Special relativity and steps towards general relativity: SR

B.F. Roukema + . . .
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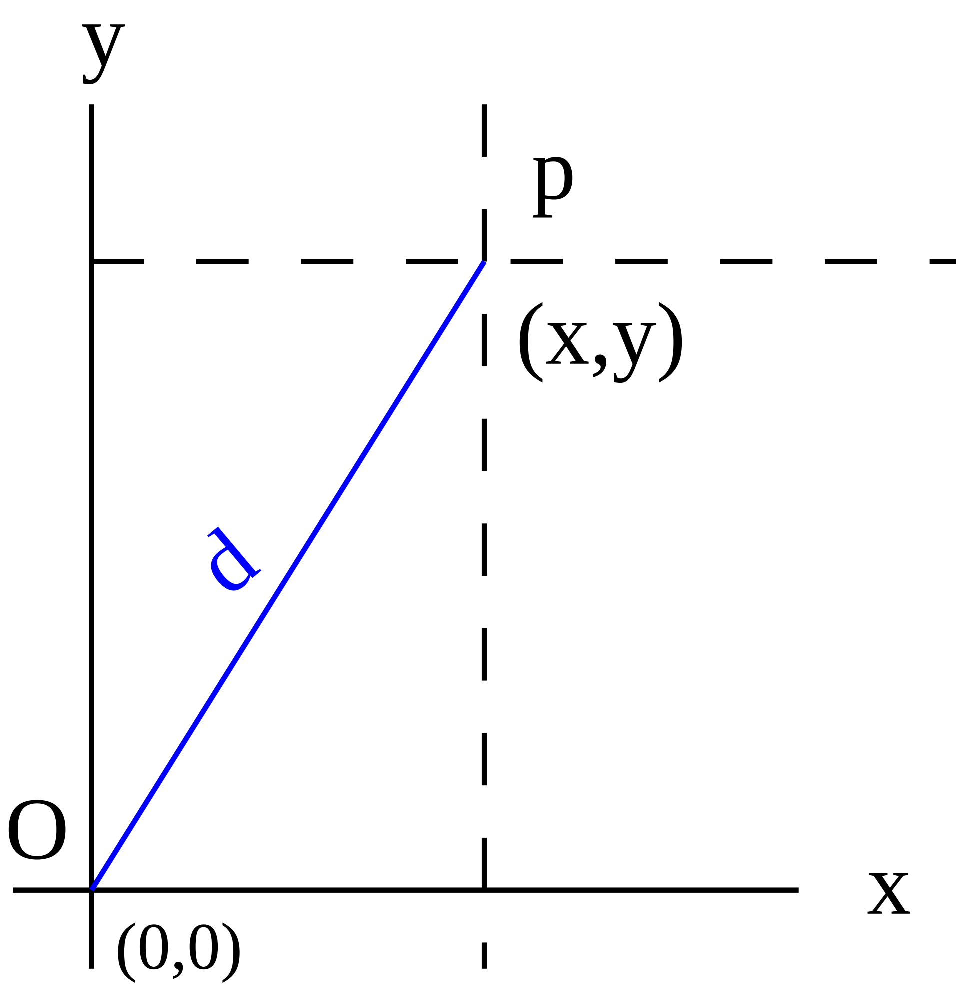
- GR: spacetime = a solution of the w:Einstein field equations

SR: Minkowski spacetime



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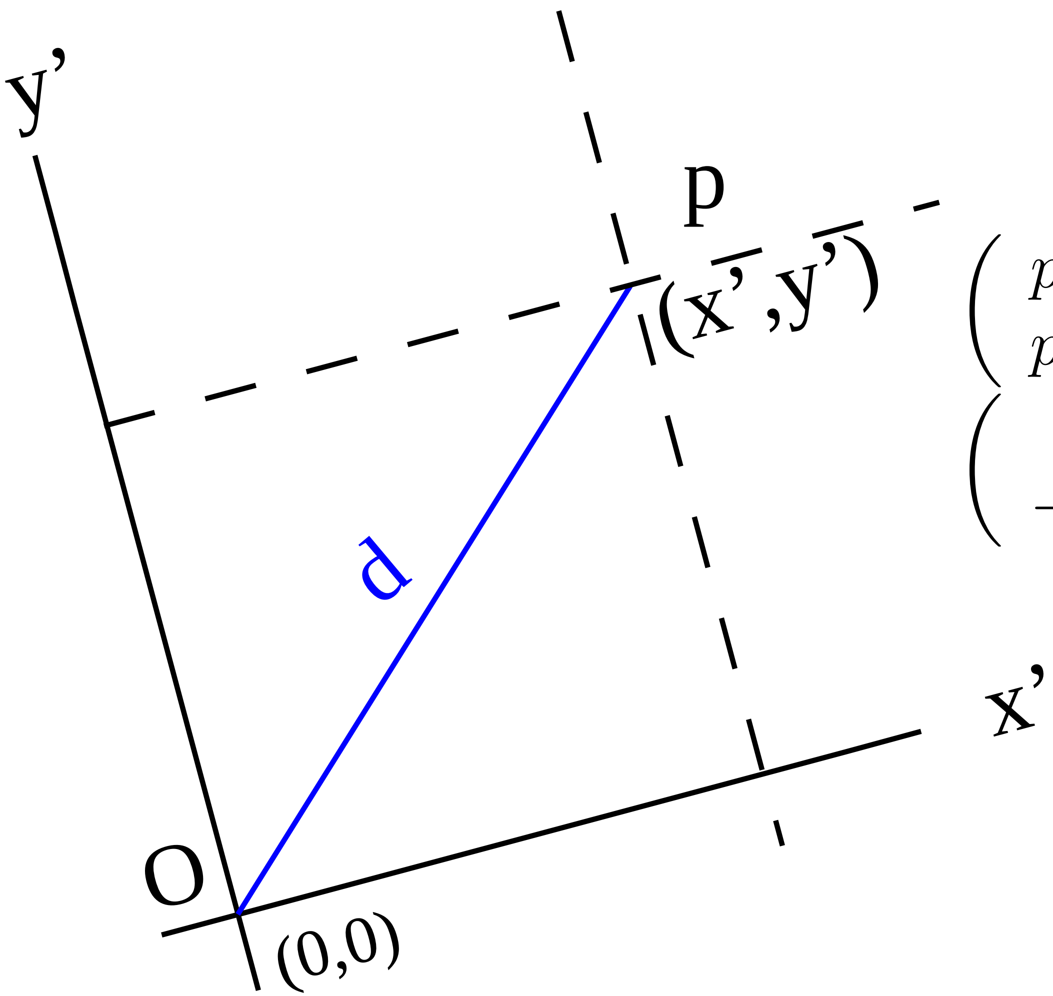

$$\begin{pmatrix} p_{\mathrm{x}'} \\ p_{\mathrm{y}'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} p_{\mathrm{x}} \\ p_{\mathrm{y}} \end{pmatrix}$$

p at (x', y'), distance from observer at O is $d =$ unchanged

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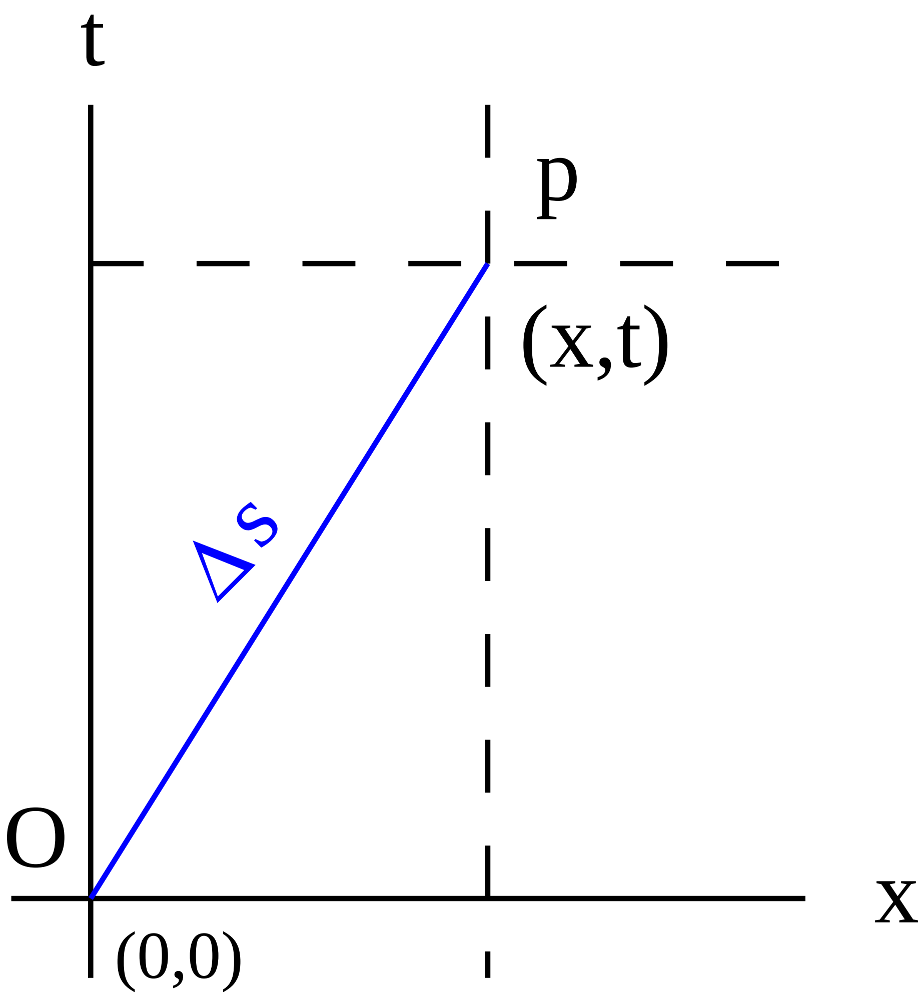

p at (x,t), w:invariant interval from observer at O is Δs where
$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$

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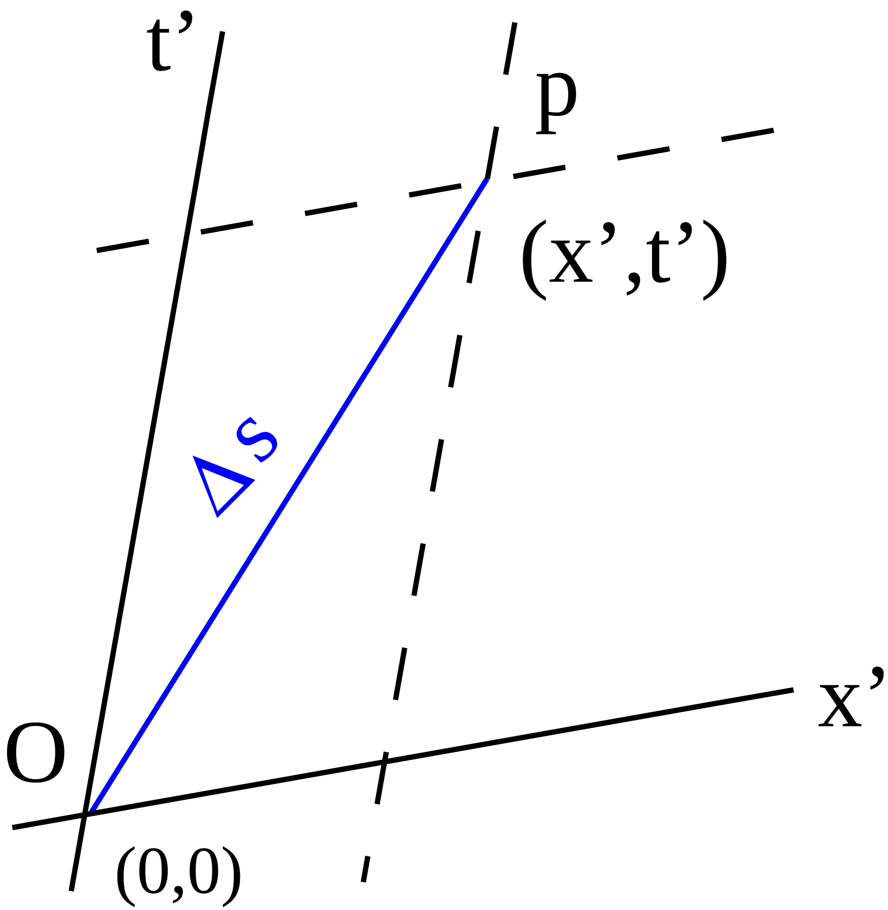

$$\begin{pmatrix} p_{\mathrm{x}'} \\ p_{\mathrm{t}'} \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} p_{\mathrm{x}} \\ p_{\mathrm{t}} \end{pmatrix}$$

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SR: Lorentz transformation

$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

$\cosh\phi := \frac{e^{\phi}+e^{-\phi}}{2}$

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$= \frac{1 \text{ s}}{1 \text{ s}} = 1$ (dimensionless)

SR: rapidity ϕ vs velocity β

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B's worldline in B's coordinate system is:

$$\begin{pmatrix} 0 \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} x \\ t \end{pmatrix}$$

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observer B has worldline $(x', t') = (0, t')$

in A's coordinate system, B's worldline is:

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where velocity $\boxed{\beta := v/c \equiv v = \tanh \phi}$

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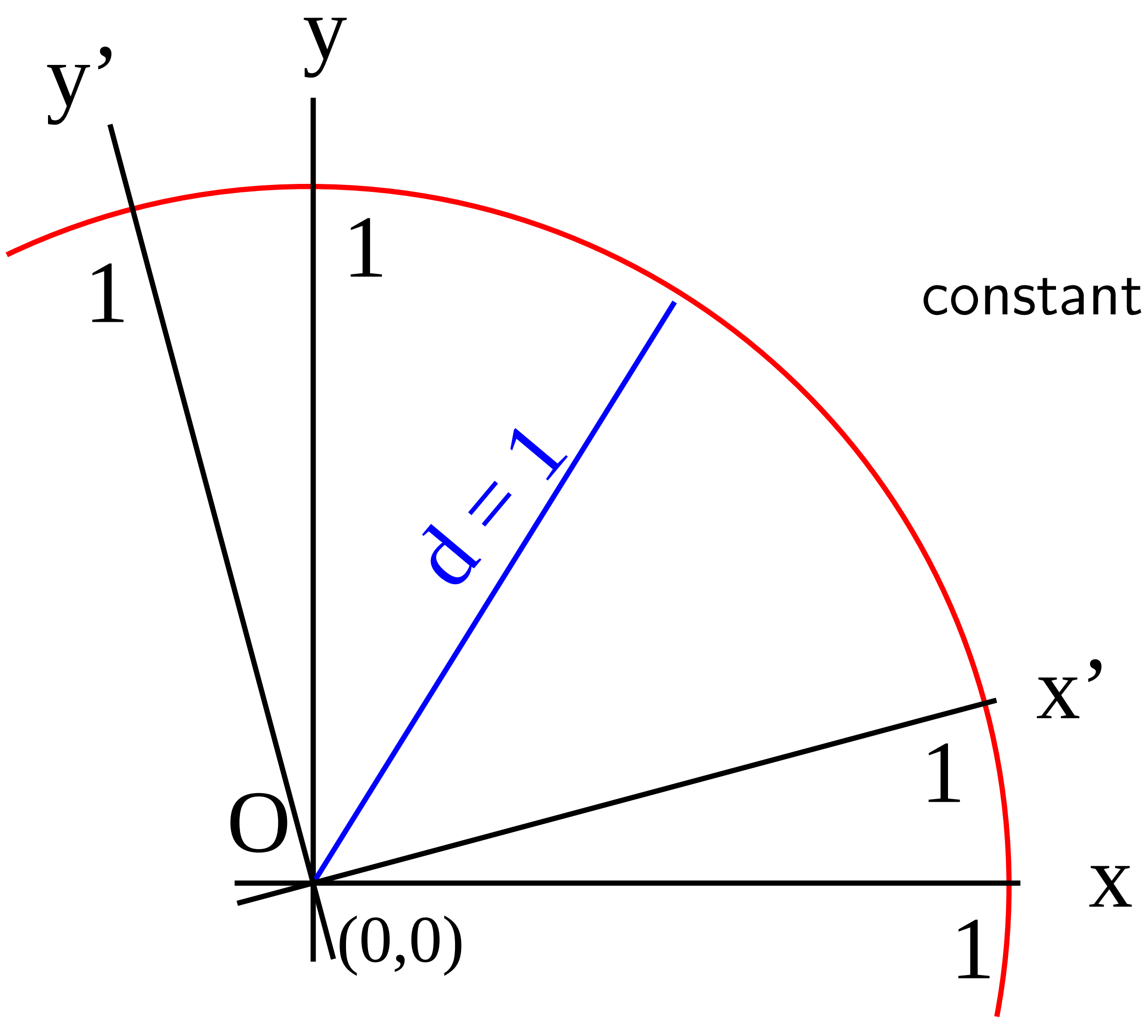

constant distance $d = 1$ from $(0, 0)$

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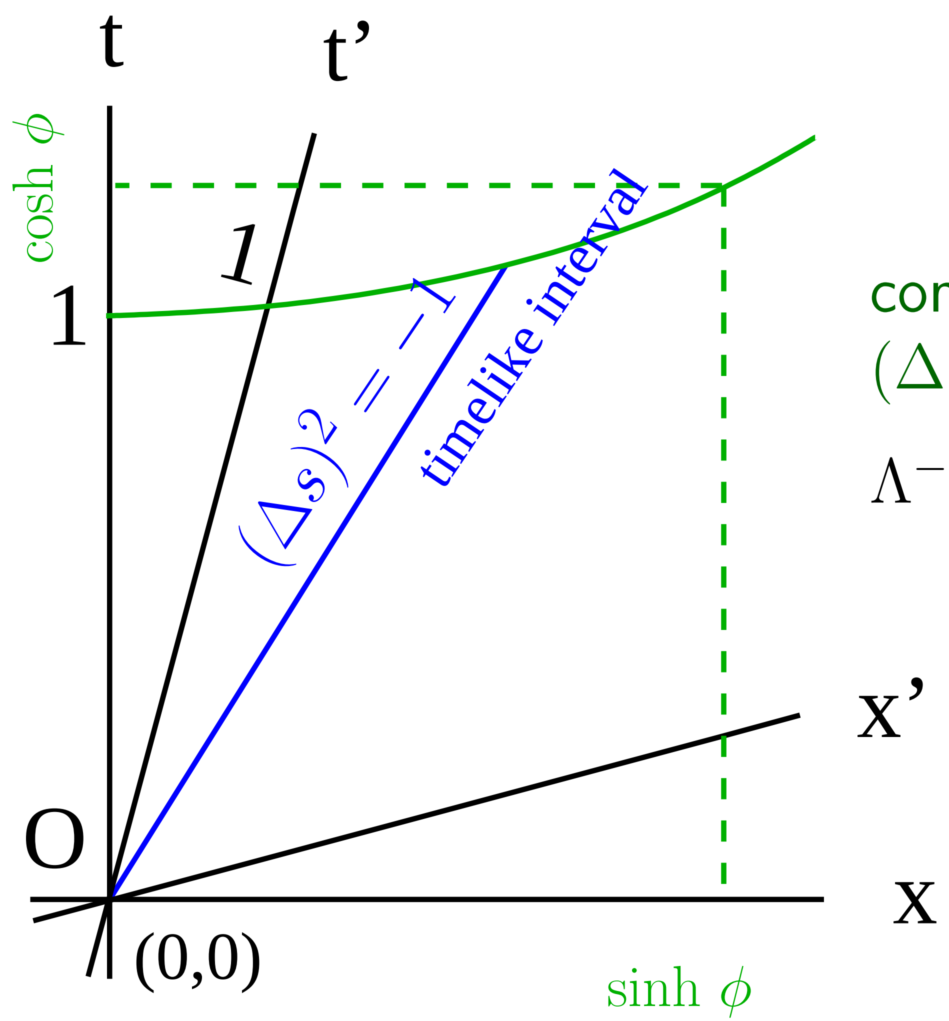

constant interval

$(\Delta s)^2 = -1$ from $(0, 0)$

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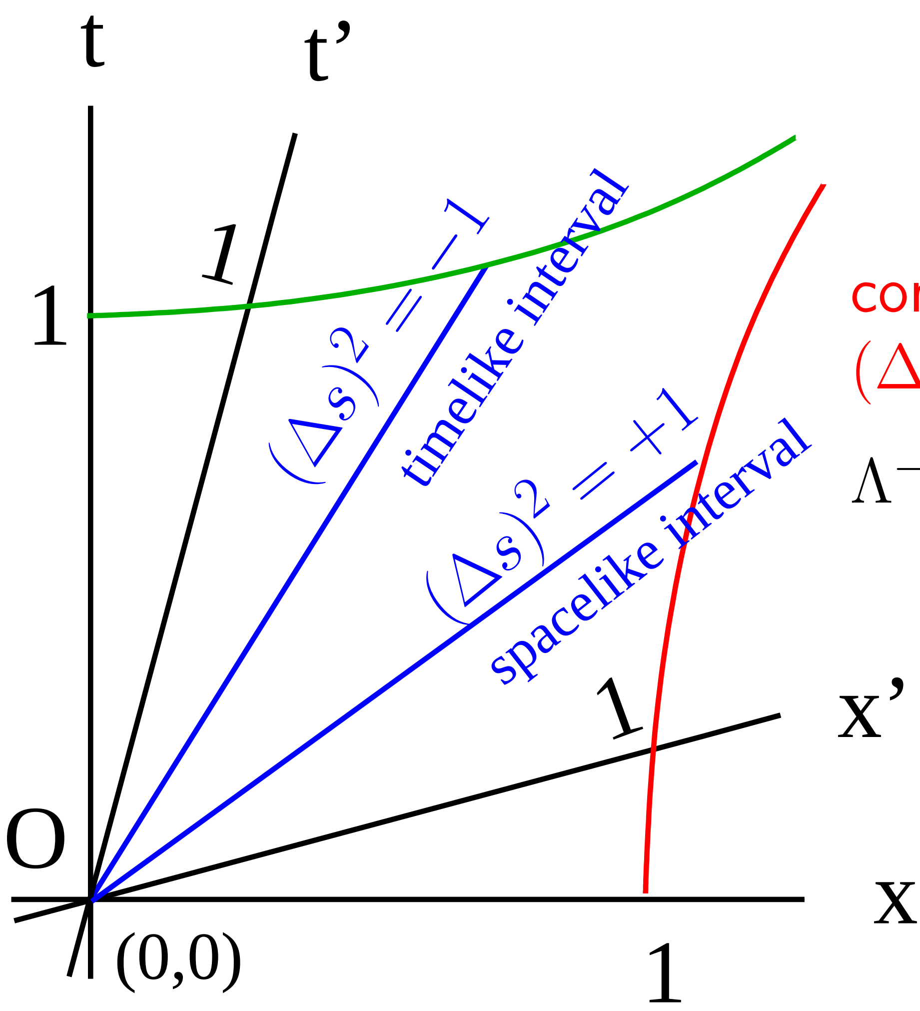

constant interval
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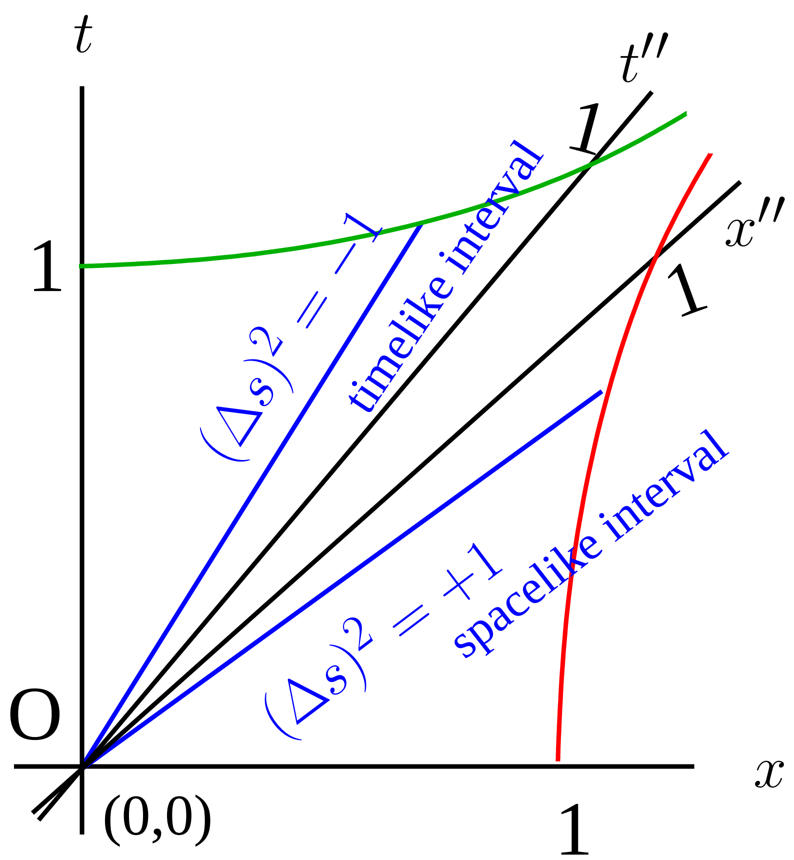

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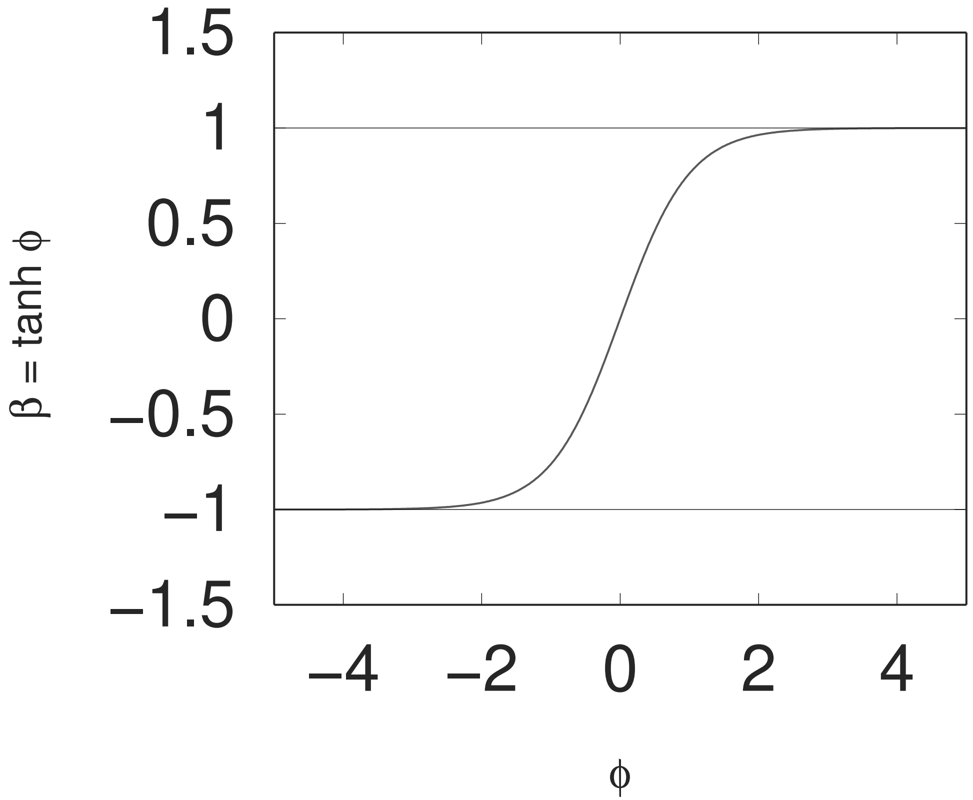

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w:Michelson-Morley experiment (1887)

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interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?

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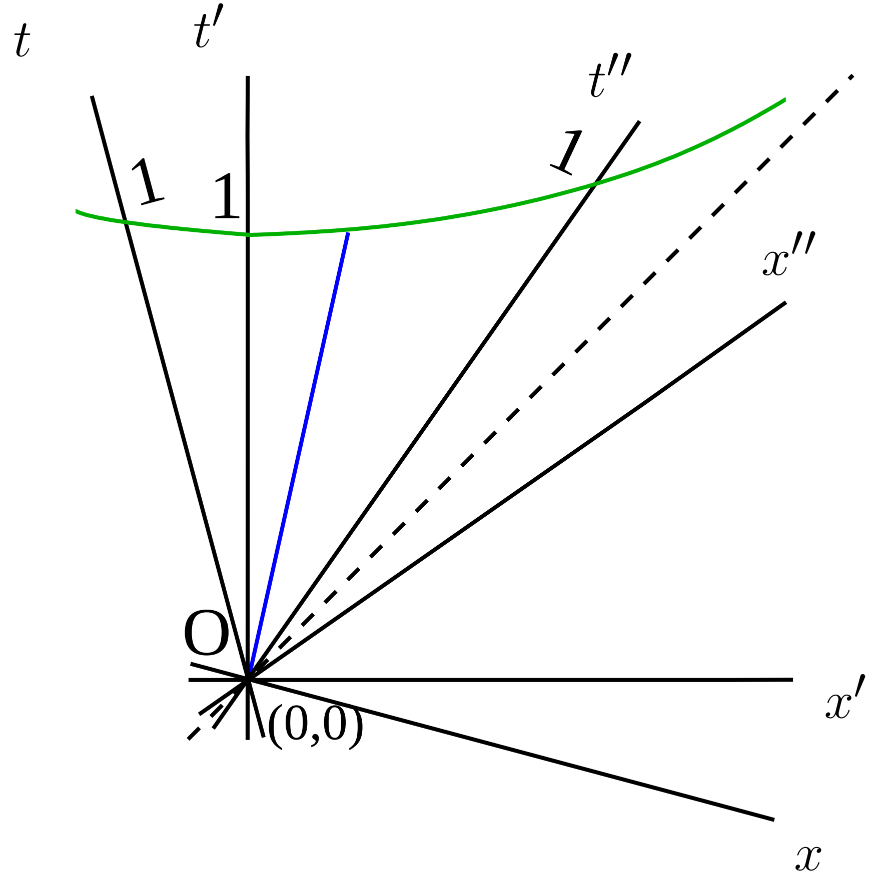

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where $\tanh\phi_1 = \beta_1 = 0.1$

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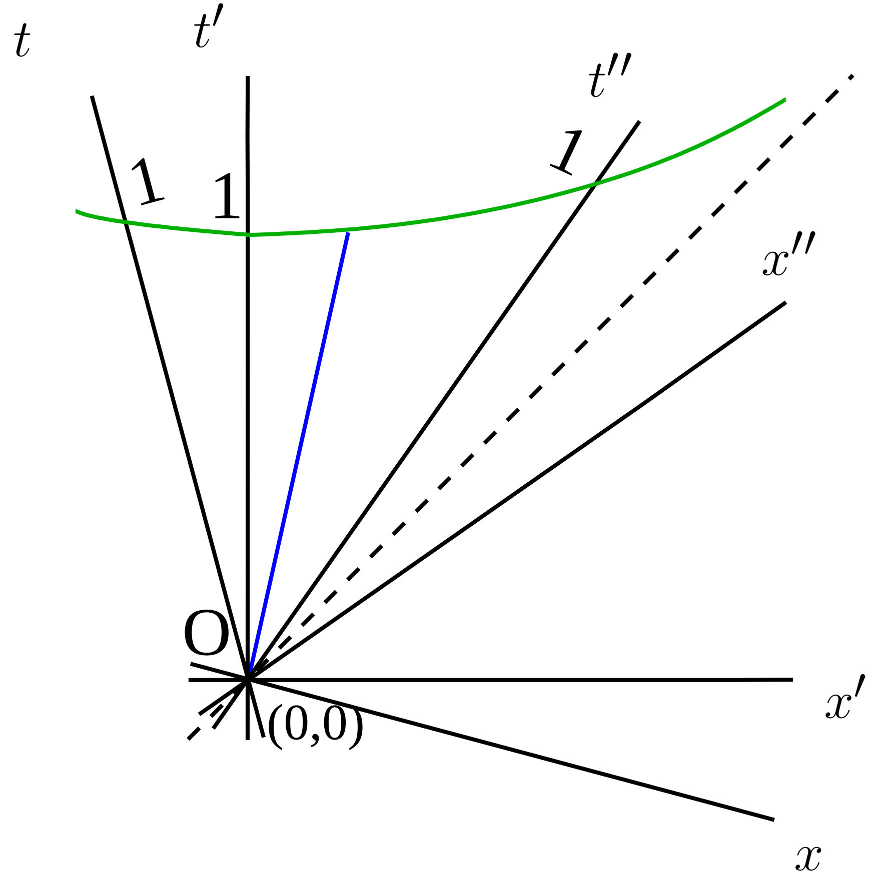

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} =$$

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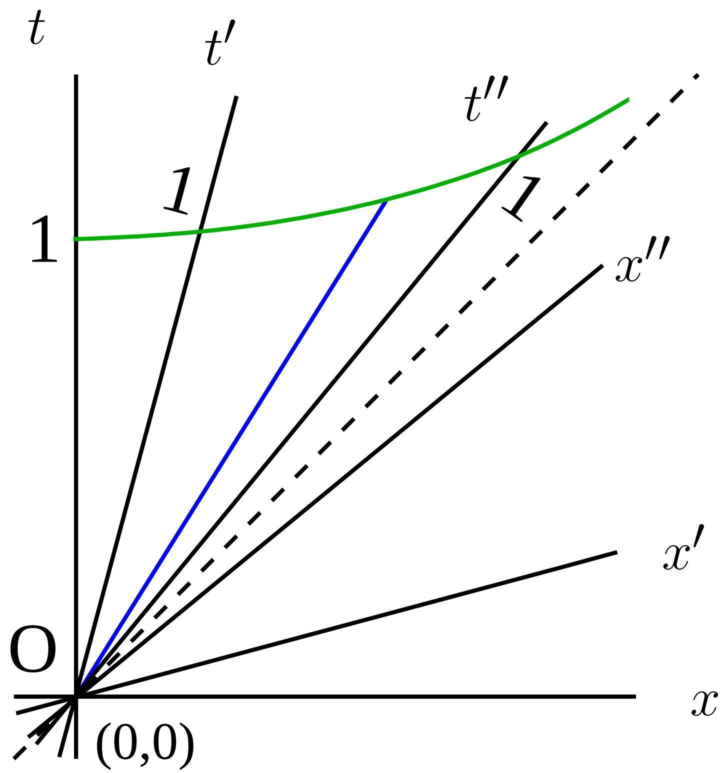

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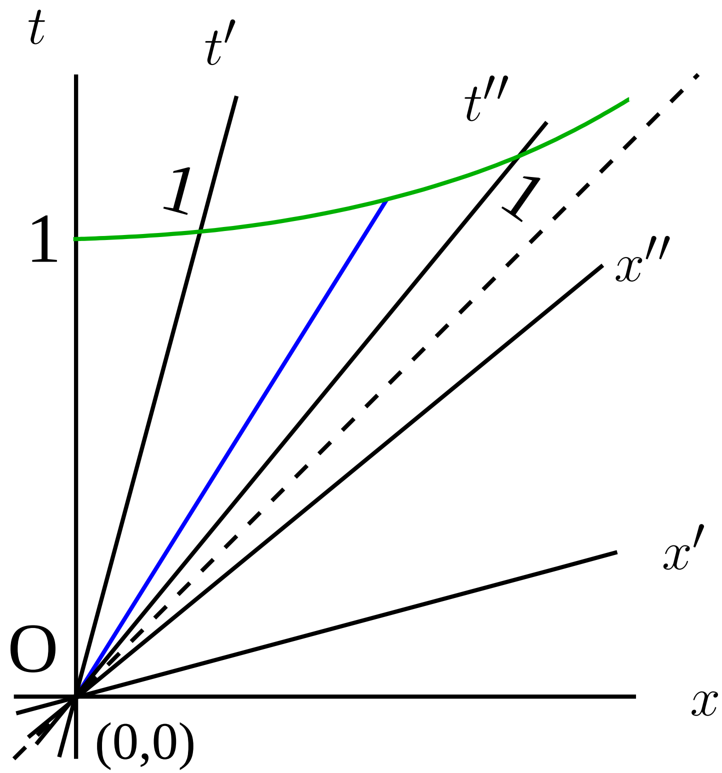

$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = \Lambda(\phi_2)\left[\Lambda(\phi_1) \begin{pmatrix} x \\ t \end{pmatrix}\right]$$

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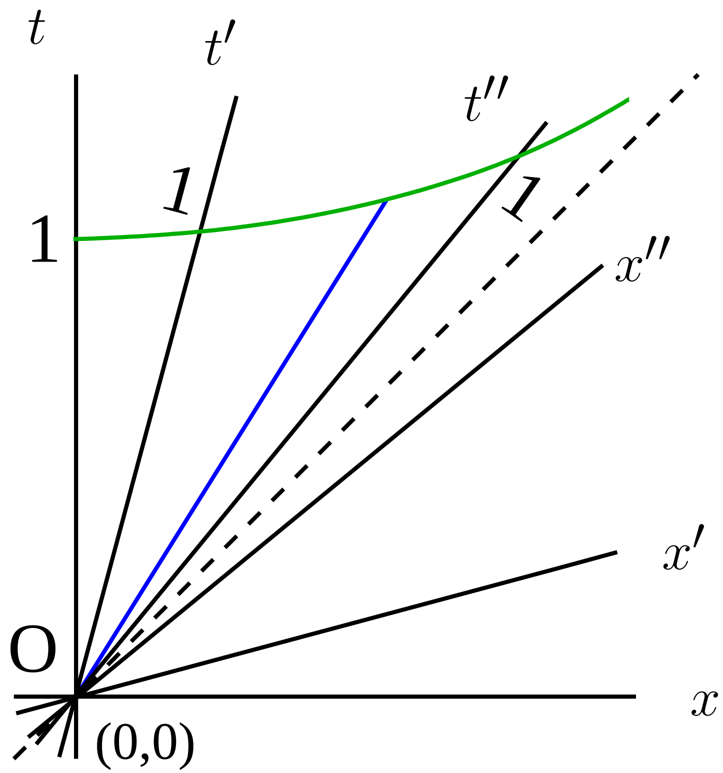

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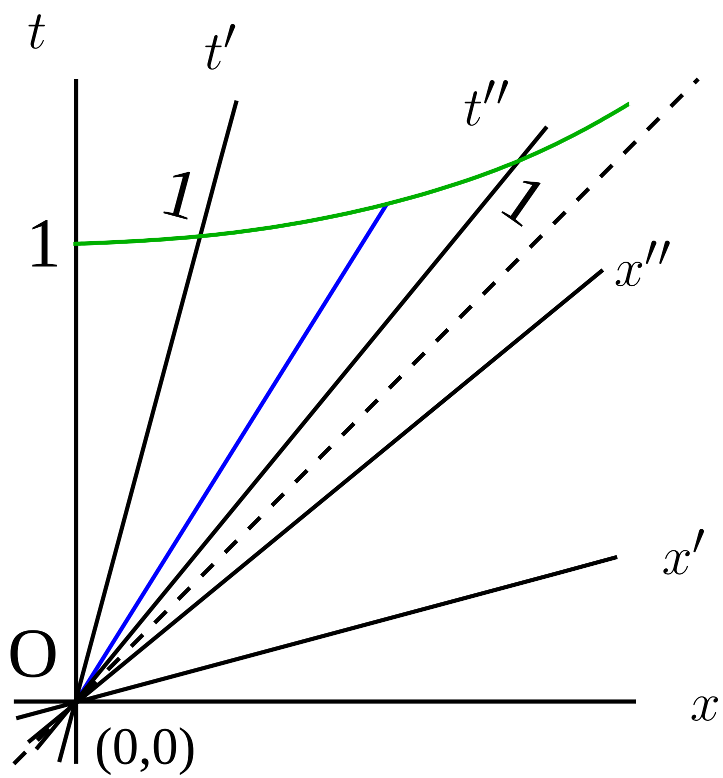

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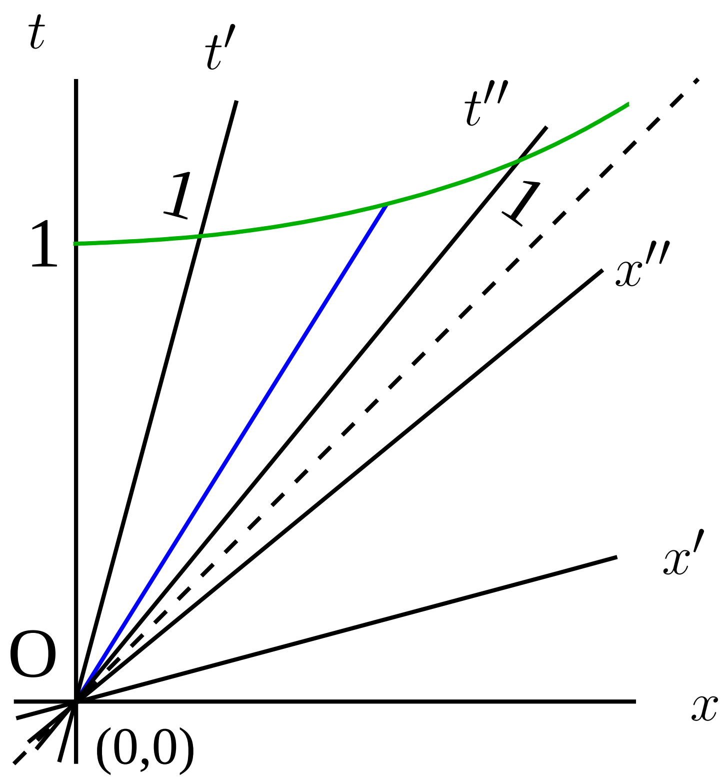

