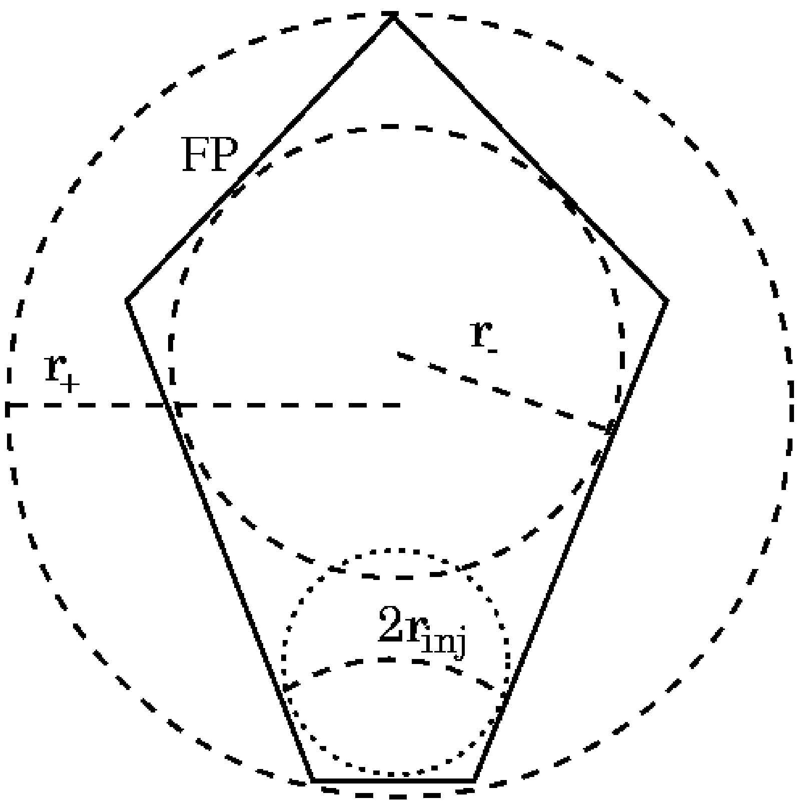

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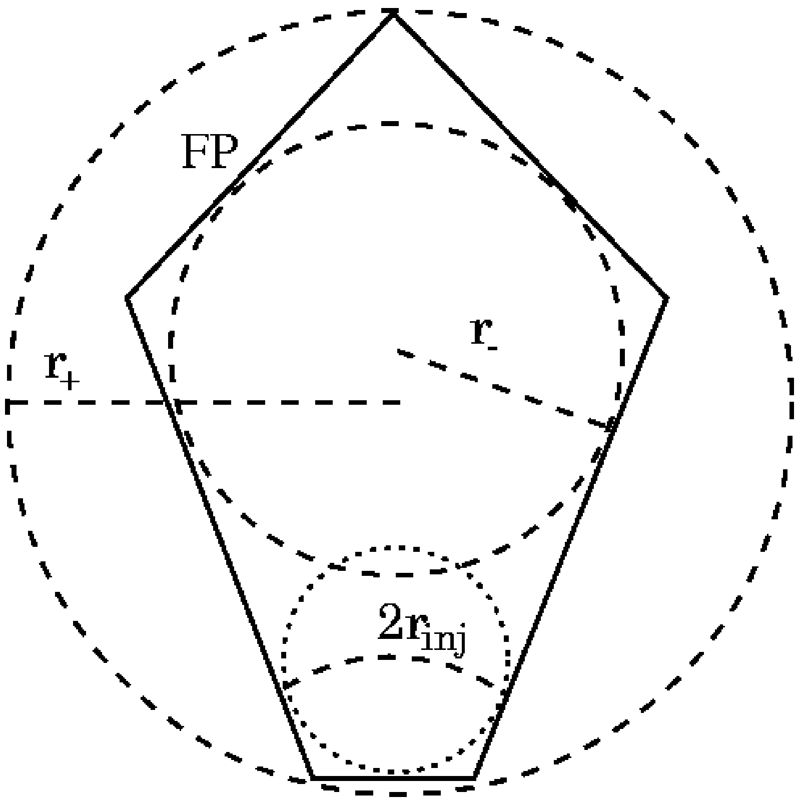

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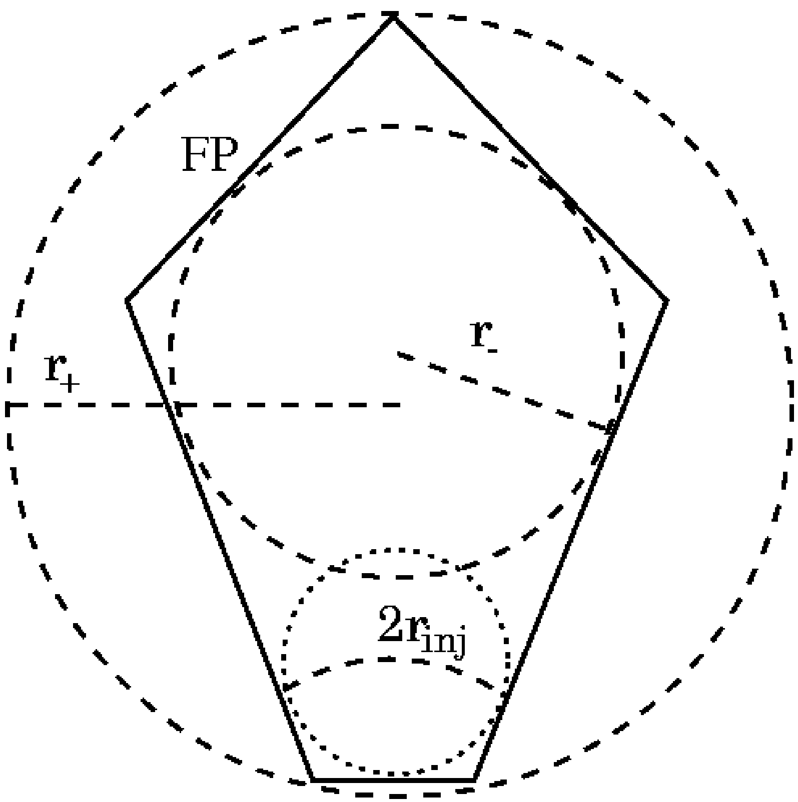