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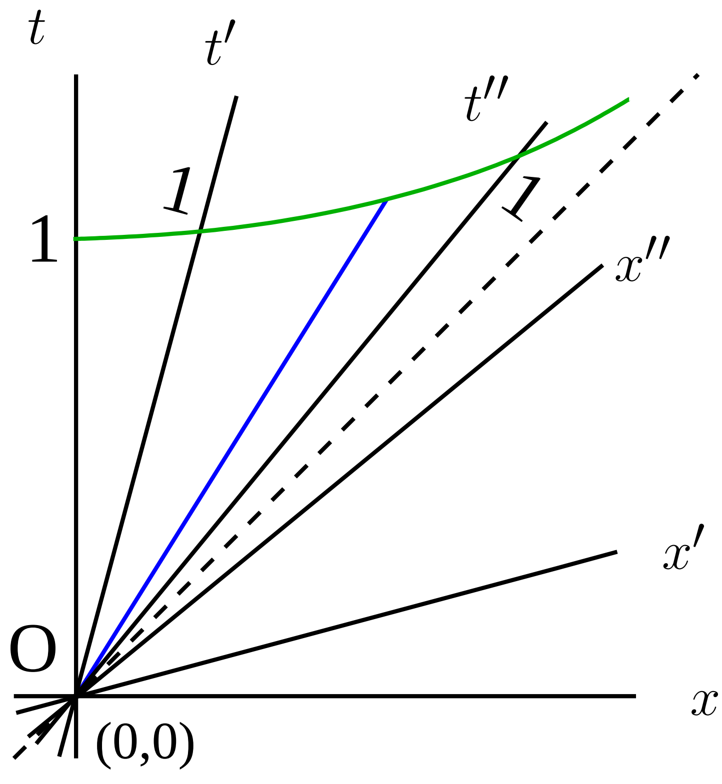
but $\Lambda(\phi_2)\Lambda(\phi_1) = \Lambda(\phi_1 + \phi_2)$ (property of this matrix group)
cf. rotation by θ_1 "plus" rotation by θ_2 = rotation by $(\theta_1 + \theta_2)$

SR: adding velocities

interstellar ark travels at $\beta_1 = 0.1$ from Sun, sends out rocket at $\beta_2 = 0.5$; rocket's speed β_3 in Sun frame =?



$$\begin{pmatrix} x'' \\ t'' \end{pmatrix} = [\Lambda(\phi_2)\Lambda(\phi_1)] \begin{pmatrix} x \\ t \end{pmatrix}$$

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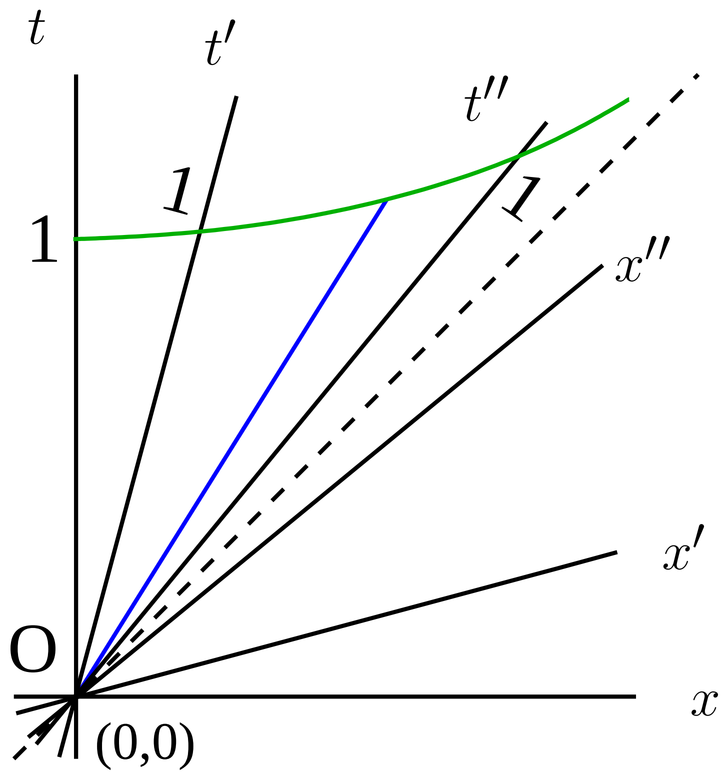

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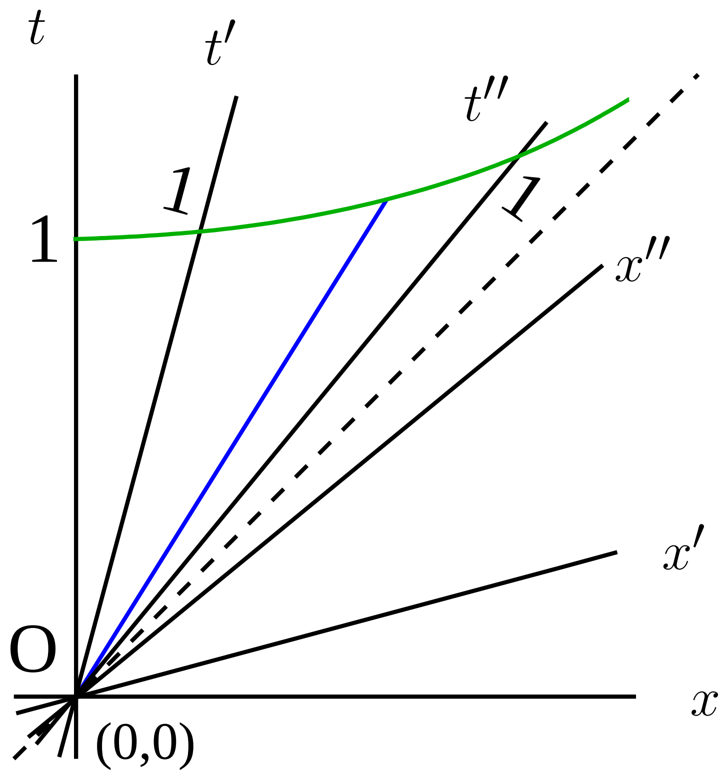

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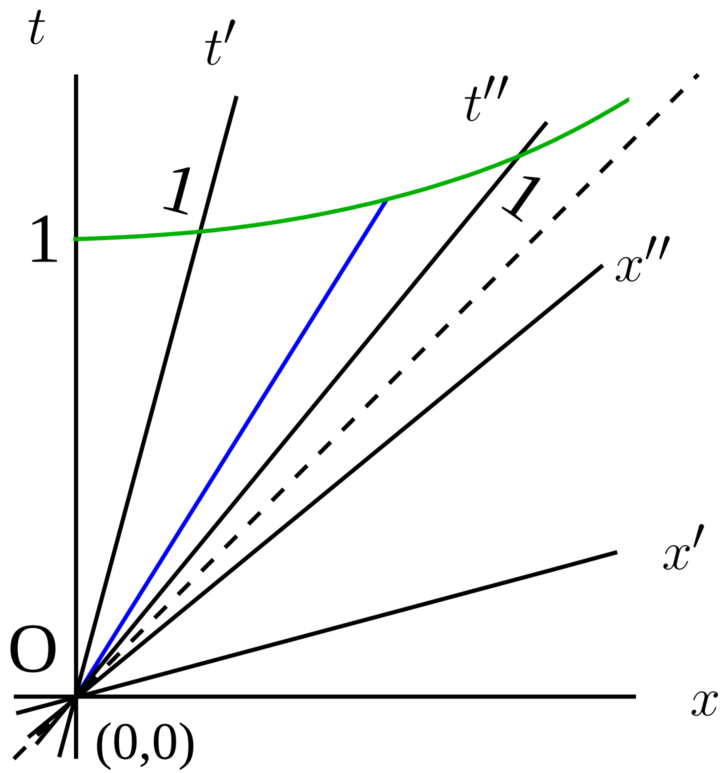

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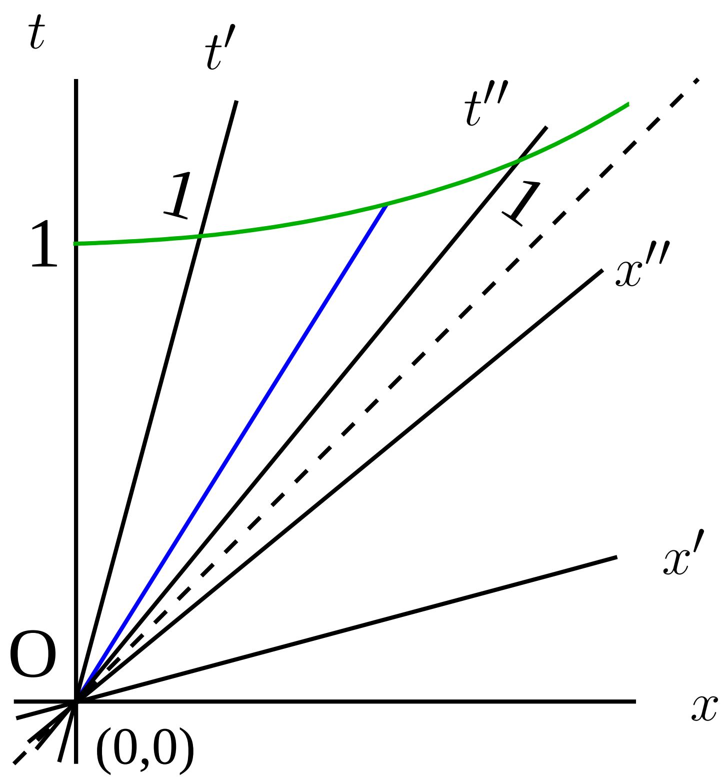

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SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

$$\Lambda(\phi) := \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix}$$

$\cosh\phi := \frac{e^{\phi}+e^{-\phi}}{2}$

$\sinh\phi := \frac{e^{\phi}-e^{-\phi}}{2}$

w:hyperbolic function

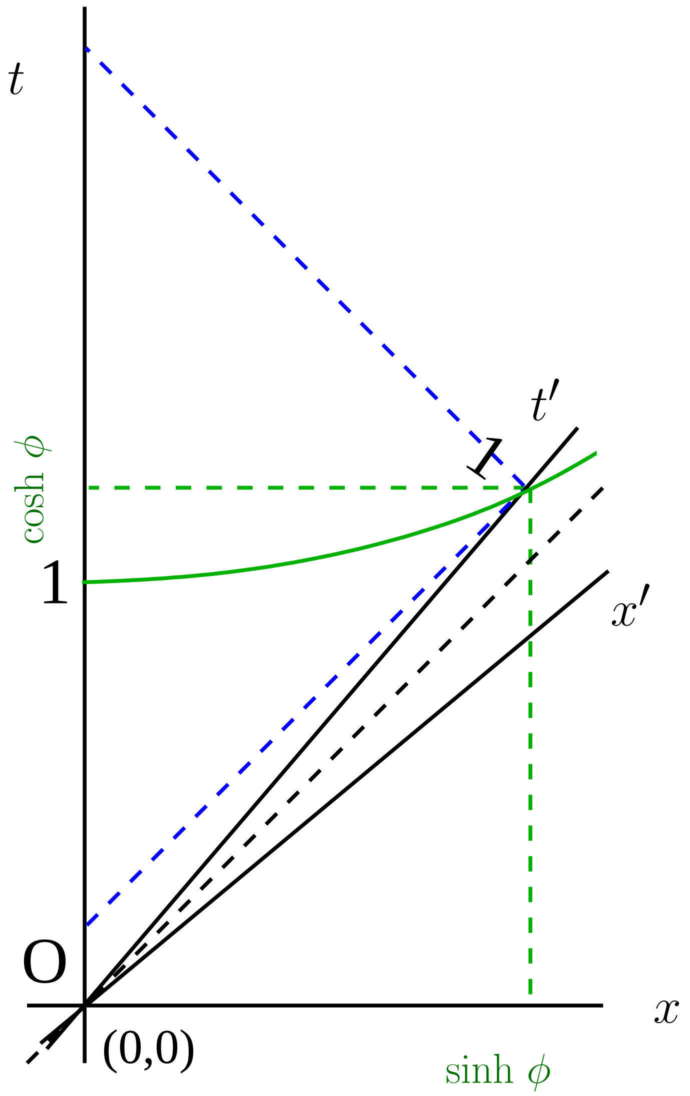
SR: Lorentz factor

Λ: alternative to hyperbolic trig functions

$$\beta = \tanh\phi$$
$$\gamma := (1-\beta^2)^{-1/2} = \text{Lorentz factor}$$
$$\gamma = \cosh\phi$$
$$\beta\gamma = \sinh\phi$$

$$\Lambda(\beta) := \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix}$$

SR: worldline time dilation



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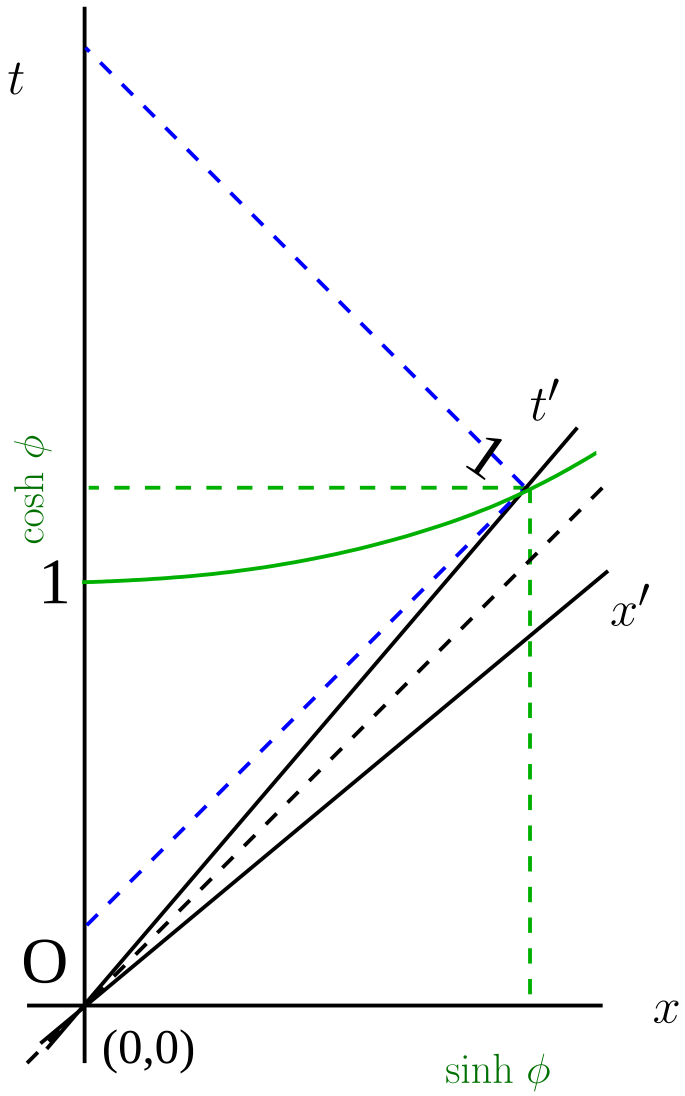

$$\cosh\phi \quad \equiv \quad \gamma$$

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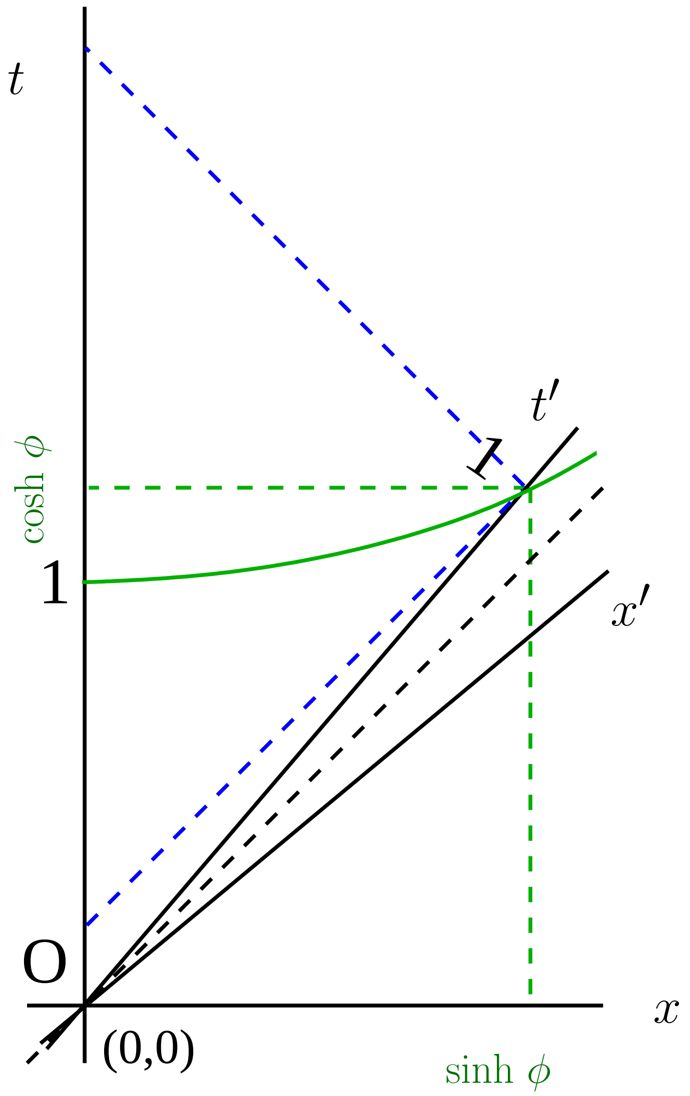

$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

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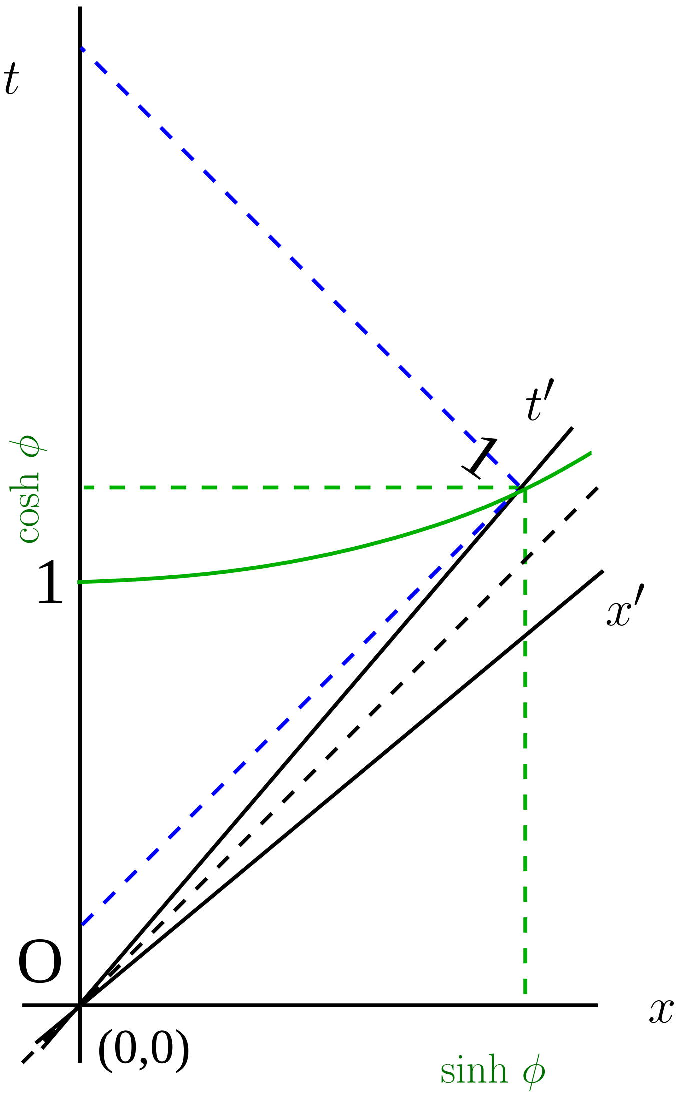

$$\cosh\phi \equiv \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} > 1$$

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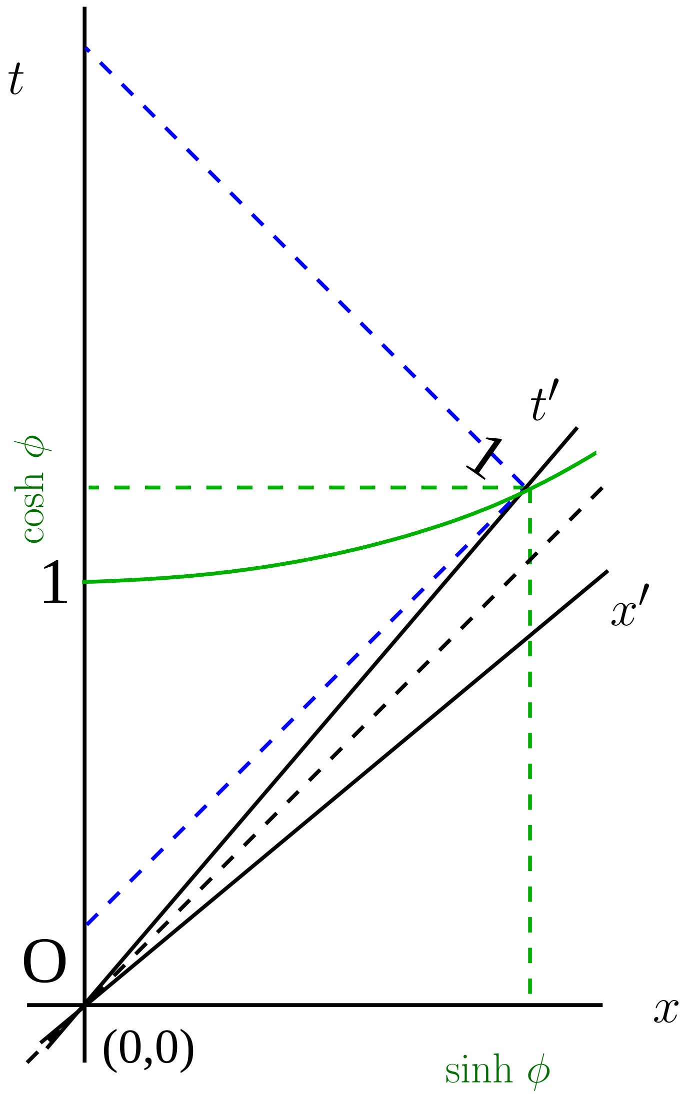

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worldline “time dilation"

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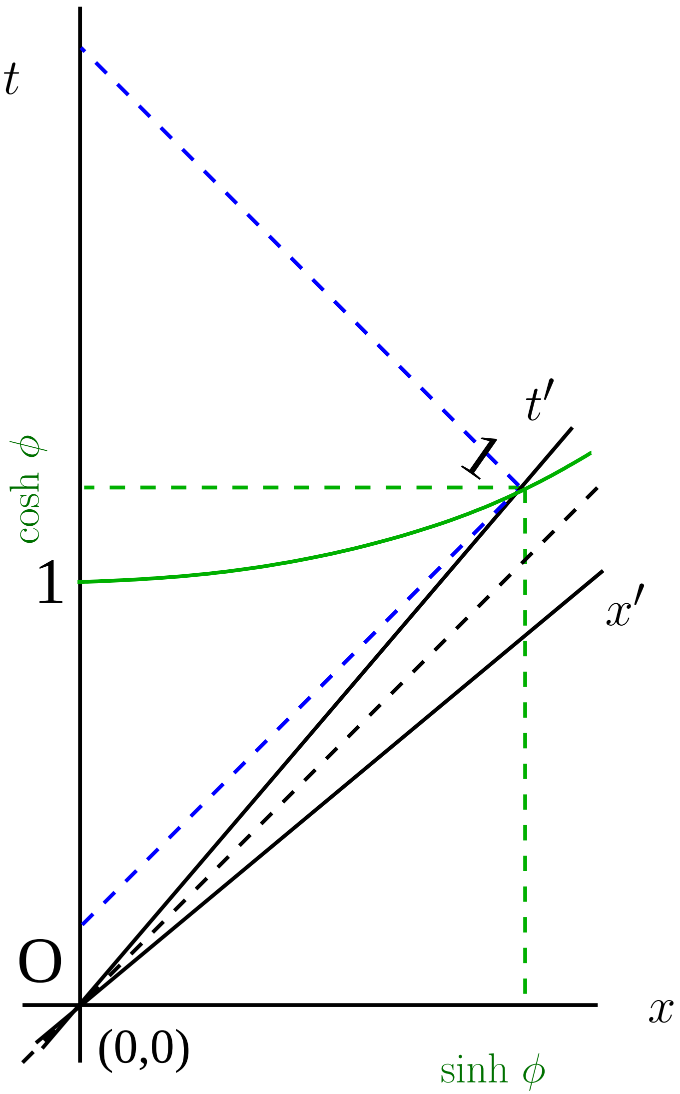

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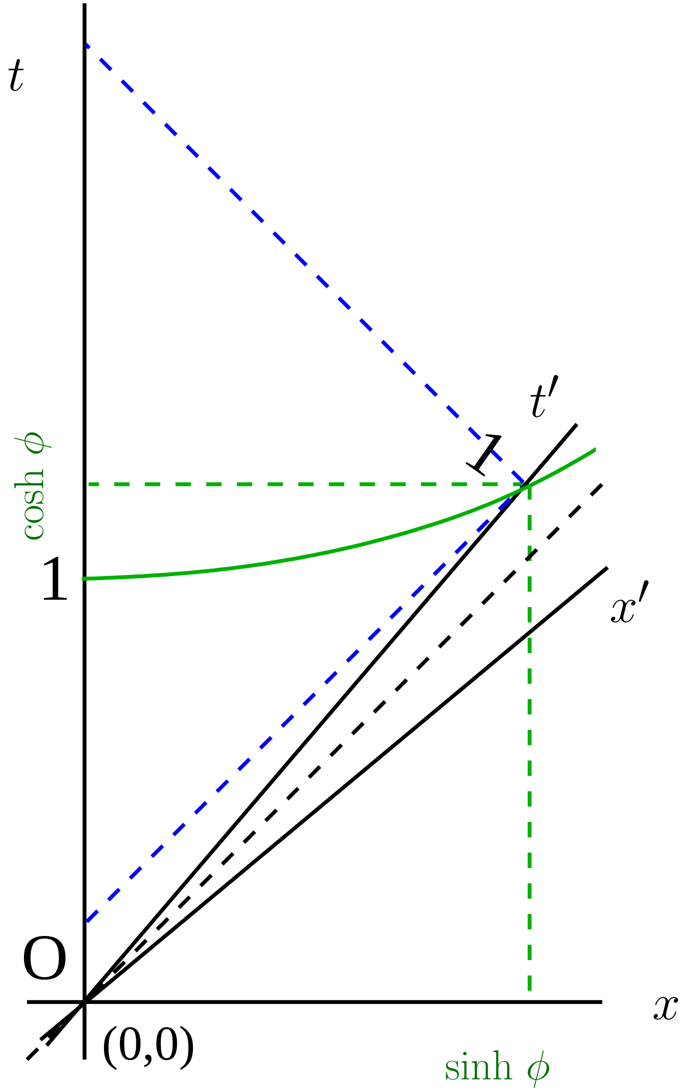
worldline “time dilation"

muons: mean lifetime 2197 ns $\ll 15$ km

SR: worldline time dilation



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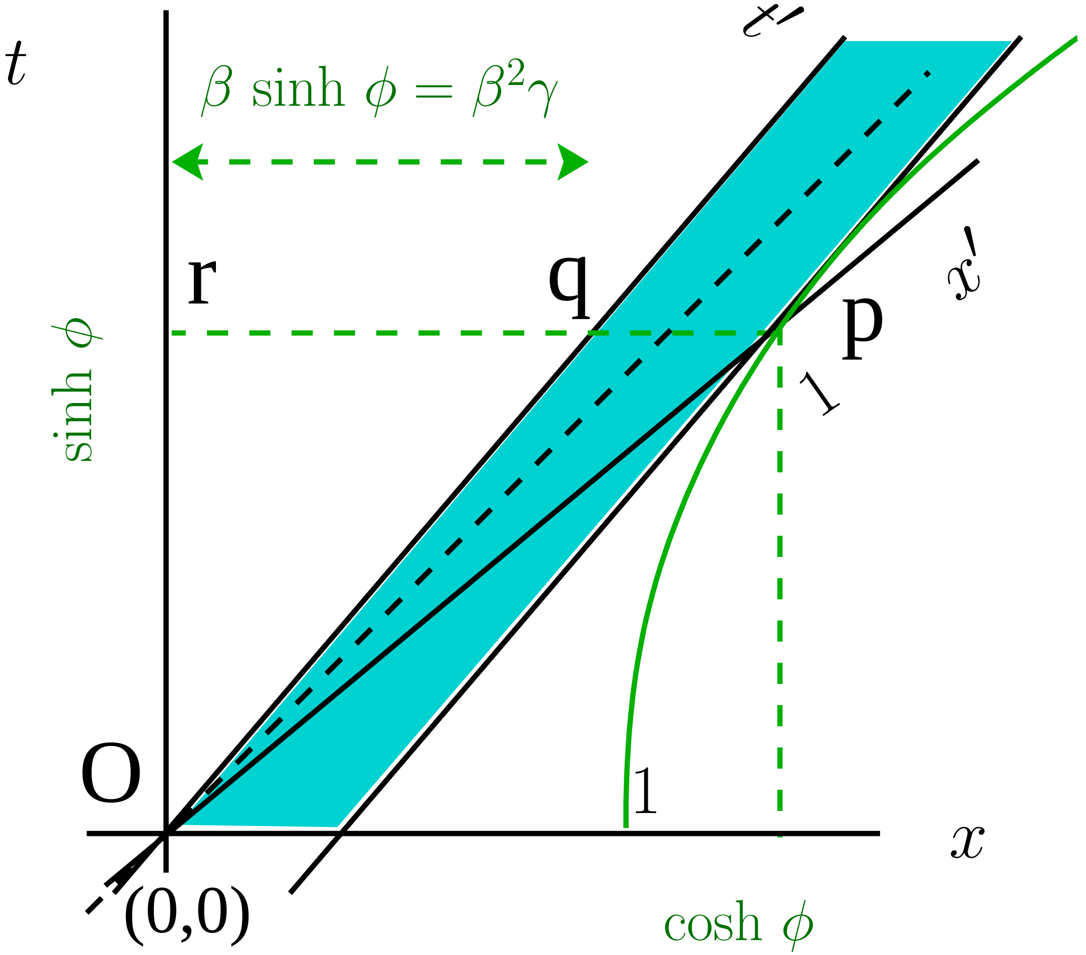
worldline “time dilation"

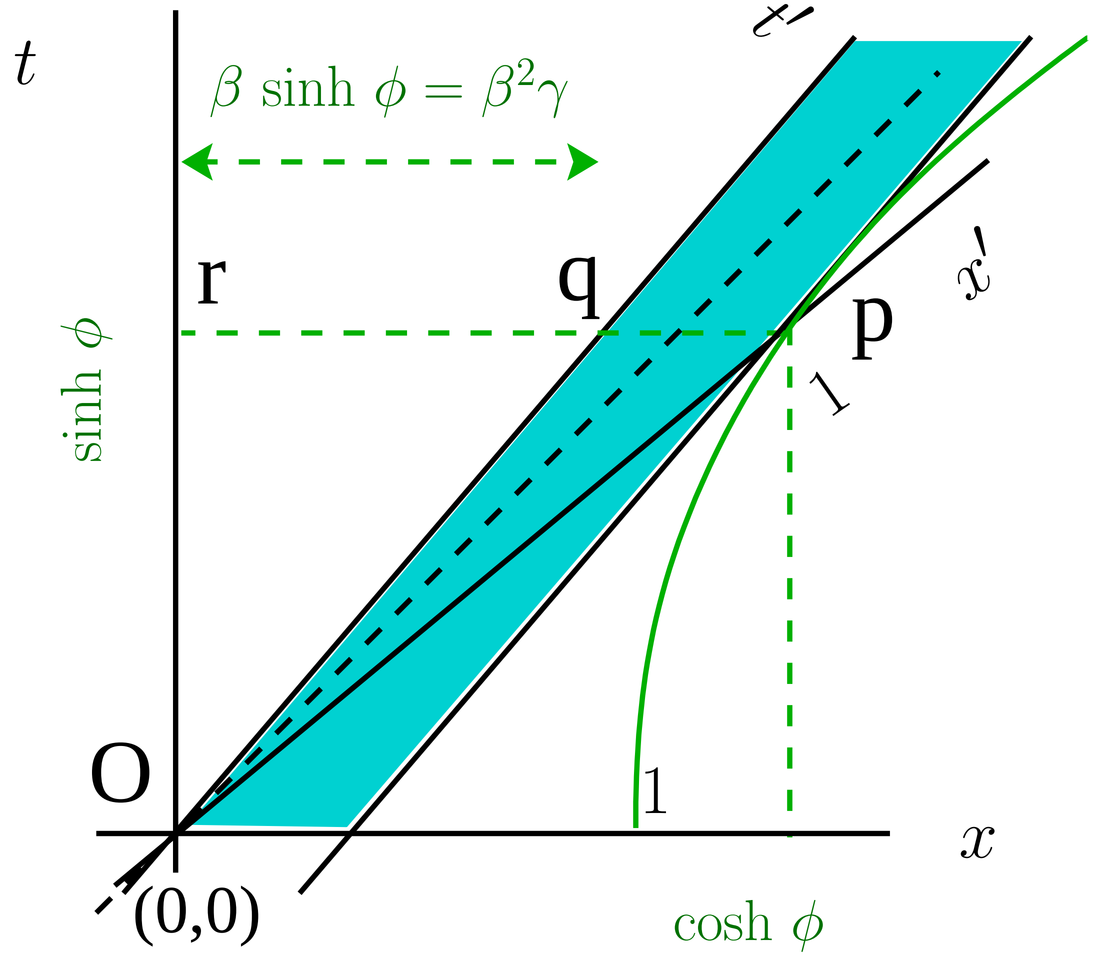
muons: mean lifetime 2197 ns $\ll 15$ km

time dilation $\Rightarrow$ muons can hit the ground

SR: worldsheet space contraction

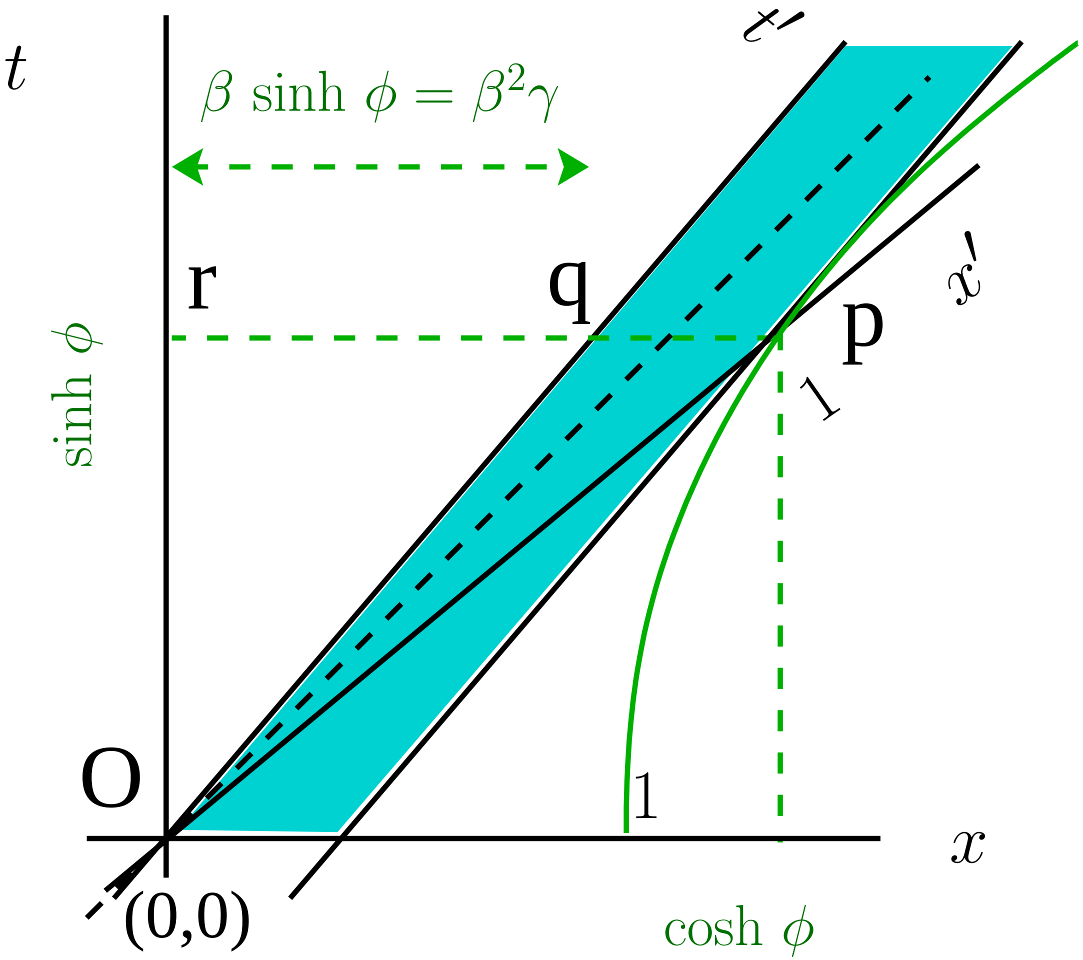


SR: worldsheet space contraction



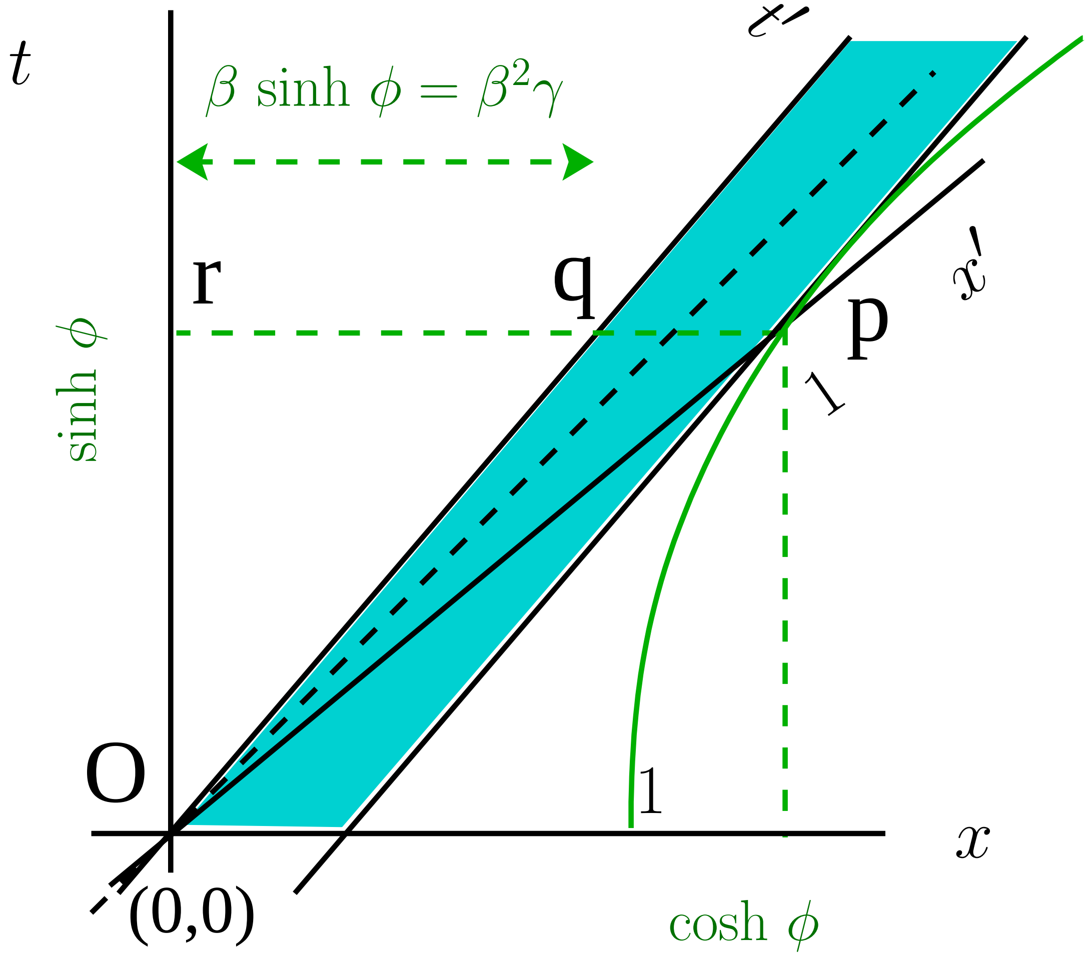
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} =$$

SR: worldsheet space contraction



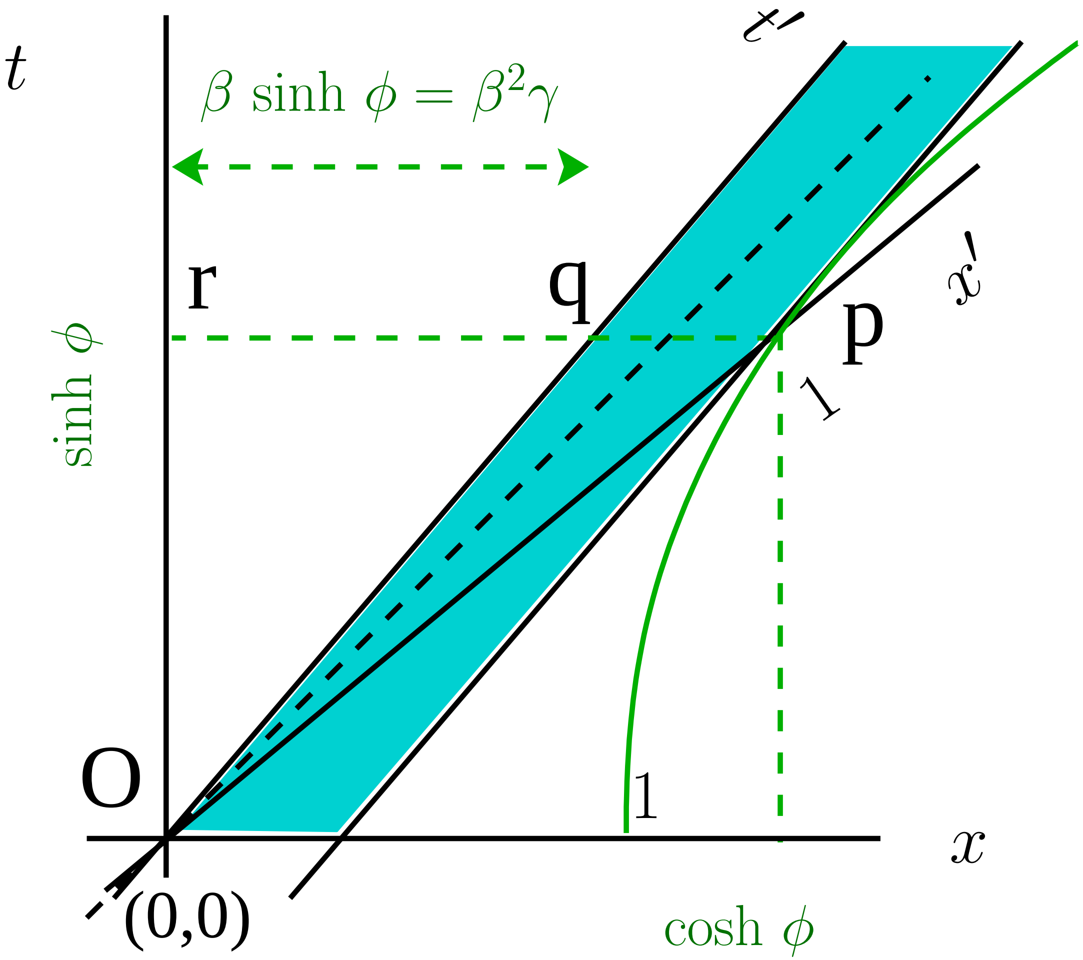
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \cosh\phi - \beta\sinh\phi$$

SR: worldsheet space contraction



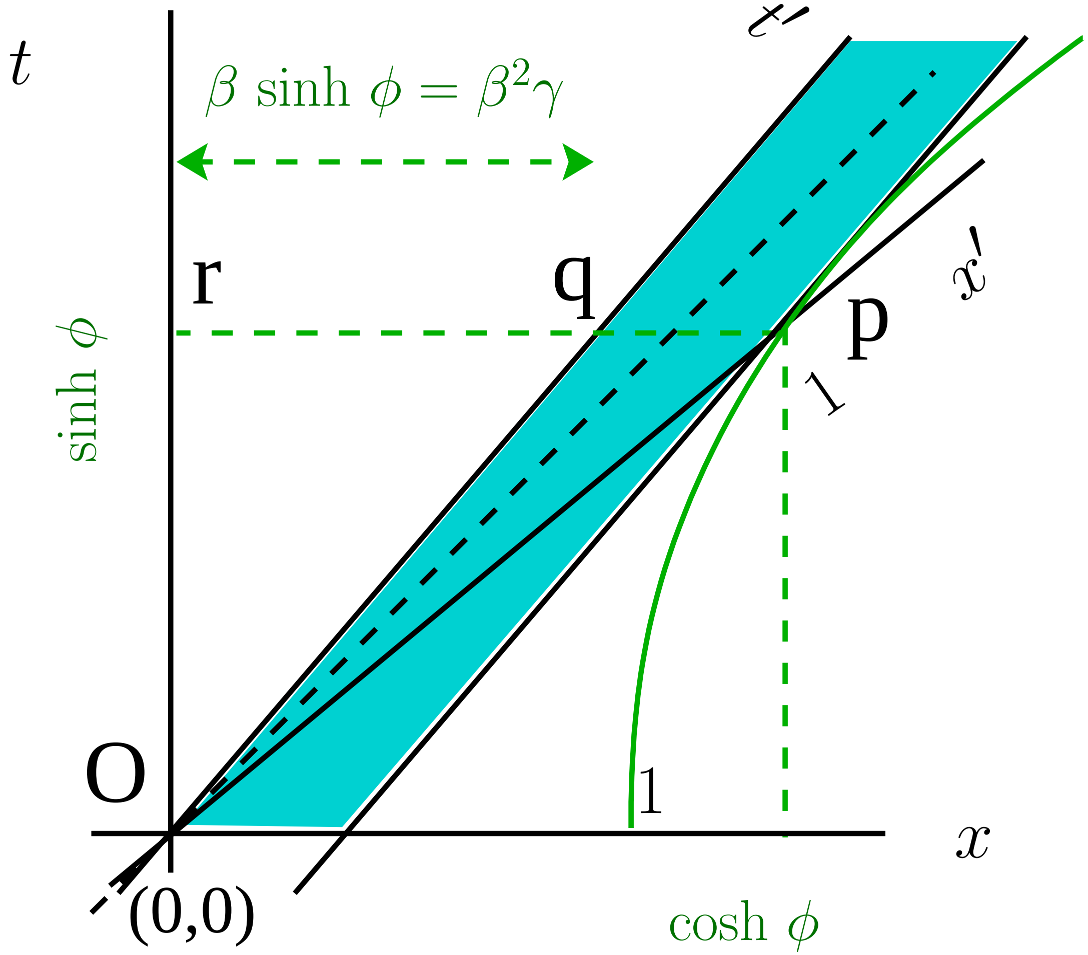
$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma - \beta\beta\gamma$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)\gamma$$

SR: worldsheet space contraction



$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = (1-\beta^2)^{1+(-1/2)}$$

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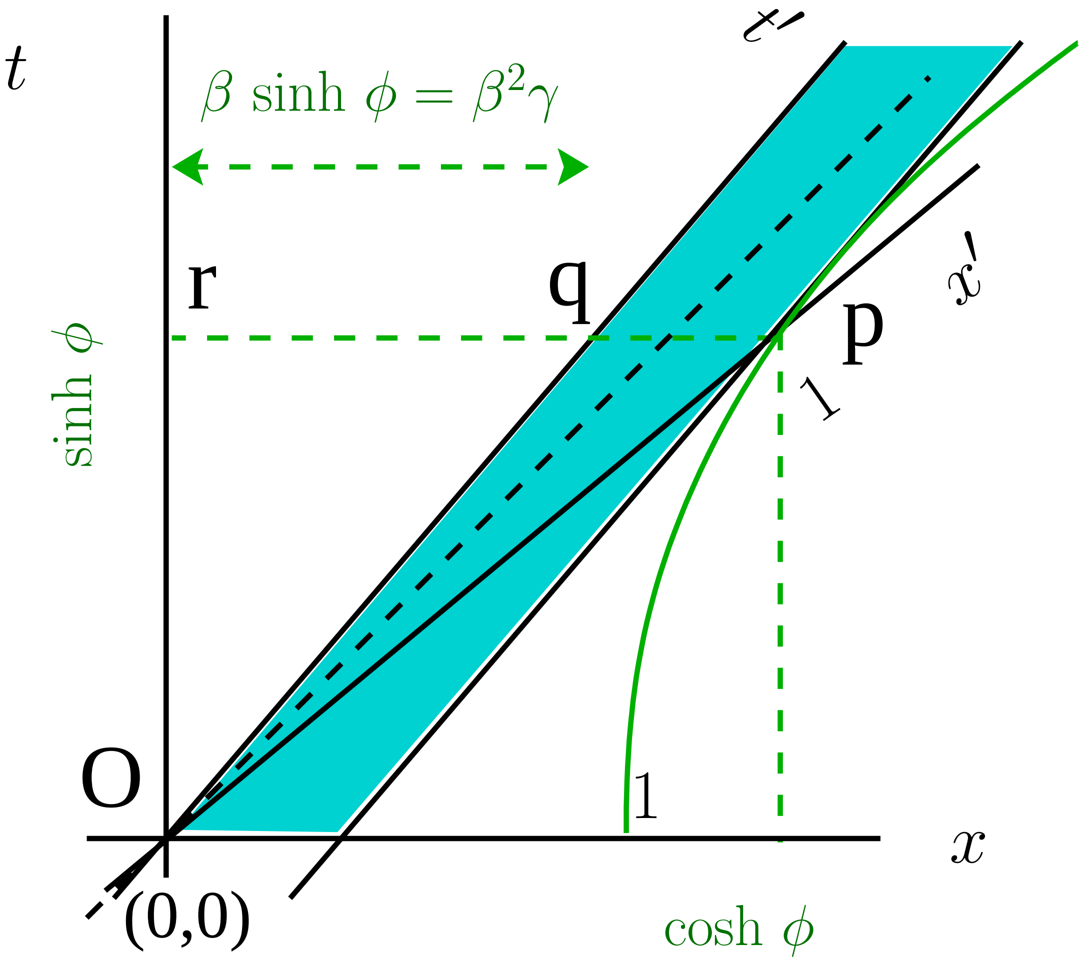

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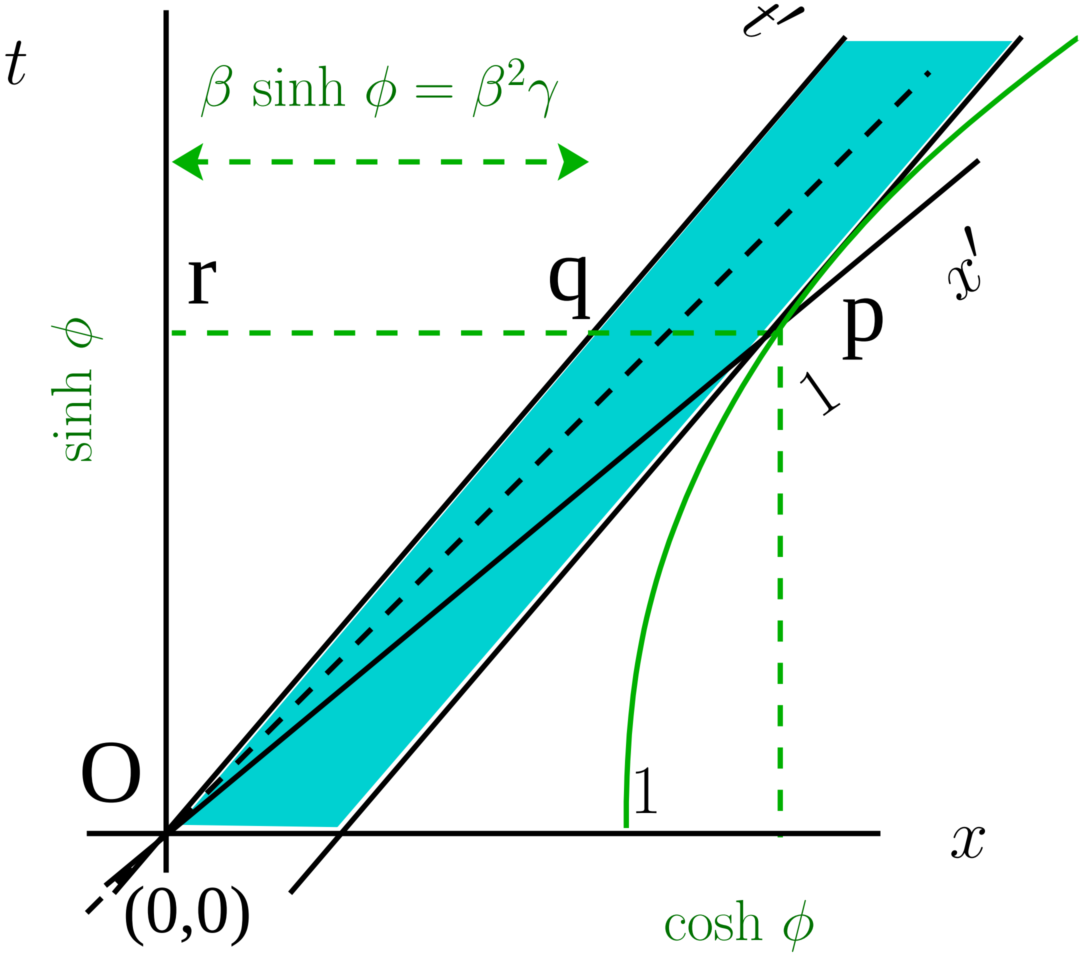
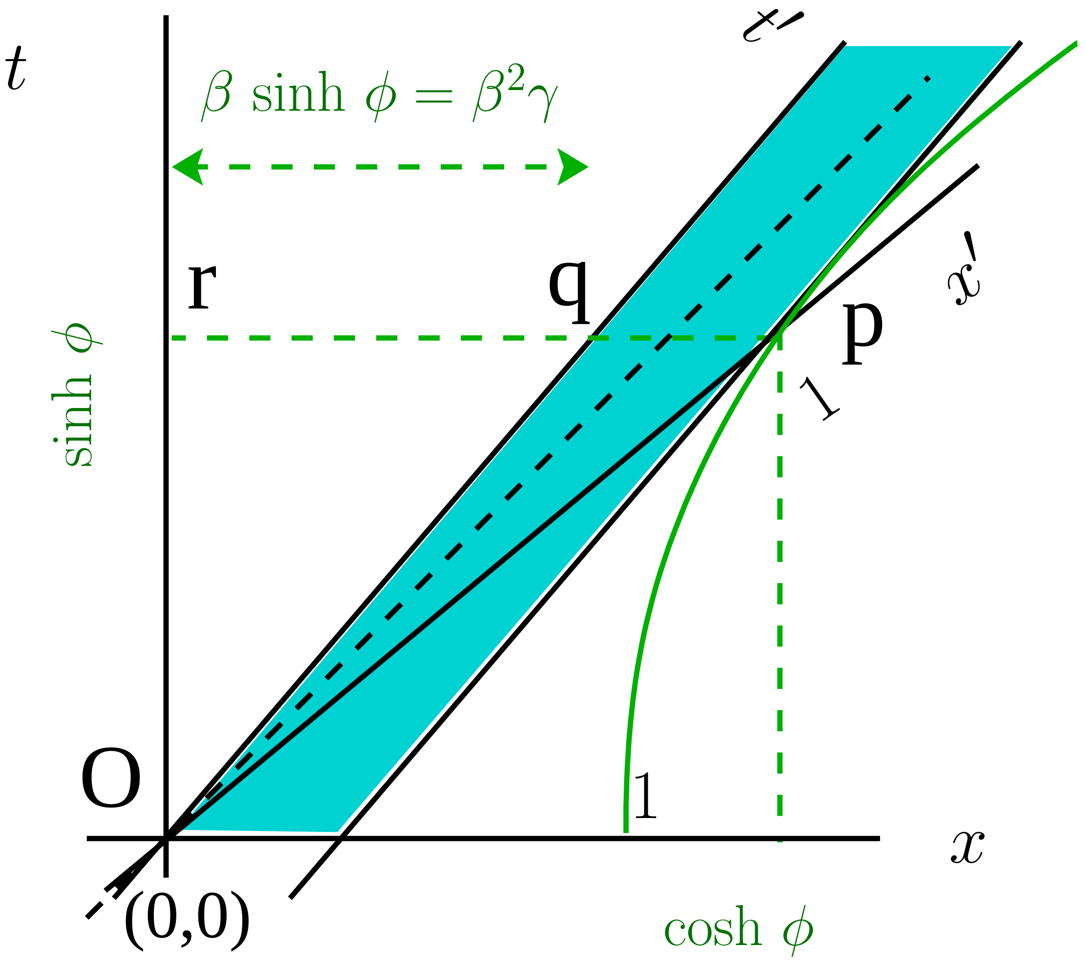


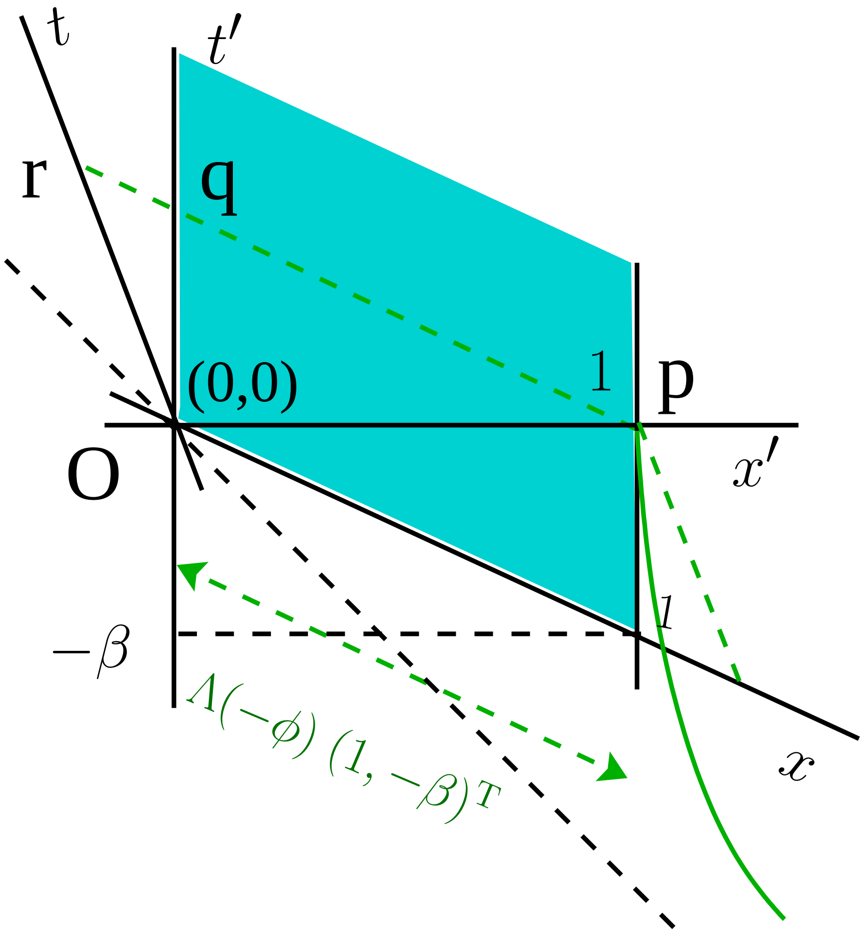
$$\sqrt{\Delta s^2(\mathrm{q}, \mathrm{p})} = \gamma^{-1} < 1$$

SR: worldsheet space contraction

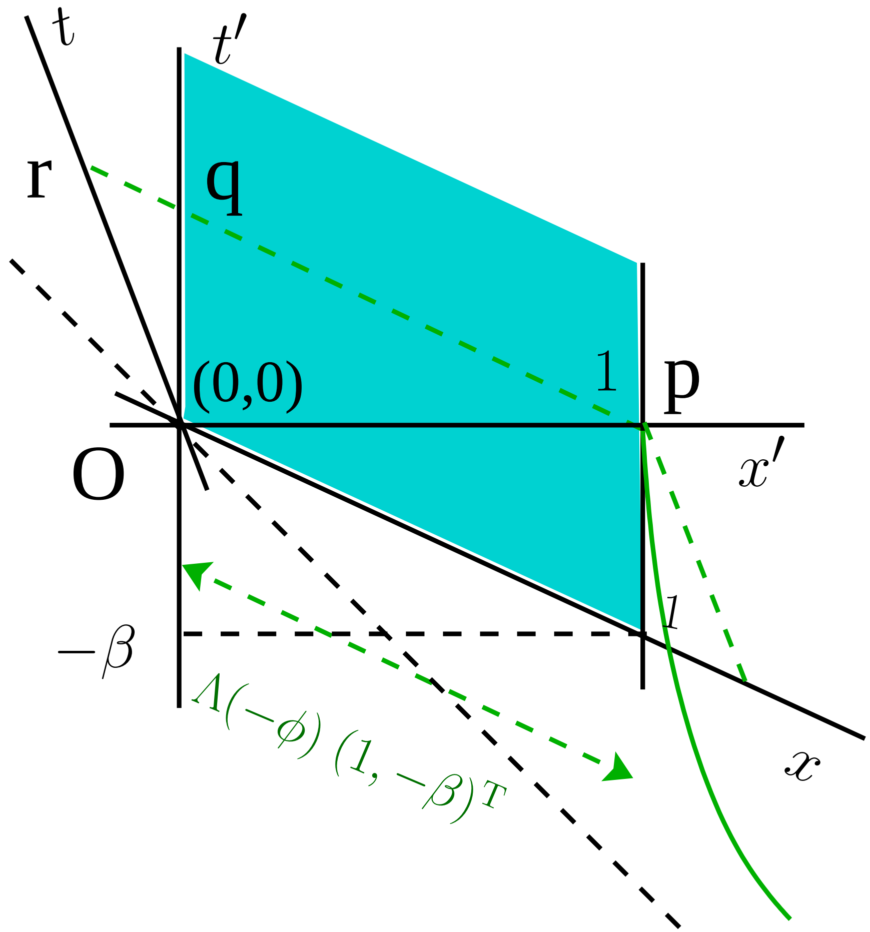


$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$ worldsheet "space contraction"

SR: worldsheet space contraction



SR: worldsheet space contraction



$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\cosh\phi-\beta\sinh\phi\\ \sinh\phi-\beta\cosh\phi\end{pmatrix}$$

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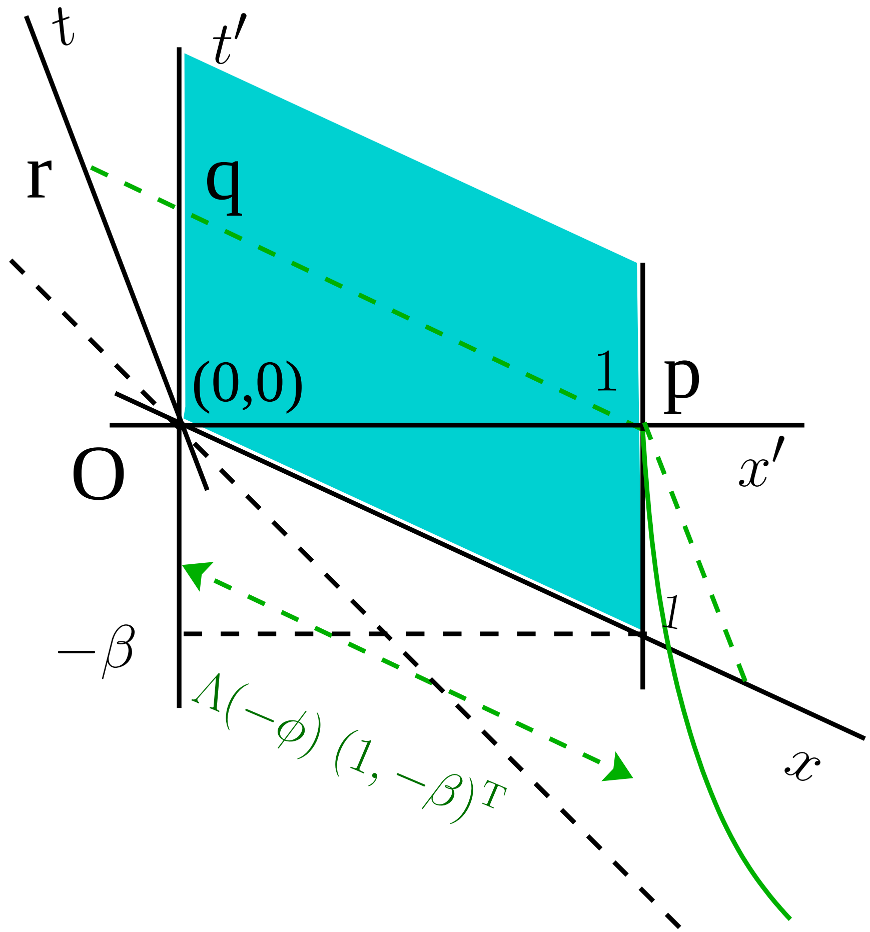

$$\Lambda^{-1}\begin{pmatrix}1\\-\beta\end{pmatrix}=\begin{pmatrix}\cosh\phi(1-\beta^2)\\ \cosh\phi(\tanh\phi-\tanh\phi)\end{pmatrix}$$

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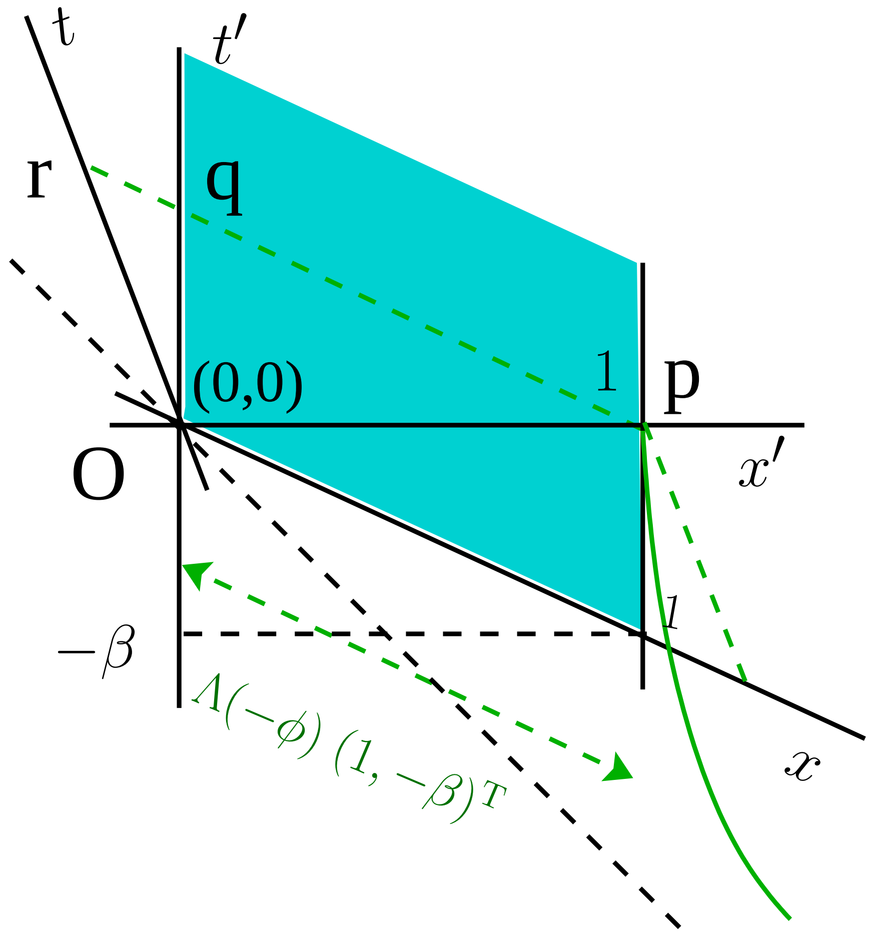

$$\Lambda^{-1} \begin{pmatrix} 1 \\ -\beta \end{pmatrix} = \begin{pmatrix} \gamma(1-\beta^2) \\ 0 \end{pmatrix}$$

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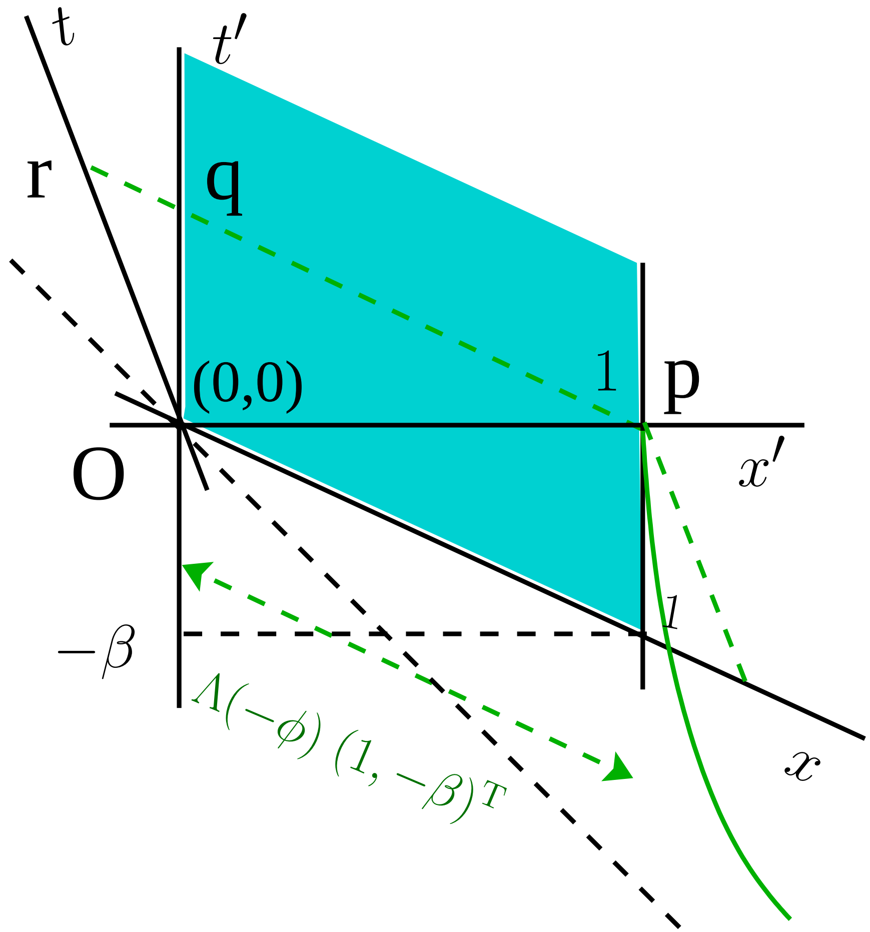

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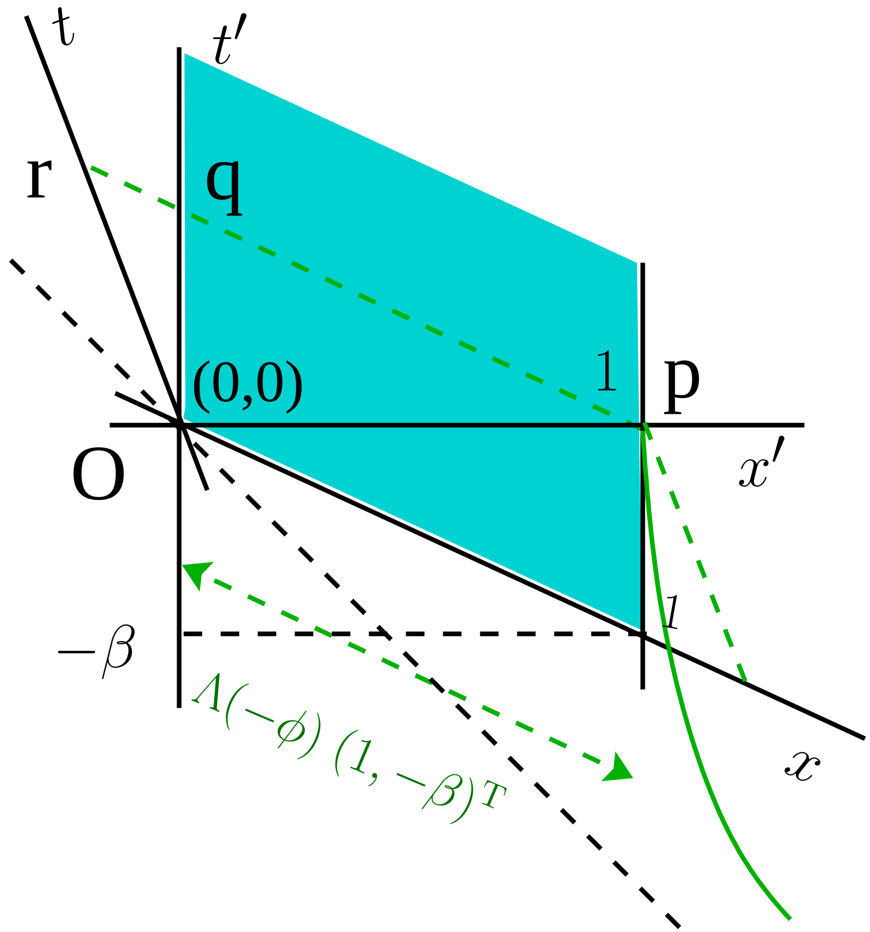

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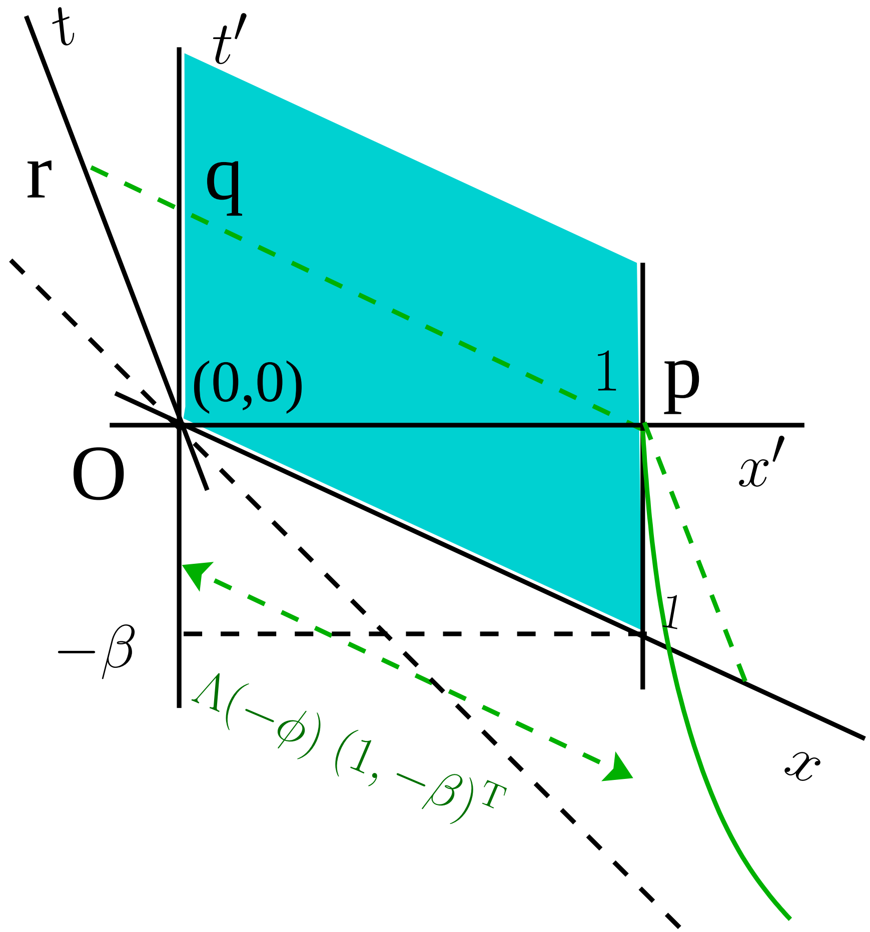
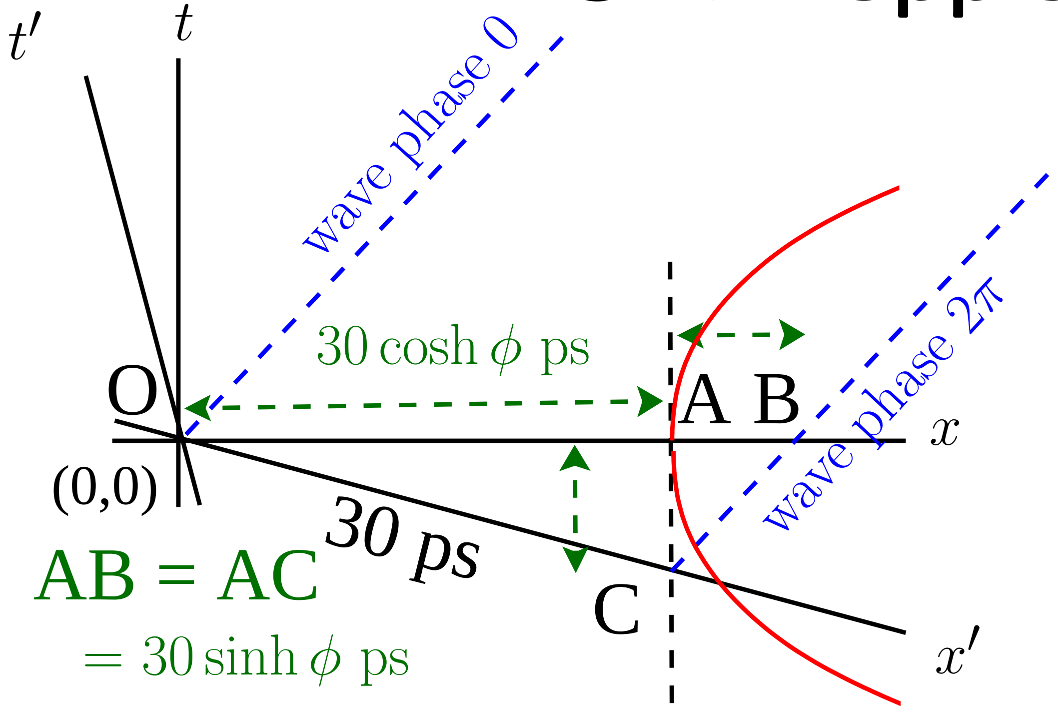


$$\sqrt{\Delta s^2(\mathrm{q},\mathrm{p})} = \gamma^{-1} < 1$$ worldsheet "space contraction"

SR: Doppler shift



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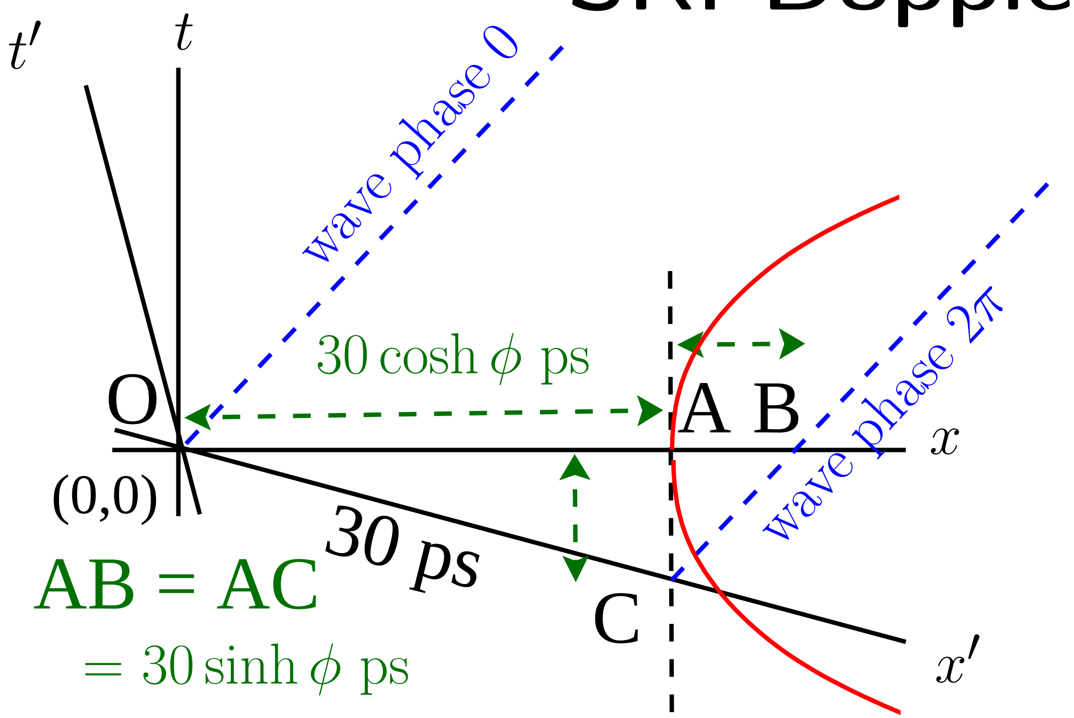
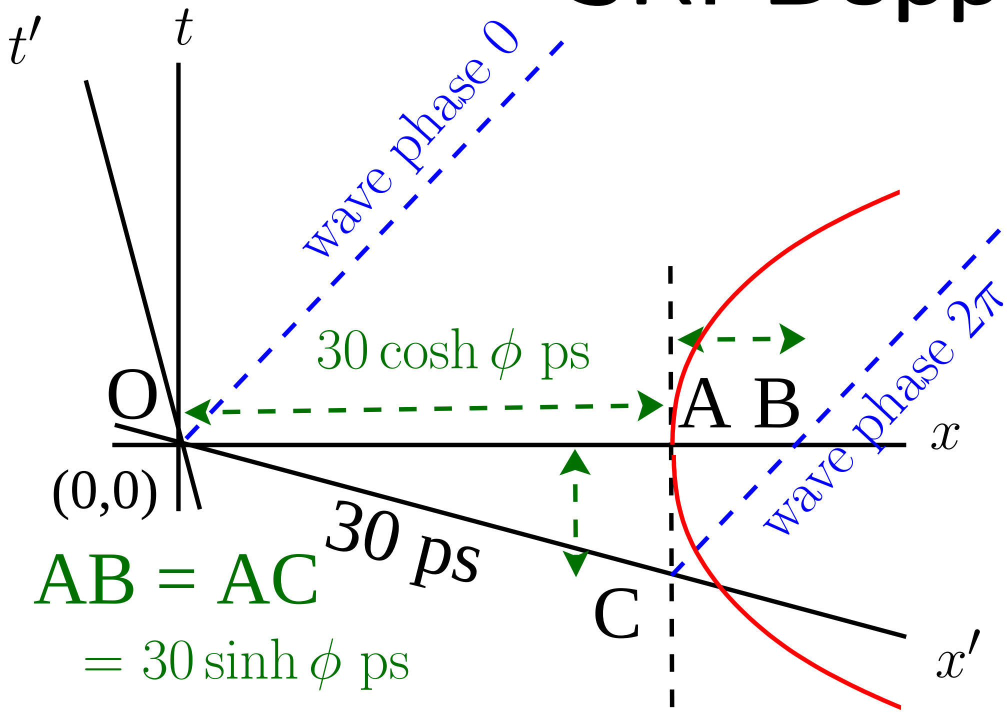


receding galaxy (x', t') frame: $\lambda_{\rm em} =$ OC $= 30$ ps

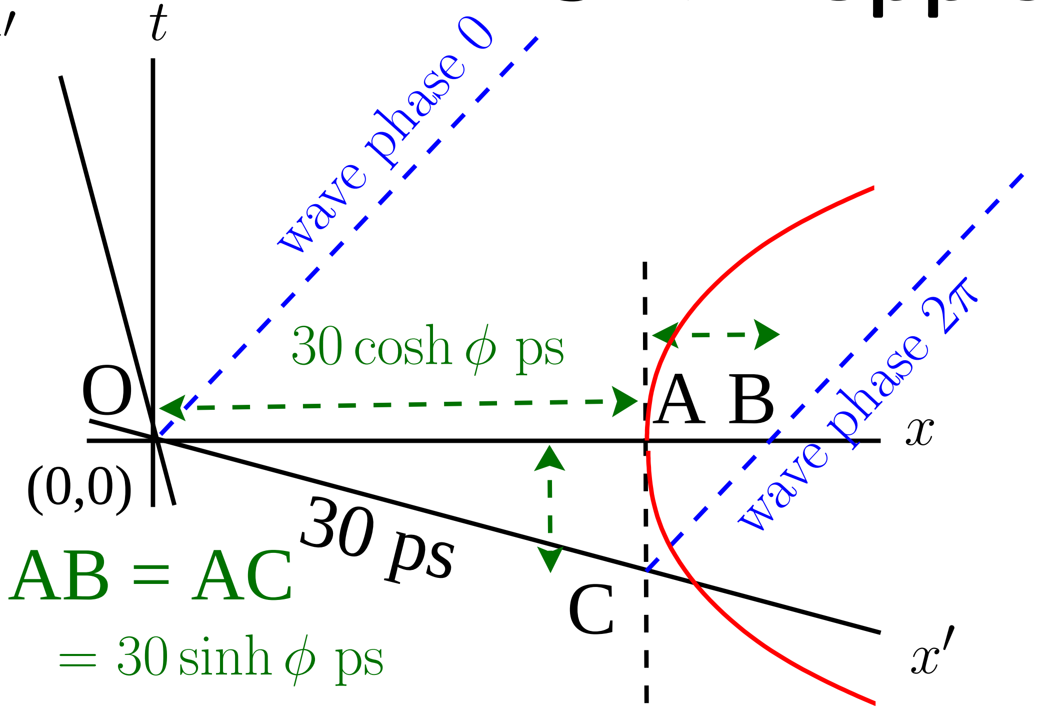
SR: Doppler shift



receding galaxy (x', t') frame: $\lambda_{\rm em} = \text{OC} = 30$ ps

Sun (x, t) frame: $\lambda_{\rm obs} = \text{OB}$,

SR: Doppler shift



receding galaxy (x', t') frame: $\lambda_{\rm em} =$ OC $= 30$ ps

Sun (x, t) frame: $\lambda_{\rm obs} =$ OB, OA $= 30\cosh\phi$ ps,

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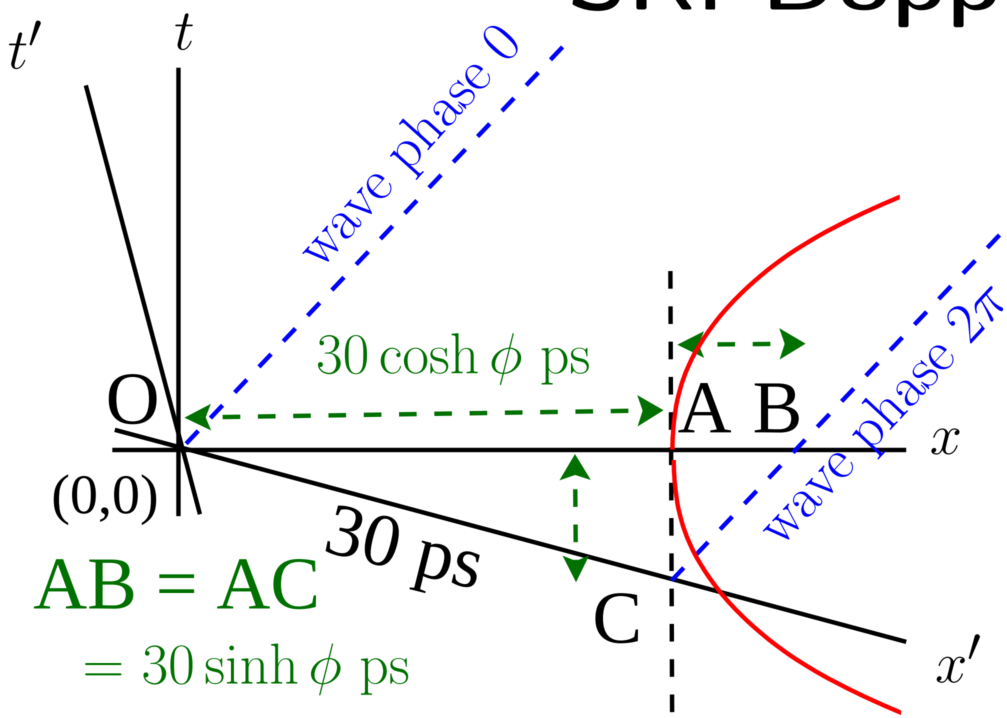