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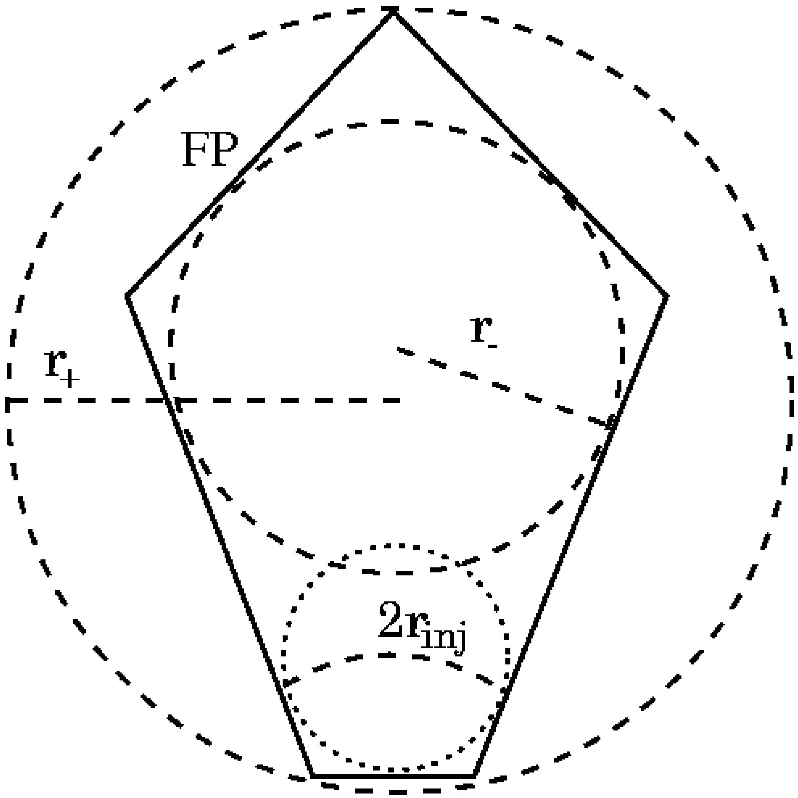

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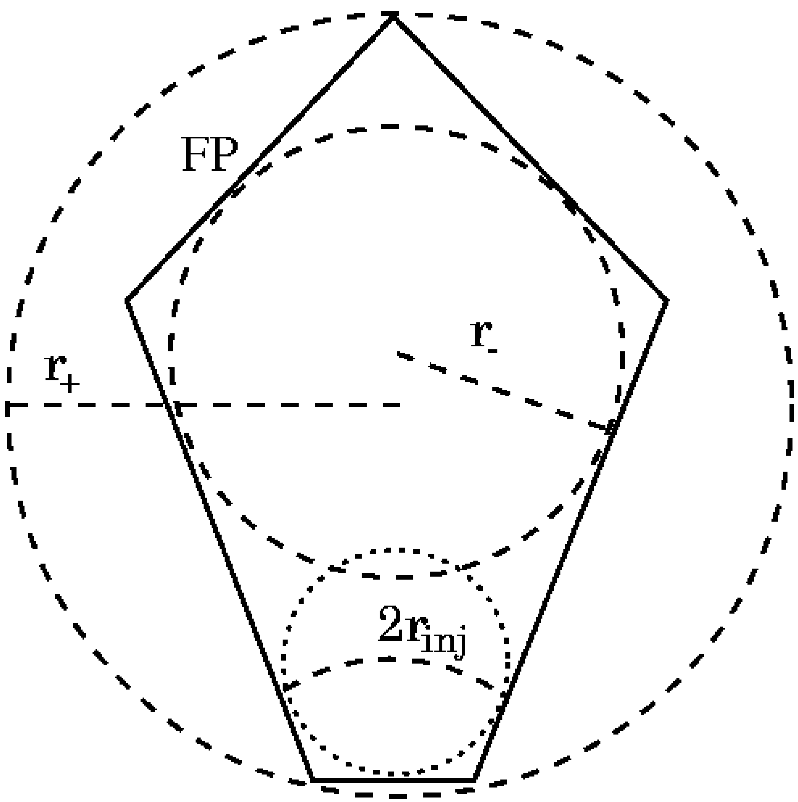

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- $r_{\rm inj} < r_-$ or $r_{\rm inj} \ll V_{\rm FD}^{1/3}$ possible

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  - w:Poincaré Conjecture *"Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere."* w:Grigori Perelman, arXiv:math/0211159 + arXiv:math/0303109 + arXiv:math/0307245

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  - active research area, e.g. arXiv:0705.4325 min. vol.
- 5 other Thurston classes – w:Geometrization conjecture

Cosmic topol: obs. strategies

empirical strategies: arXiv:astro-ph/0010189

A. multiple topological images:

  A.i 3D (grav collapsed objects):

   A.i.1 local isometries—collect "type I pairs" or "local pairs" (pair types)

   A.i.2 cosmic crystallography—collect "type II pairs" or "holonomy pairs"

   A.i.3 successive filters (obs)

   A.i.4 characteristics of individual objects

Cosmic topol: obs. strategies

A. multiple topological images:

  A.ii 2D (e.g. microwave background = CMB):

    A.ii.1 cutoff of large-scale power [also 3D] (obs)

    A.ii.2 identified circles principle, (obs)

    A.ii.3 matched discs corollary

    A.ii.4 patterns of spots

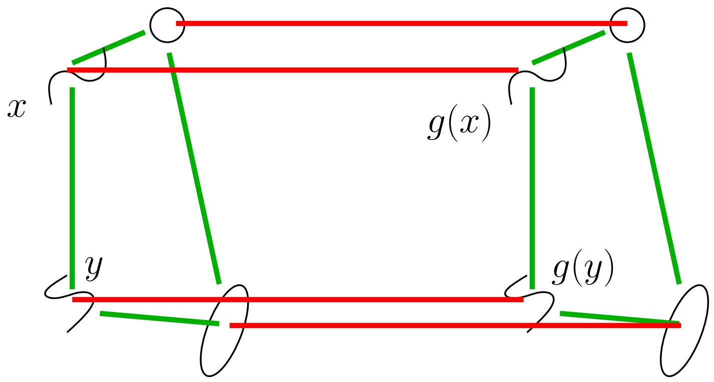
    A.ii.5 perturbation statistics assumptions

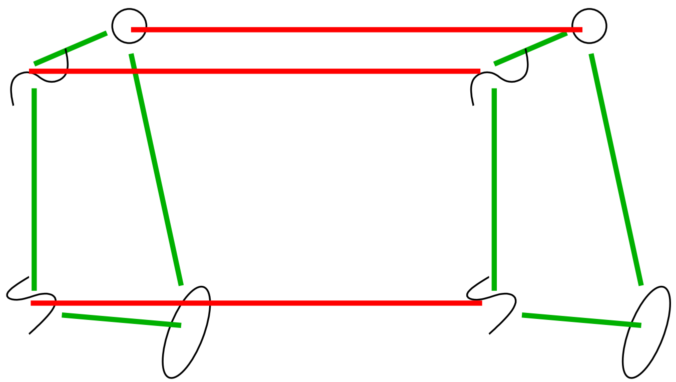
B. other:

  B.i cosmic strings

  B.ii topological acceleration

3D strategies—pair types





Type I pairs = local pairs or n-tuples

Type II pairs = fundamental length pairs

PSH = pair separation histogram

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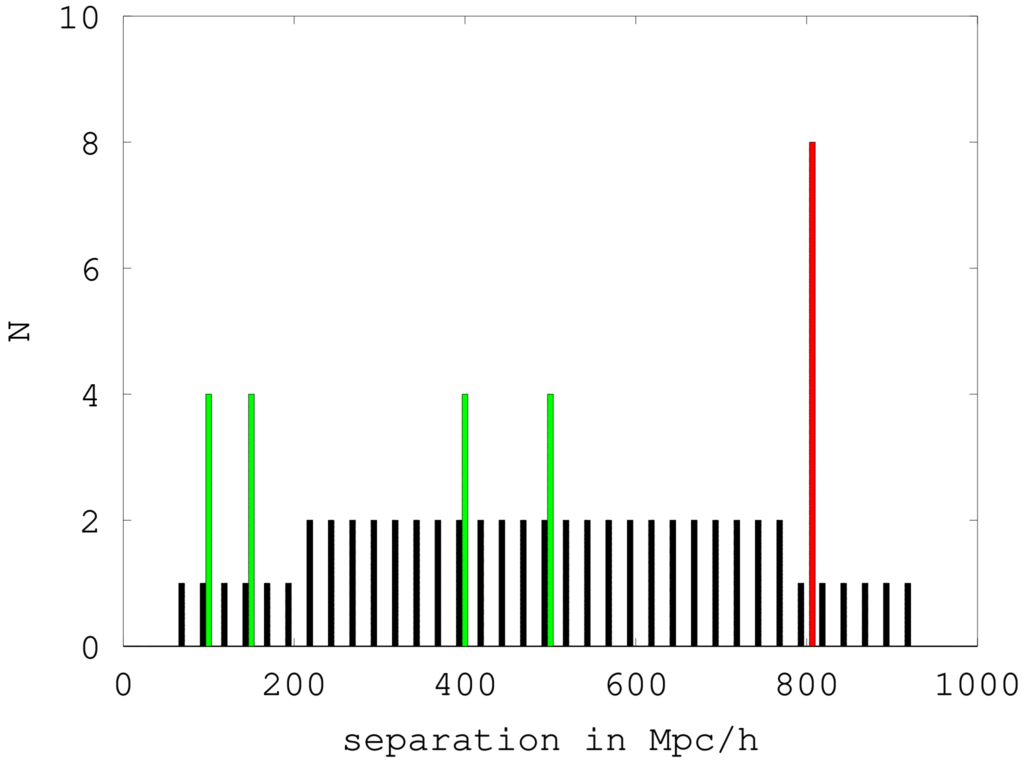

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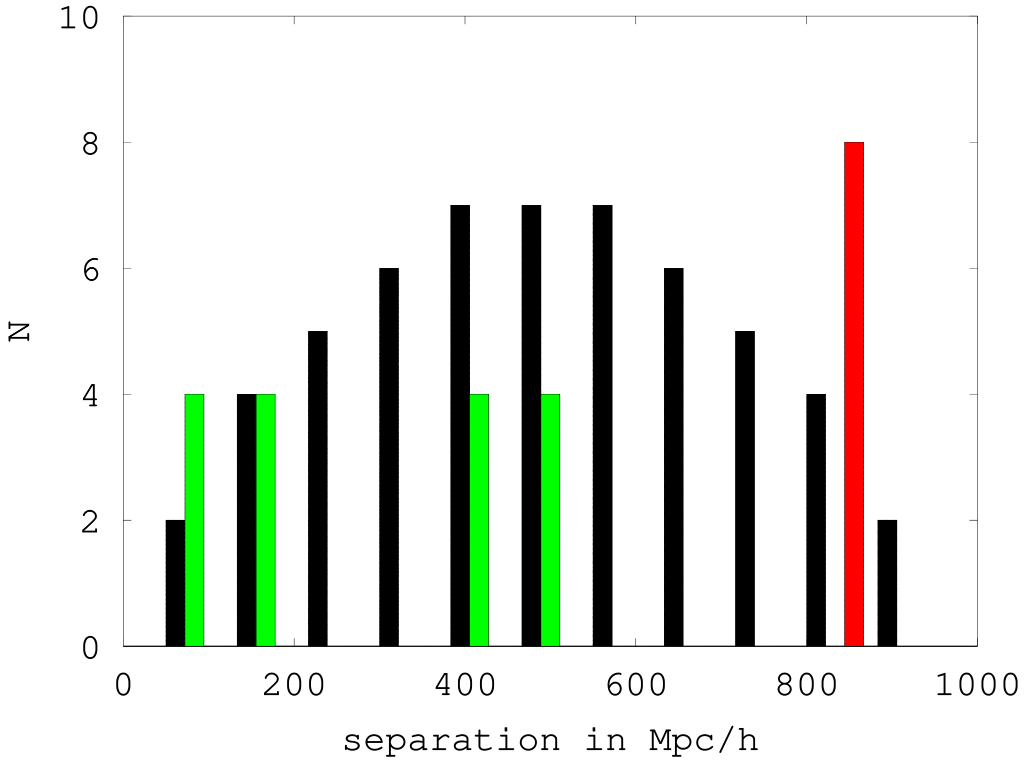

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3D strategies—history

- quasar–galaxy alignments, Fagundes (1985) ADS:1985ApJ...291..450F;
- opposite QSO pairs: Demiański & Lapucha (1987); Fagundes & Wichoski (1987)
- type II pair collection: "cosmic crystallography"—Lehoucq, Lachièze-Rey, Luminet (1996) arXiv:gr-qc/9604050
- type I pair or n-tuple collection: Roukema (1996) arXiv:astro-ph/9603052
- "type I, type II" terminology: Lehoucq, Luminet, Uzan (1999) arXiv:astro-ph/9811107
- successive filters: Marecki, Roukema, Bajtlik (2005)
- quadruples + successive filters + collect membership s of quadruples Fujii & Yoshii (2013)

3D strategies—pair types

- Type I pairs = local pairs or n-tuples

  occur for any curvature

- Type II pairs = fundamental length pairs

  occur for some flat and spherical cases:

  require a holonomy (mapping) $g : x \to g(x)$ for which

$$\forall x, y, \;\; d(x, g(x)) = d(y, g(y))$$

  where d = comoving distance
  this defines a: *Clifford translation*

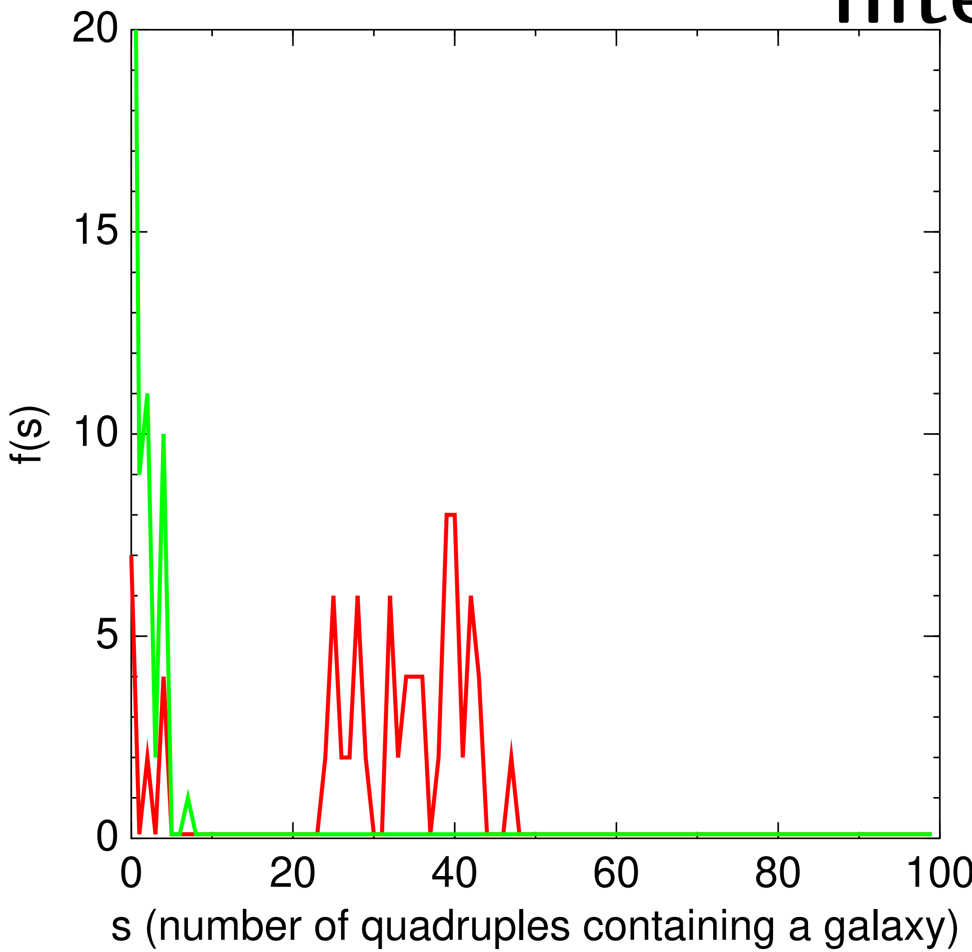
- Clifford translation examples: T^3: yes; S^2: rotations = no; S^3: pair of orthogonal rotations = possible

AGNs—successive filters

Fujii & Yoshii (2013) arXiv:1103.1466

- method valid for compact flat spaces:
- pairs of Type II pairs = quadruples +
- require type I pairs (2nd filter) +
- δt filter—short QSO lifetimes +
- collect n-tuples:
- each i-th object $\in s_i$ quadruples
- plot histogram of frequency of s values

AGN Catalogues—successive filters

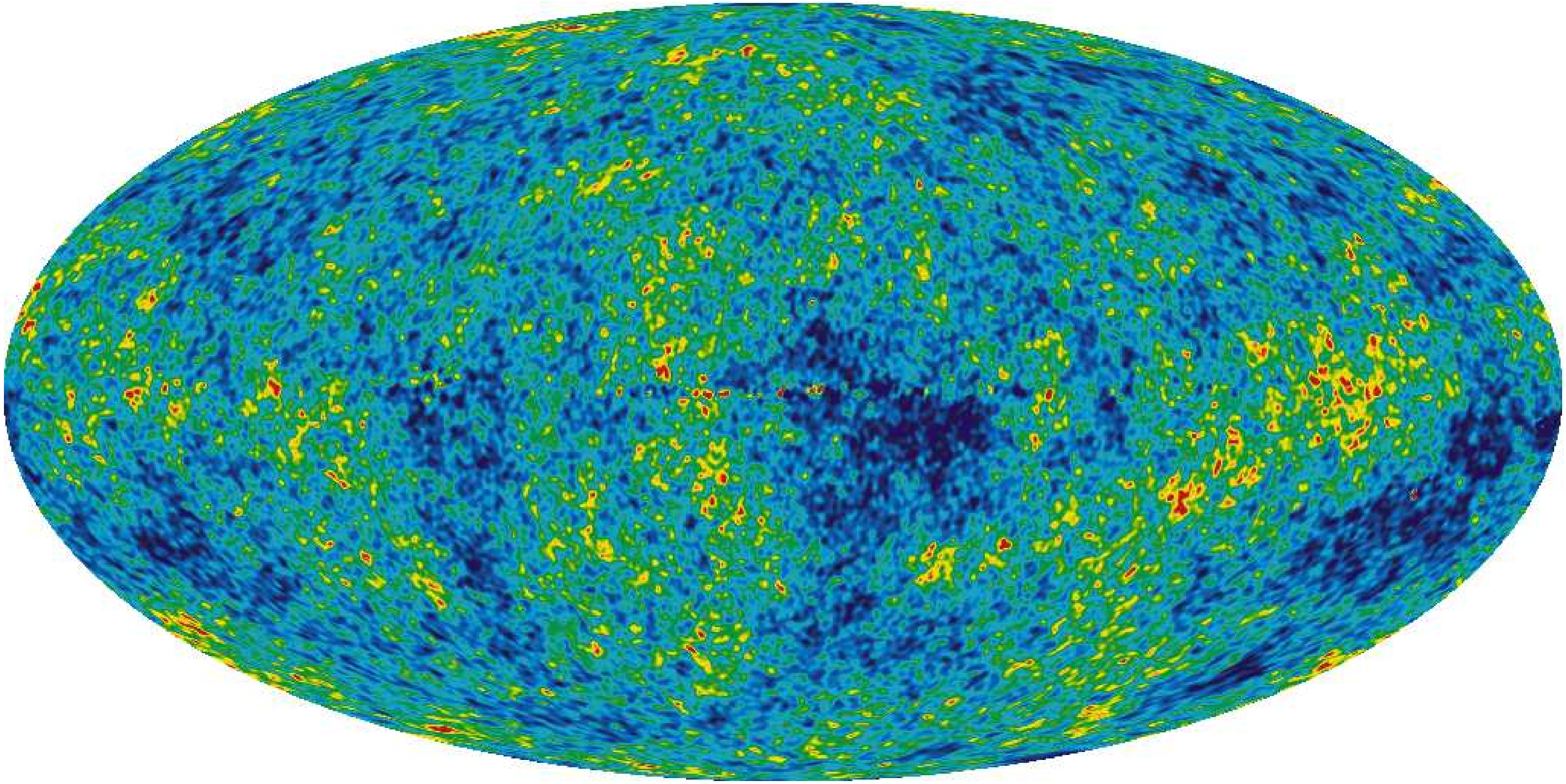


simulation of s histogram for Lyman break galaxies (LBGs) at $z \approx 6$

green: simply connected; red: T^3

ADS:2014MNRAS.437.1096R

2D methods: structure cutoff



WMAP 5yr ILC (internal linear combination)