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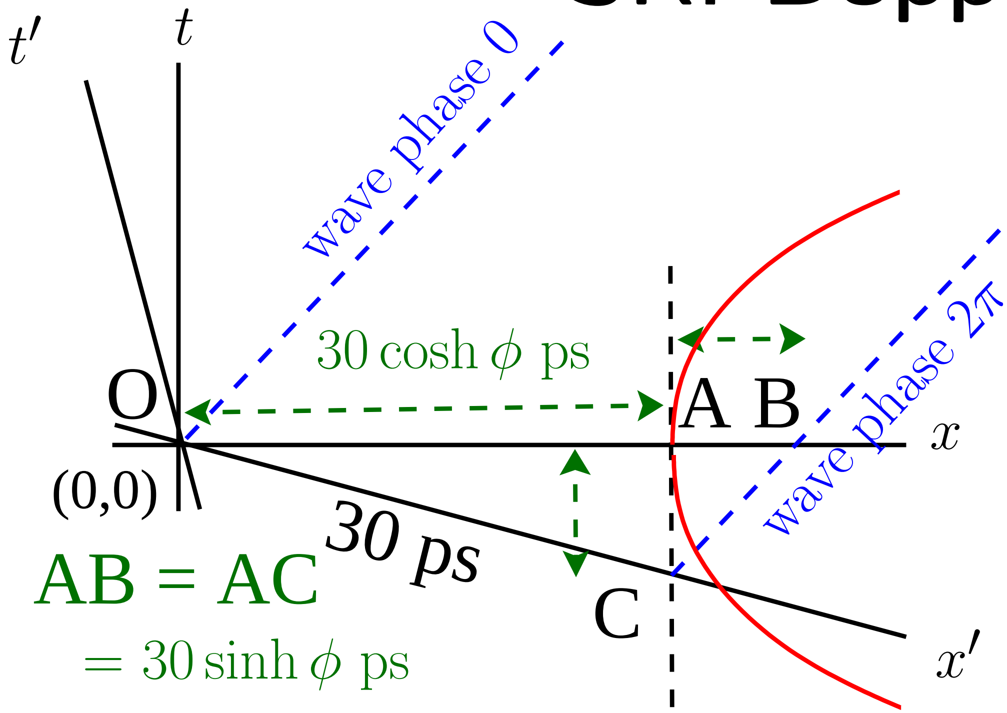

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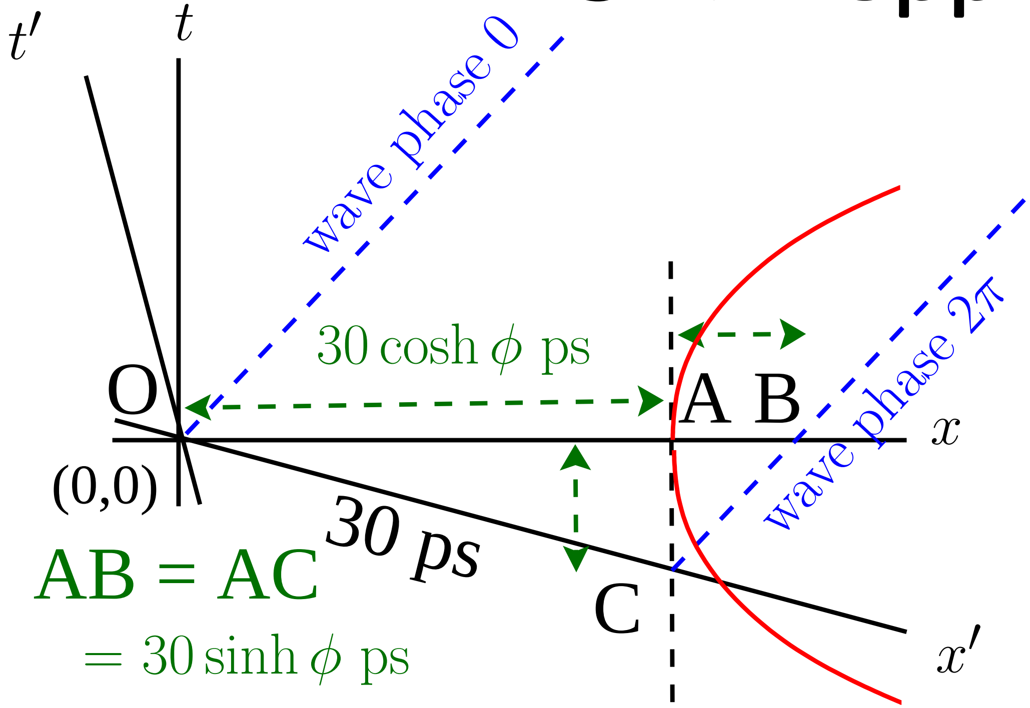
$$1 + z := \lambda_{\rm obs}/\lambda_{\rm em} = \text{OB/OC} = \cosh\phi + \sinh\phi$$

SR: Doppler shift



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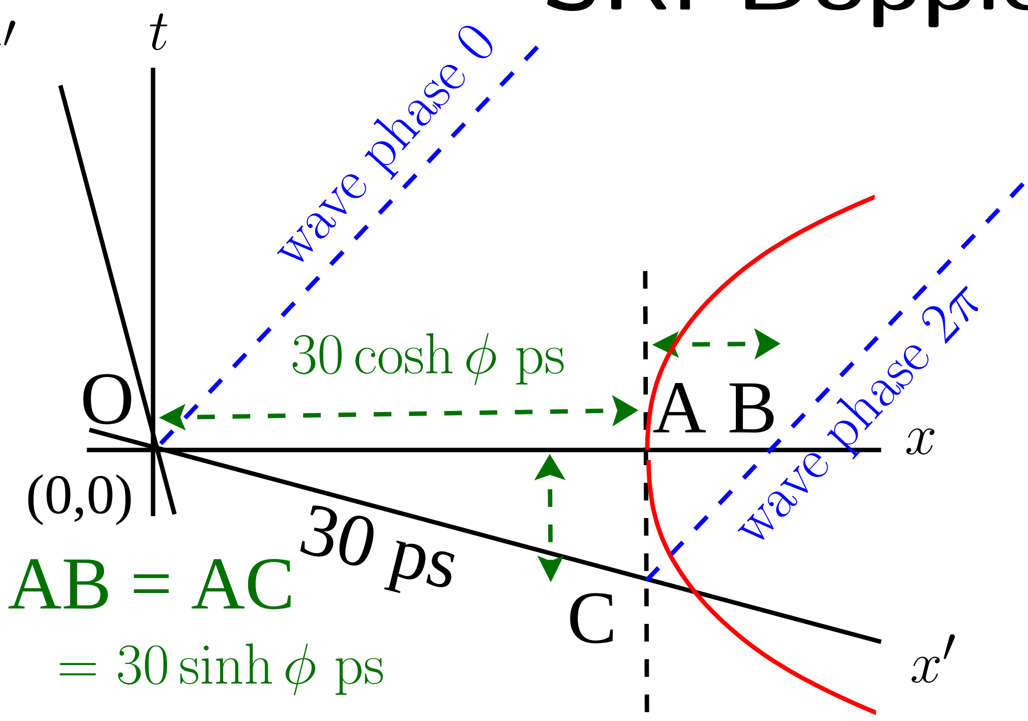

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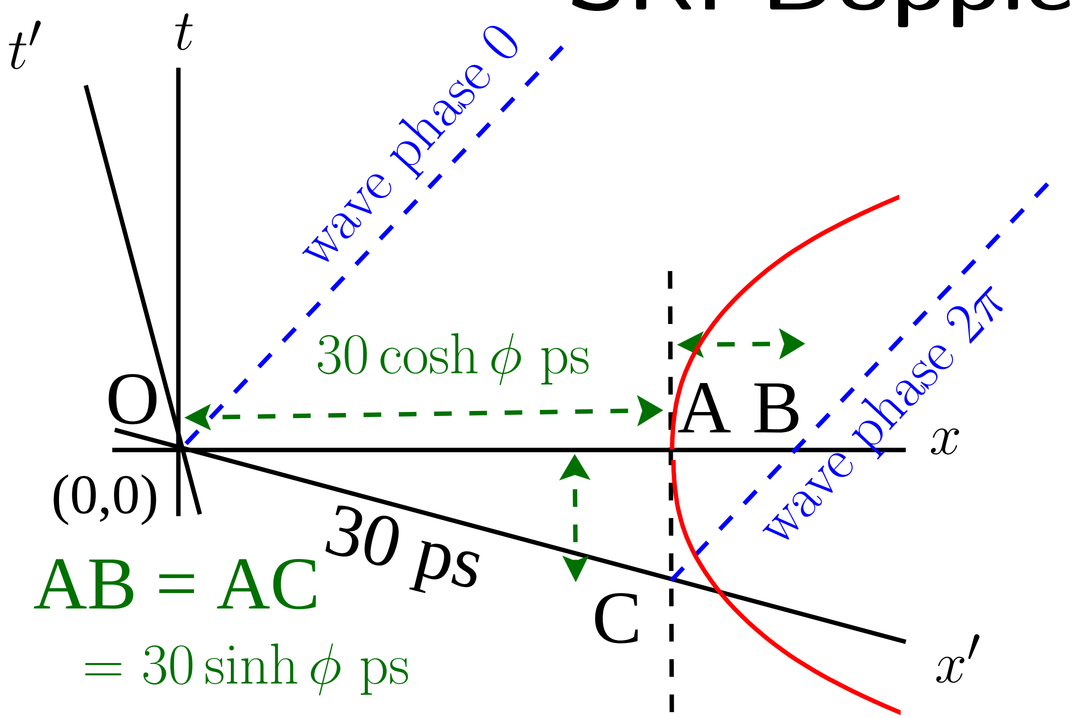

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redshift $\boxed{1 + z = {\rm e}^{\phi} = \sqrt{\frac{1+\beta}{1-\beta}}}$

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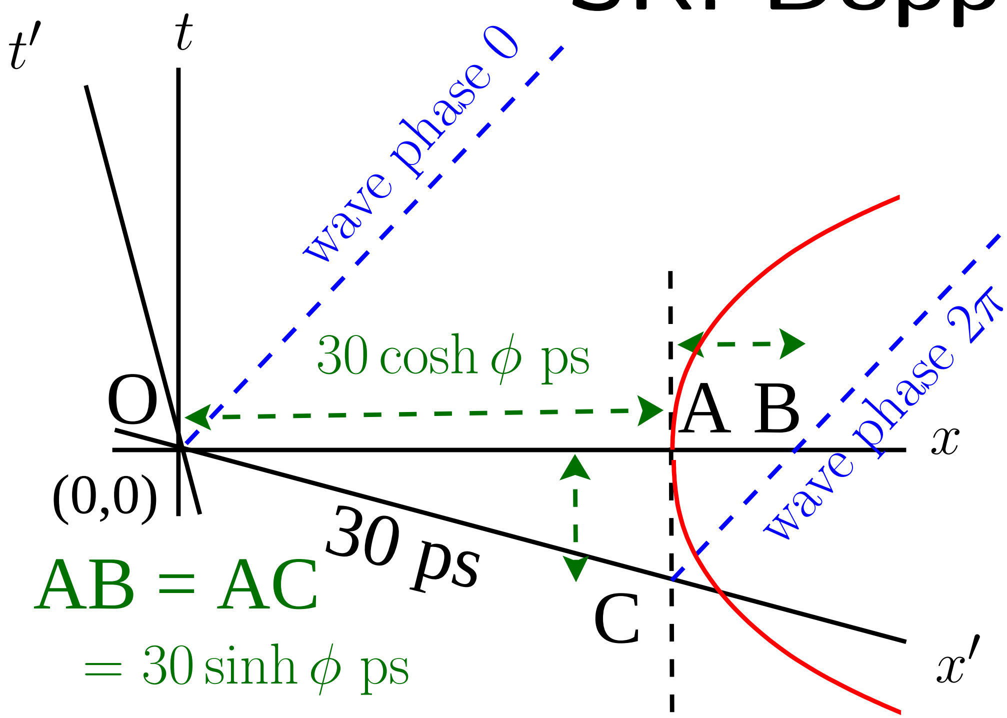

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Sun (x, t) frame: $\lambda_{\mathrm{obs}} = \mathrm{OB}$, $\quad \mathrm{OA} = 30 \cosh\phi$ ps,

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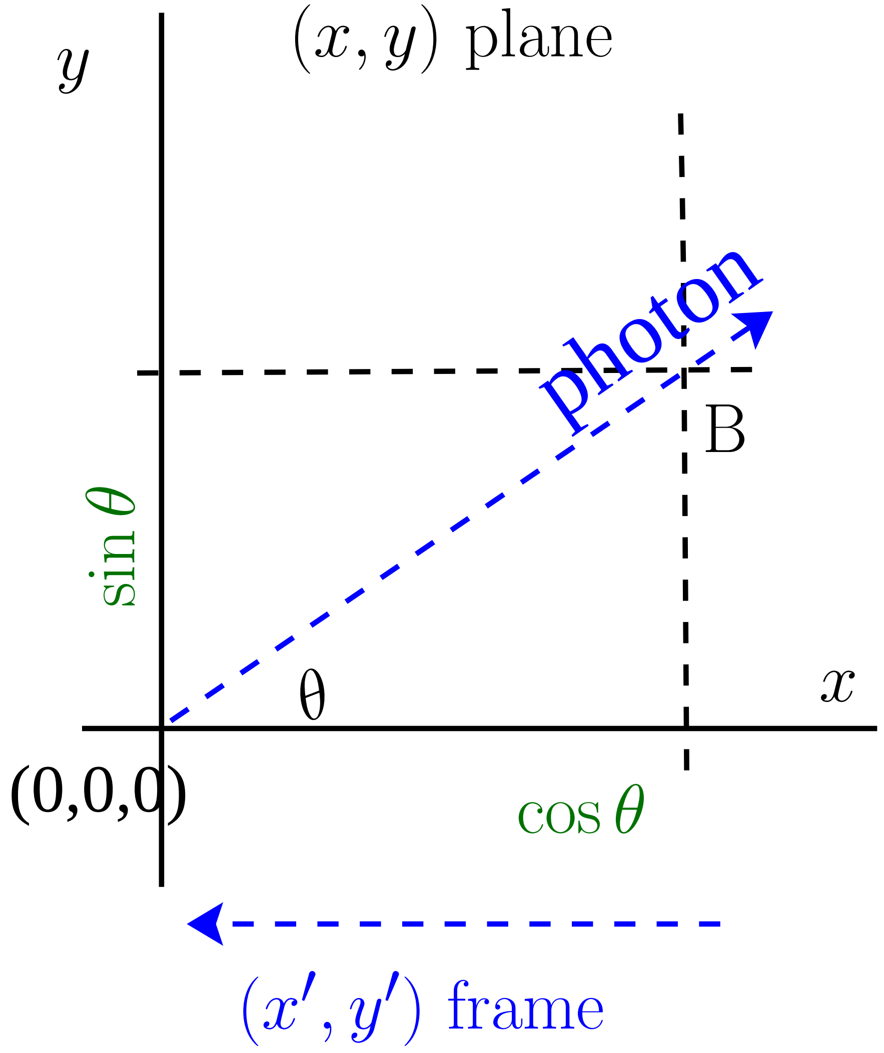
$$1 + z := \lambda_{\mathrm{obs}}/\lambda_{\mathrm{em}} = \mathrm{OB}/\mathrm{OC} = \cosh\phi + \sinh\phi = \mathrm{e}^{\phi} = \gamma + \beta\gamma = \sqrt{\tfrac{1+\beta}{1-\beta}}$$

redshift $\boxed{1 + z = \mathrm{e}^{\phi} = \sqrt{\tfrac{1+\beta}{1-\beta}}}$

$\Rightarrow$ when $\phi \ll 1$, $z \approx \phi \approx \beta$ (w:Taylor series)

SR: relativistic aberration



SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

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$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma & 0 & \beta\gamma \\ 0 & 1 & 0 \\ \beta\gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma\cos\theta + \beta\gamma \\ \sin\theta \\ \beta\gamma\cos\theta + \gamma \end{pmatrix}$$

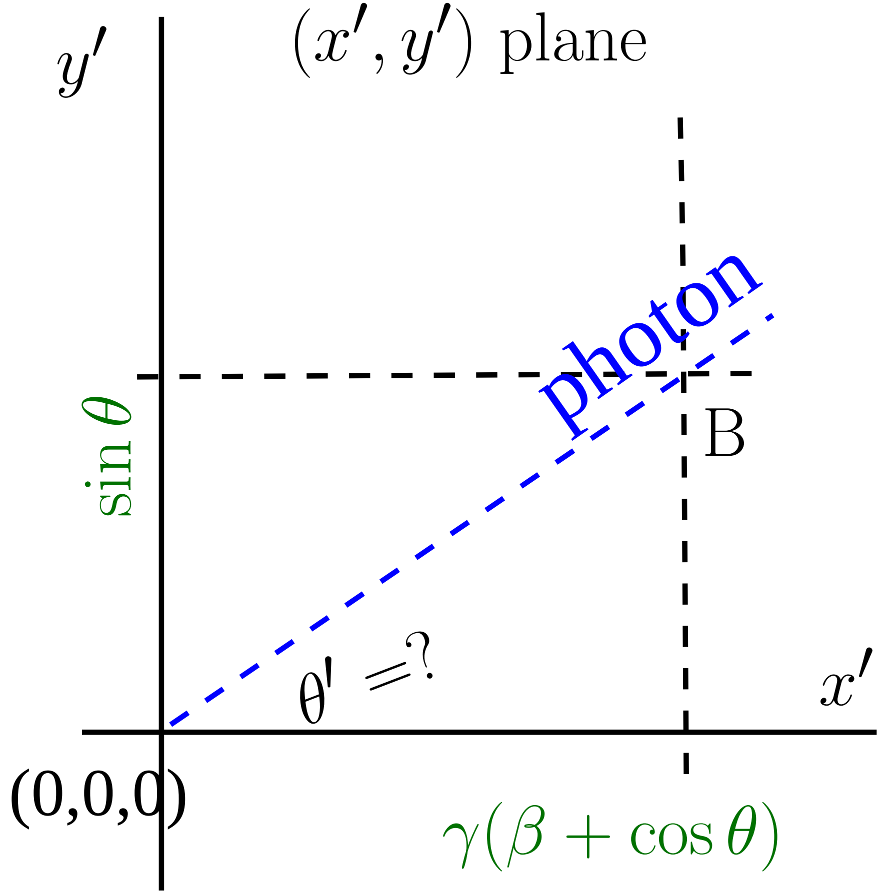
SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$

$$\Lambda^{-1} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 1 \end{pmatrix} = \begin{pmatrix} \gamma(\cos\theta + \beta) \\ \sin\theta \\ \gamma(1 + \beta\cos\theta) \end{pmatrix}$$

SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$



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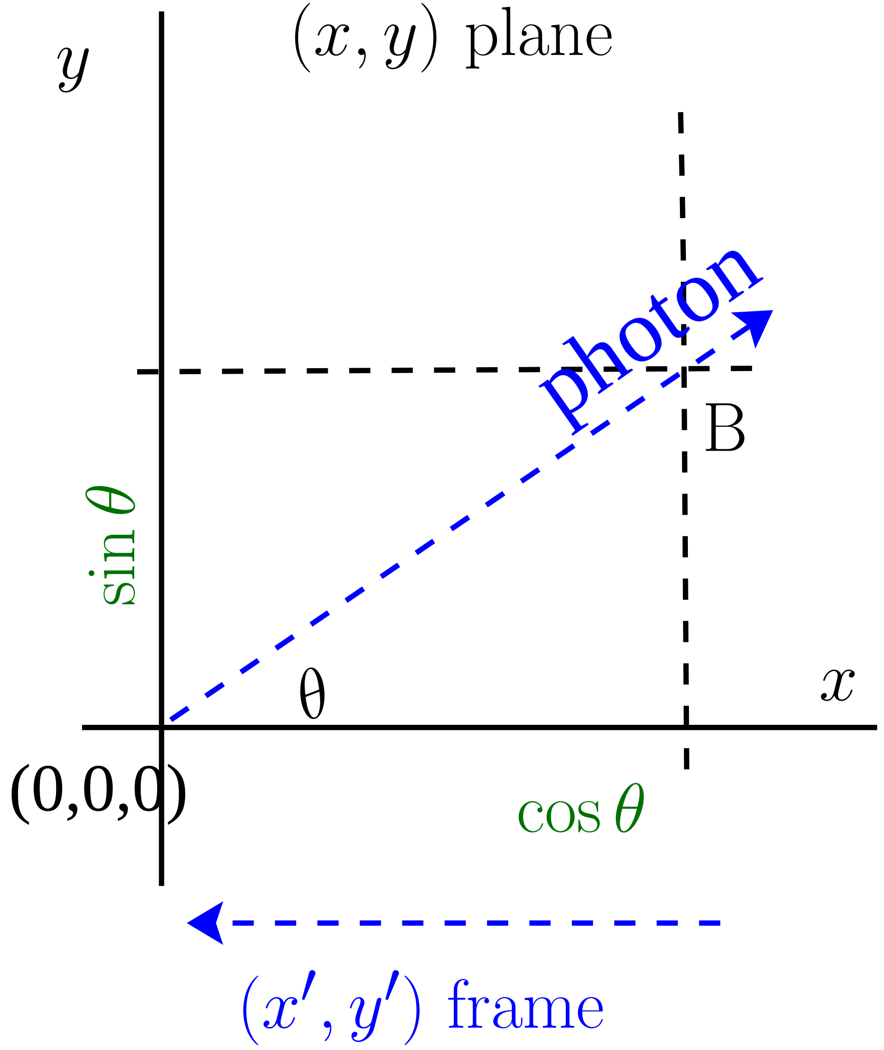

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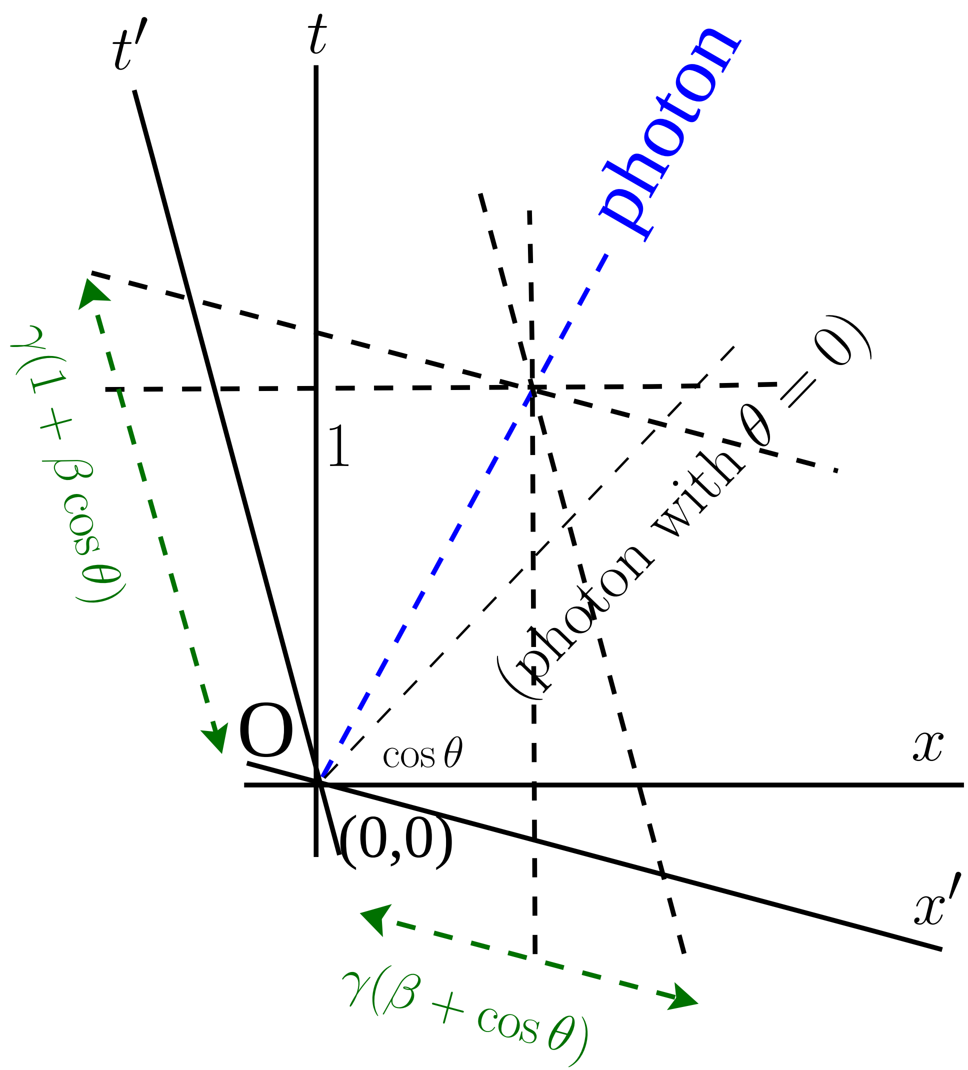

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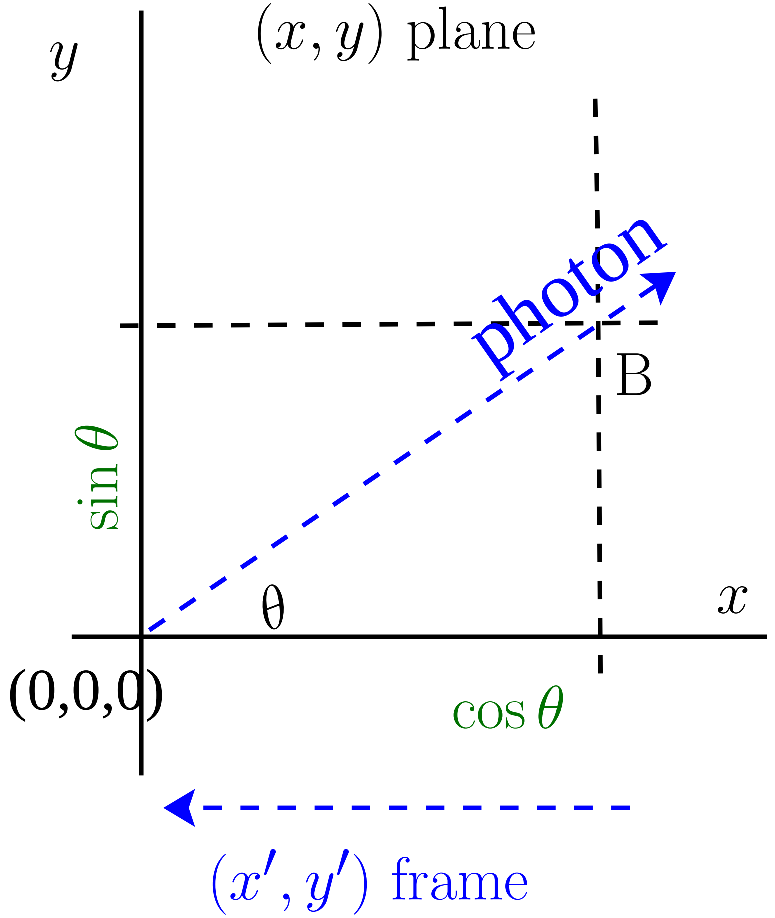
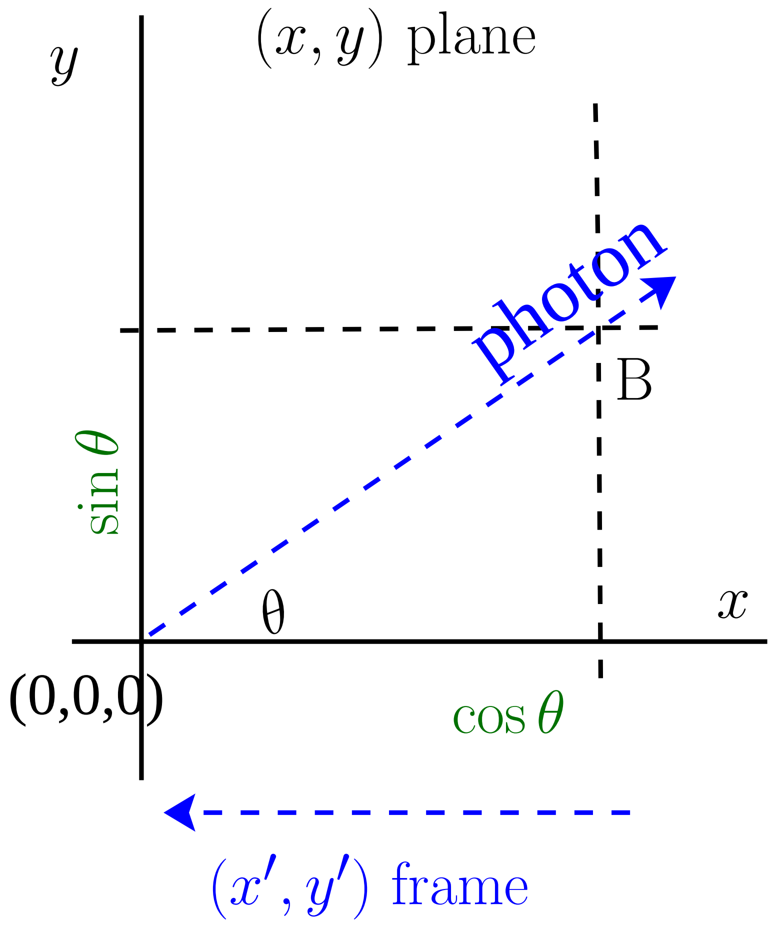


SR: relativistic aberration

event B: $(x, y, t) = (\cos\theta, \sin\theta, 1)$



$$\tan\theta' = \frac{\sin\theta}{\gamma(\beta+\cos\theta)}$$

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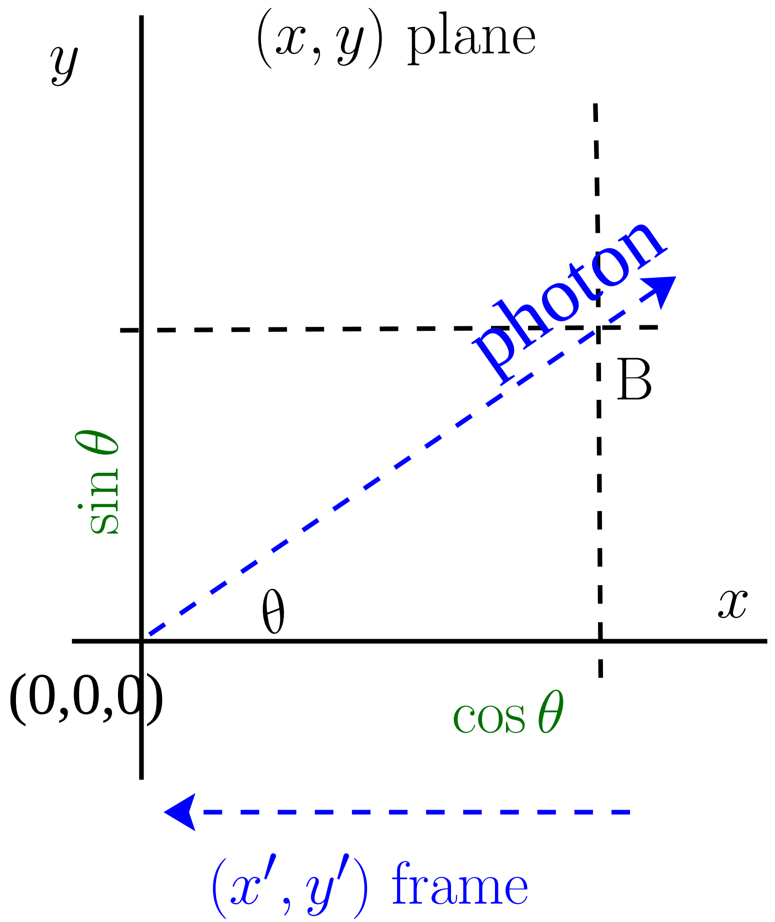

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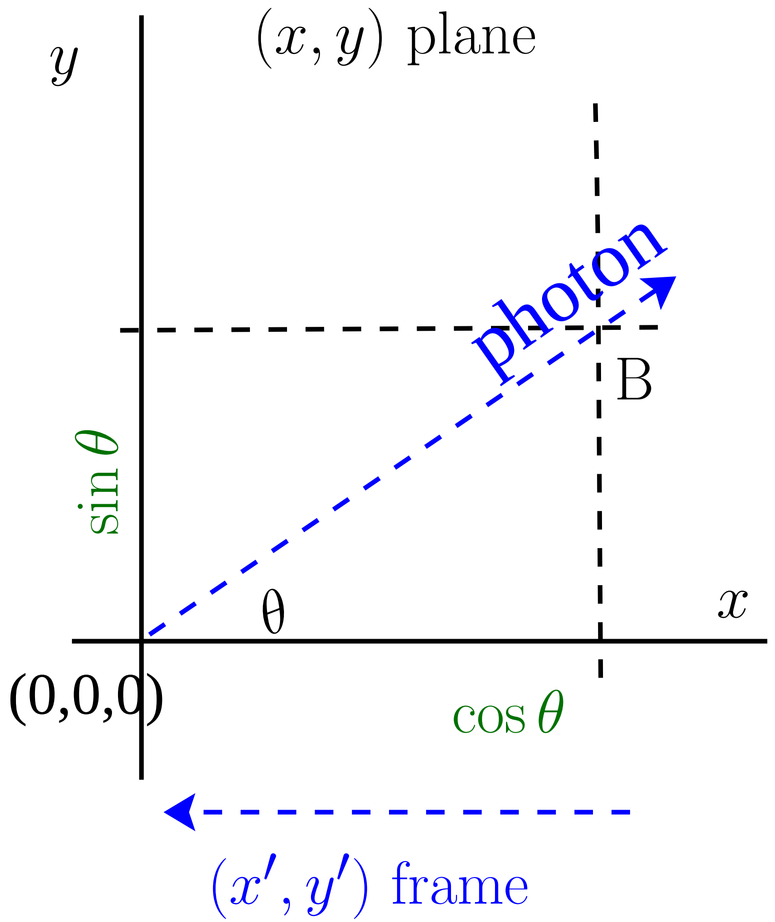

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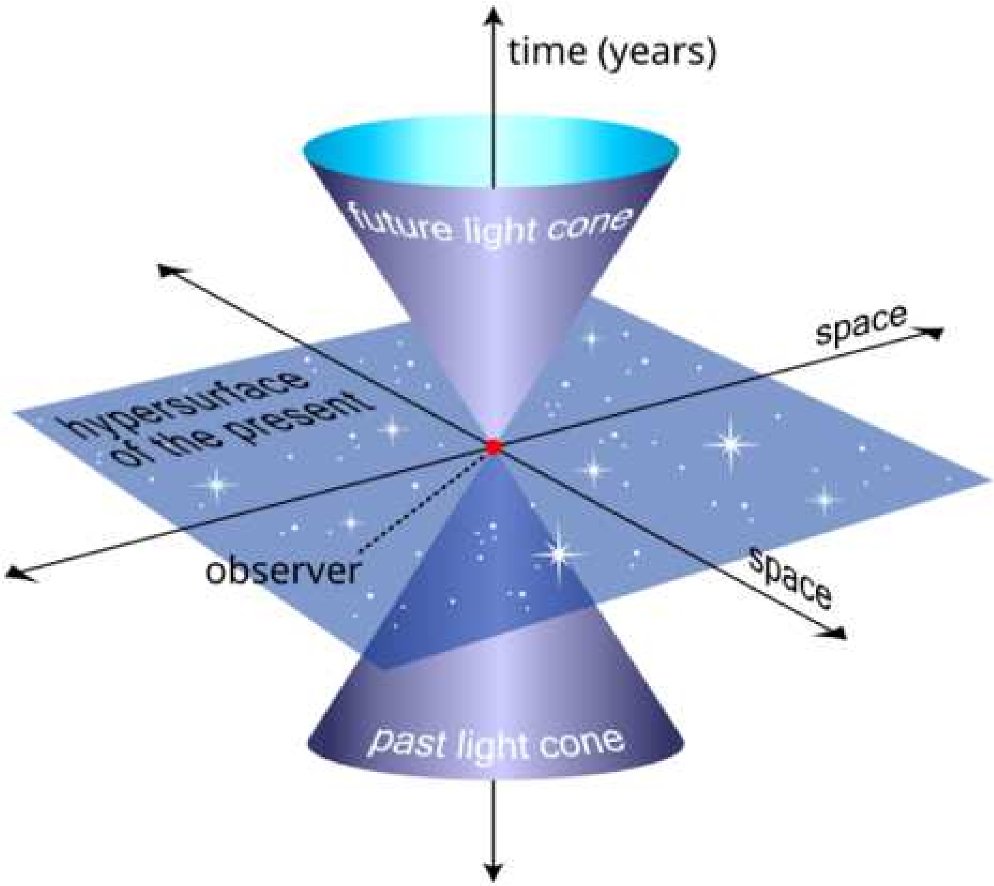
$\Rightarrow$ relativistic beaming, e.g. AGN jets