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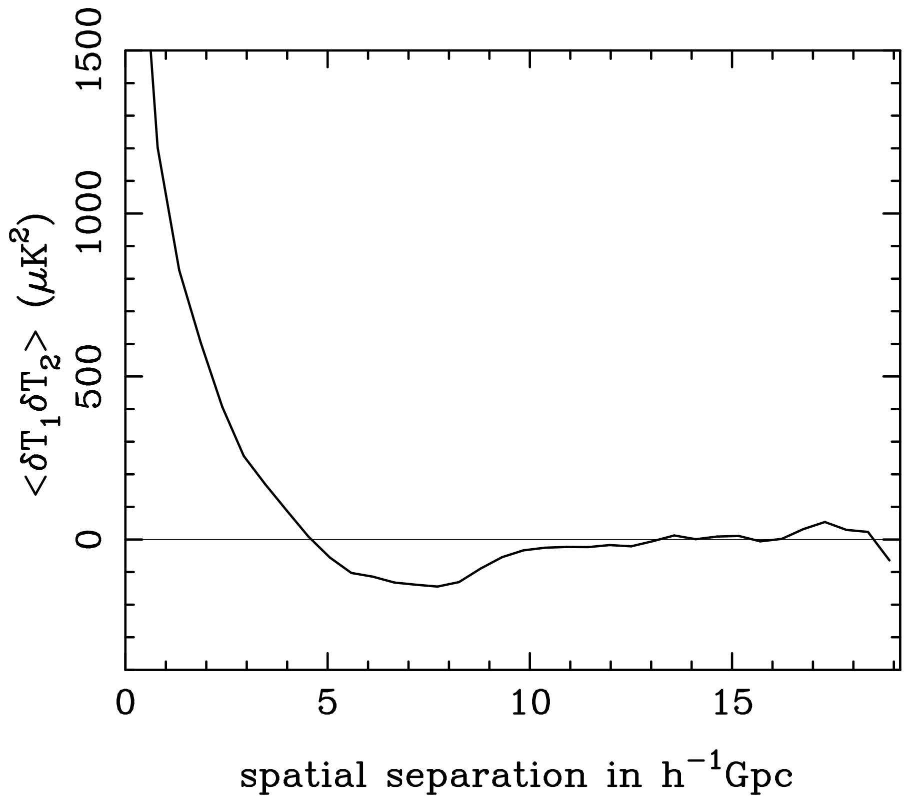

3D: structures bigger than FD cannot exist

roughly $\Rightarrow$ 2D structure cutoff

Starobinsky (1993); Stevens et al. (1993)

The Identified Circles Principle

- discovery of principle: Cornish, Spergel & Starkman (1996)
- original article only as preprint: arXiv:gr-qc/9602039
- closed access peer-reviewed article: CQG, 15, 2657 (1998)

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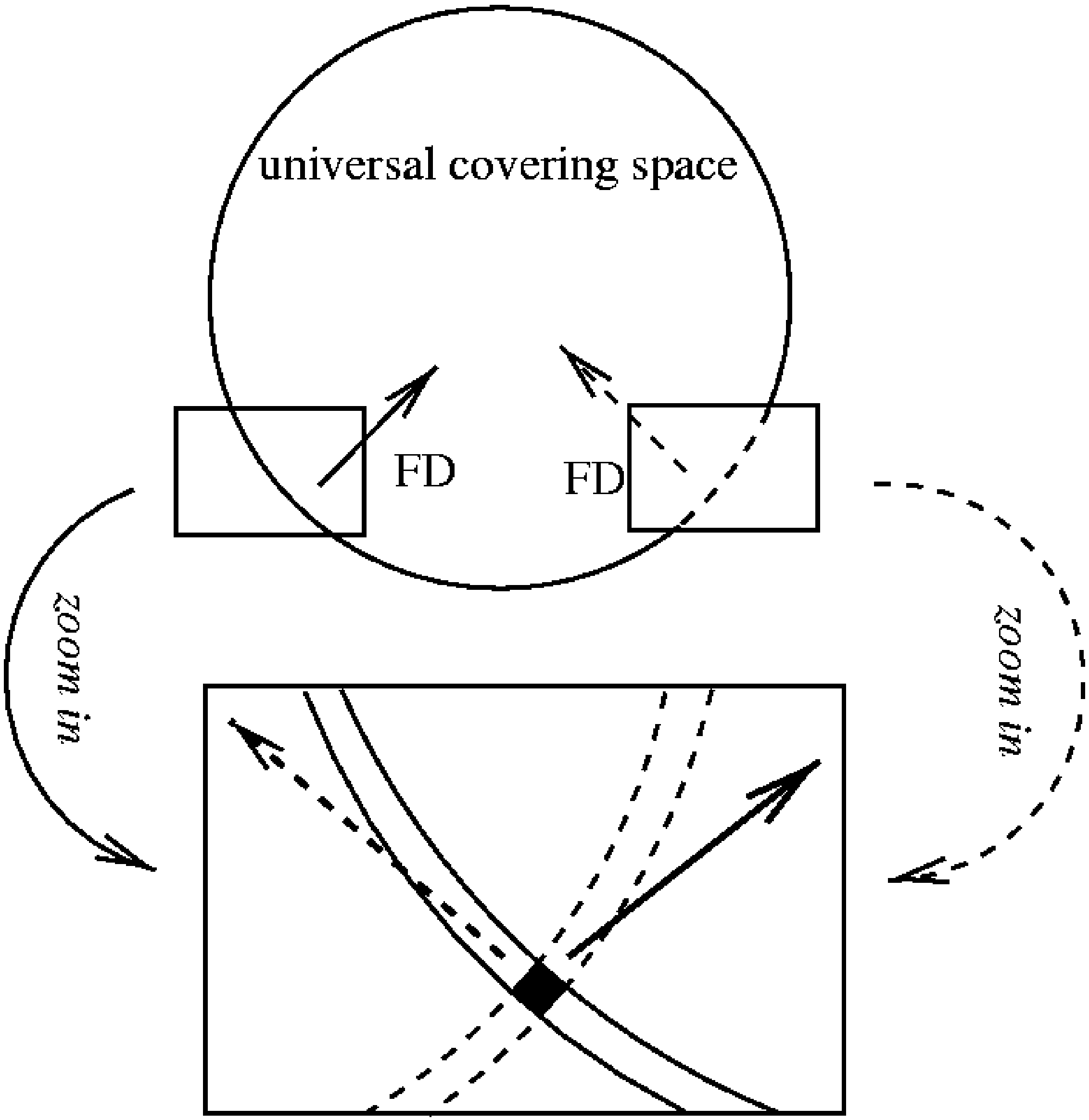
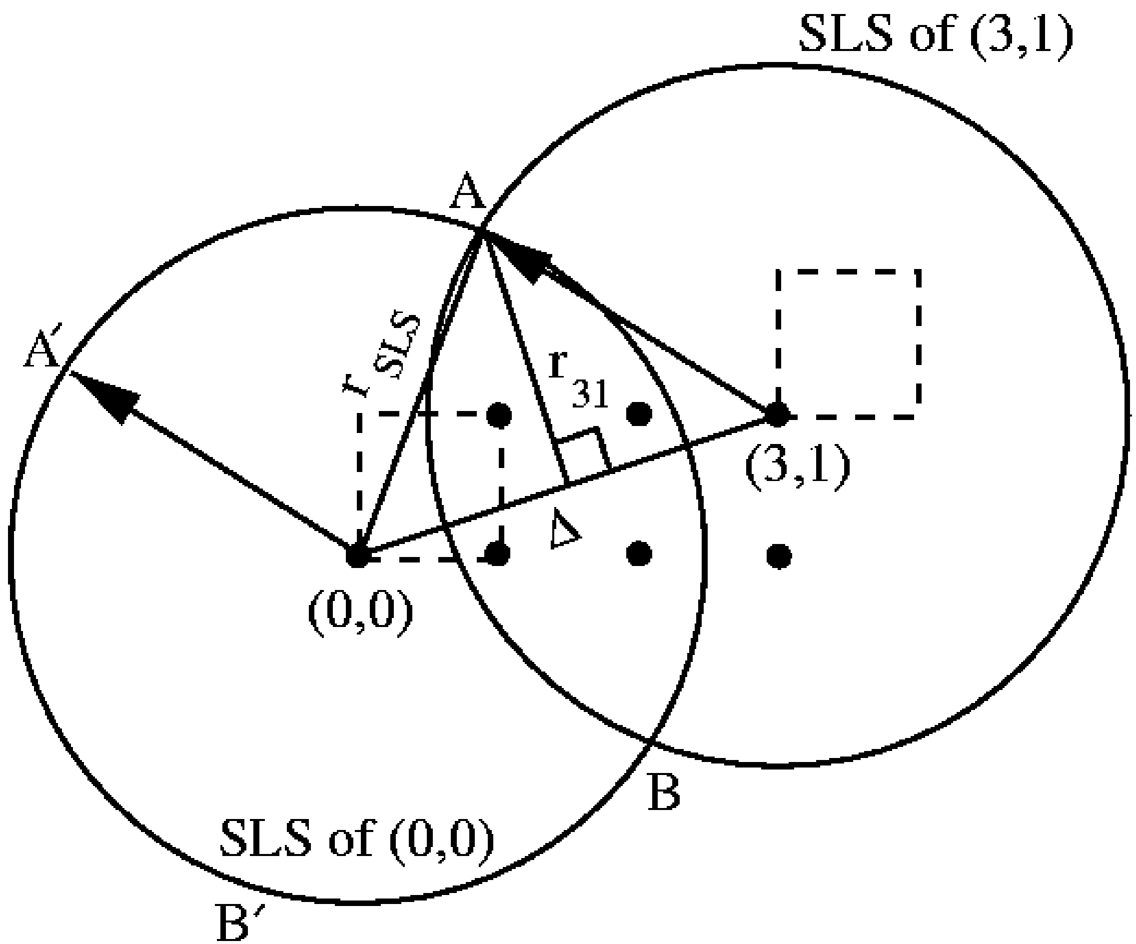


FD (fundamental domain)

The Identified Circles Principle



Cosmic topol: top. accel.

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- (quantum gravity arguments)

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- global spatial topology: topological acceleration (patching away black holes): Roukema+2007 (A&A 2007) arXiv:astro-ph/0602159

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- scalar averaging and dynamical topology change (e.g. black holes): Brunswic & Buchert (CQG, 2020) arXiv:2002.08336

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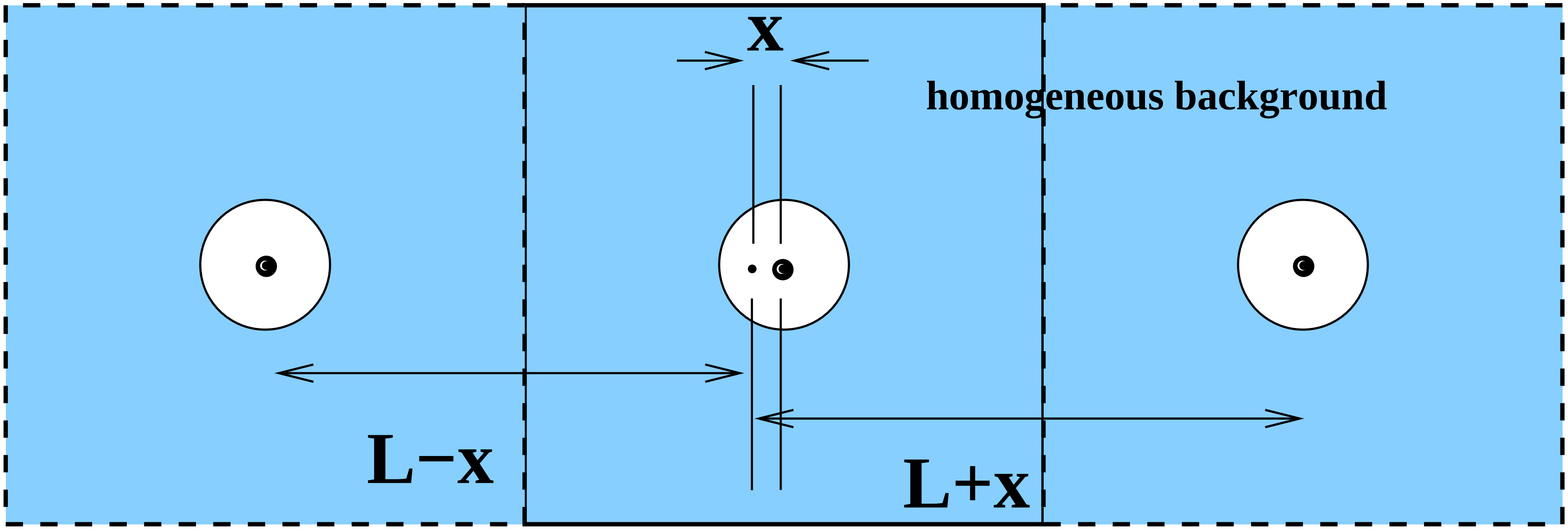