SR: world line



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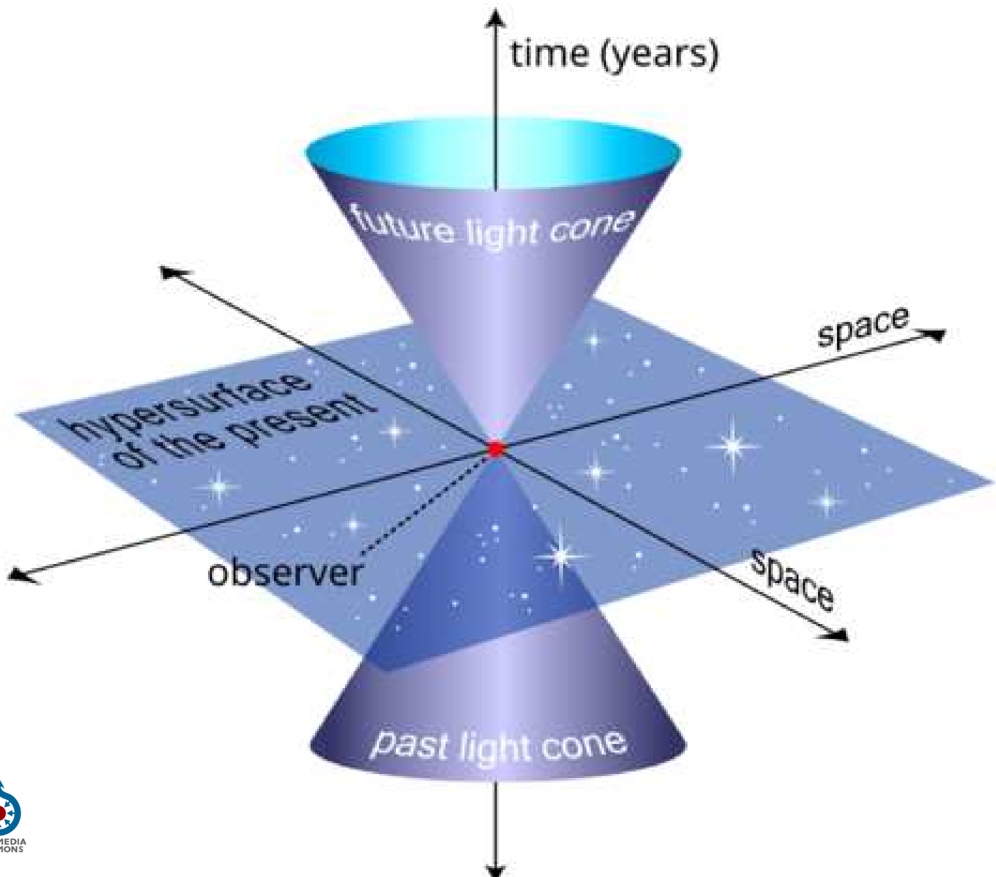

lightlike interval = null interval: $(\Delta s)^2 = 0$

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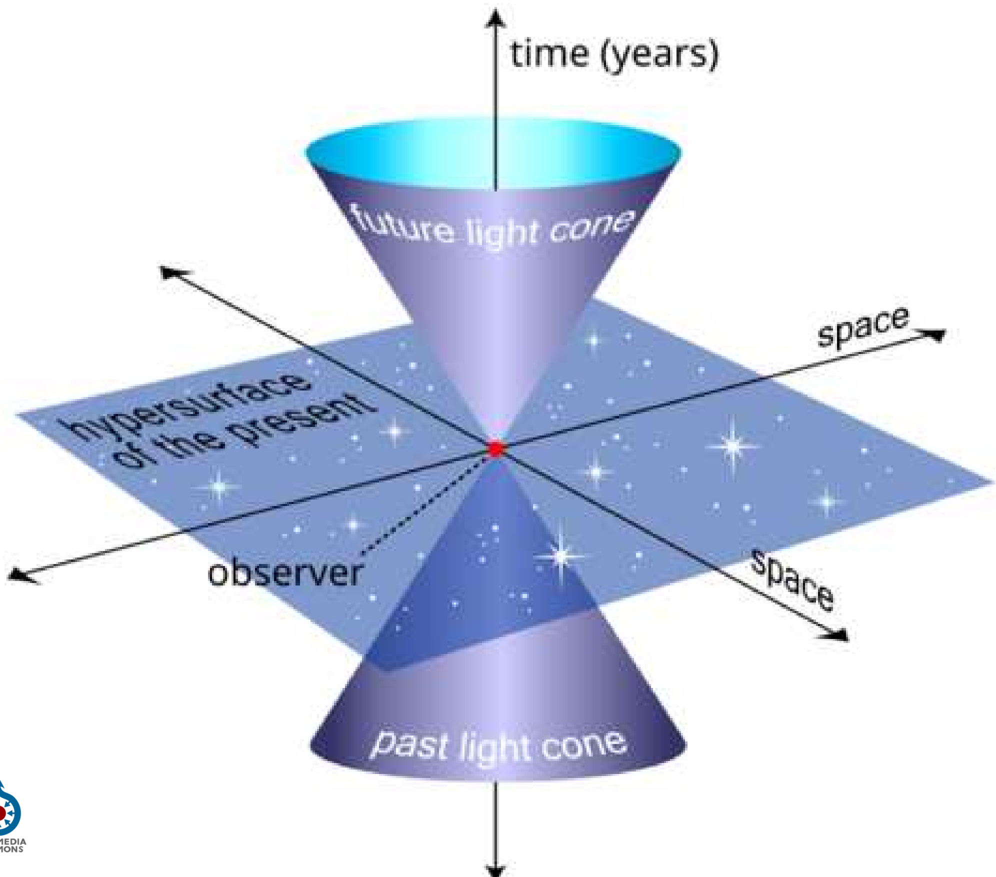

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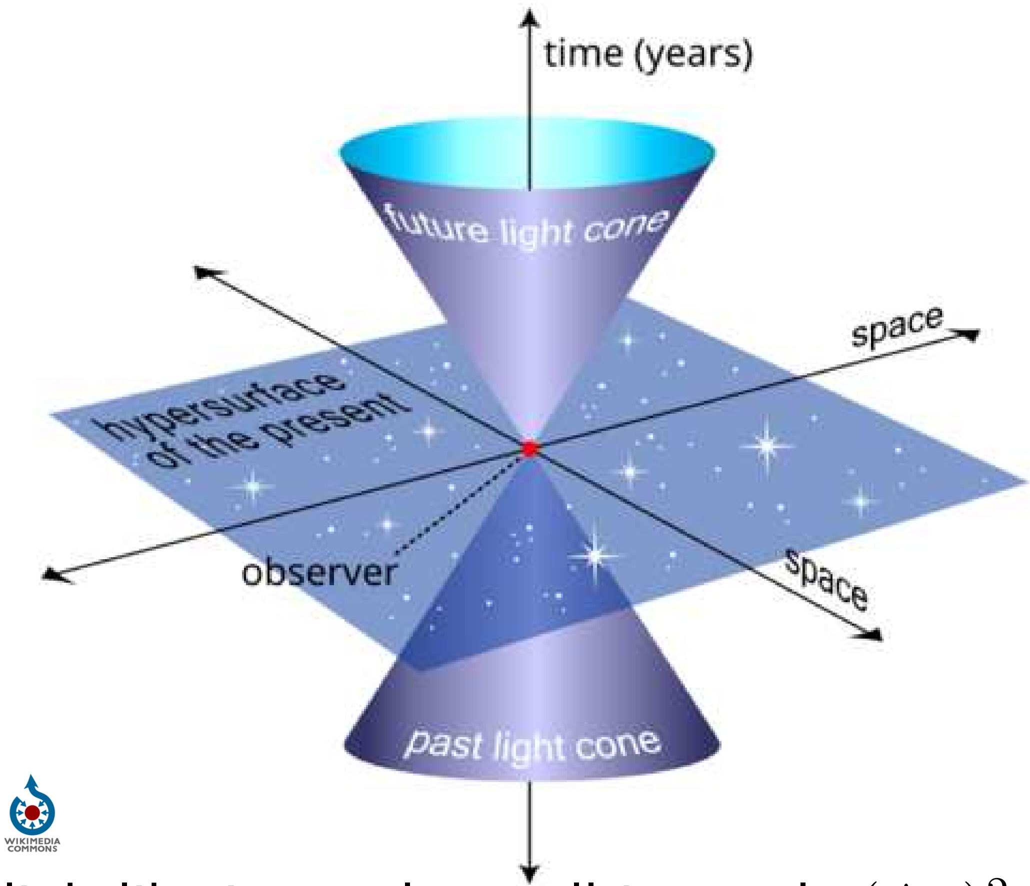

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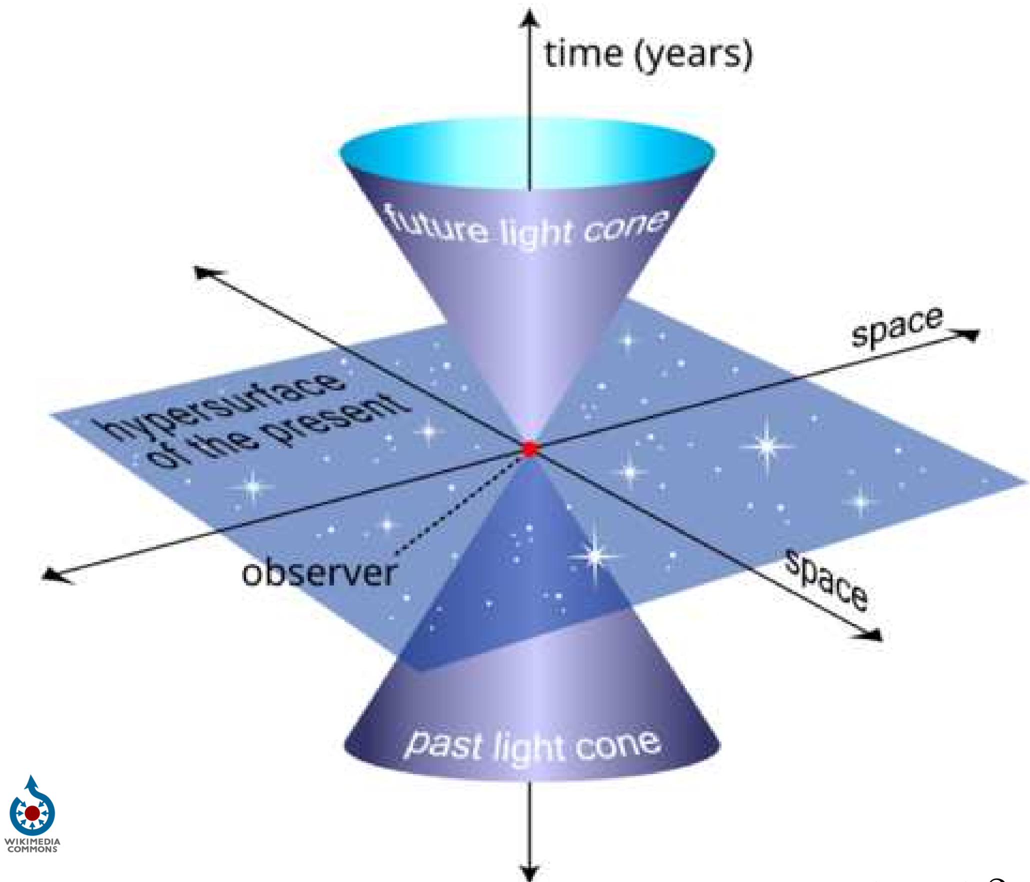

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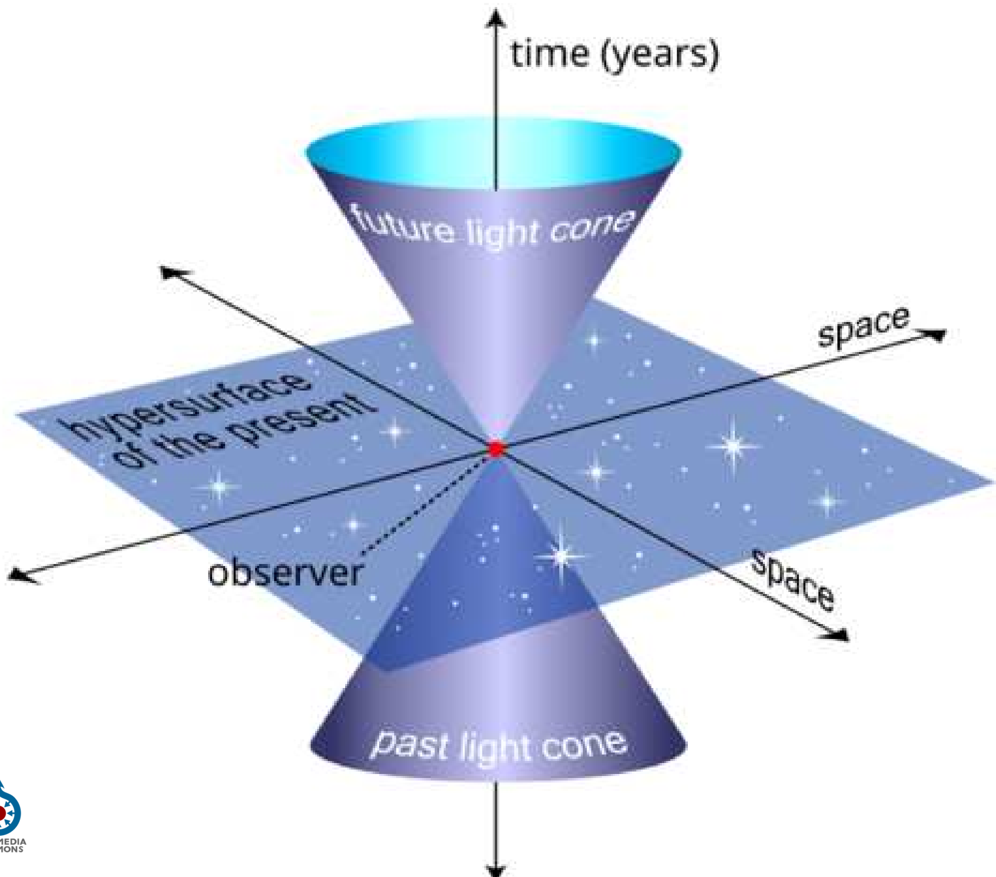

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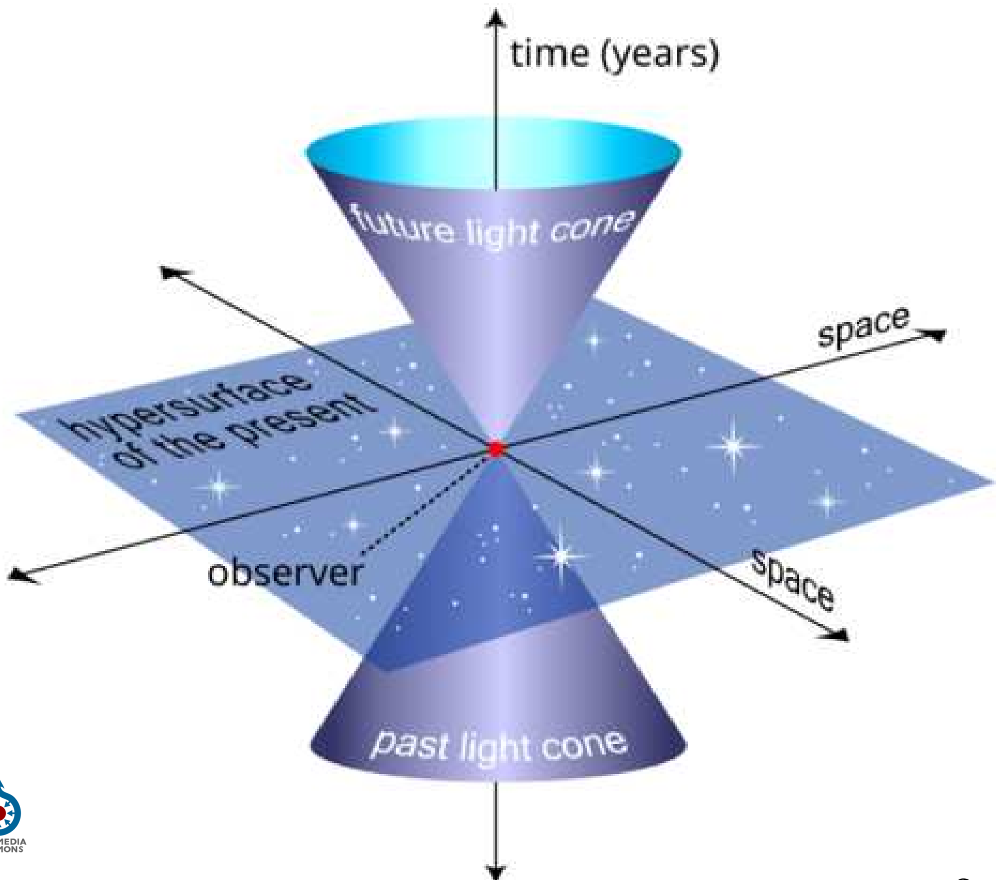

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+ elsewhere (subset: *reference-frame-dependent* "now")

+ here-now

SR: world line

Lorentz transform of world line

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Lorentz transform of world line

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SR: world line

Lorentz transform of world line

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w:proper time $\tau :=$ time along a worldline measured by clock following that worldline

often $\mathrm{d}\tau$ is useful for integrating

SR: paradoxes; verb tenses

- Where in a sentence is “now”, the “past” or the “future” hidden?

SR: paradoxes; verb tenses

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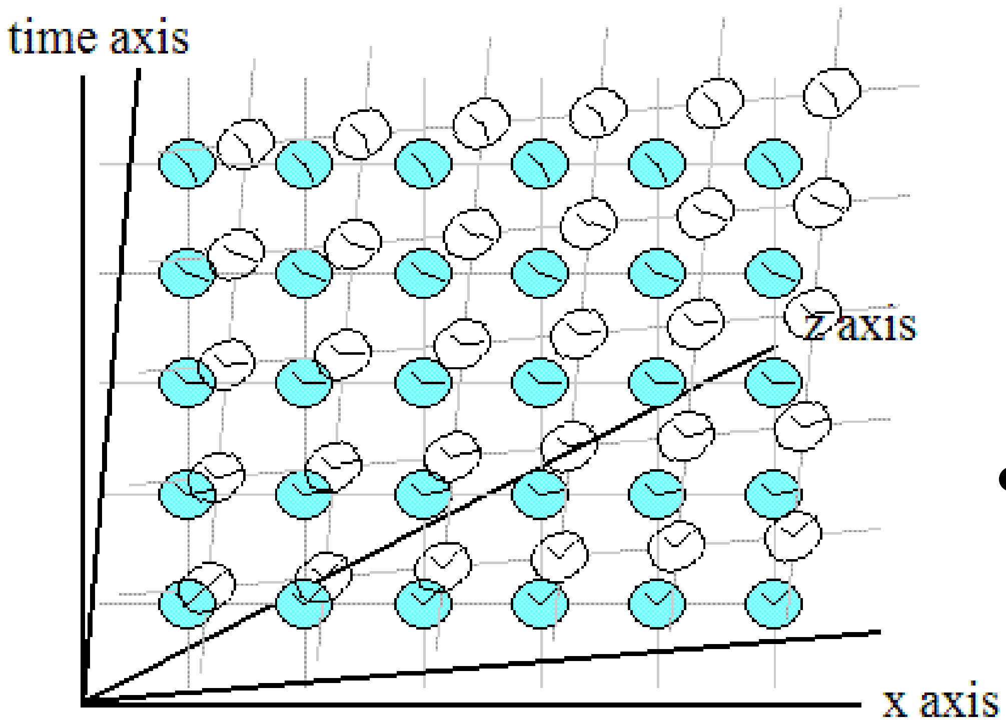
- Which of the above assume tachyonic communication?

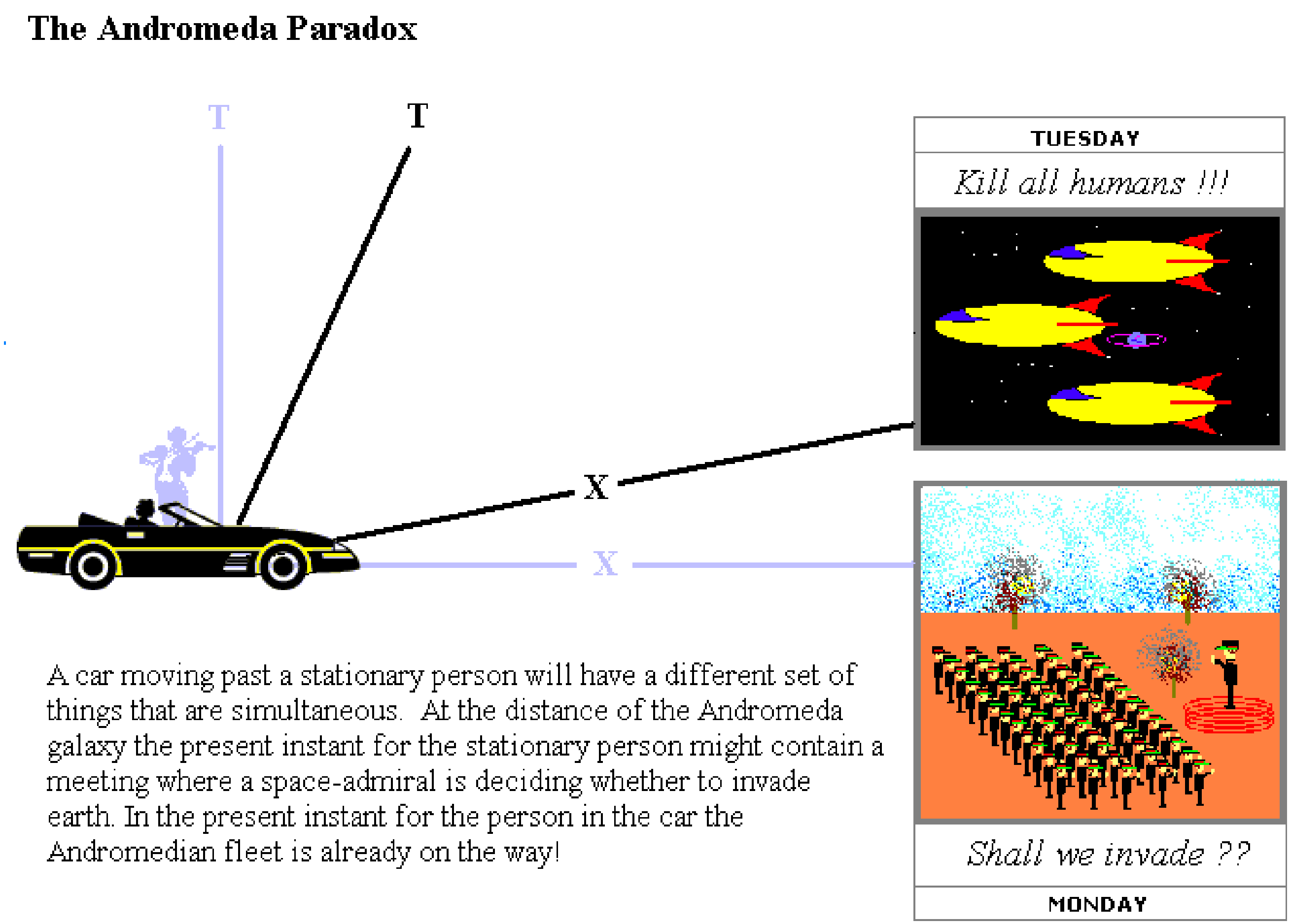
SR: Rietdijk–Putnam argument



Relativity shows that the inertial frames of reference of relatively moving objects do not overlie each other.

- each observer can synchronise clocks + rods

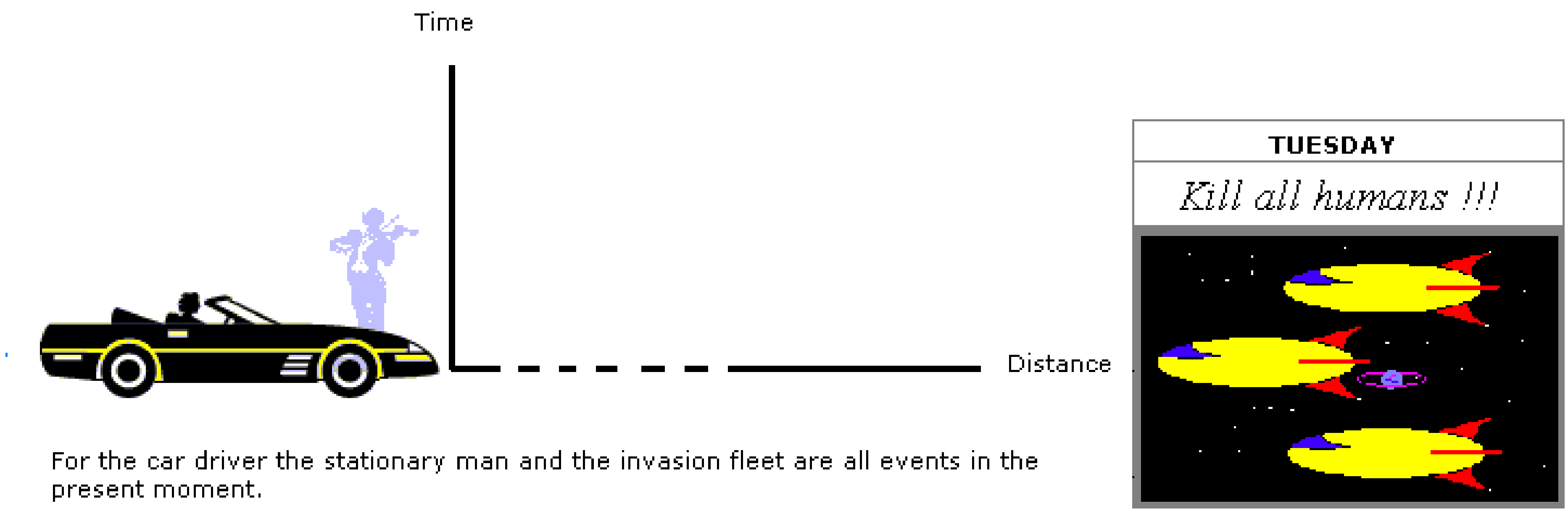
SR: Rietdijk–Putnam argument



w:Rietdijk-Putnam argument

Roger Penrose version: the “Andromeda paradox”

SR: Rietdijk–Putnam argument



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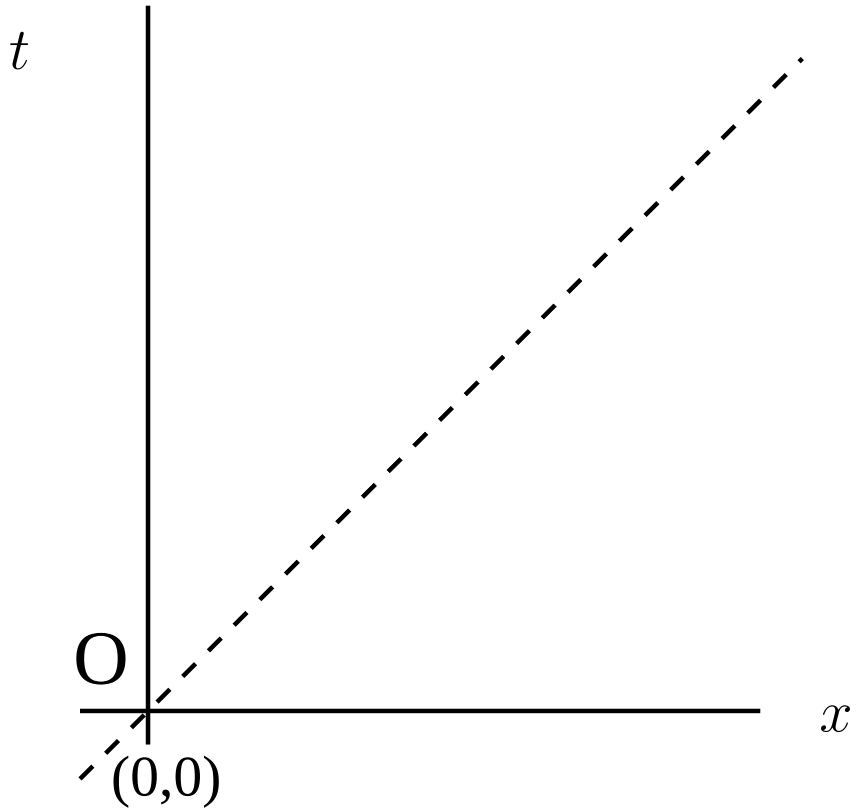
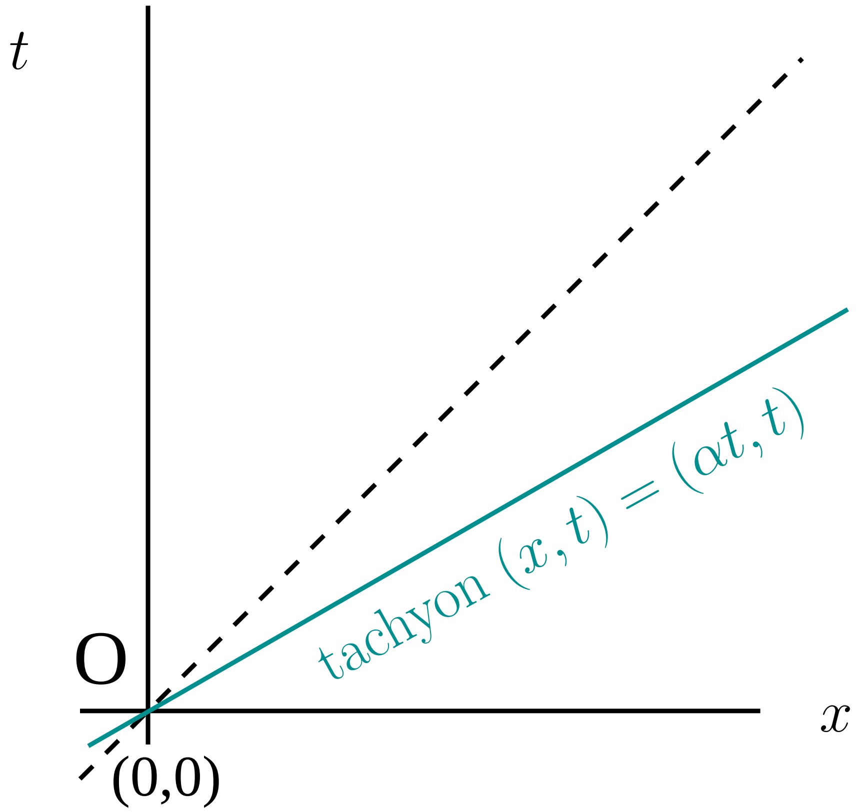
SR: tachyons and causality



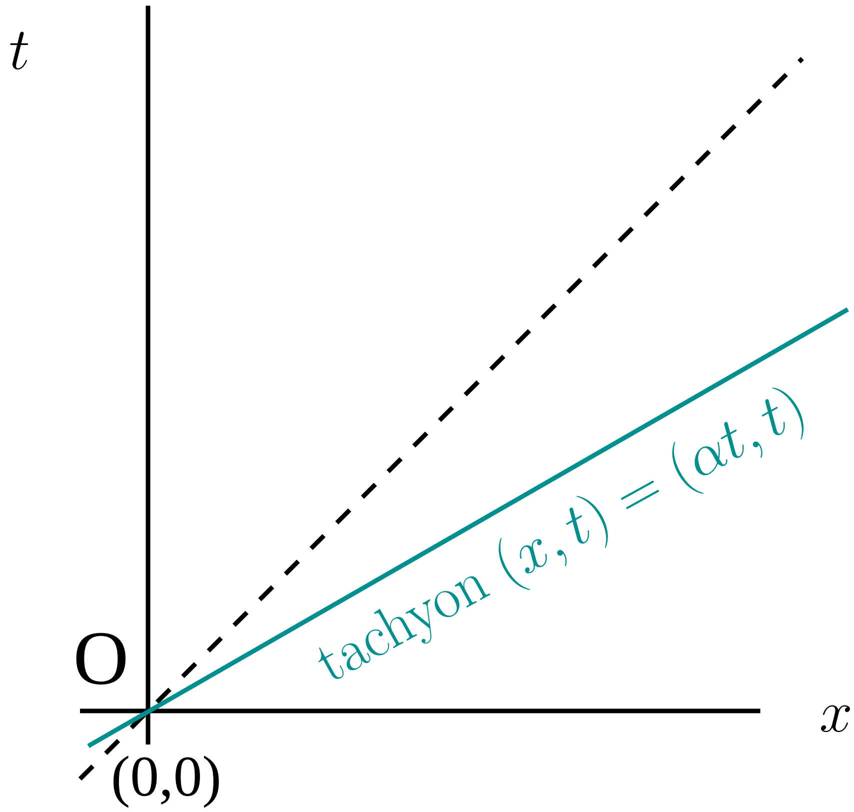
observer “at rest"

SR: tachyons and causality



add a tachyon with speed $\alpha > 1$

SR: tachyons and causality



add a tachyon with speed $\alpha > 1$

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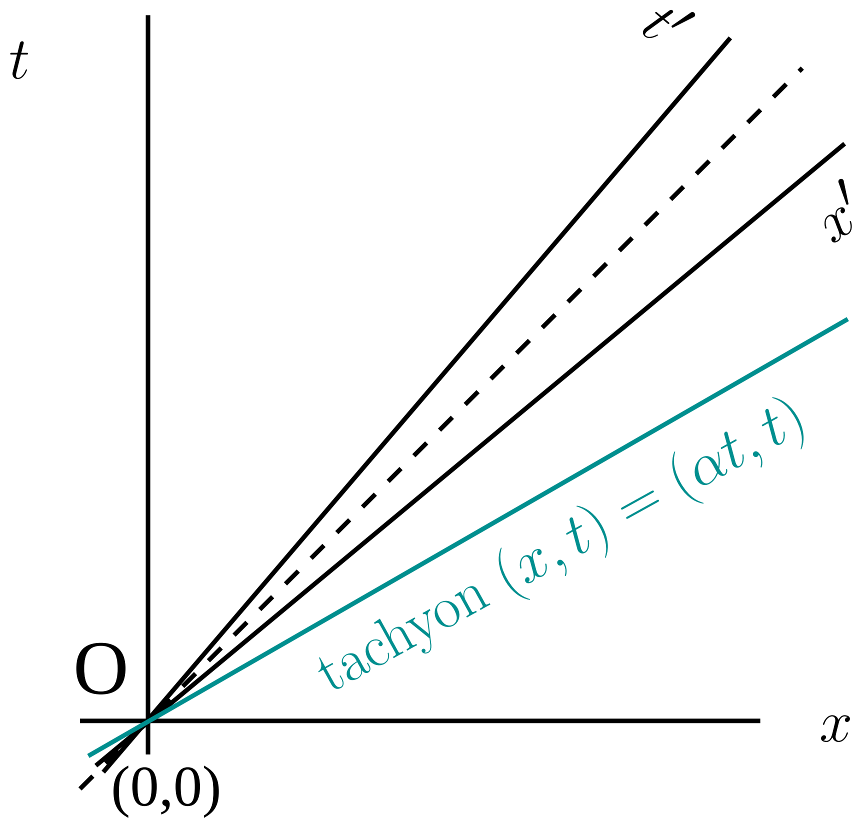

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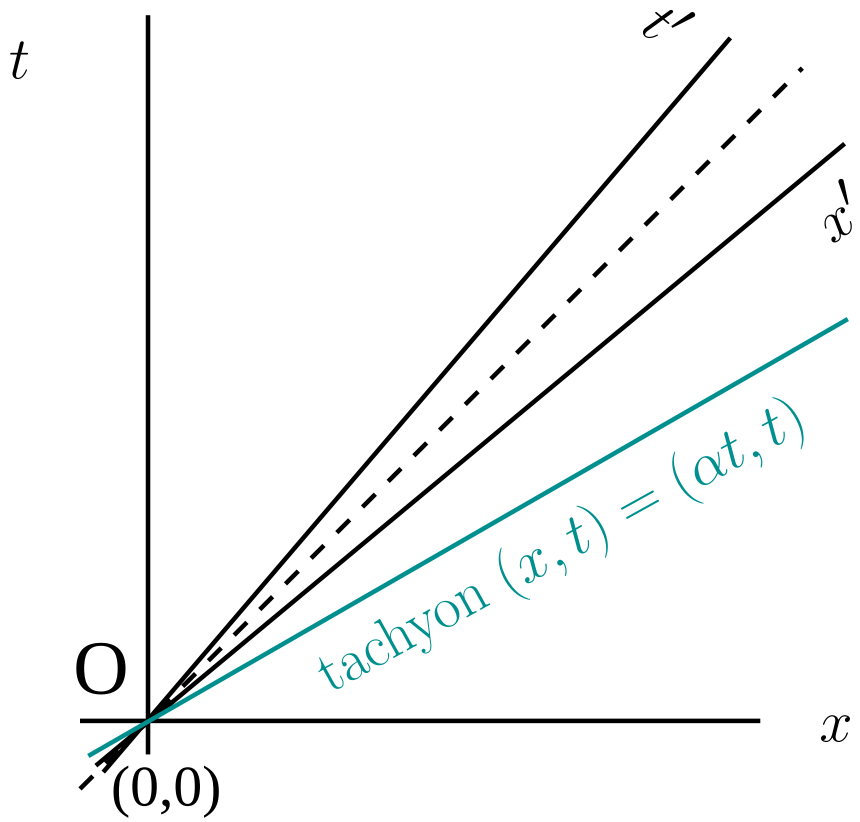

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rocket frame: $(\alpha t, t)$ becomes $\Lambda\ (\alpha t, t)^{\mathrm{T}}$

SR: tachyons and causality

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SR: tachyons and causality

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t \\ t \end{pmatrix} = \begin{pmatrix} \gamma\alpha t - \beta\gamma t \\ -\alpha\beta\gamma t + \gamma t \end{pmatrix}$$

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same sequence of spacetime events = tachyon spacetime path:

t increases for observer “at rest",

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- observer at rest: tachyon emitted at origin
- rocket: tachyon absorbed at origin

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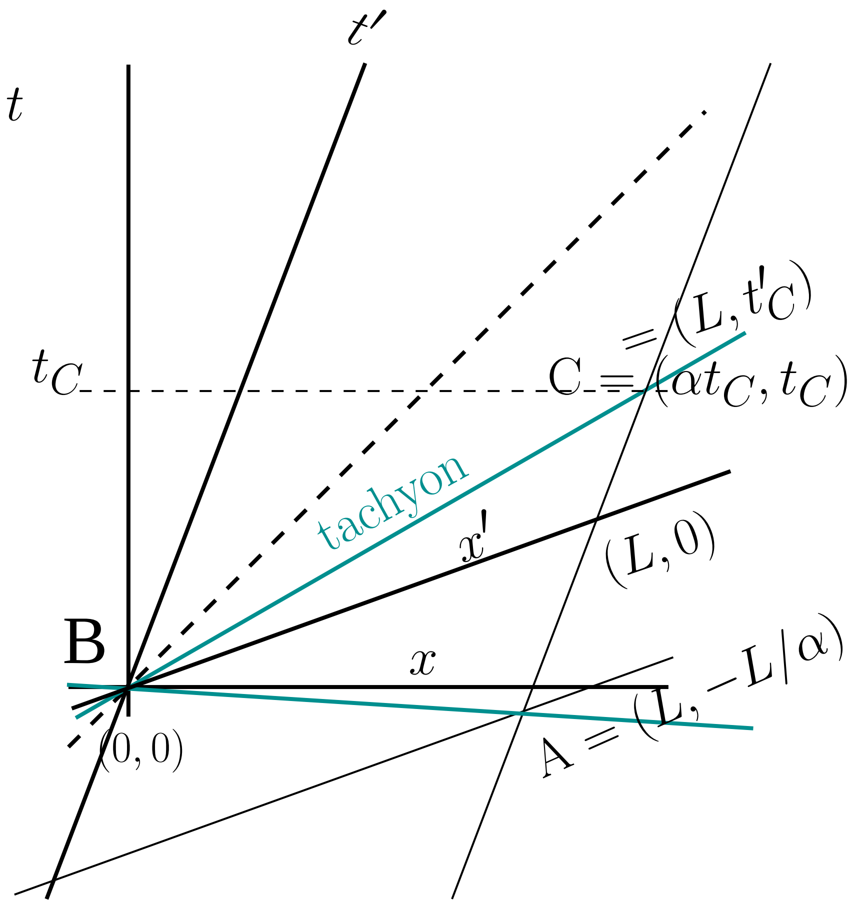
- observer at rest: tachyonic neutrino emitted at CERN?
- rocket: tachyonic neutrino absorbed at CERN?

w:2011 OPERA faster-than-light neutrino anomaly:

CERN $\rightarrow$ Gran Sasso

SR: tachyonic antitelephone



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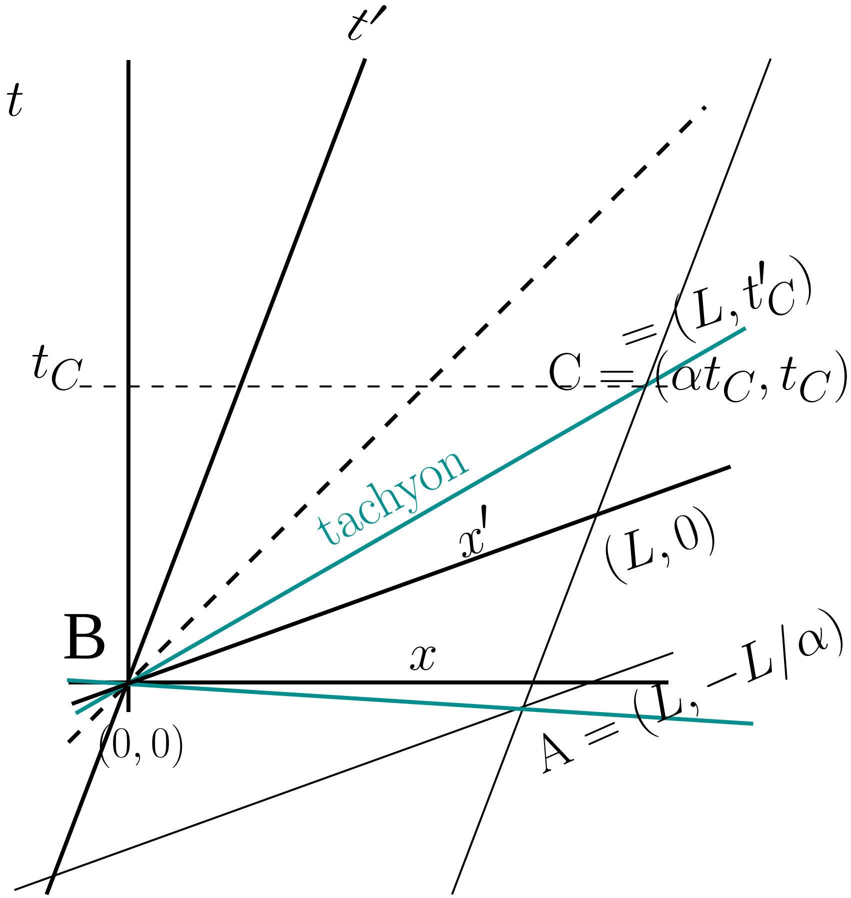

B stationary: (x, t) frame

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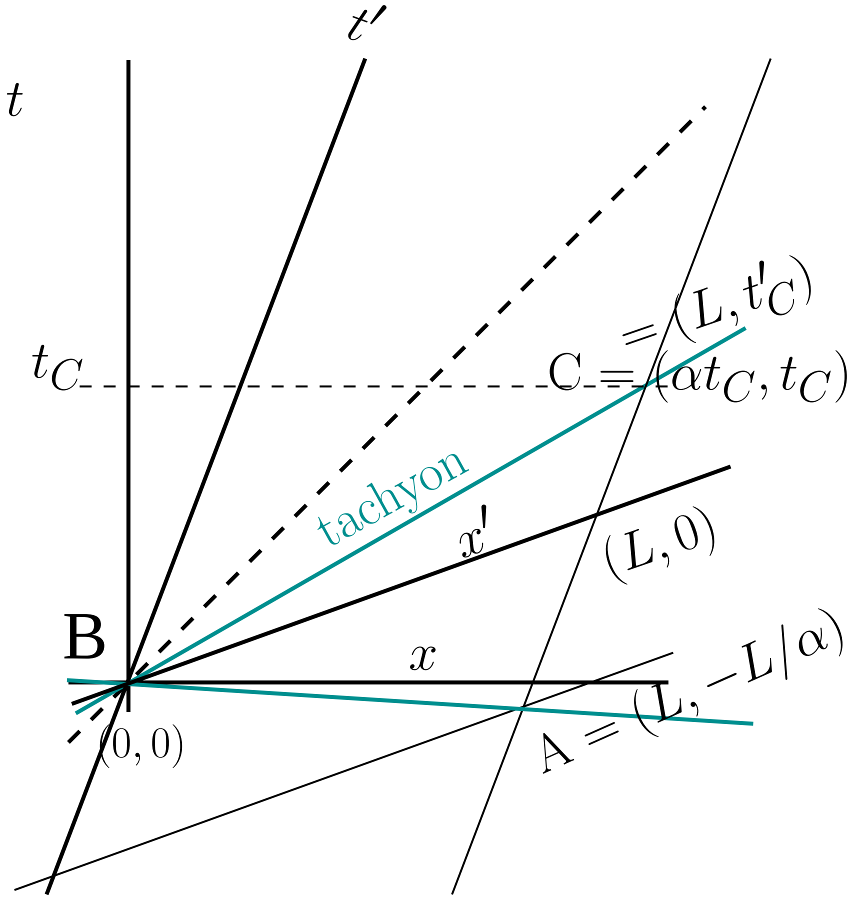