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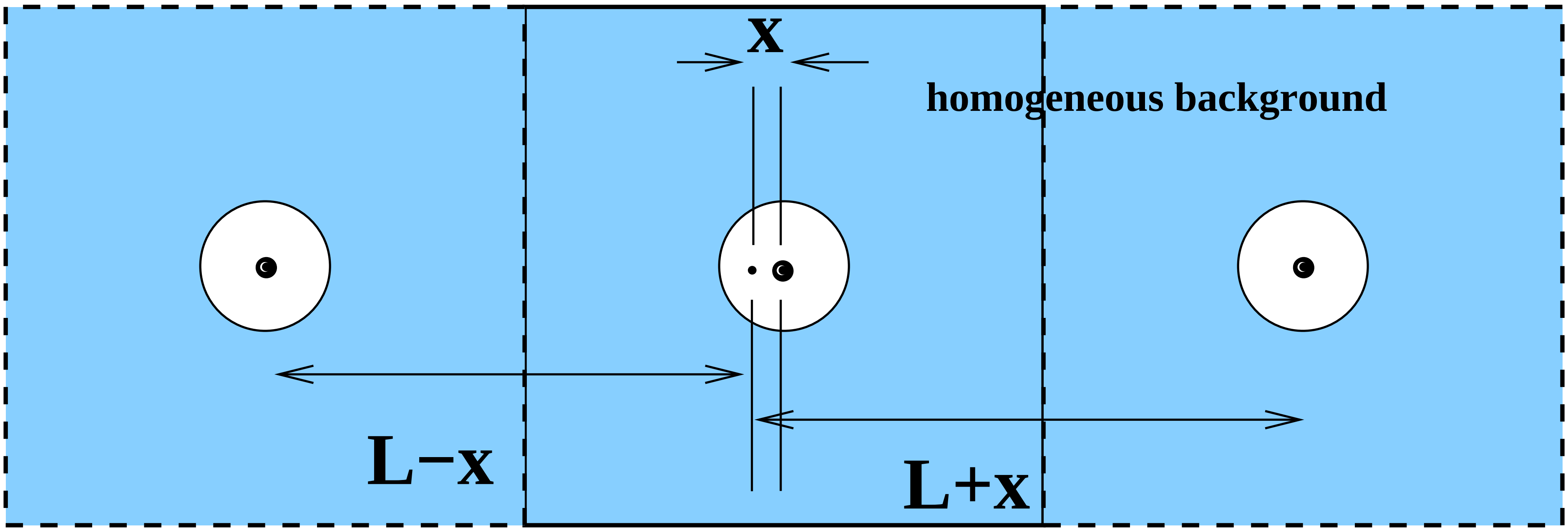

$$\ddot{x} \approx -G\frac{m}{x^2} + Gm\left[\frac{1}{(L-x)^2} - \frac{1}{(L+x)^2}\right]$$

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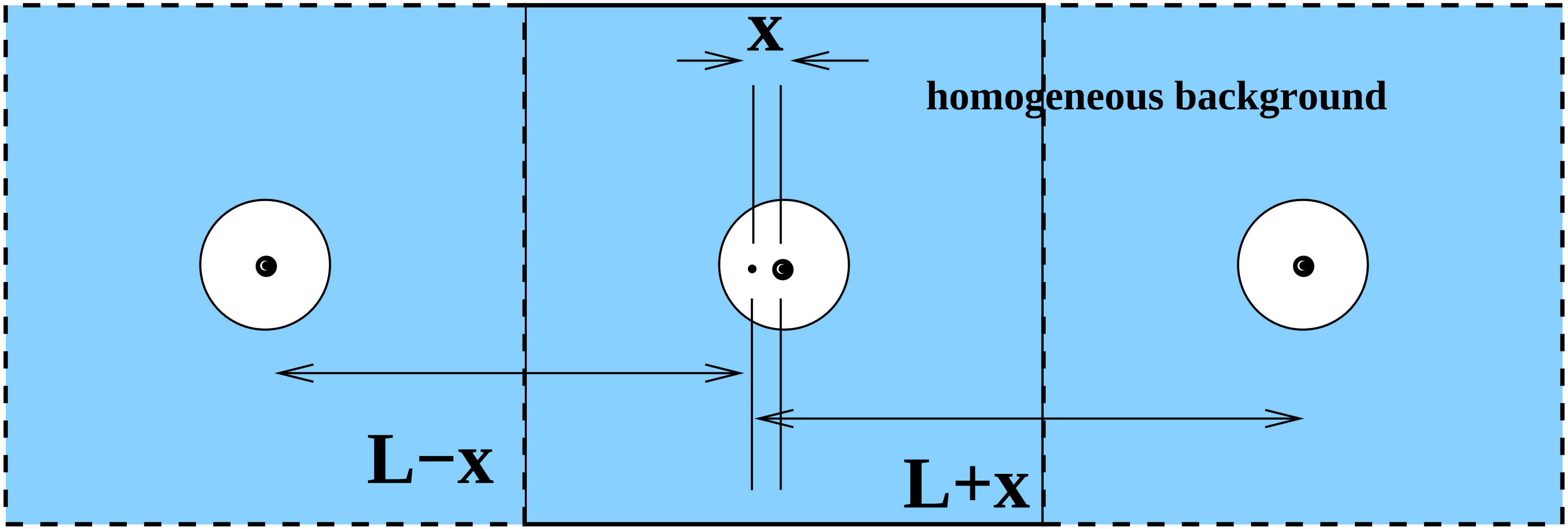

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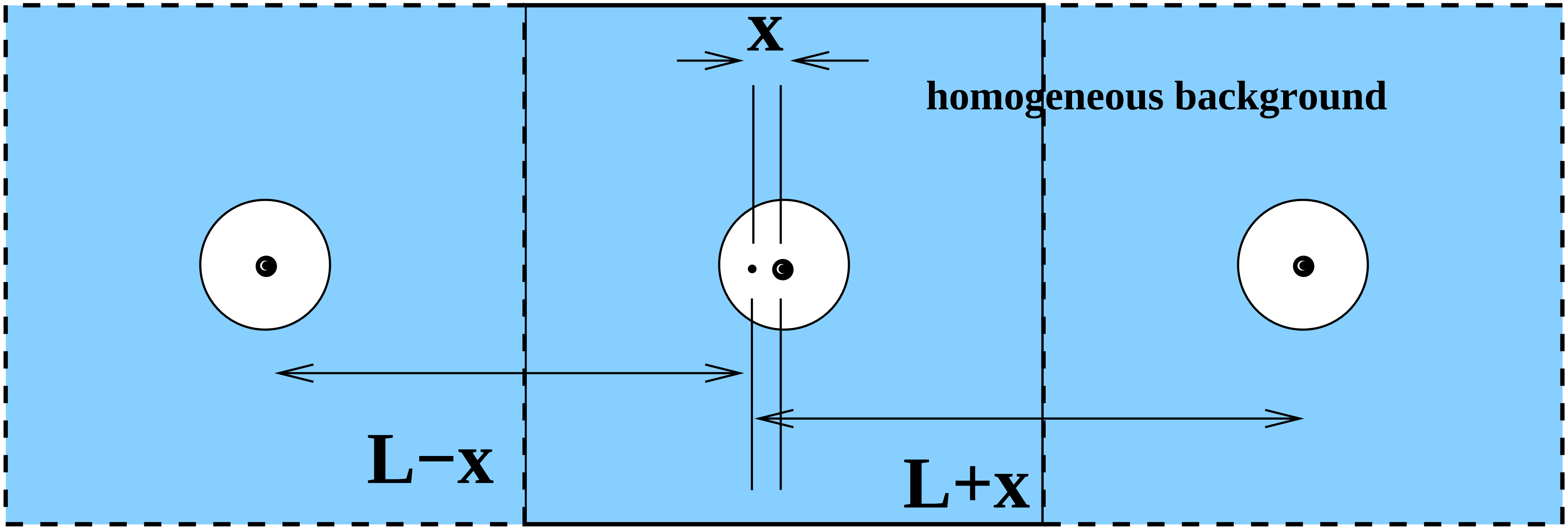

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$$\boxed{\ddot{x}_{\mathrm{resid}} \propto (x/L)^1 + \ldots}$$

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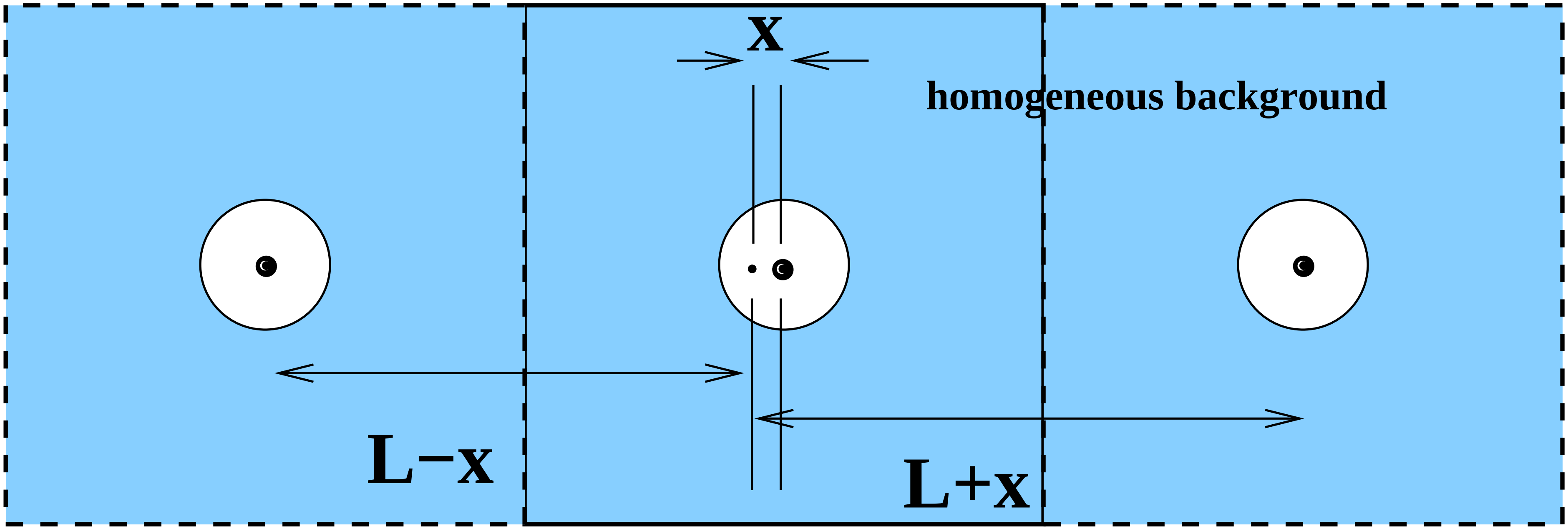

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topological acceleration— arXiv:astro-ph/0602159

Is topolog. acceleration relativistic?

- Korotkin & Nikolai (1994) arXiv:gr-qc/9403029 solution: Schwarzschild-like BH in $S^1 \times E^2$ (slab space = T^1)
- outside event horizon, inside topology scale:

$$\ddot{x} = 4\zeta(3)G\,\frac{M}{L^3}x \propto x$$

Ostrowski, Roukema & Buliński (2012) arXiv:1109.1596

$\Rightarrow$ Yes.

Heuristic top. accel.

original heuristic — Roukema+2007 A&A arXiv:astro-ph/0602159

- weak-field gravity of distant, multiple images
- covering space $\mathbb{E}^3$ or $\mathbb{S}^3$
- calculations made in covering space
- consider only first layer of topological images (e.g. particle horizon)

Heuristic top. accel.

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- $\mathbb{T}^3 = \mathbb{E}^3/\mathbb{Z}^3 \quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/L)^3 + \ldots$
- $\mathbb{S}^3/T^* \equiv M_6$ (octahedral space) $\quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/R_C)^3 + \ldots$
- $\mathbb{S}^3/O^*$ (truncated cube space) $\quad \Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/R_C)^3 + \ldots$
- $\mathbb{S}^3/I^* \equiv M_8$ (Poincaré dodecahedral space)
  $\Rightarrow \quad \ddot{x}_{\mathrm{resid}} \propto (x/R_C)^5 + \ldots$

- *topological acceleration is manifold-dependent*
  Roukema & Różański arXiv:0902.3402, A&A, 502, 27

Newt. non-Euclid. top.accel.

NEN: $\Phi_{\mathbb{S}}(\xi) \propto -\cot\xi\,(1-\xi/\pi) + A$

Topology	N_Σ	Φ_{-1}	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5
Euclidean (infinite or Thurston-type)							
$\mathbb{E}^3$		-1	0	0	0	0	0
$\mathbb{T}^3$		-1	0	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	0	–	0
Spherical							
$\mathbb{S}^3$	1	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	$-\frac{2\pi}{45}\frac{\mathcal{R}/6}{V_\Sigma}$	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
M_3	8	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	–	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
M_6	24	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	–	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
M_7	120	-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	$-\frac{2\pi}{3}\frac{1}{V_\Sigma}$	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	$-\frac{2\pi}{45}\frac{\mathcal{R}/6}{V_\Sigma}$	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$
Hyperbolic (infinite)							
$\mathbb{H}^3$		-1	$\frac{1}{3}\frac{\mathcal{R}}{6}$	0	$\frac{1}{45}\left(\frac{\mathcal{R}}{6}\right)^2$	0	$\frac{2}{945}\left(\frac{\mathcal{R}}{6}\right)^3$

even terms $\Rightarrow$ closed; odd terms $\Rightarrow$ curved

Vigneron & Roukema (2022) arXiv:2201.09102

$T^3, S^3/\Gamma$

- Some spaces are more equal than others.
- Roukema & Różański arXiv:0902.3402, A&A, 502, 27
- Newton–Cartan approach for preparing for full GR approach: Vigneron (2020, PRD) arXiv:2010.10247; Vigneron (2021, PRD) arXiv:2012.10213; Vigneron (2022a, PRD) arXiv:2109.10336; Vigneron (2022b, CQG) arXiv:2201.02112; Vigneron & Roukema (2022) arXiv:2201.09102

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- KN94 exact GR example – Ostrowski+2012 CQG arXiv:1109.1596
- effect depends on choice of topological manifold — Roukema & Różański 2009 A&A arXiv:0902.3402
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- patterns of time-integrated effects of topological acceleration should exist at $\sim 10\text{–}1000\mathrm{h}^{-1}$ Mpc
  - very difficult to separate from artefacts
  - need detailed GR modelling
  - need excellent quality surveys
- numerical simulations — Buliński 2015 PhD thesis NCU