B stationary: (x, t) frame
A moving at speed β: (x', t') frame

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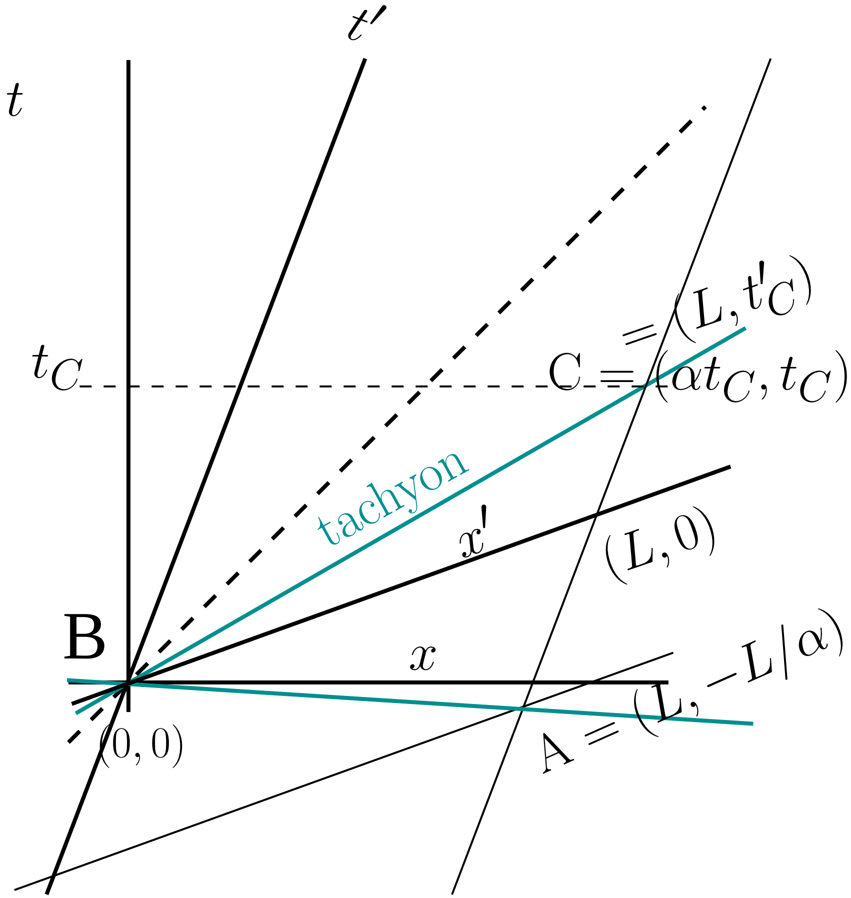

A: tachyon at $\alpha > 1$ to B

B stationary: (x, t) frame
A moving at speed β: (x', t') frame

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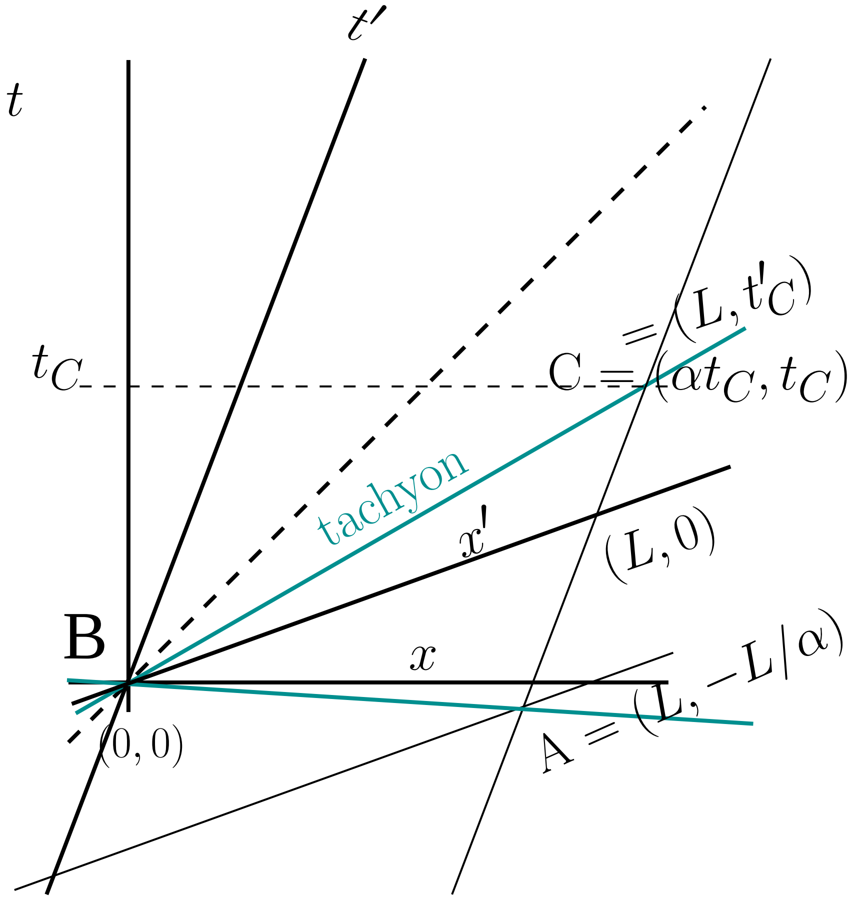

B stationary: (x,t) frame
A moving at speed β: (x',t') frame

B: tachyon at $\alpha > 1$ to C

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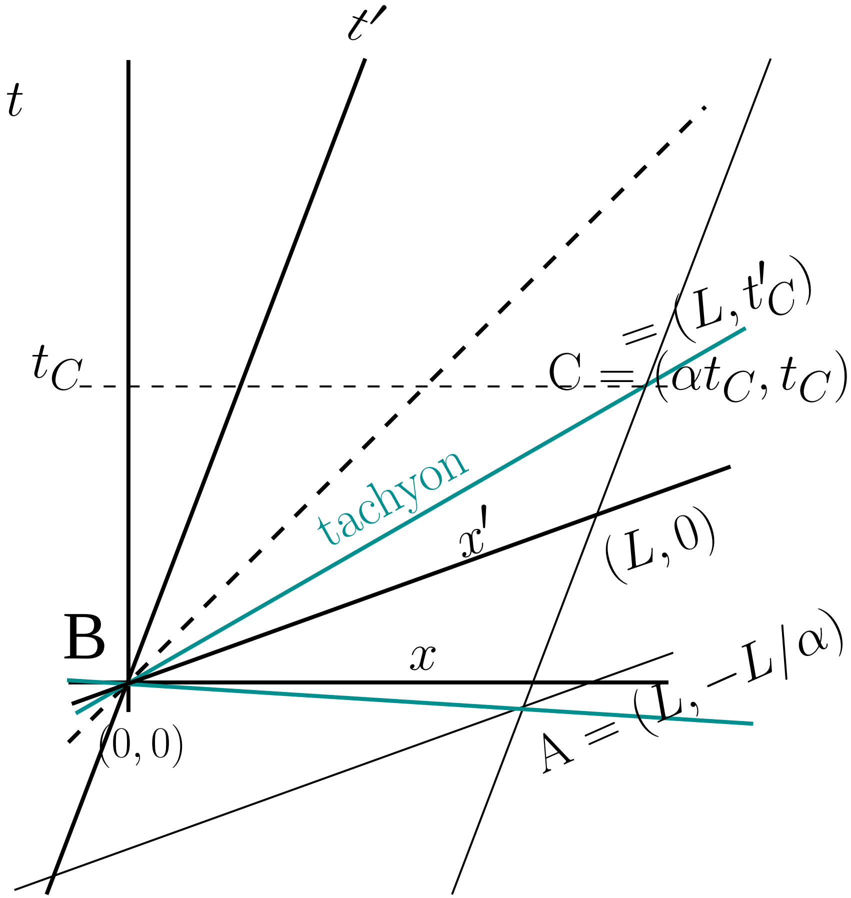
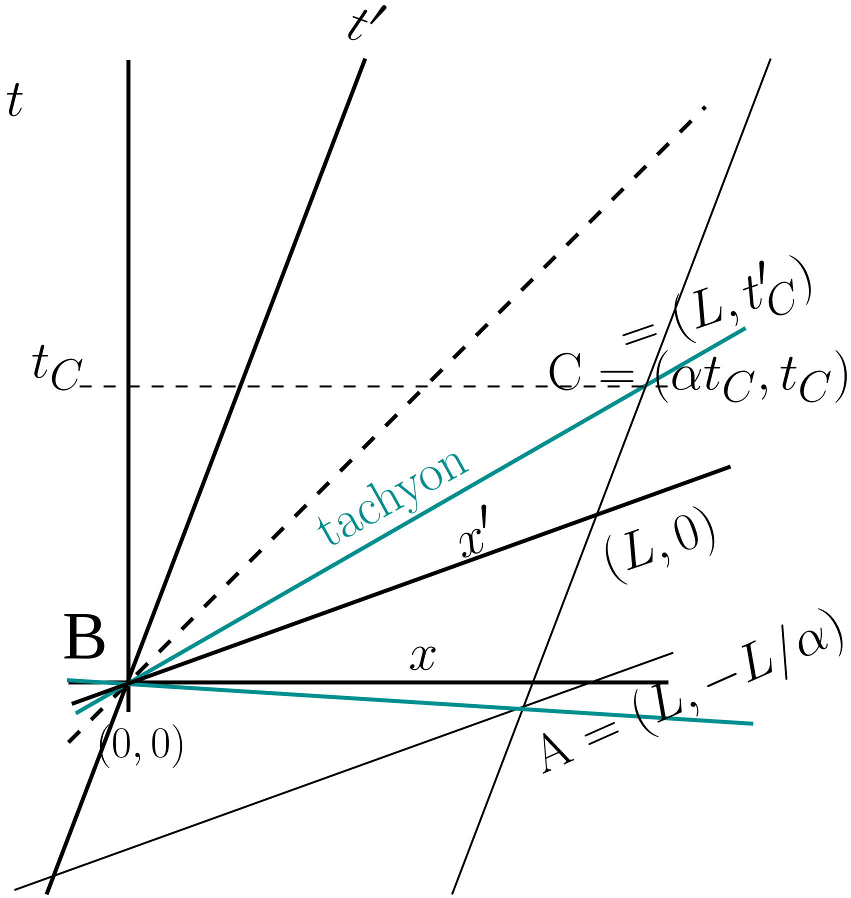


B stationary: (x, t) frame
A moving at speed β: (x', t') frame

C: $\begin{pmatrix} L \\ t'_C \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t_C \\ t_C \end{pmatrix}$

SR: tachyonic antitelephone



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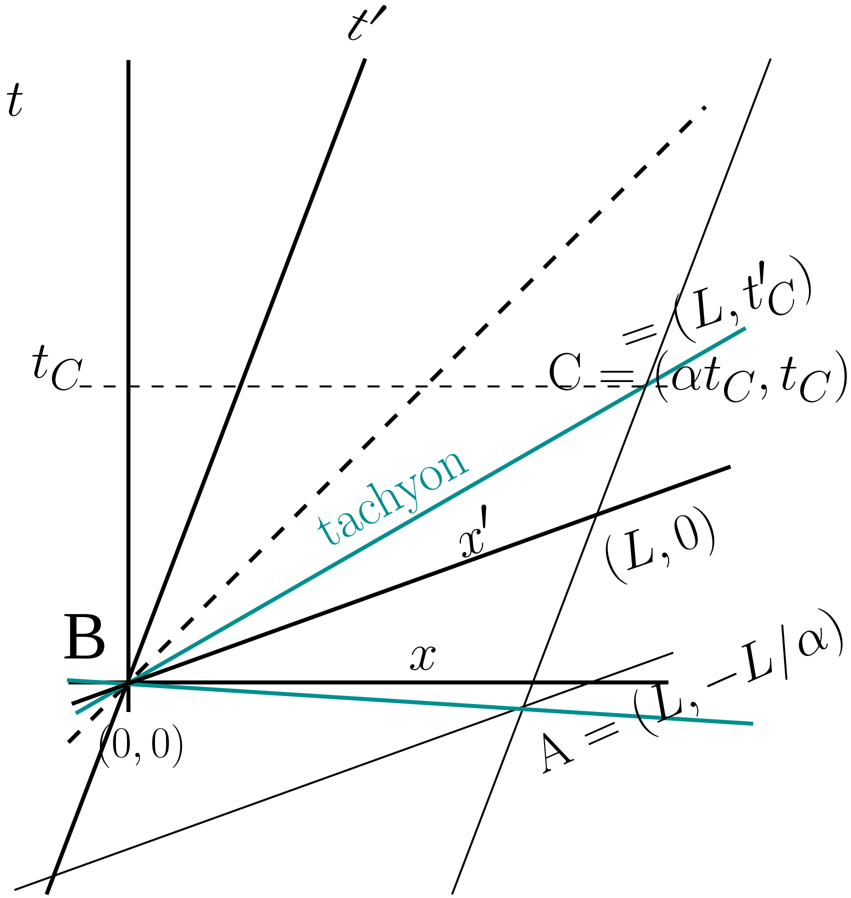

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$$t'_{\mathrm{C}} = \gamma t_{\mathrm{C}}(1 - \alpha\beta)$$

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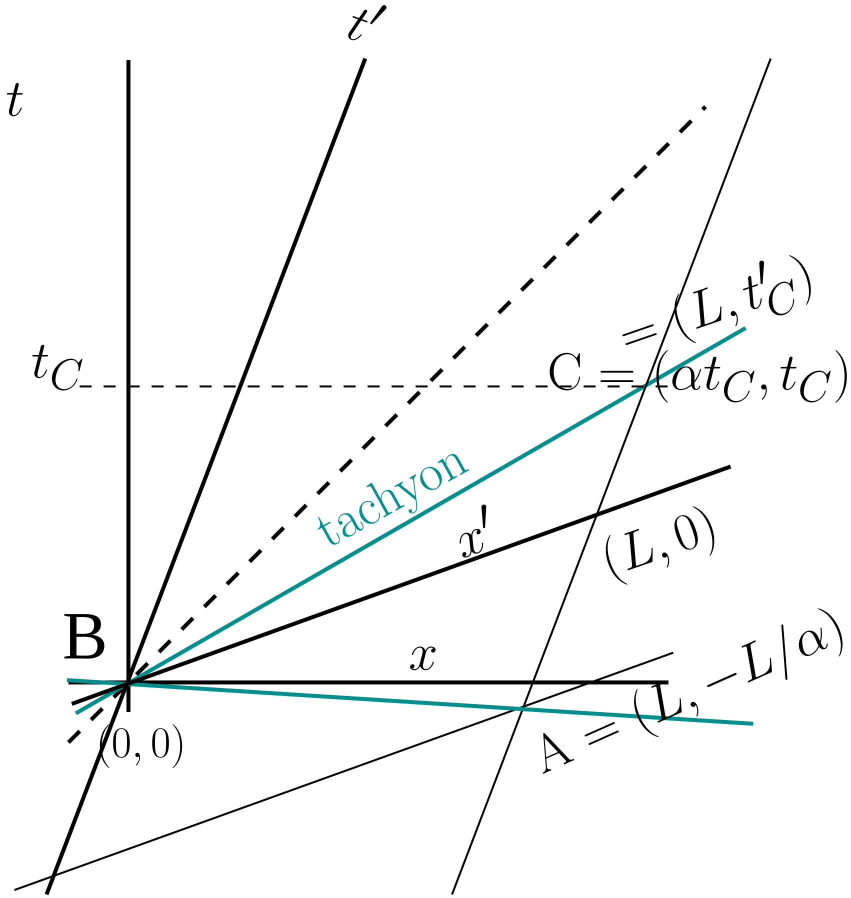

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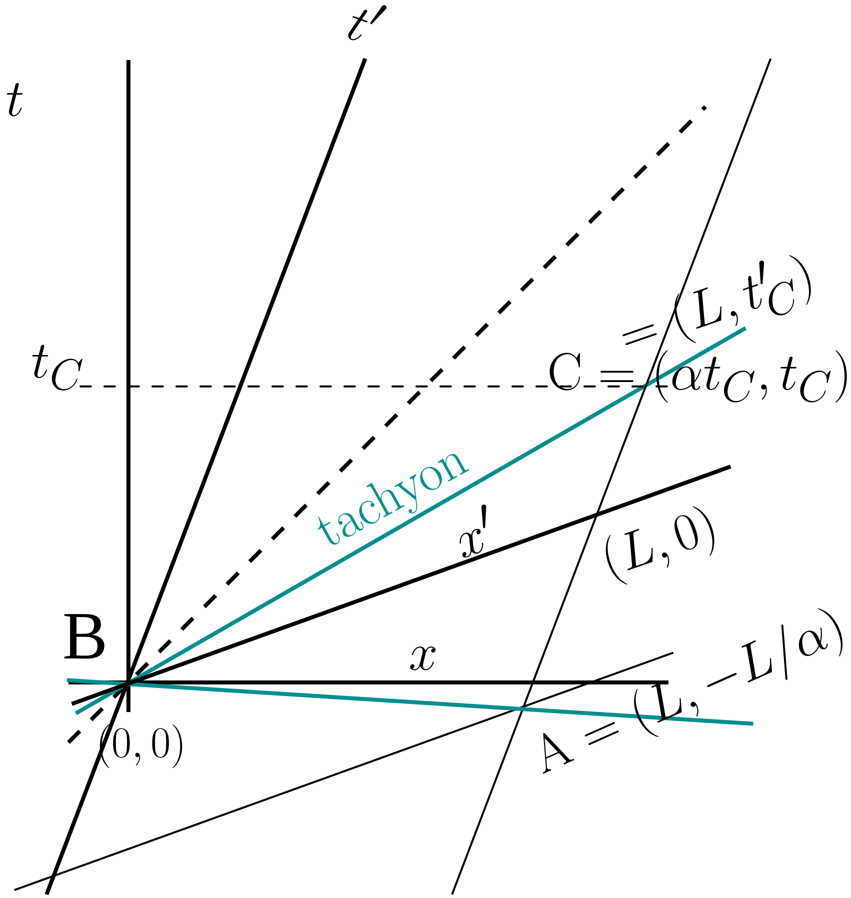
$$t'_C = \gamma \frac{L}{\gamma(\alpha - \beta)} (1 - \alpha\beta)$$

SR: tachyonic antitelephone



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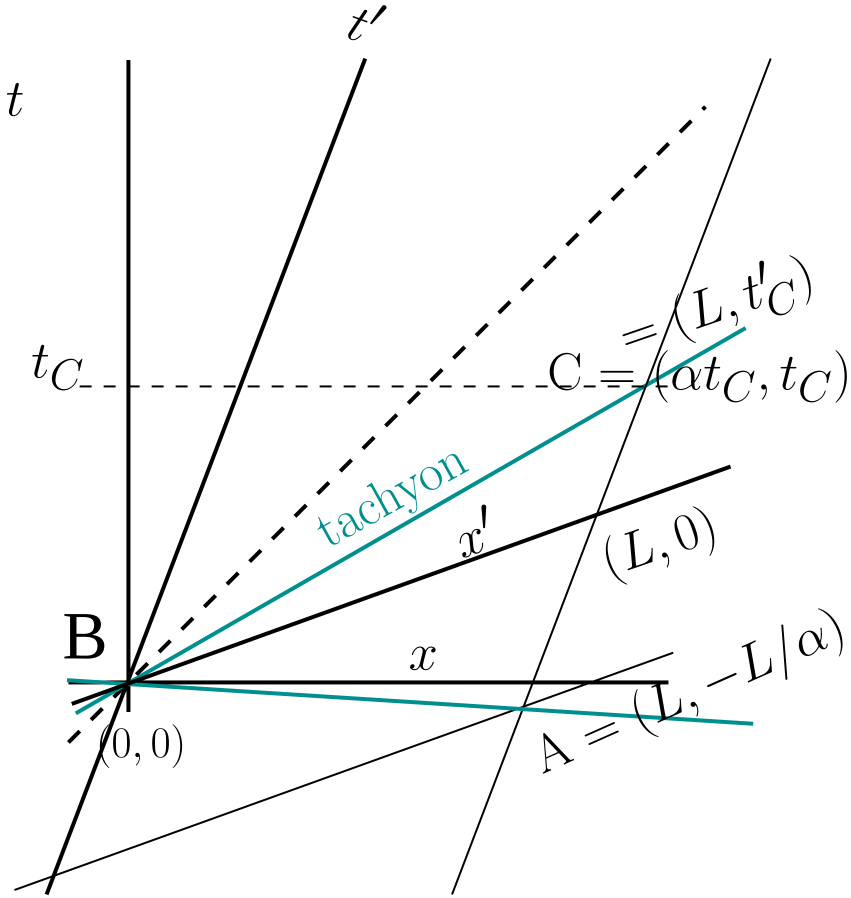

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$$t'_\mathrm{C} - t'_\mathrm{A} = L\left(\frac{1-\alpha\beta}{\alpha-\beta} + \frac{1}{\alpha}\right)$$

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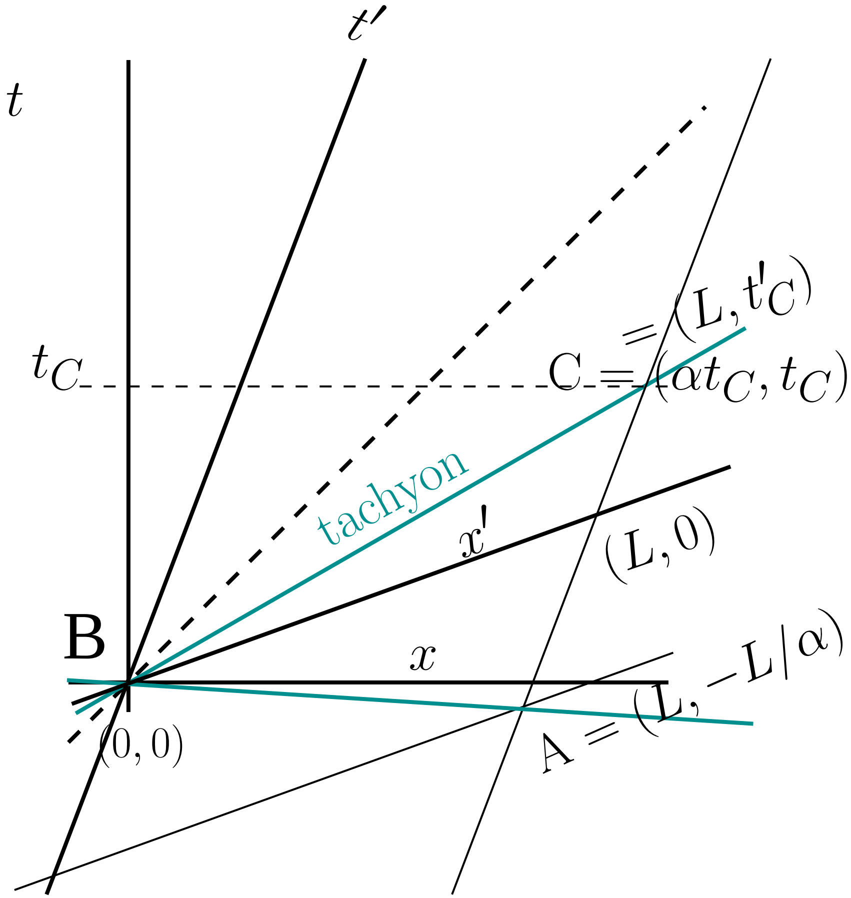

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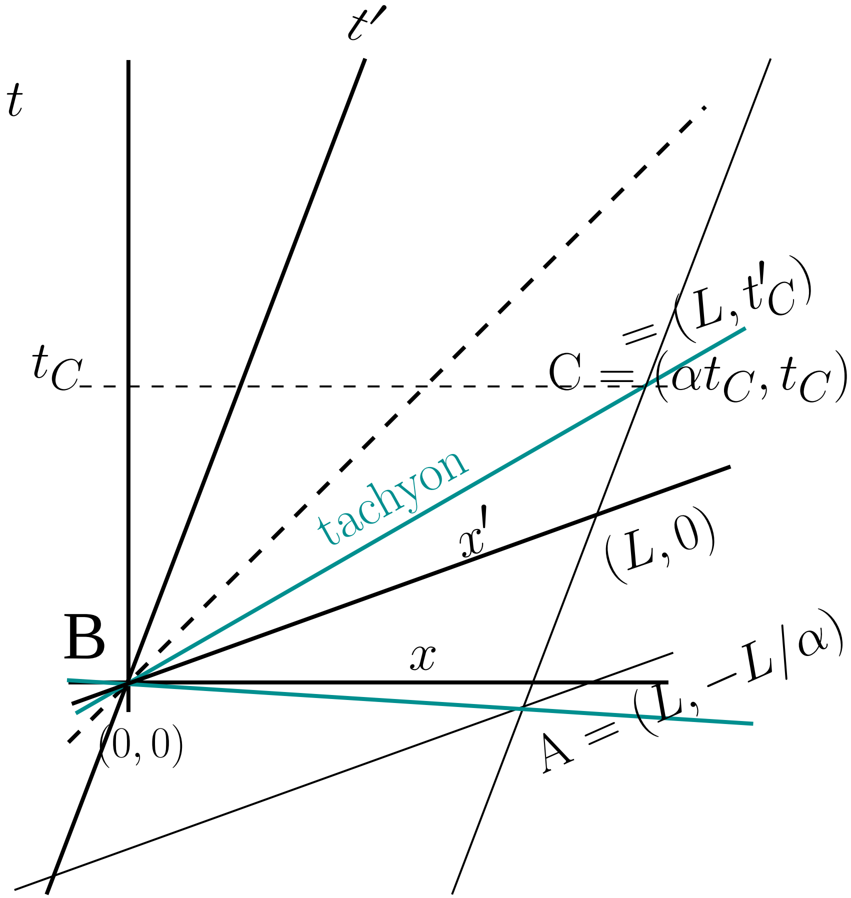

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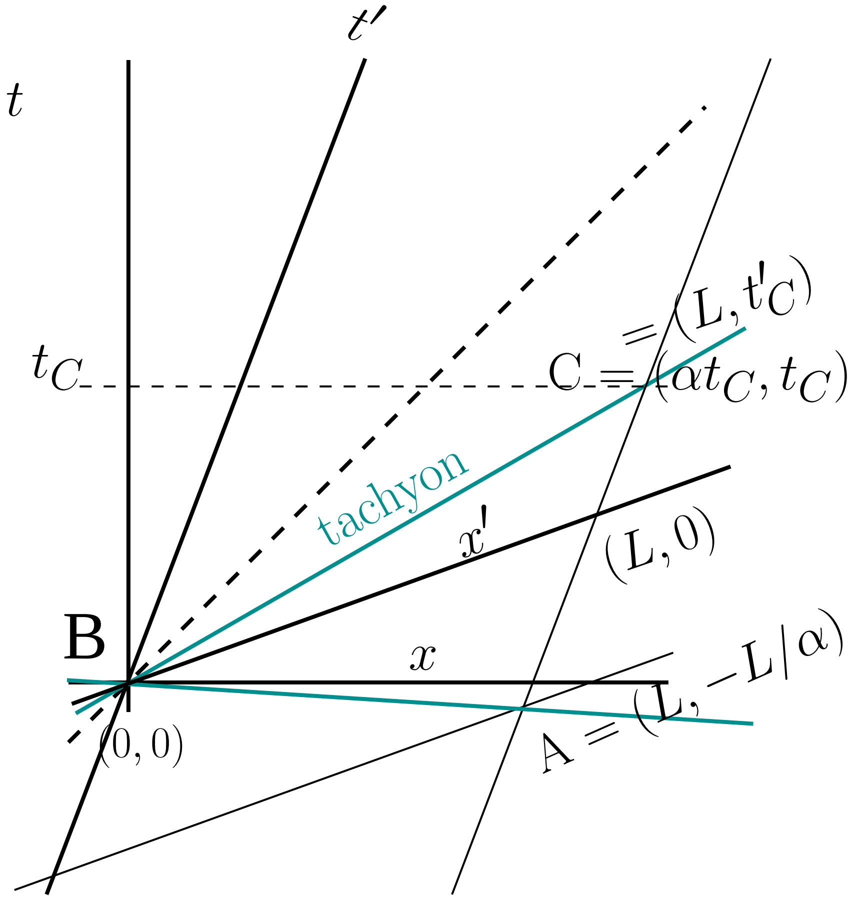

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$$t'_{\mathrm{C}} - t'_{\mathrm{A}} = L\frac{2\alpha-(\alpha^2+1)\beta}{\alpha(\alpha-\beta)}$$

< 0 if $\beta > \frac{2\alpha}{\alpha^2+1}$

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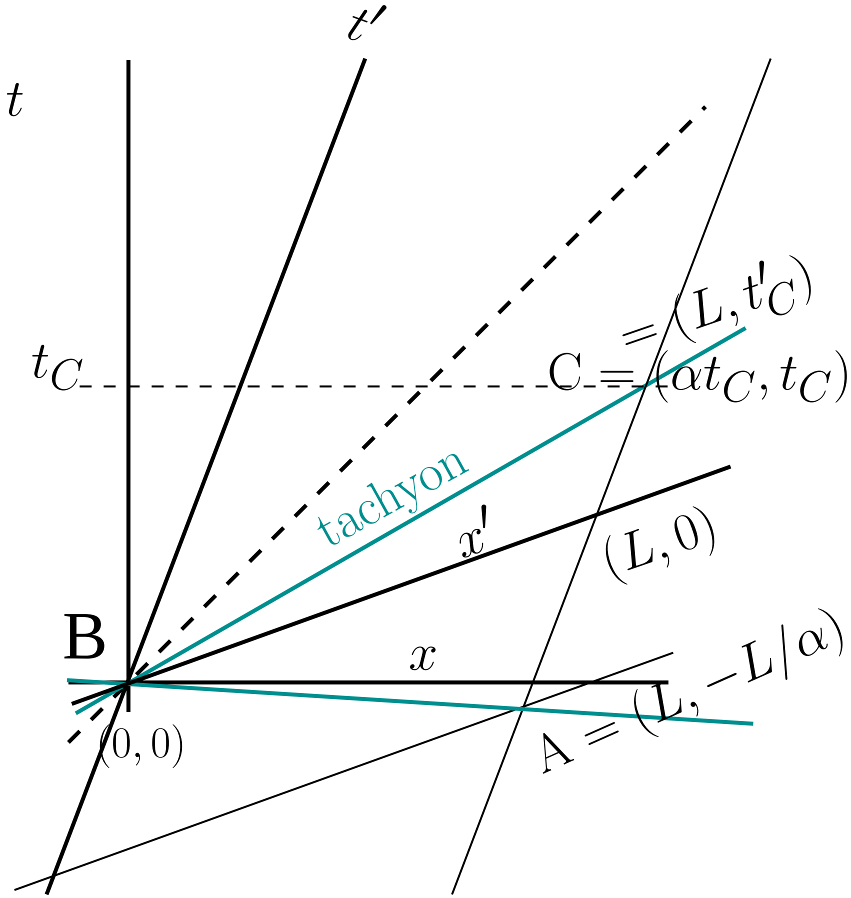
A moving at speed β: (x', t') frame

SR: tachyonic antitelephone



B stationary: (x, t) frame
A moving at speed β: (x', t') frame

C: $\begin{pmatrix} L \\ t'_{\mathrm{C}} \end{pmatrix} = \Lambda \begin{pmatrix} \alpha t_{\mathrm{C}} \\ t_{\mathrm{C}} \end{pmatrix}$

$\begin{pmatrix} L \\ t'_{\mathrm{C}} \end{pmatrix} = \gamma t_{\mathrm{C}} \begin{pmatrix} \alpha - \beta \\ -\alpha\beta + 1 \end{pmatrix}$

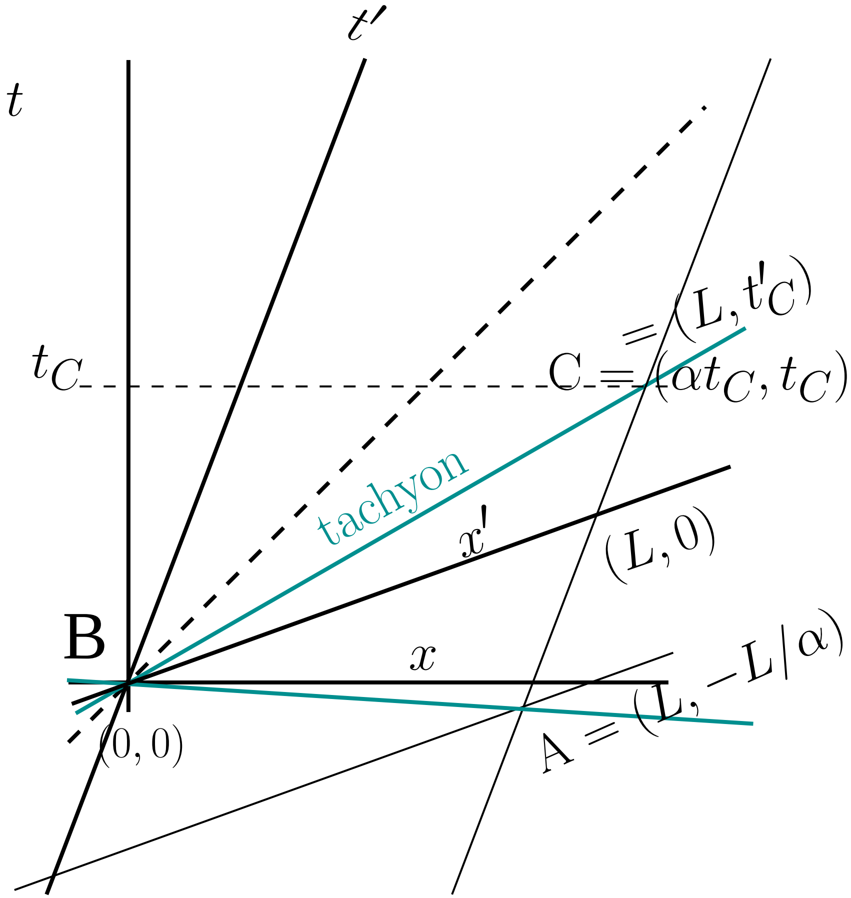
$t'_{\mathrm{C}} = L\frac{1-\alpha\beta}{\alpha-\beta}$

$t'_{\mathrm{C}} - t'_{\mathrm{A}} = L\frac{2\alpha-(\alpha^2+1)\beta}{\alpha(\alpha-\beta)}$

< 0 if $\beta > \frac{2\alpha}{\alpha^2+1}$

A receives tachyonic response at C before sending it

SR: tachyonic antitelephone



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A receives tachyonic response at C before sending it

w:tachyonic antitelephone

SR: pole-barn/ladder paradox

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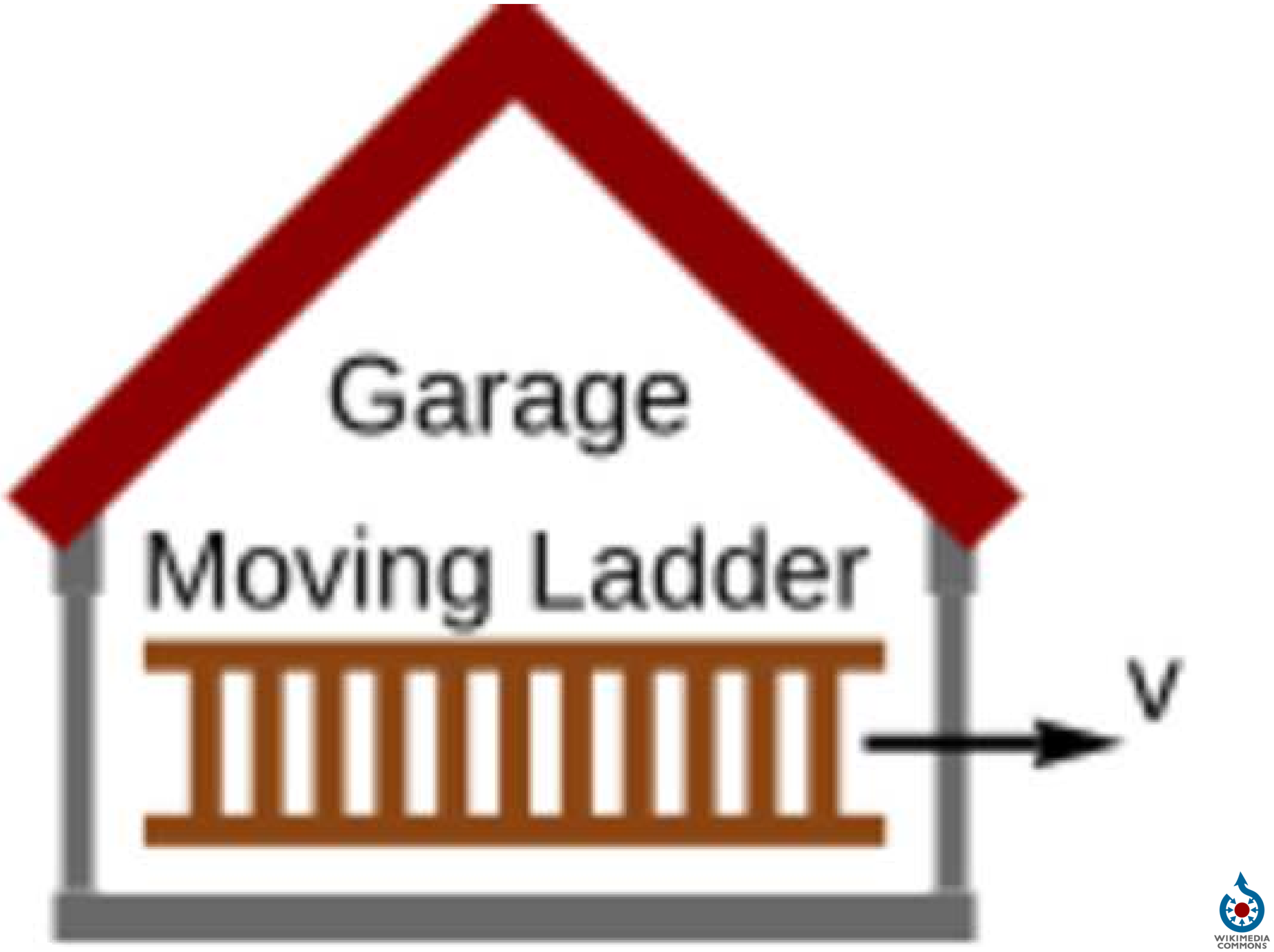
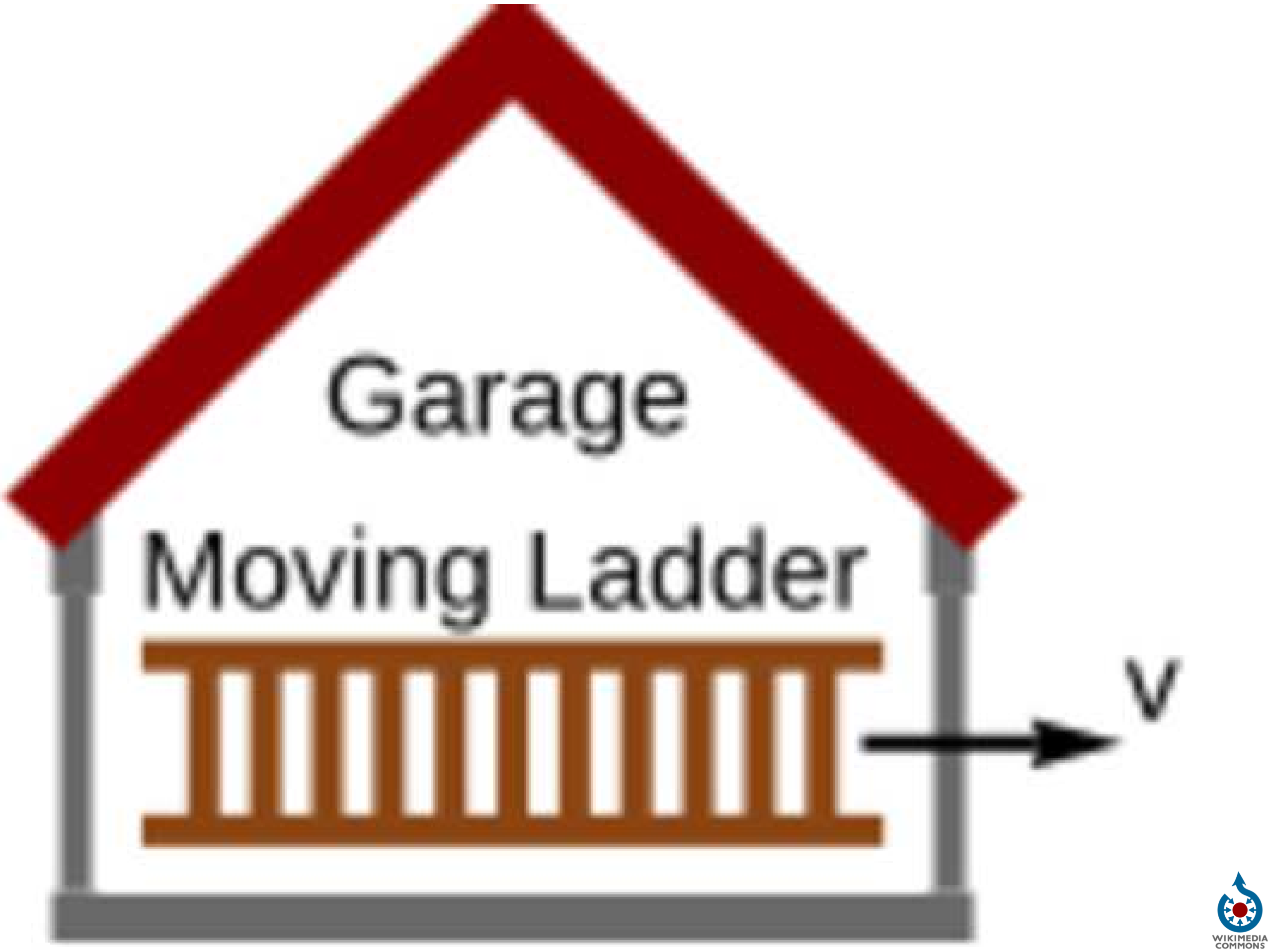
- ladder of length 29.9γ ns, garage length 30 ns (both at rest)

SR: pole-barn/ladder paradox



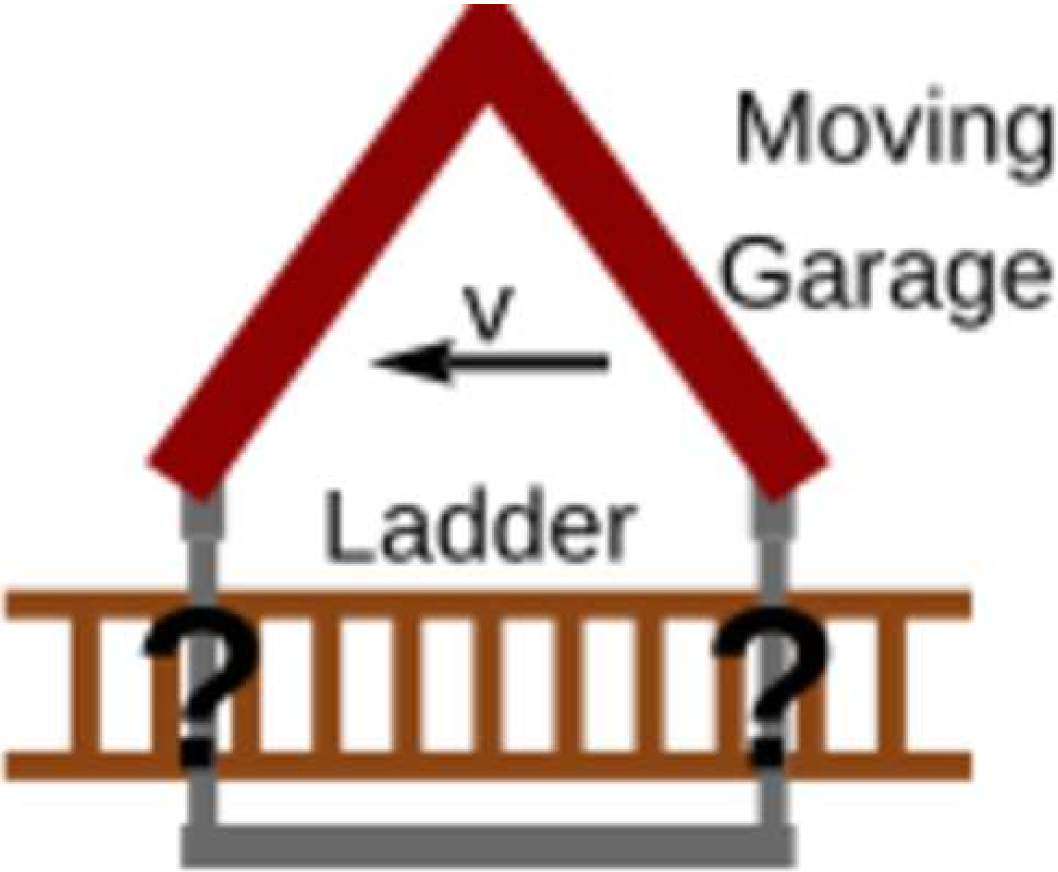
- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors

SR: pole-barn/ladder paradox



- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- 29.9γ ns $/\gamma < 30$ ns $\Rightarrow$ OK

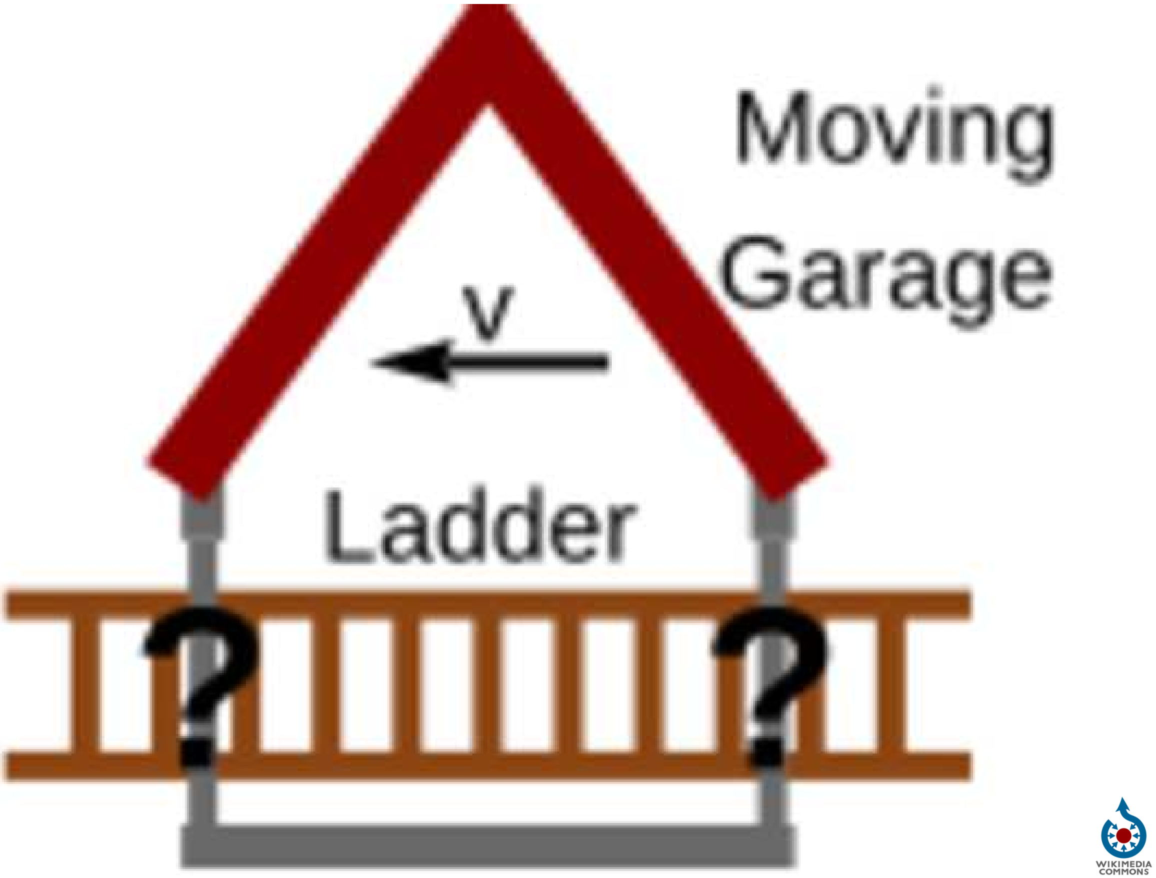
SR: pole-barn/ladder paradox



- ladder of length 29.9γ ns, garage length 30 ns
- instantaneously close front + back doors
- ladder frame: garage $30/\gamma$ ns long $\ll 29.9\gamma$ ns!!

Is this possible or not?

SR: pole-barn/ladder paradox



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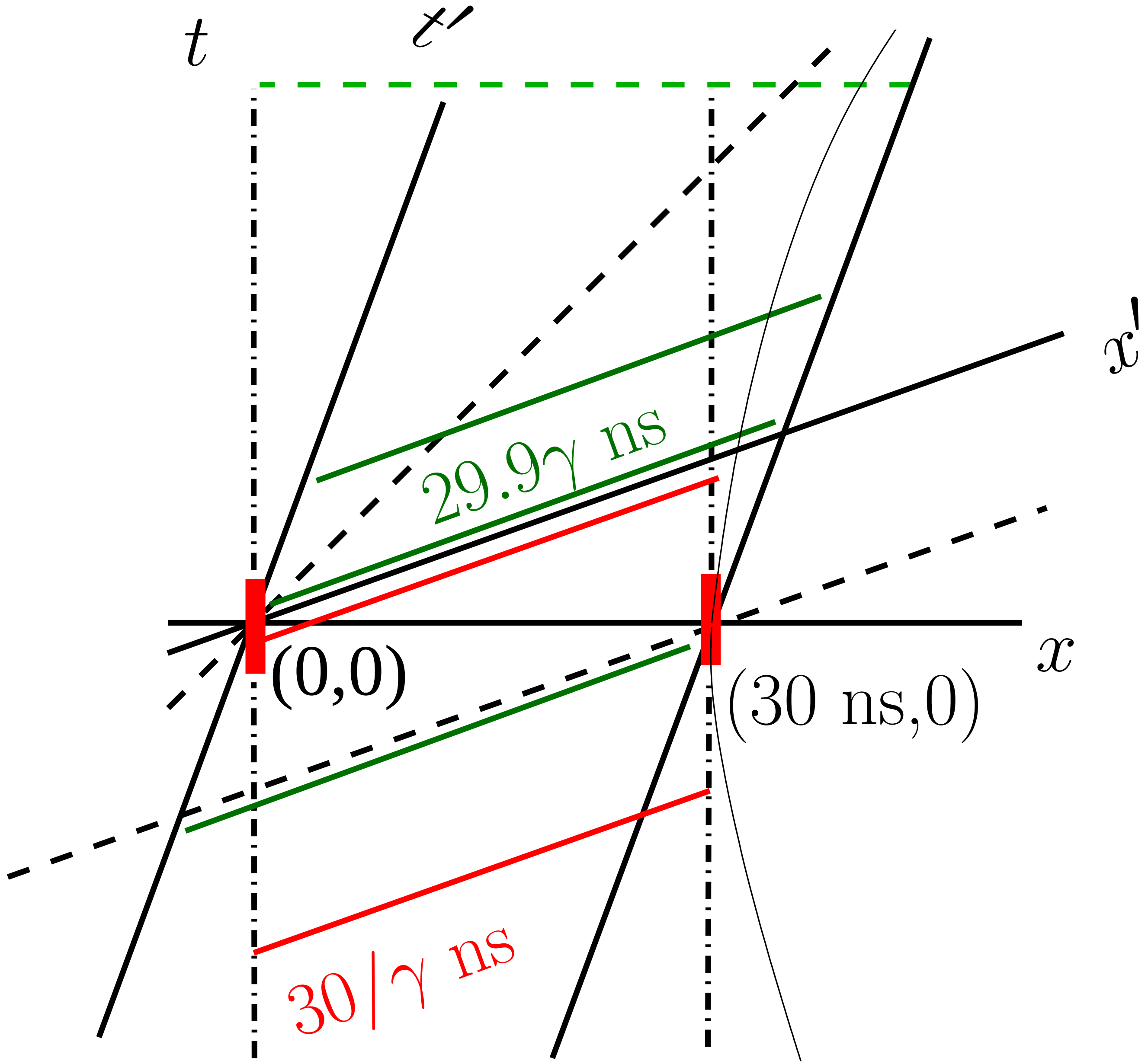
- instantaneously close front + back doors
- ladder frame: garage $30/\gamma$ ns long $\ll 29.9\gamma$ ns!!

Is this possible or not? Make a spacetime diagram.

SR: pole-barn/ladder paradox



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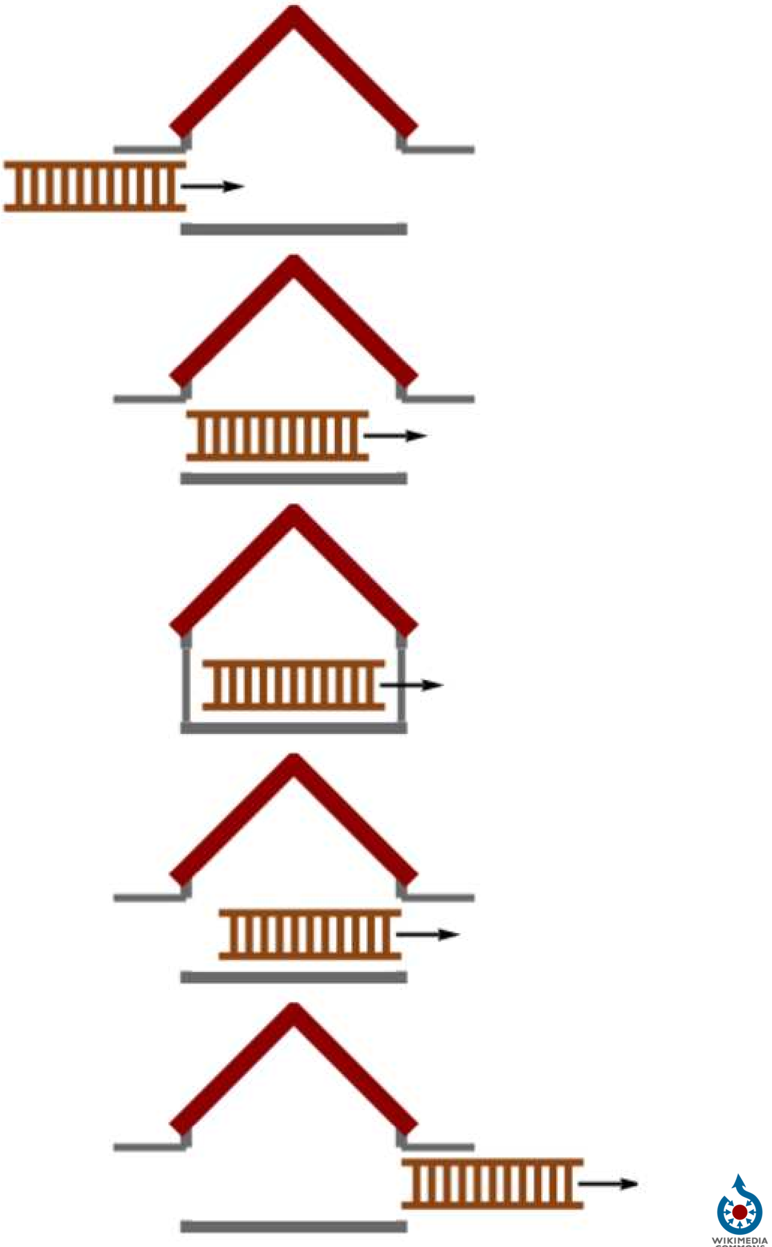

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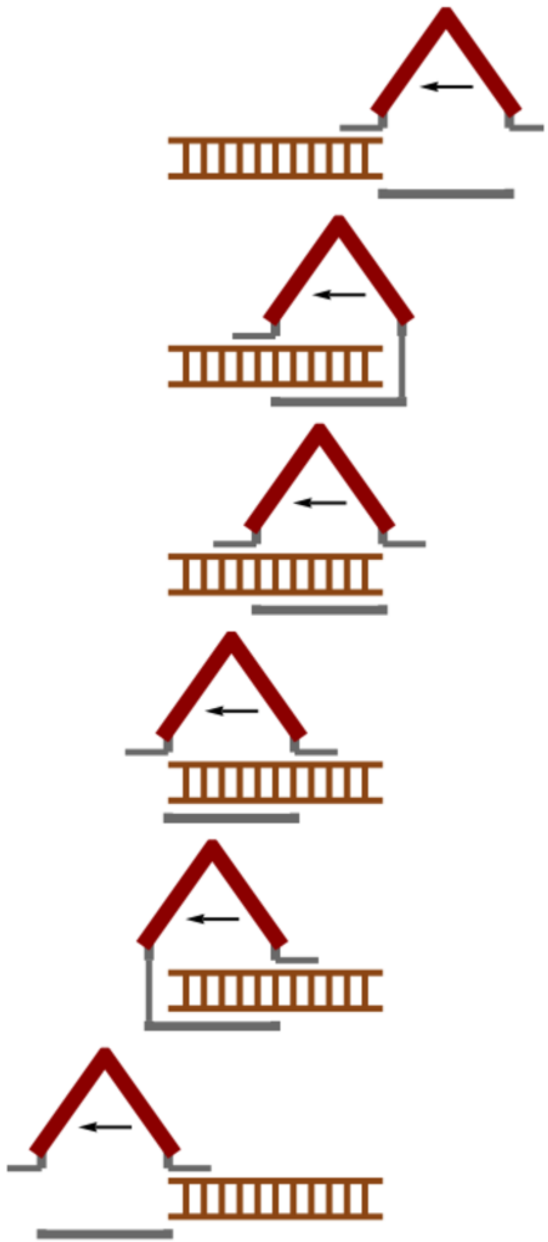

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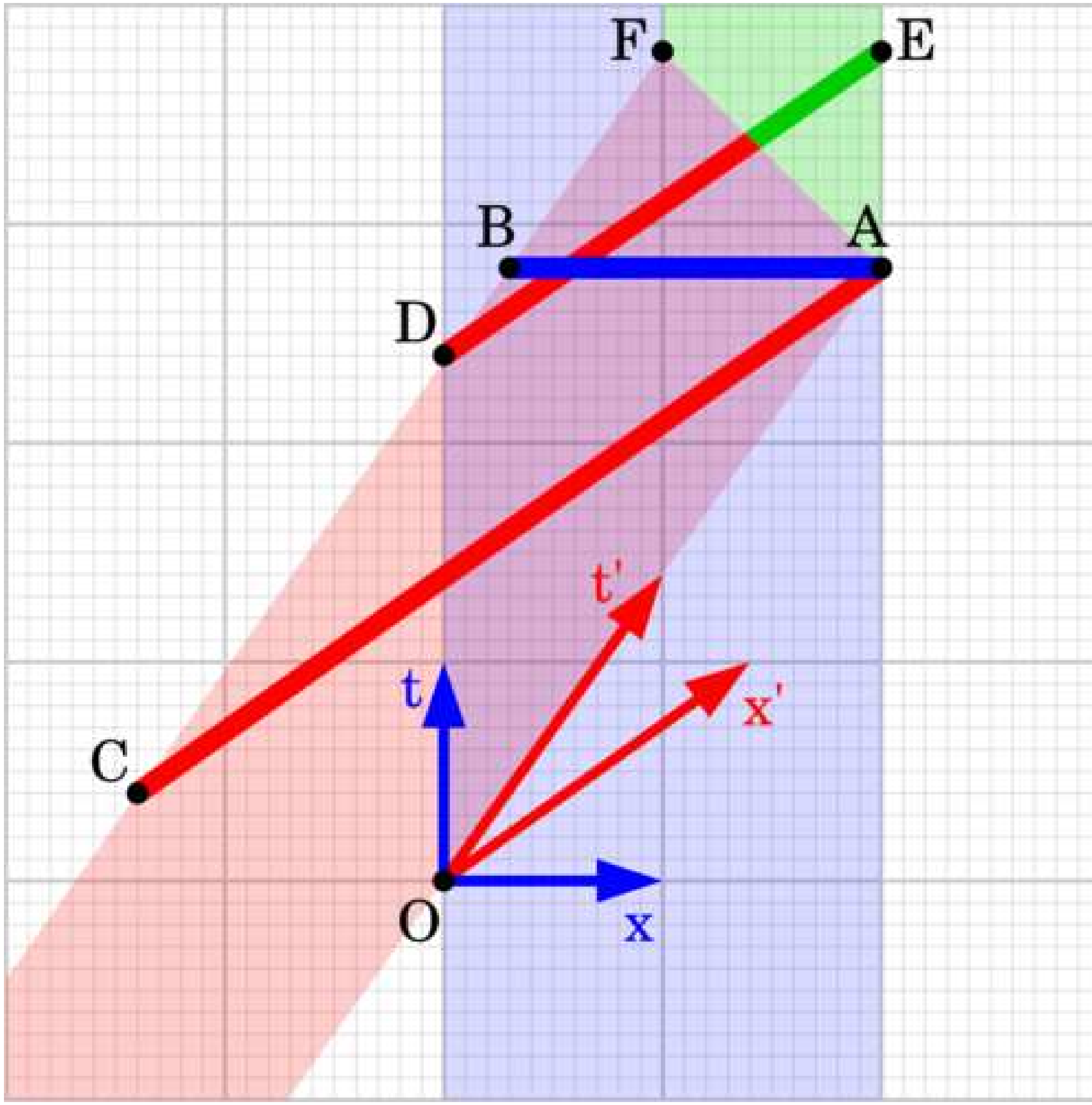
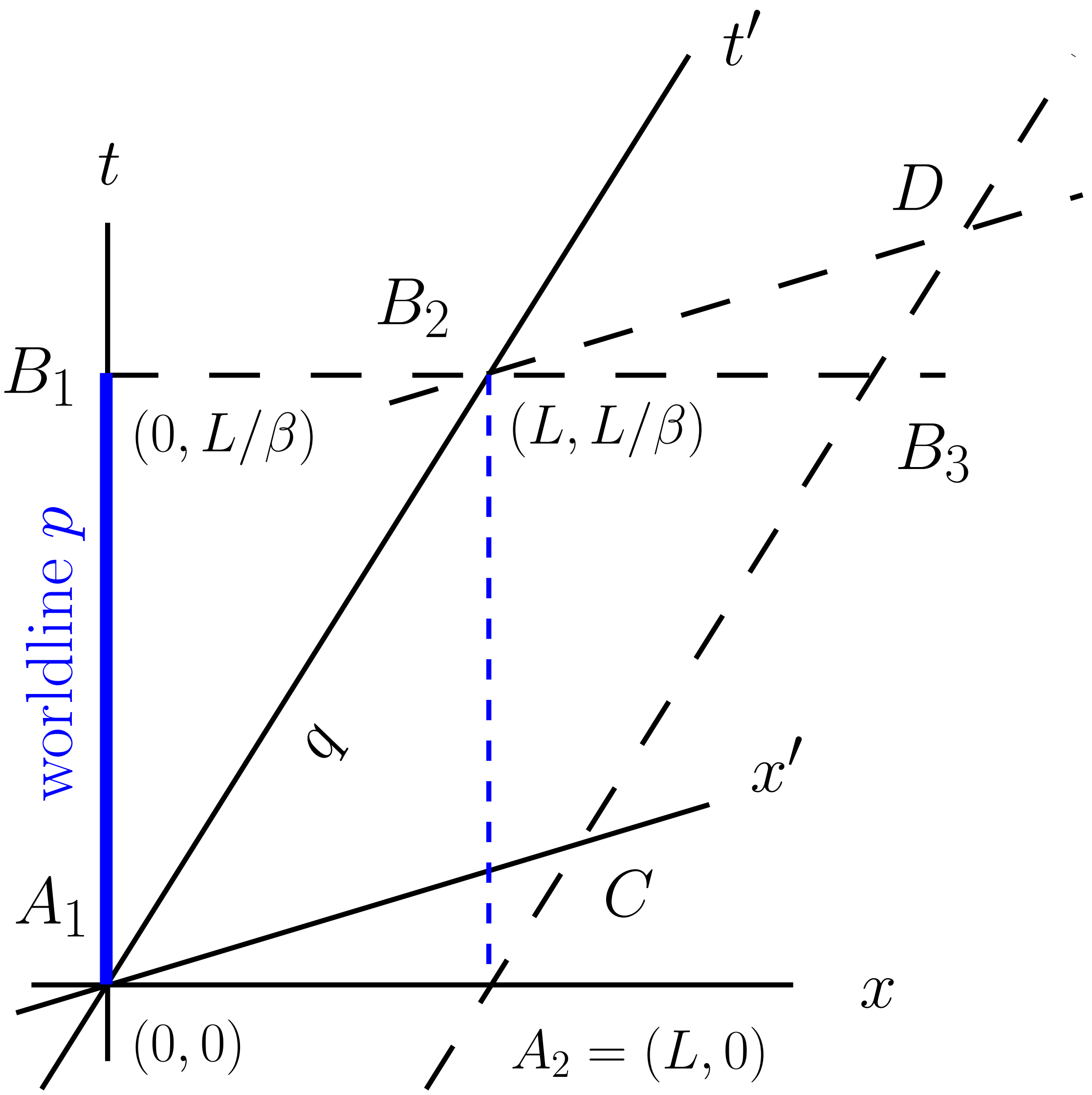


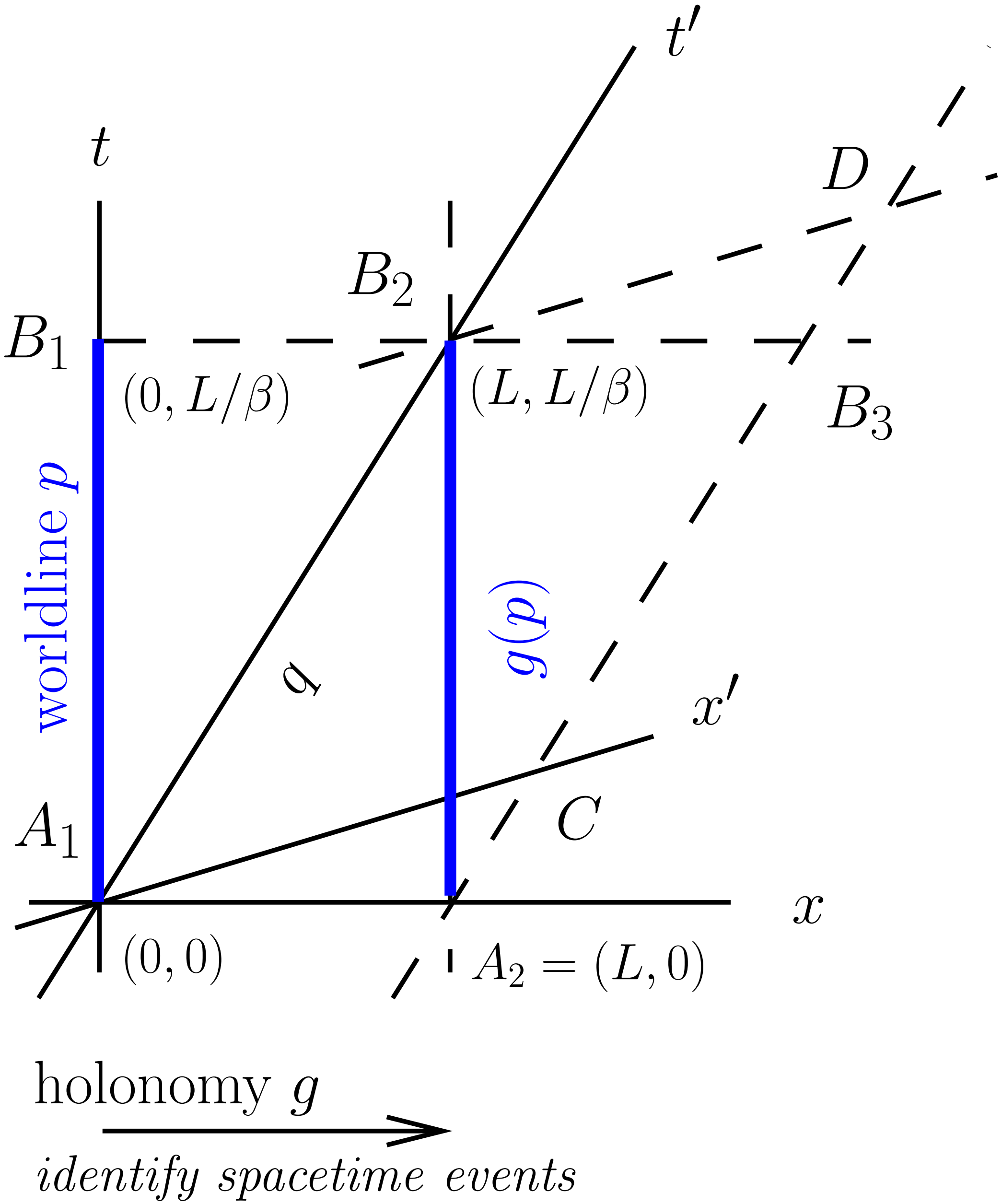
w:Ladder paradox

SR: twins paradox



simply connected Minkowski

SR: twins paradox



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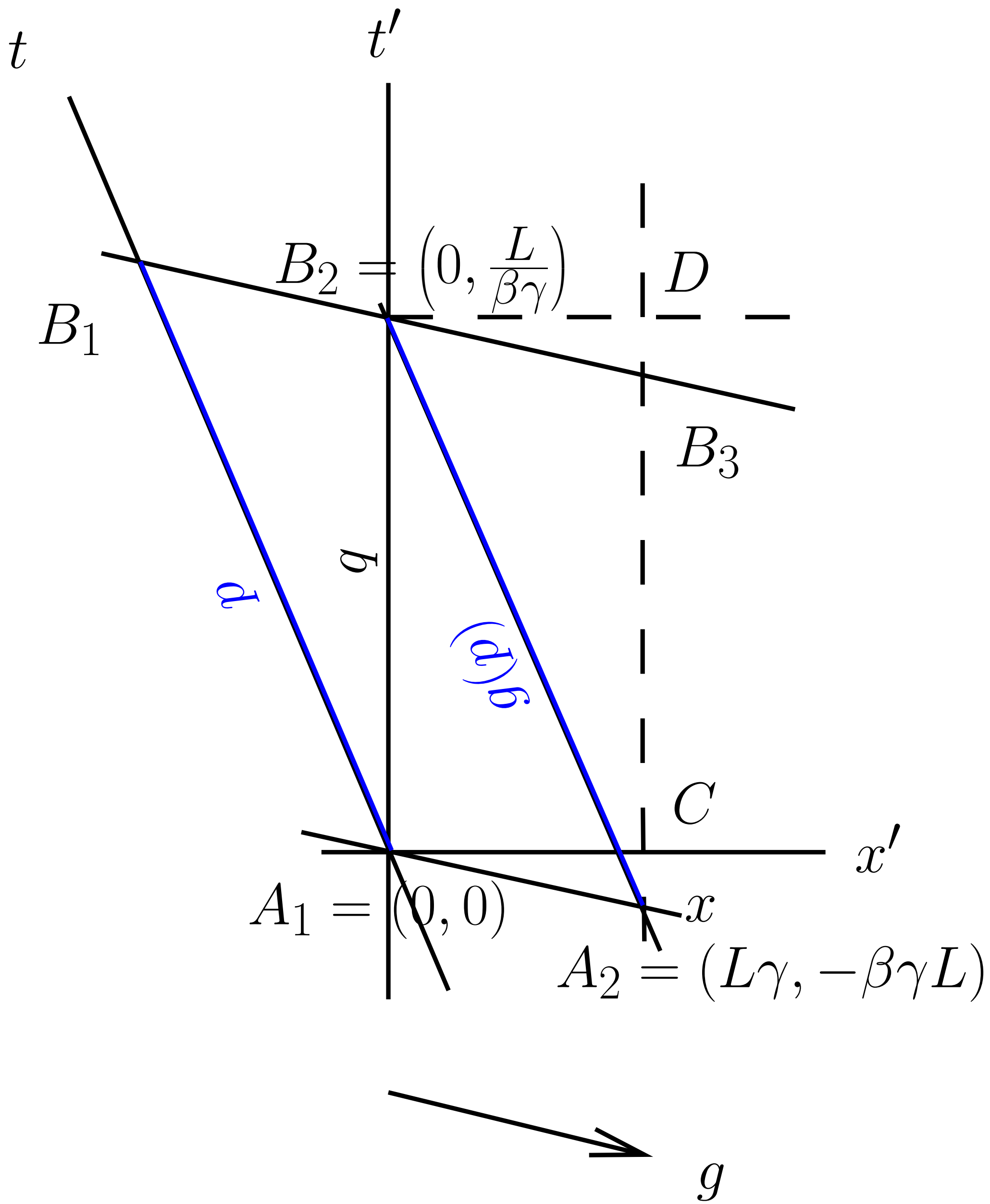

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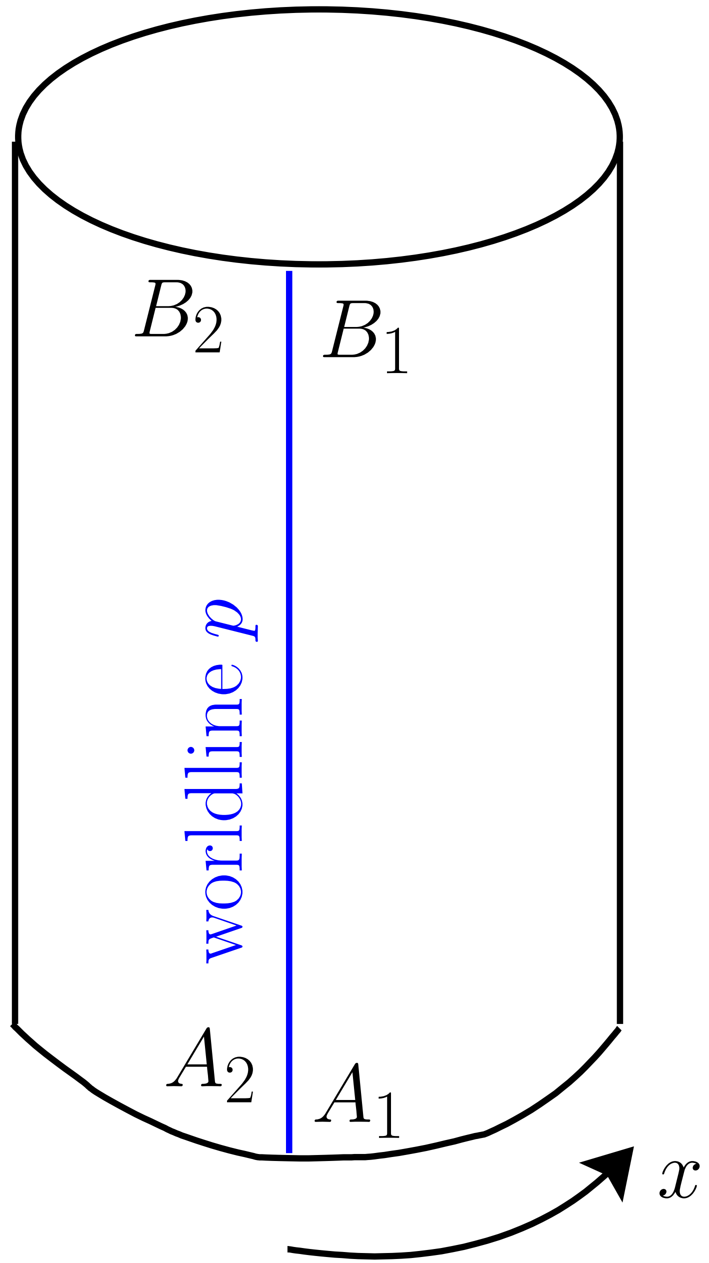

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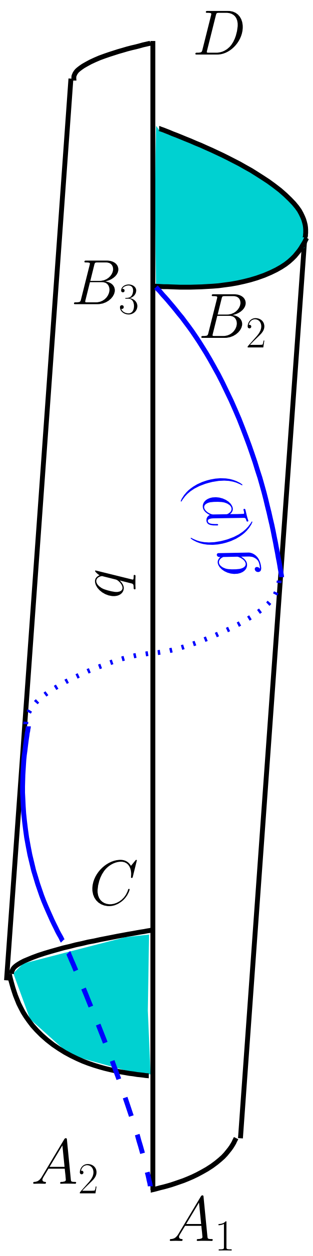

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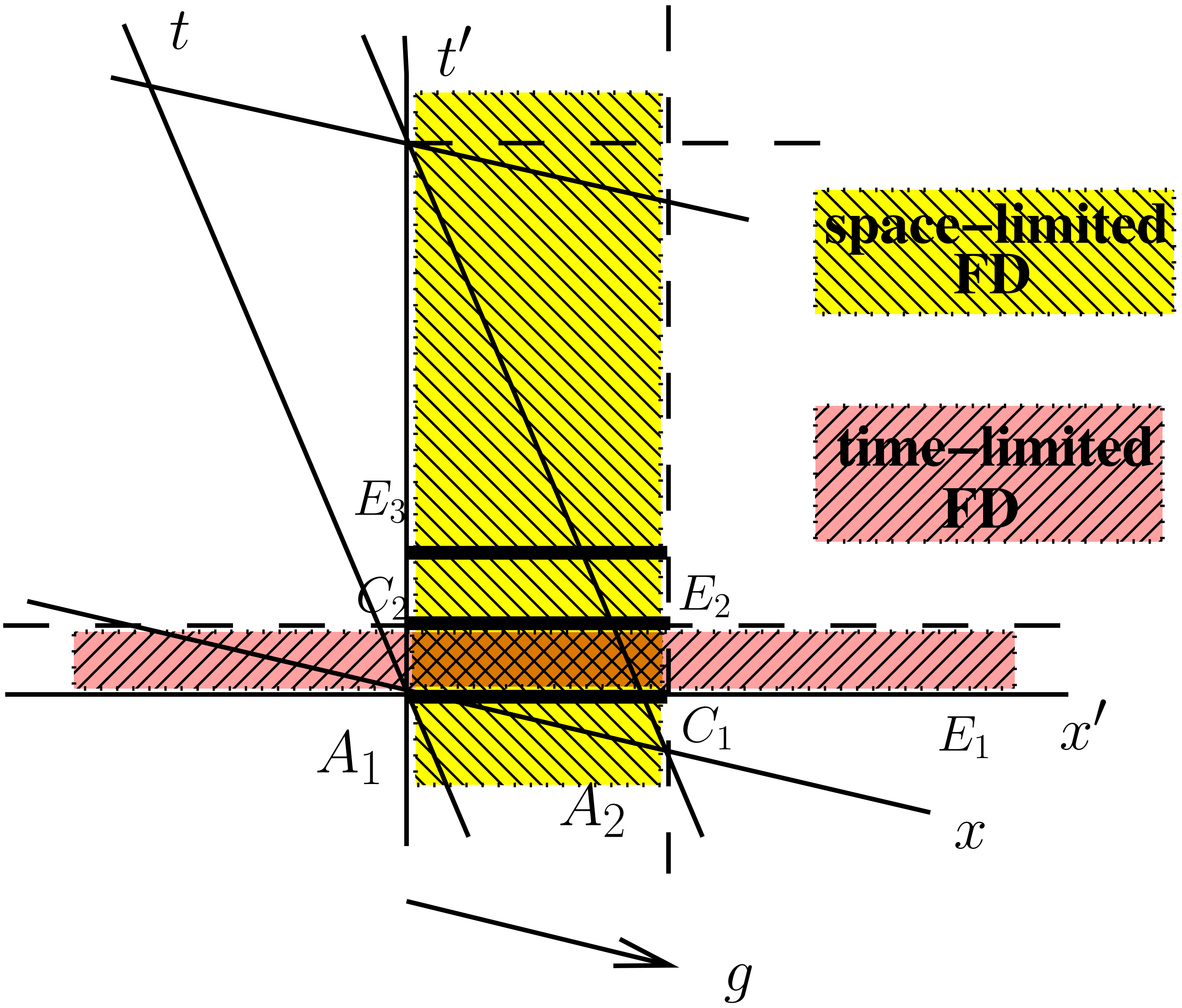
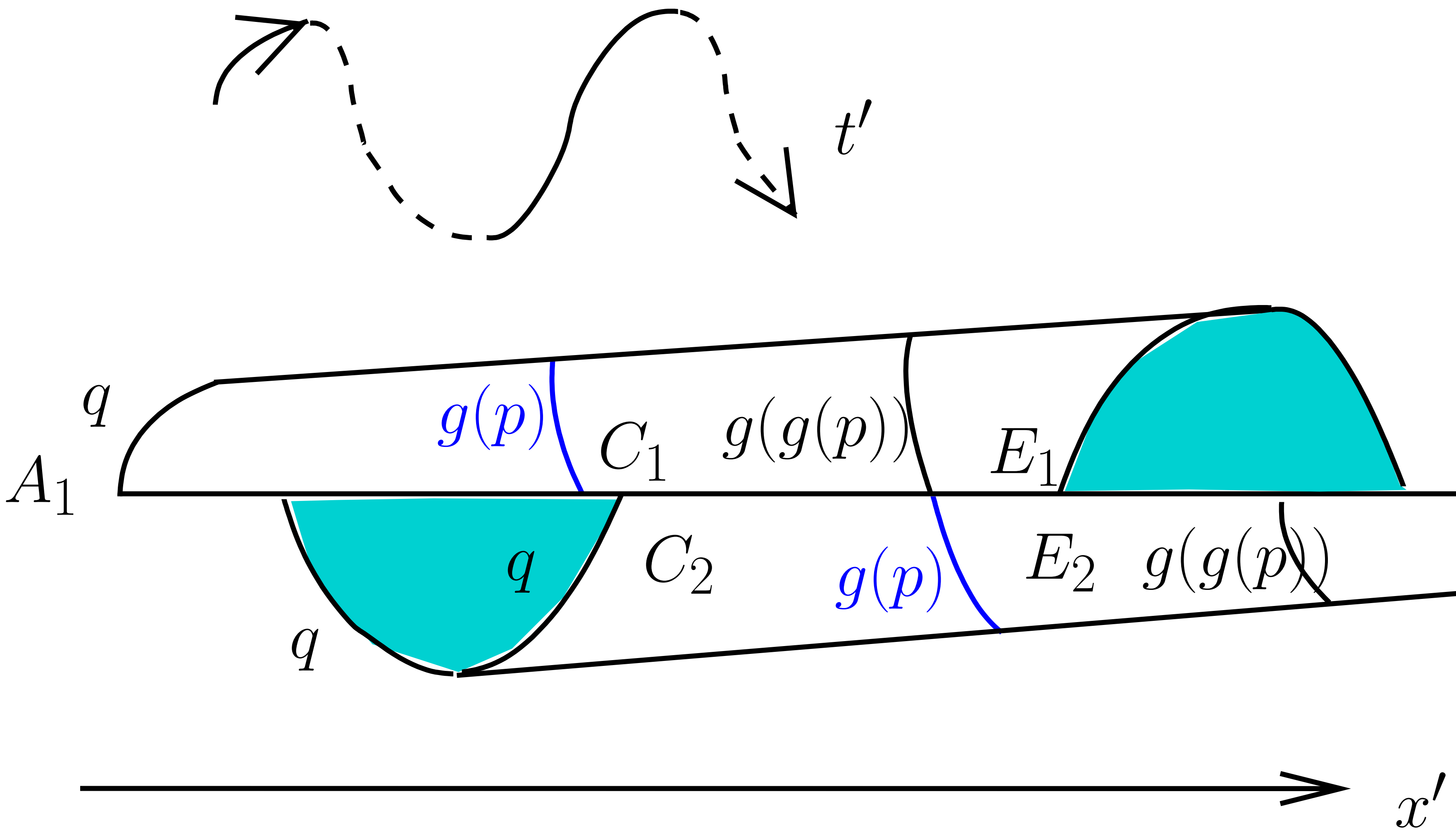


SR: twins paradox



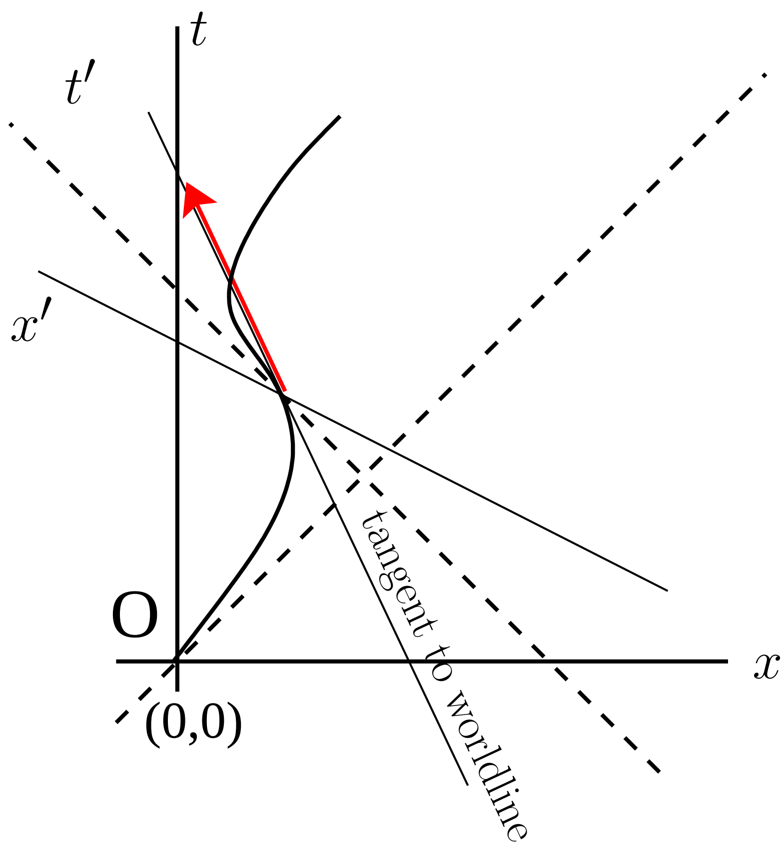
Roukema & Bajtlik 2008, MNRAS, 390, 655 arXiv:astro-ph/0606559

- helps understand w:Ehrenfest paradox

SR: 4-velocity, 4-momentum

choose x axis so that 3-velocity $u_{\mathrm{Galilean}} = (\beta, 0, 0)^{\mathrm{T}}$ for observer with $(t, x, y, z)^{\mathrm{T}}$ coord system

SR: 4-velocity, 4-momentum



- in (t,x) spacetime 2-plane, extend from scalar speed β to spacetime vector = tangent to worldline

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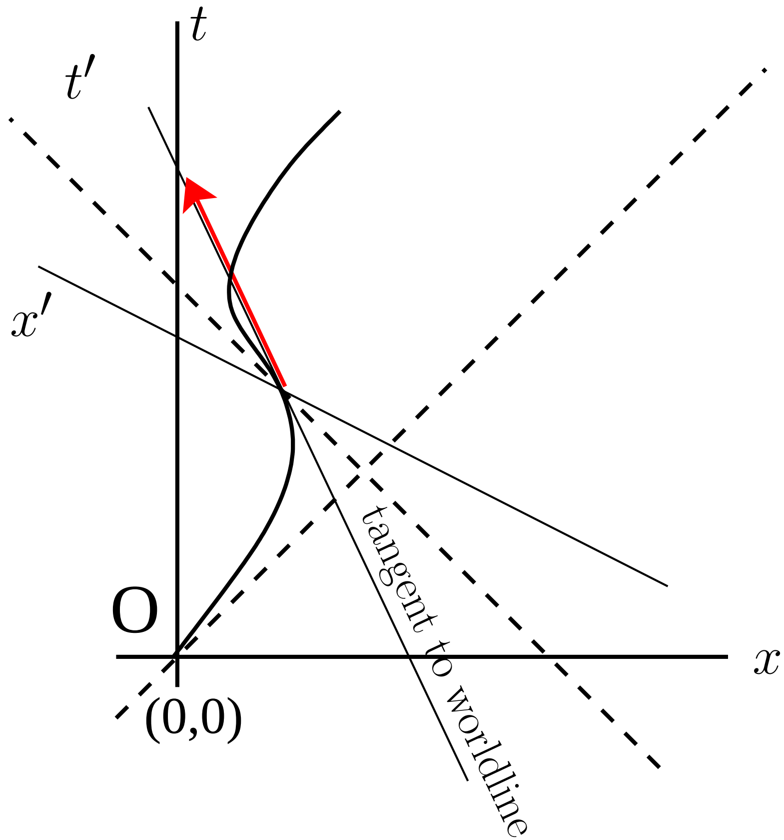

$(u^t, u^x) \quad := \quad (\frac{\mathrm{d}}{\mathrm{d}\tau}t(\tau), \frac{\mathrm{d}}{\mathrm{d}\tau}x(\tau))$

w:four-velocity

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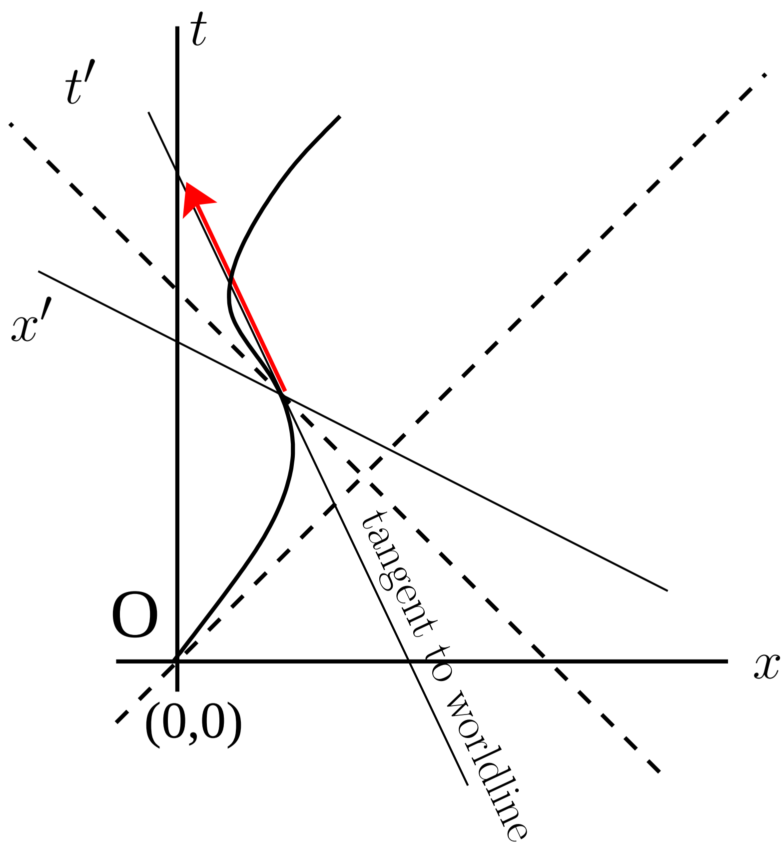

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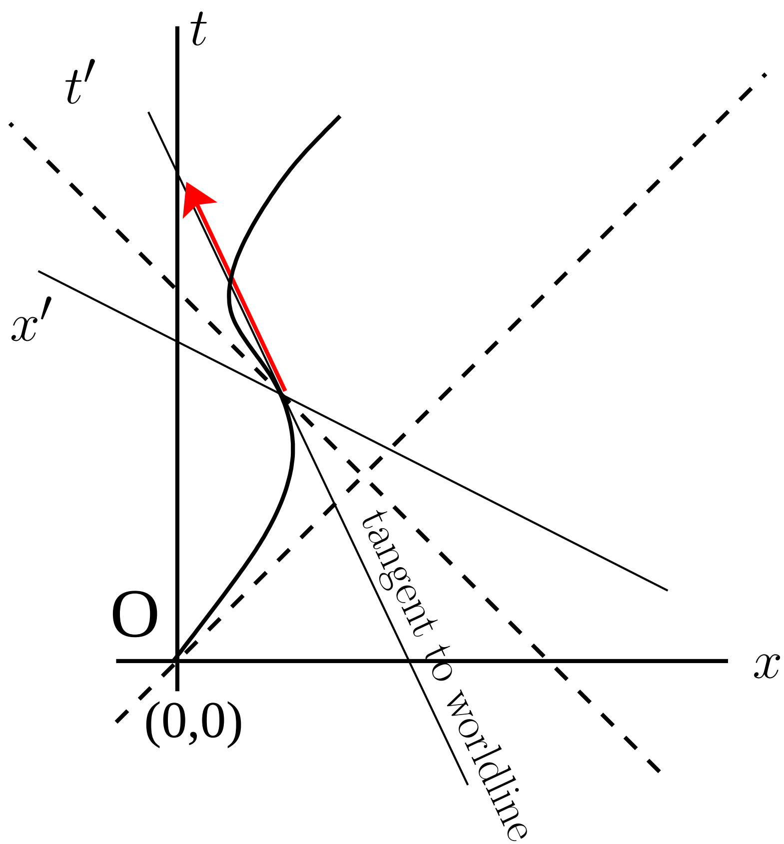

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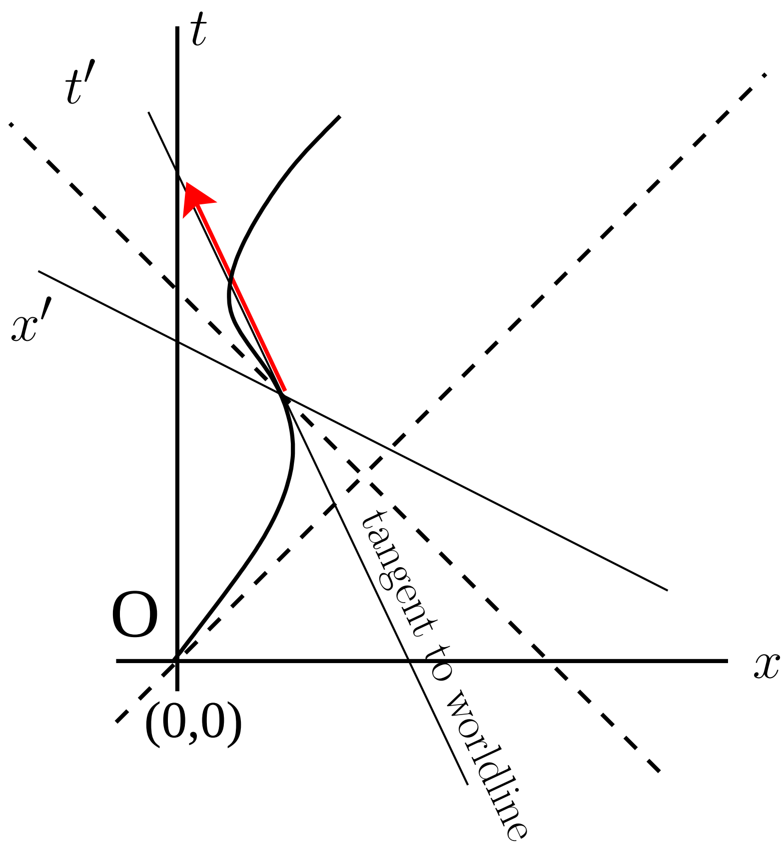

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Want $\vec{u}$ Lorentz invariant $\Rightarrow$

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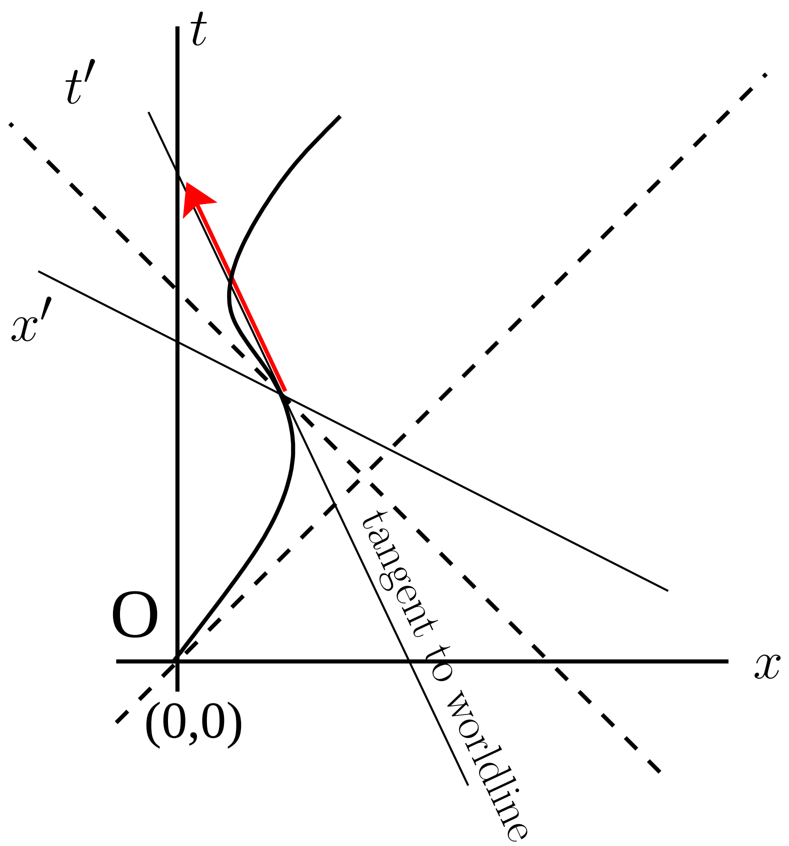

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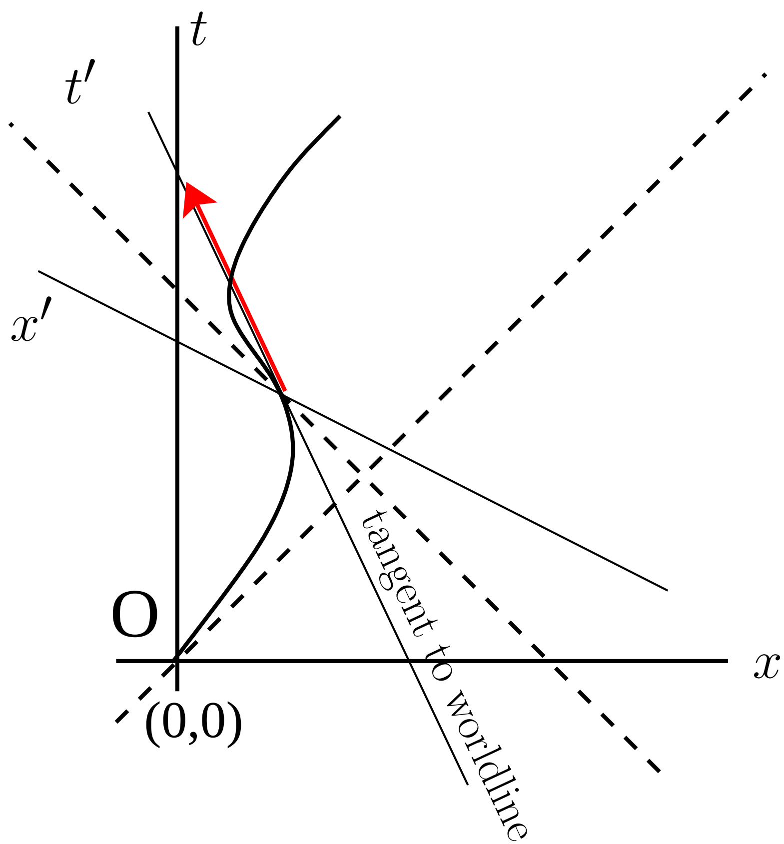

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Want $\vec{u}$ Lorentz invariant $\Rightarrow$ $(u^t, u^x)^{\mathrm{T}} = \Lambda^{-1}(1, 0)^{\mathrm{T}} = \gamma(1, \beta)^{\mathrm{T}}$

4D: $\vec{u} = \gamma(1, \beta^x, \beta^y, \beta^z)^{\mathrm{T}}$

notation in this pdf:

$\vec{u}$ = 4-vector, ${}^{(3)}\vec{u}$ = spatial component

SR: 4-velocity, 4-momentum

Is the $^{(3)}$-component (spatial component) of $\vec{u}$ the same as the non-relativistic velocity?

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$\neq \frac{\mathrm{d}}{\mathrm{d}t}(x, y, z)^{\mathrm{T}}$ except if $\beta = 0 \Leftrightarrow \gamma = 1$

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momentum: $\vec{p} := m\vec{u} = m\gamma(1, \beta^x, \beta^y, \beta^z)^{\mathrm{T}}$, where m = constant

w:invariant mass

^{x} $\ldots =$ tensor-style component notation, not powers

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What does the time component of momentum $= p^0 = m\gamma$ mean physically?

- first look at spatial component in a given ref. frame

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SR: 4-velocity, 4-momentum

let us define 4-acceleration, 4-force

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${}^{(4)}\vec{f} := m \; {}^{(4)}\vec{a}$ defn w:four-force

$= \frac{\mathrm{d}}{\mathrm{d}\tau}\vec{p}$

SR: invariance of ${}^{(4)}u, {}^{(4)}a, {}^{(4)}f$

Euclidean norm: $\|\vec{x}\|^2 = \sum_\mu (x^\mu)^2$

SR: invariance of ${}^{(4)}u, {}^{(4)}a, {}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = \sum_{\mu,\nu} \eta_{\mu\nu} x^\mu x^\nu$

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w:Einstein summation sum is implicit

SR: invariance of ${}^{(4)}u, {}^{(4)}a, {}^{(4)}f$

Minkowski pseudo-norm: $\|\vec{x}\|^2 = -x^0x^0 + \delta_{ij}x^ix^j$

$\delta_{ij} = 1$ if $i = j$, otherwise $= 0$; $i, j \in 1, 2, 3$

invariance: $\|\vec{x}\|^2 =$ same in all reference frames

sign convention: $(-, +, +, +)$ or $(+, -, -, -)$ are common

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non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2 \beta^2$

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similarly: $\|\vec{a}\|^2$, $\|\vec{f}\|^2$ invariant

SR: energy: varies with ref frame

Newtonian $K = (1/2)m\beta^2 = 0$ in rest frame

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4-force $\vec{f}$ is invariant, but

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in (x, t) frame,
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$K =$ work done

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$$= \int_0^{\vec{x}_2} \frac{\mathrm{d}}{\mathrm{d}t}\, {}^{(3)}\vec{p} \cdot \mathrm{d}\vec{x}$$

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$$= \int_0^{\vec{x}_2} \frac{\mathrm{d}}{\mathrm{d}t}(m\beta\gamma)\mathrm{d}x$$

(assume ${}^{(3)}\vec{f}/\gamma \parallel \vec{x}$)

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$$= m \int_0^{\gamma_2} (\beta \mathrm{d}\gamma)\beta + m \int_0^{\beta_2} (\gamma \mathrm{d}\beta)\beta$$

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SR: energy: varies with ref frame

in (x,t) frame,
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so $p^0 =$ kinetic energy + rest mass

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$= m[1-(1/2)(-\beta^2)+\mathcal{O}(\beta^4)]$ if $\beta \ll 1$ (w:Taylor series)

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Yes.

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non-rest frame: $\|\vec{u}\|^2 = -\gamma^2 + \gamma^2\beta^2 = -1$

rest frame: $\|\vec{u}\|^2 = -1^2 + 0^2 = -1$ invariant

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WARNING: assume that 4-momentum vectors at different space-time positions can be translated; NOT the case in curved spacetime

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vector space $\Rightarrow p^i + q^i = r^i \; (i = 1, 2, 3)$

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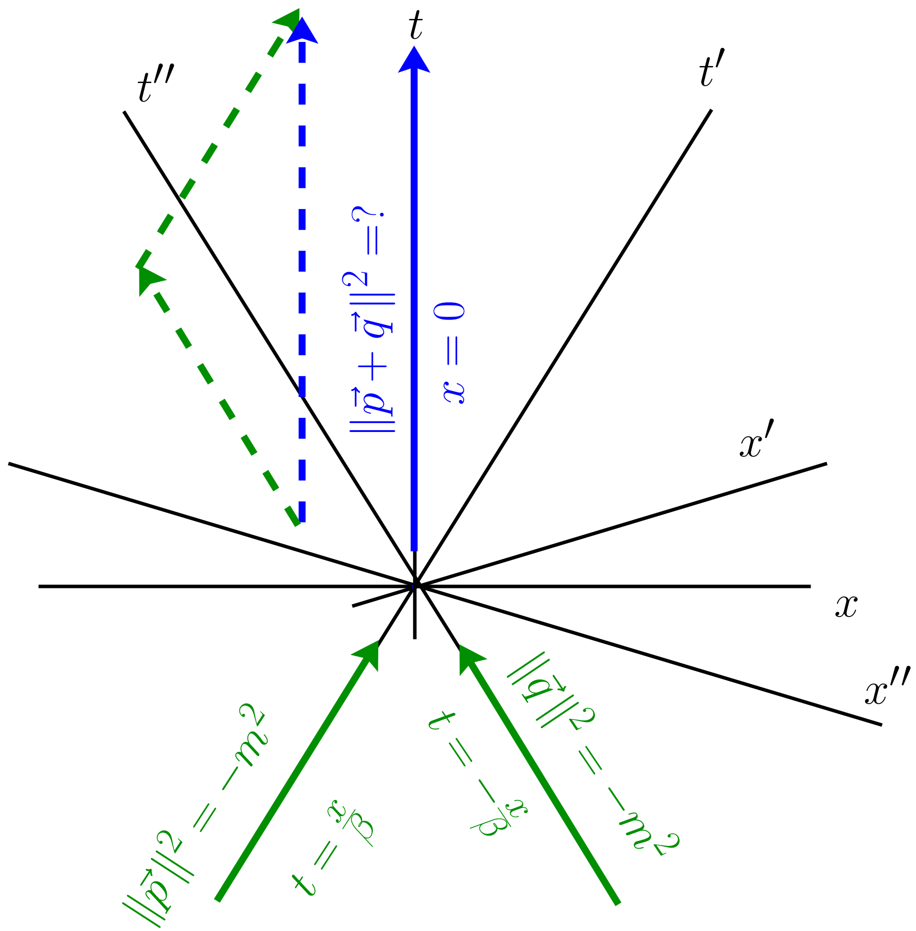
vector space $\Rightarrow p^0 + q^0 = r^0$

= conservation of (relativistic) “total energy" $= m + K$ (Newtonian: m conserved, K not conserved, $K+$ potential energy conserved)

but NOT invariant (Newtonian: m invariant, K not invariant)

SR: $\vec{p}\ldots$: invariant or not?



$$\vec{p} = m(\gamma, +\beta\gamma, 0, 0)^{\mathrm{T}}$$

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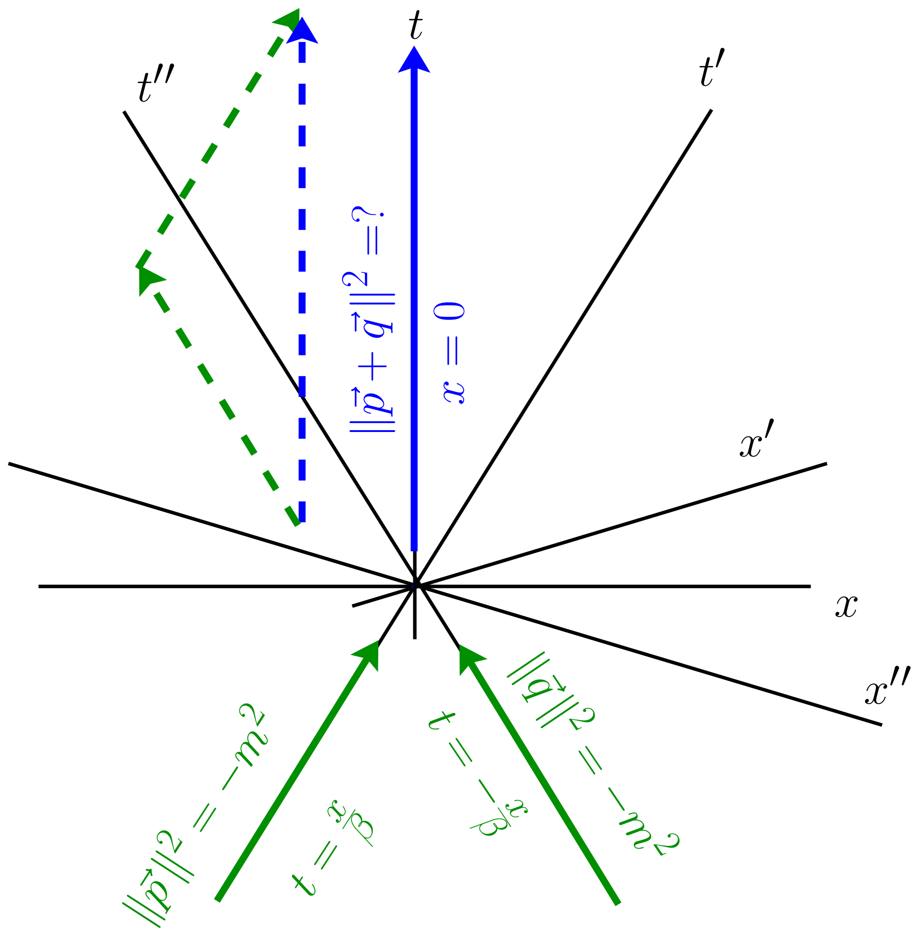

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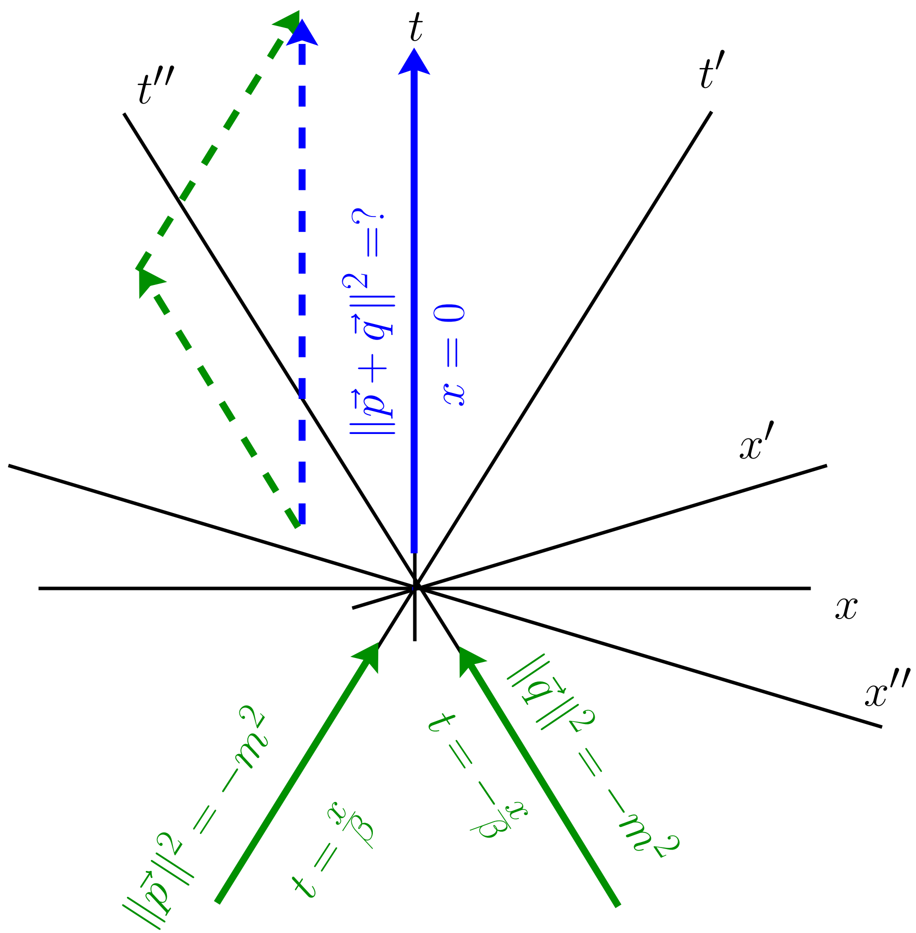

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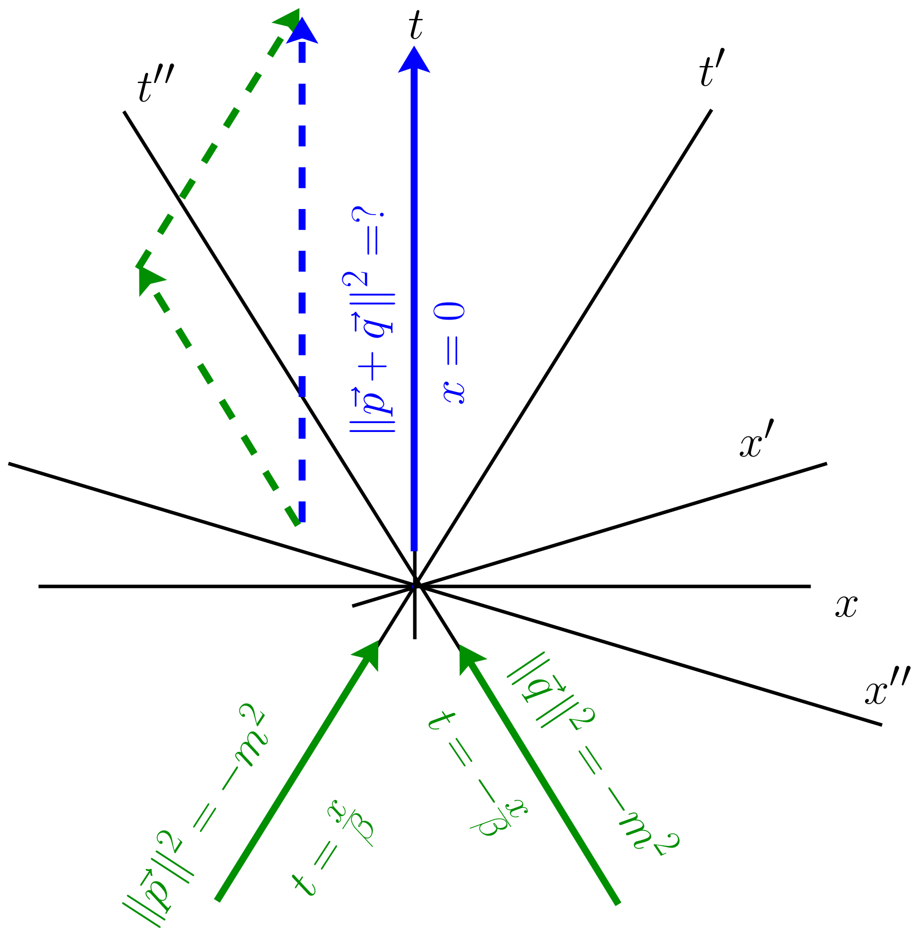

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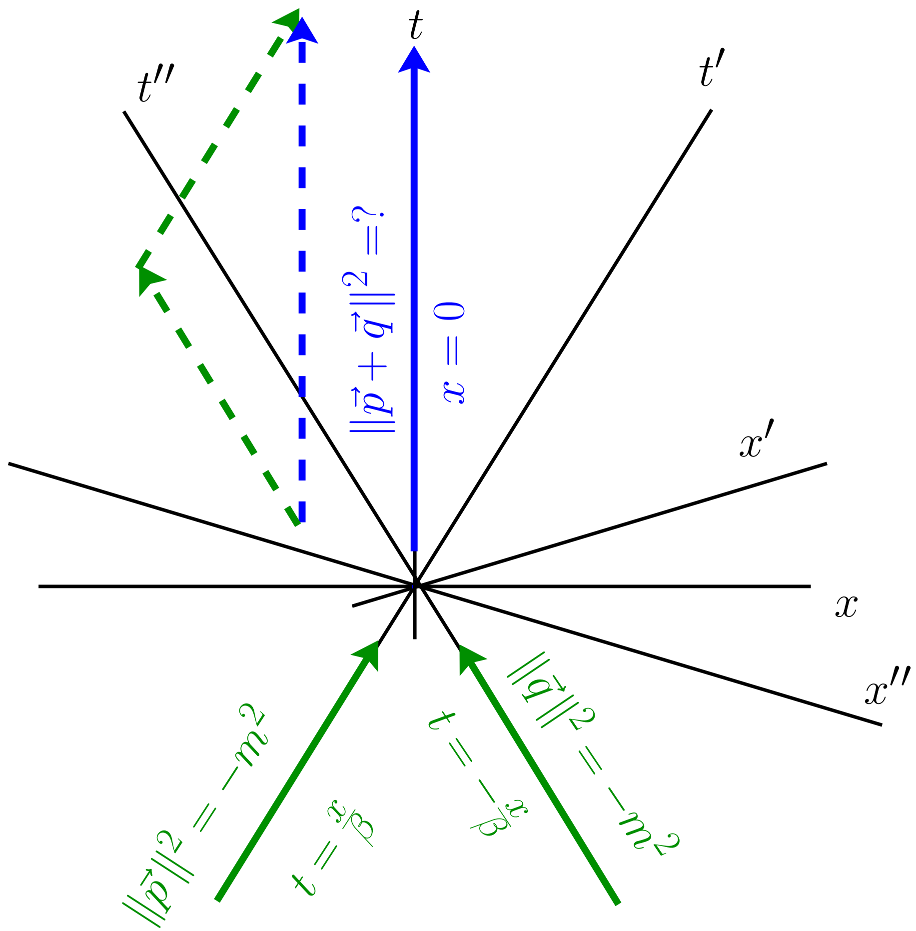

system rest mass before and after:
$2m\gamma$

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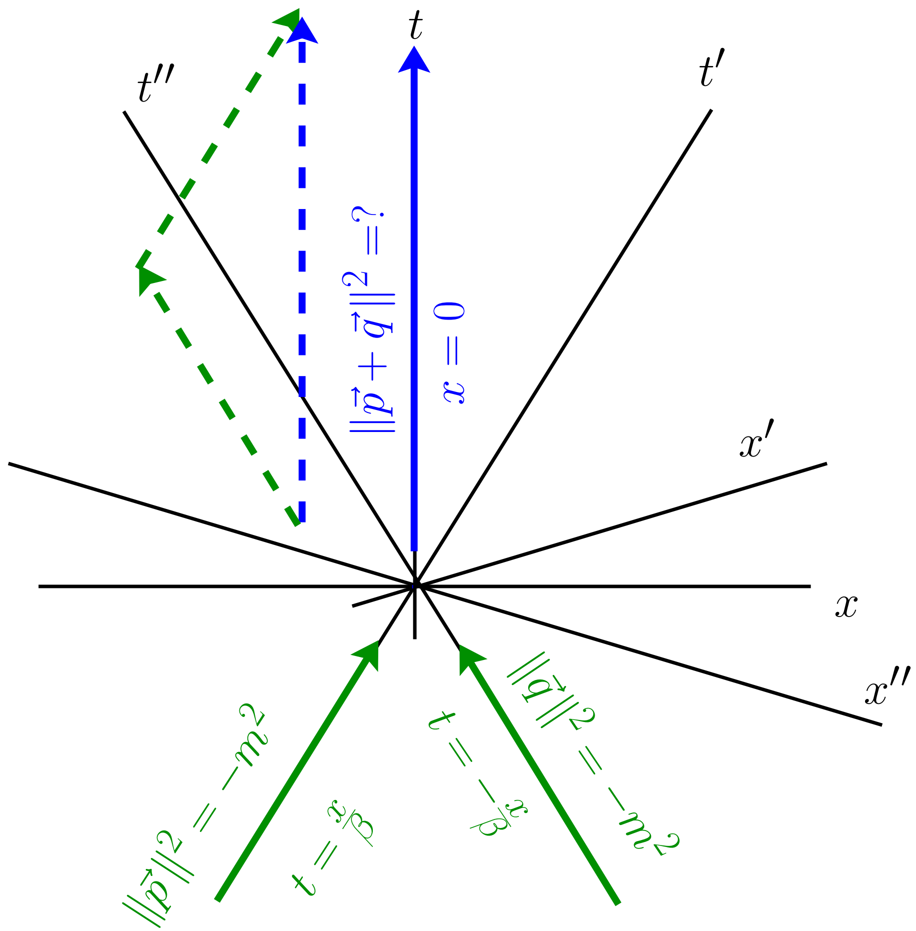

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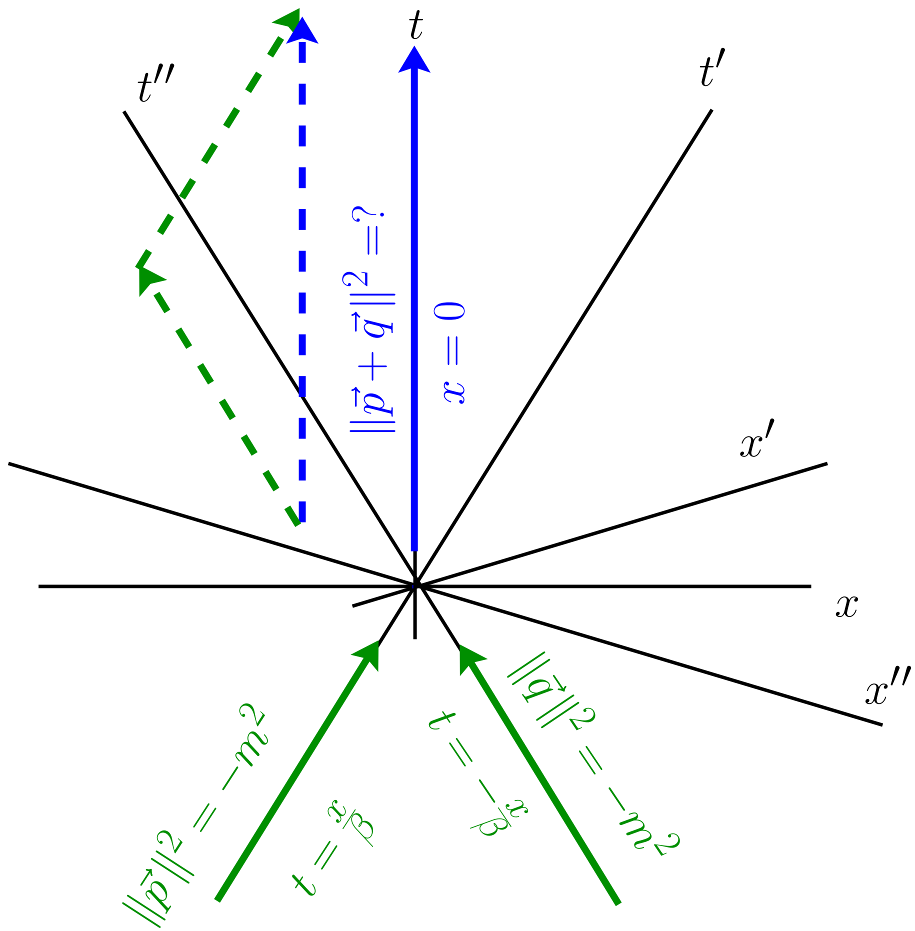

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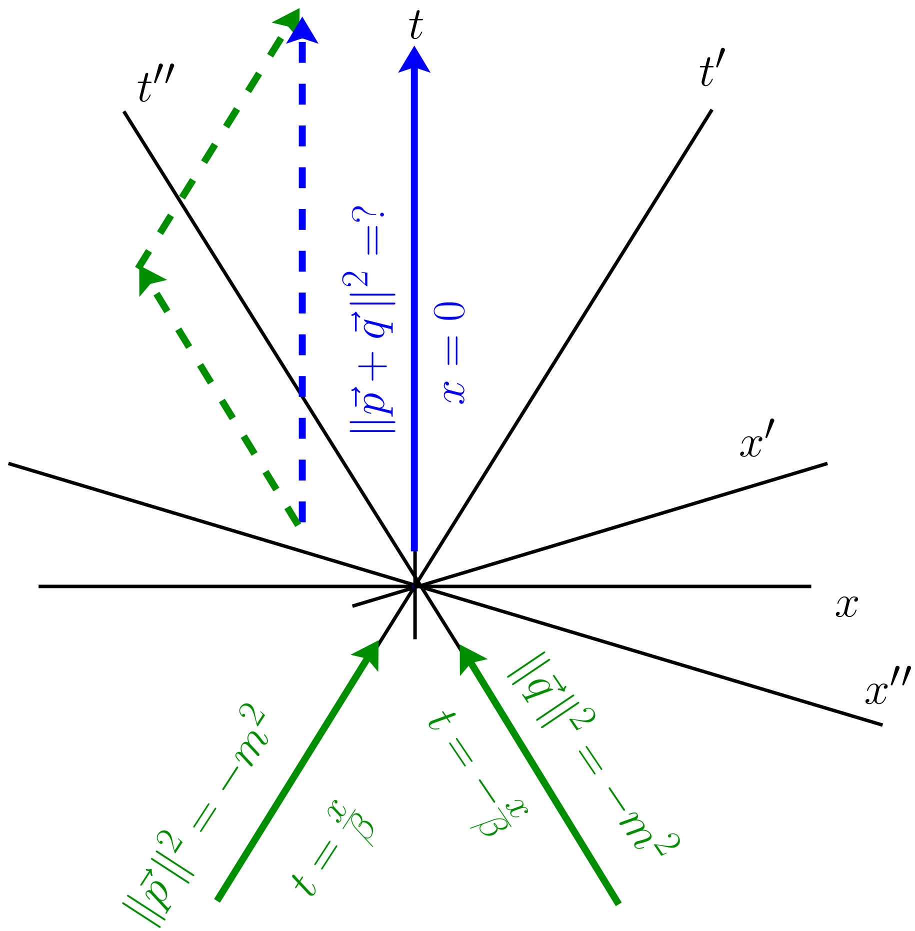

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system mass is invariant, but can be divided into p^0 and $p^i, i \in \{1, 2, 3\}$ components in many different ways

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reword hidden assumptions of absolute simultaneity (time)